

5.6 Fluctuations and noise

Thermodynamic fluctuations^a

Fluctuation probability	$\text{pr}(x) \propto \exp[S(x)/k]$ $\propto \exp\left[\frac{-A(x)}{kT}\right]$	(5.130) (5.131)	pr x S A	probability density unconstrained variable entropy availability
General variance	$\text{var}[x] = kT \left[\frac{\partial^2 A(x)}{\partial x^2} \right]^{-1}$	(5.132)	var[.] k T	mean square deviation Boltzmann constant temperature
Temperature fluctuations	$\text{var}[T] = kT \frac{\partial T}{\partial S} \Big _V = \frac{kT^2}{C_V}$	(5.133)	V C_V	volume heat capacity, V constant
Volume fluctuations	$\text{var}[V] = -kT \frac{\partial V}{\partial p} \Big _T = \kappa_T V kT$	(5.134)	p κ_T	pressure isothermal compressibility
Entropy fluctuations	$\text{var}[S] = kT \frac{\partial S}{\partial T} \Big _p = kC_p$	(5.135)	C_p	heat capacity, p constant
Pressure fluctuations	$\text{var}[p] = -kT \frac{\partial p}{\partial V} \Big _S = \frac{K_S kT}{V}$	(5.136)	K_S	adiabatic bulk modulus
Density fluctuations	$\text{var}[n] = \frac{n^2}{V^2} \text{var}[V] = \frac{n^2}{V} \kappa_T kT$	(5.137)	n	number density

^aIn part of a large system, whose mean temperature is fixed. Quantum effects are assumed negligible.

Noise

Nyquist's noise theorem	$dw = kT \cdot \beta \epsilon (e^{\beta \epsilon} - 1)^{-1} dv$ $= kT_N dv$ $\simeq kT dv \quad (h\nu \ll kT)$	(5.138) (5.139) (5.140)	w k T T_N $\beta \epsilon$ ν h	exchangeable noise power Boltzmann constant temperature noise temperature $= h\nu/(kT)$ frequency Planck constant
Johnson (thermal) noise voltage ^a	$v_{\text{rms}} = (4kT_N R \Delta\nu)^{1/2}$	(5.141)	v_{rms} R $\Delta\nu$	rms noise voltage resistance bandwidth
Shot noise (electrical)	$I_{\text{rms}} = (2eI_0 \Delta\nu)^{1/2}$	(5.142)	I_{rms} $-e$ I_0	rms noise current electronic charge mean current
Noise figure ^b	$f_{\text{dB}} = 10 \log_{10} \left(1 + \frac{T_N}{T_0} \right)$	(5.143)	f_{dB} T_0	noise figure (decibels) ambient temperature (usually taken as 290 K)
Relative power	$G = 10 \log_{10} \left(\frac{P_2}{P_1} \right)$	(5.144)	G P_1, P_2	decibel gain of P_2 over P_1 power levels

^aThermal voltage over an open-circuit resistance.

^bNoise figure can also be defined as $f = 1 + T_N/T_0$, when it is also called “noise factor.”