Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The least positive integer divisible by 20 and 24 is **[1]**

a) 480 b) 240

c) 360 d) 120

2. If a = ($2^2 \times 3^3 \times 5^4$) and b = ($2^3 \times 3^2 \times 5$) then HCF (a, b) = ? **[1]**

a) 360 b) 90

c) 180 d) 540

3. The product of two consecutive integers is 240. The quadratic representation of the above situation is **[1]**

a) x(x + 1) = 240 b) $x(x + 1)^2 = 240$

c) x + (x + 1) = 240 d) $x^2 + (x + 1) = 240$

4. In△ABC, if ∠C = 50° and ∠A exceeds ∠B by 44^o, then ∠A = **[1]**

a) 87^o b) 43^o

c) 67^o d) 40^o

5. Let b = a + c. Then the equation $ax^2 + bx + c = 0$ has equal roots if **[1]**

a) a = -c b) a = c

c) a = -2c d) a = 2c

6. If (3, –6) is the mid-point of the line segment joining (0, 0) and (x, y), then the point (x, y) is: **[1]**

a) (6, - 6)

b) (6, -12)

c) $\left(\frac{3}{2}, -3\right)$

d) (- 3, 6)

7. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high? **[1]**

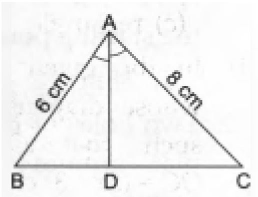
a) 13.5 m

b) 1.35 m

c) 1.5 m

d) 2.4 m

8. In a $\triangle$ ABC it is given that AB = 6 cm, AC = 8 cm and AD is the bisector of $\angle$A. Then, BD : DC = ? **[1]**



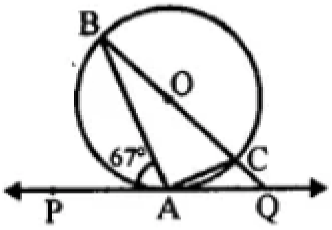
a) 3 : 4

b) 9 :16

c) $\sqrt{3} : 2$

d) 4 : 3

9. In the given figure, PQ is a tangent to a circle with centre O. A is the point of contact. If $\angle$PAB = 67°, then the measure of $\angle$AQB is **[1]**



a) 53°

b) 64°

c) 44°

d) 73°

10. $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$ = ? **[1]**

a) $\frac{75}{8}$

b) $\frac{73}{8}$

c) $\frac{83}{8}$

d) $\frac{81}{8}$

11. From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is **[1]**

a) 20 m

b) 40 m

c) 80 m

d) 60 m

12. If a cot θ + b cosec θ = p and b cot θ + a cosec θ = q, then $p^2 - q^2$ = **[1]**

a) $a^2 + b^2$

b) $a^2 - b^2$

c) $b^2 - a^2$

d) b - a

13. If the perimeter of a sector of a circle of radius 6.5 cm is 29 cm, then its area is **[1]**

a) 56 cm^2

b) 58 cm^2

c) 52 cm^2

d) 25 cm^2

14. In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. The area of the sector formed by the arc is: **[1]**

a) $231\ cm^2$
b) $250\ cm^2$
c) $220\ cm^2$
d) $200\ cm^2$

15. The probability expressed as a percentage of a particular occurrence can never be **[1]**

a) anything but a whole number
b) greater than 1
c) less than 100
d) less than 0

16. In the given data if n = 44, l = 400, cf = 8, h = 100, f = 20, then its median is **[1]**

a) 400
b) 480
c) 470
d) 460

17. The volume of a cylinder of radius r is 1/4 of the volume of a rectangular box with a square base of side length x. If the cylinder and the box have equal heights, what is r in terms of x? **[1]**

a) $\frac{x}{2\sqrt{\pi}}$
b) $\frac{x^2}{2\pi}$
c) $\frac{\pi}{2\sqrt{x}}$
d) $\frac{\sqrt{2x}}{\pi}$

18. For the following distribution: **[1]**

Marks Below	10	20	30	40	50	60
Number of Students	3	12	27	57	75	80

the modal class is:

a) 50 – 60
b) 40 – 50
c) 20 – 30
d) 30 – 40

19. **Assertion (A):** The value of y is 6, for which the distance between the points P(2, -3) and Q(10, y) is 10. **[1]**
Reason (R): Distance between two given points A(x_1, y_1) and B(x_2, y_2) is given by
AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

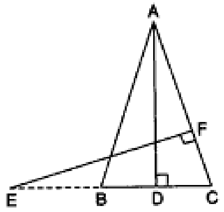
20. **Assertion (A):** 2 is a rational number. **[1]**
Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

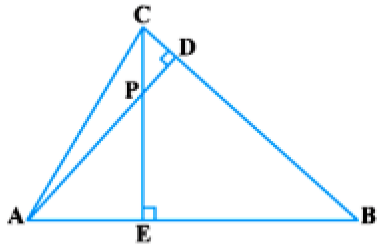
21. The age of a father is equal to the sum of the ages of his 5 children. After 15 years, sum of the ages of the children will be twice the age of the father. Find the age of father. **[2]**

22. In the figure, E is the point on side CB produced on an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF$\perp$ AC, prove that $\triangle$ABD $\sim$ $\triangle$ECF. **[2]**

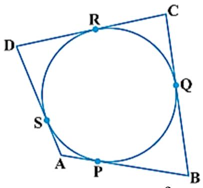


OR

In the figure, altitudes AD and CE of △ABC intersect each other at the point P. Show that: $\triangle\ AEP \sim \triangle\ ADB$

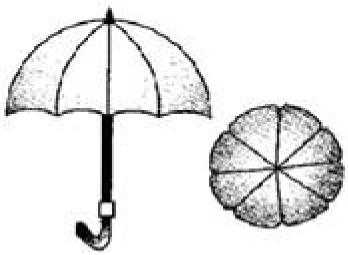


23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC **[2]**



24. Evaluate: $\frac{3\tan^2\ 30^\circ\ +\ \tan^2\ 60^\circ\ + cosec\ 30^\circ\ -\ \tan\ 45^\circ}{\cot^2\ 45^\circ}$. **[2]**

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**



OR

What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm.

Section C

26. Prove that $3 + 2\sqrt{5}$ is irrational. **[3]**

27. Read the following statement carefully and deduce about the sign of the constants p, q, and r. **[3]**

"The zeroes of a quadratic polynomial $px^2\ +\ qx\ +\ r$ are both negatives."

28. The difference between the two numbers is 26 and one number is three times the other. Find them by substitution method. **[3]**

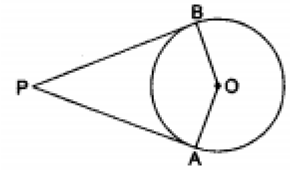
OR

Solve algebraically the following pair of linear equations for x and y

31x + 29y = 33

29x + 31y = 27

29. In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic. **[3]**



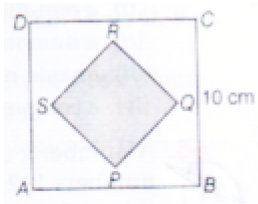
30. Prove the identity: **[3]**

$\frac{\cos A}{1-\sin A}+\frac{\sin A}{1-\cos A}+1=\frac{\sin A\cos A}{(1-\sin A)(1-\cos A)}$

OR

If tan A = n tanB and sin A = m sinB, then prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$

31. A square of side 5 cm is drawn in the interior of another square of side 10 cm and shaded as shown in the figure . A point is selected at random from the interior of square ABCD . What is the probability that the point will be chosen from the shaded part ? **[3]**



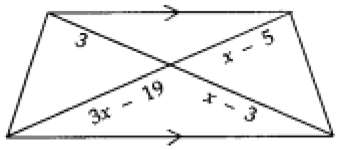
Section D

32. Solve: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x\neq-\frac{1}{2},1$ **[5]**

OR

A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed?

33. If a line is drawn parallel to one side of a trianlge, prove that the other two sides are divided in the same ratio. Using the above result, find x from the adjoining figure. **[5]**



34. The interior of a building is in the form of cylinder of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building. (Use π = 3.14). **[5]**

OR

A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub. (Useπ= $\frac{22}{7}$)

35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. **[5]**

Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17

500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Section E

36. **Read the text carefully and answer the questions:** **[4]**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Find the production during first year.

(ii) Find the production during 8th year.

OR

In which year, the production is ₹ 29,200.

(iii) Find the production during first 3 years.

37. **Read the text carefully and answer the questions:** **[4]**

To raise social awareness about the hazards of smoking, a school decided to start a ‘No smoking’ campaign. 10 students are asked to prepare campaign banners in the shape of a triangle. The vertices of one of the triangles are P(-3, 4), Q(3, 4) and R(-2, -1).

(i) What are the coordinates of the centroid of $\triangle PQR$?

(ii) If T be the mid-point of the line joining R and Q, then what are the coordinates of T?

OR

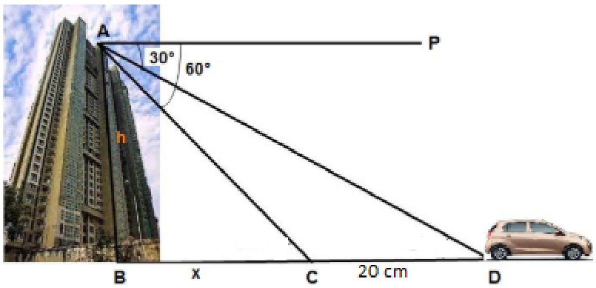
What are the coordinates of centroid of $\triangle STU$?

(iii) If U be the mid-point of line joining R and P, then what are the coordinates of U?

38. **Read the text carefully and answer the questions:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60°. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to

30°.



(i) Find the value of x.

(ii) Find the height of the building AB.

OR

Find the distance between top of the building and a car at position C?

(iii) Find the distance between top of the building and a car at position D?

Solution

Section A

1.
   (d) 120

   Explanation: Least positive integer divisible by 20 and 24 is
   LCM of (20, 24).

   $20 = 2^2 \times 5$

   $24 = 2^3 \times 3$

   $\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5 = 120$

   Thus 120 is divisible by 20 and 24.

2.
   (c) 180

   Explanation: It is given that: a = $(2^2 \times 3^3 \times 5^4)$ and b = $(2^3 \times 3^2 \times 5)$

   $\therefore$ HCF (a, b) = Product of smallest power of each common prime factor in the numbers = $2^2 \times 3^2 \times 5 = 180$

3. **(a)** x(x + 1) = 240

   Explanation: Let one of the two consecutive integers be x
   then the other consecutive integer will be (x + 1)

   $\therefore$ According to question, $(x) \times (x + 1) = 240$

   $\Rightarrow$ x(x + 1) = 240

4. **(a)** 87^o

   Explanation: Let x and y be the measures of $\angle A$ and $\angle B$ respectively.

   Now, $\angle A + \angle B + \angle C = 18^o$ [By angle sum property]

   $\Rightarrow x + y + 50^o = 180^o$ [Given, $\angle C = 50^o$]

   $\Rightarrow x + y = 130^o$ ...(i)

   Also, $\angle A - \angle B = 44^o \Rightarrow x - y = 44^o$ ...(ii)

   Adding (i) and (ii), we get

   $2x = 174^o \Rightarrow x = 87^o \Rightarrow \angle A = 87^o$

5.
   (b) a = c

   Explanation: Since, If $ax^2 + bx + c = 0$ has equal roots, then

   $b^2 - 4ac = 0$

   $\Rightarrow (a + c)^2 - 4ac = 0$ ... [Given: b = a + c]

   $\Rightarrow a^2 + c^2 + 2ac - 4ac = 0$

   $\Rightarrow a^2 + c^2 - 2ac = 0$

   $\Rightarrow (a - c)^2 = 0$

   $\Rightarrow$ a - c = 0

   $\Rightarrow$ a = c

6.
   (b) (6, -12)

   Explanation: If (a, b) and (c, d) be the coordinates of any two points, then the coordinates of the mid-point joining those points be $\left(\frac{(a+c)}{2}, \frac{(b+d)}{2}\right)$.

   The line segment is formed by points are (0, 0) and (x, y), whose mid-point is (3, -6).

   Then,

   $\frac{(0+x)}{2} = 3$ and $\frac{(0+y)}{2} = -6$

or, $\frac{x}{2}$ = 3 or, $\frac{y}{2}$ = -6

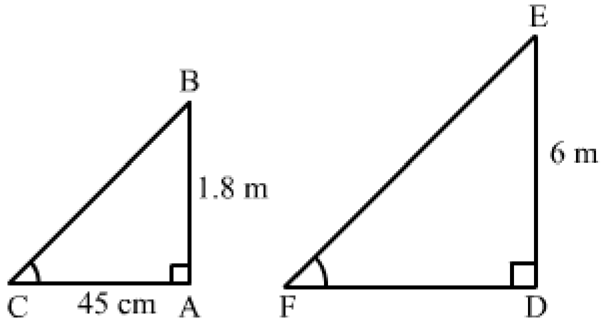
or, x = 6 or, y = -12

Therefore the required point is (6, -12).

7.

(c) 1.5 m

Explanation:



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

AB = 1.8 m

AC = 45 cm = 0.45 m

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

DE = 6 m

DF = ?

Now, in right-angled triangles ABC and DEF, we have:

$\angle$BAC = $\angle$EDF = 90°

$\angle$ACB = $\angle$DFE (Angular elevation of the Sun at the same time)

Therefore, by AA similarity theorem,

we get:$\triangle$ABC$\sim\triangle$DEF

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{1.8}{0.45} = \frac{6}{DF} \Rightarrow DF = \frac{6\times 0.45}{1.8} = 1.5\text{m}$

8. **(a)** 3 : 4

Explanation: $\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$ [by angle-bisector theorem]

9.

(c) 44°

Explanation: In the given figure, PQ is the tangent to the circle at A.

$\angle$PAB = 67°, $\angle$AQB = ?

Join BC.

$\angle$BAC = 90° (Angle in a semi circle)

But, $\angle$PAB + $\angle$BAC + $\angle$CAQ = 180°

$\Rightarrow$ 67° + 90° + $\angle$CAQ = 180°

$\Rightarrow$ 157° + $\angle$CAQ = 180°

$\angle$CAQ = 182° – 157° = 23°

$\angle$ACB = $\angle$PAB (Angles in the alternate segment)

$\angle$ACB = 67°

In $\triangle$ACQ,

Ext. $\angle$ACB = $\angle$CAQ + $\angle$AQC

$\Rightarrow$ 67° = 23° + $\angle$AQC

$\Rightarrow\angle$AQC = 67° – 23° = 44°

$\Rightarrow$ $\angle$AQB = 44°

10.

(c) $\frac{83}{8}$

Explanation: $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + \left(4 \times 2^2\right) + \left(\frac{1}{2} \times 0^2\right) - 2 \times (\sqrt{3})^2$
$= \left(\frac{3}{4} \times \frac{1}{2}\right) + 16 + 0 - 6 = \frac{3}{8} + 10 = \frac{83}{8}$

11.

(b) 40 m

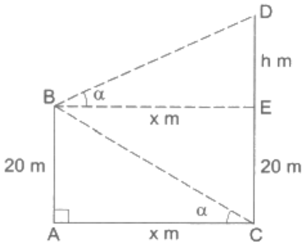
Explanation: Let AB be the cliff and CD be the tower. Draw BE $\perp$ CD.

Let $\angle$ ACB = $\angle$EBD = α and let DE = h metres.

ALso, AB = 20 m, Let AC = BE = x m. Then

$\frac{x}{h}$ = cot α and $\frac{x}{20}$ = cot α

Thus, $\frac{x}{h} = \frac{x}{20} \Rightarrow$ h = 20 m.



The height of the tower is = CD = 20 + 20 = 40 m

12.

(c) b^2 - a^2

Explanation: Given,

a cot θ + b cosec θ = p

b cot θ + a cosec θ = q

Squaring and subtracting above equations, we get

$p^2 - q^2$ = (a cot θ + b cosec θ)^2 - (b cot θ + a cosec θ)^2

= a^2 cot^2 θ + b^2 cosec^2 θ + 2ab cot θ cosec θ - (b^2 cot^2 θ + a^2 cosec^2 θ + 2ab cot θ cosec θ)

= a^2 cot^2 θ + b^2 cosec^2 θ + 2ab cot θ cosec θ - b^2 cot^2 θ - a^2 cosec^2 θ - 2ab cot θ cosec θ

= a^2 (cot^2 θ - cosec^2 θ) + b^2 (cosec^2 θ - cot^2 θ)

= -a^2 (cosec^2 θ - cot^2 θ) + b^2 (cosec^2 θ - cot^2 θ)

= -a^2 $\times 1$ + b^2 $\times$ 1

= b^2 - a^2

13.

(c) 52 cm^2

Explanation: We know that perimeter of a sector of radius, $r = 2r + \frac{\theta}{360} \times 2\pi r$ ...(1)

Therefore, substituting the corresponding values of perimeter and radius in equation (1), we get,

$29 = 2 \times 6.5 + \frac{\theta}{360} \times 2\pi \times 6.5$ ...(2)

$29 = 2 \times 6.5 \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2\times6.5} = \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2\times6.5} - 1 = \frac{\theta}{360} \times \pi$ ....(3)

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$

From equation (3), we get

Area of the sector $= \left(\frac{29}{2\times6.5} - 1\right) r^2$

Substituting r = 6.5 we get,

Area of the sector $= \left(\frac{29}{2\times6.5} - 1\right) 6.5^2$

$= \left(\frac{29\times6.5^2}{2\times6.5} - 6.5^2\right)$

$= \left(\frac{29\times6.5}{2} - 6.5^2\right)$

$= \left(\frac{29\times6.5}{2} - 6.5^2\right)$

$= (94.25 - 42.25)$

= 52

Therefore, area of the sector is 52 cm^2.

14. **(a)** 231 cm^2

Explanation: The angle subtended by the arc = 60^o

So, area of the sector = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

= $(\frac{441}{6}) \times (\frac{22}{7})cm^2$

= 231 cm^2

15.

(d) less than 0

Explanation: We know that the probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

16.

(c) 470

Explanation: Median = $l + \frac{\frac{n}{2} - c}{f} \times h$

= 400 + $\frac{\frac{44}{2} - 8}{20} \times 100$

= 400 + $\frac{14}{20} \times 100$

= 400 + 14 $\times$ 5

= 400 + 70

= 470

17. **(a)** $\frac{x}{2\sqrt{\pi}}$

Explanation: Let V_1 be the volume of the cylinder with radius r and height h, then

$V_1 = \pi r^2 h$ .... (i)

Now, let V_2 be the volume of the box, then

$V_2 = x^2h$

It is given that $V_1 = 1/4\ V_2$. Therefore,

$\pi r^2 h = \frac{1}{4} x^2 h$

$\Rightarrow r^2 = \frac{x^2}{4\pi} \Rightarrow r = \frac{x}{2\sqrt{\pi}}$

18.

(d) 30 – 40

Explanation: According to the question,

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Freq	3	9	15	30	18	5

Here Maximum frequency is 30.

Therefore, the modal class is 30 – 40.

19.

(d) A is false but R is true.

Explanation: PQ = 10

$PQ^2 = 100$

$(10 - 2)^2 + (y + 3)^2 = 100$

$(y + 3)^2 = 100 - 64 = 36$

y + 3 = $\pm$6

y = -3 $\pm$6

y = 3, -9

20.

(c) A is true but R is false.

Explanation: Here reason is not true. $\sqrt{4} = \pm 2$, which is not an irrational number.

Section B

21. Let age of father = x $years$ and

sum of the ages of 5 children = y $years$

$\Rightarrow x = y$ ..(i)

After 15 years, father's age = $x + 15$ and sum of ages of 5 children = $y + 75$

ATQ,

$y + 75 = 2(x + 15)$

$\Rightarrow 2x - y = 45$ ..(ii)

Using eq. (i), we get

$2x - x = 45 \Rightarrow x = 45$

therefore, Age of father =45 $years$

22. E is the point on side CB produced on an isosceles triangle ABC with AB=AC.AD-BC$\perp$ and EF$\perp$AC. with AB=AC. Also, AD $\perp$ BC and EF$\perp$AC.

To prove: $\triangle$ABD $\sim$ $\triangle$ECF

Proof: In $\triangle$ABD and $\triangle$ECF,

$\therefore$ AB = AC ....Given

$\therefore$ $\angle$ACB = $\angle$ABC ......Angle opposite to equal sides of a triangle are equal

$\Rightarrow \angle ABC = \angle ACB$

$\Rightarrow \angle ABD = \angle ECF$ ....(1)

$\angle ADB = \angle EFC$ ....(2) [Each equal to 90^0 In view of (1) and (2)]

$\triangle$ABD $\sim$ $\triangle$ECF.............AA similarity criterion

OR

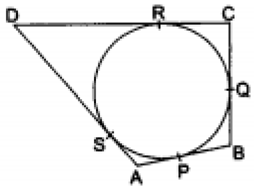
In $\triangle$AEP and $\triangle$ADB, we have

AEP= $\angle$ ADB ....(1) [Each equal to 90^0]

$\angle$EAP=$\angle$DAB ..... (2) [Common angle]

In view of (1) and (2),

$\triangle$AEP$\sim$$\triangle$ADB [AA similarity criterion]

23. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

AP = AS, ... (i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]

DR = DS. ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC.

Hence, AB + CD = AD + BC.

24. $= \frac{3\tan^2 30^\circ + \tan^2 60^\circ + cosec 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$

$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$

$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$

= 1 + 3 + 2 - 1

= 6 - 1

= 5

25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm ^{2}

OR

Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 5 cm+ Area of a circle of radius 12 cm

$\Rightarrow \pi R^2 = \left(\pi \times 5^2 + \pi \times 12^2\right)$

$\Rightarrow \pi R^2 = (25\pi + 144\pi)$

$\Rightarrow \pi R^2 = 169\pi$

$\Rightarrow R^2 = 169$

$\Rightarrow$ R = 13 cm

$\Rightarrow$ Diameter = 2R

= 26 cm

Section C

26. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b $(b \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$ Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$

Since a and b are integers,

We get $\frac{a}{2b} - \frac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect

assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

27. Sum of zeroes = $\frac{-q}{p}$ <0 [as zeroes are negative means sum of zeroes is negative]

So that $\frac{q}{p}$ > 0

$\Rightarrow\ q > 0,\ p > 0\ or\ q < 0,\ \ p <\ 0$ …….. (i)

Product of zeros = $\frac{r}{p}$ > 0 [as zeroes are negative means product of zeroes is positive]

$\Rightarrow\ r > 0,\ p > 0\ or\ r < 0,\ p\ < 0$ ………. (ii)

∴ From (i) and (ii), p, q and r will have same signs i.e.

Either $p > 0,\ q > 0,\ \ r > 0$

Or $p\ < 0,\ q < 0,\ r\ < 0$.

28. Let the two numbers be x and y (x > y) then, according to the question,

the pair of linear equations formed is:

x - y = 26.........(1)

x = 3y.............(2)

Substitute the value of x from equation (2) in equation (1), we get

3y - y = 26

$\Rightarrow \quad 2y = 26$

$\Rightarrow \quad y = \frac{26}{2}$

$\Rightarrow \quad y = 13$

Substituting this value of y in equation (2), we get

x = 3(13) = 39

Hence, the required numbers are 39 and 13.

verification: Substituting x = 39 and y = 13, we find that both

the equation (1) and (2) are satisfied as shown below:

x - y = 39 - 13 = 26

3y = 3(13) = 39 = x.

This verifies the solution.

OR

$31x + 29y = 33$ ----------(1)

$29x + 31y = 27$ ------------ (2)

Multiply (1) by 29 and (2) by 31 (Since 29,31 are primes and Lcm is 29×31)

(1) becomes 31x×29 + 29×29y = 33×29 ------- (3)

(2) becomes 29x×31+31×31y=27×31 ------- (4)

Subtracting (3) from (4),

$(312 - 292)y = 27 \times 31 - 33 \times 29 = -120$

$(31 - 29)(31 + 29)y = -120$

$120y = -120$

$y = -1$

Substituting in (1),

$31x - 29 = 33$

$31x = 62$

Hence,

$x = 2 \; and \; y = -1$

29. Here, OA = OB

And OA ⊥ AP, OB ⊥ BP (Since tangent is perpendicular to the radius at the point of contact)

$\therefore \angle OAP$ = 90°,

$\angle OBP$ = 90°

$\therefore \angle OAP$ + $\angle OBP$ =90° + 90° = 180°

$\therefore \angle AOB$ + $\angle APB$ = 180°

(Since,$\angle AOB$+ $\angle OAP$ + $\angle OBP$ + $\angle APB$ = 360°)

Thus, sum of opposite angle of a quadrilateral is 180°.

Hence, A, O, B and P are concyclic.

30. We have,

$$\Rightarrow \quad \text{LHS} = \frac{\cos A}{1-\sin A} + \frac{\sin A}{1-\cos A} + 1$$

$$\Rightarrow \quad \text{LHS} = \frac{\cos A(1-\cos A)+\sin A(1-\sin A)+(1-\sin A)(1-\cos A)}{(1-\sin A)(1-\cos A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\cos A-\cos^2 A+\sin A-\sin^2 A+1-\sin A-\cos A+\sin A\cos A}{(1-\sin A)(1-\cos A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{(\cos A+\sin A)-\left(\cos^2 A+\sin^2 A\right)+1-(\cos A+\sin A)+\sin A\cos A}{(1-\sin A)(1-\cos A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{(\cos A+\sin A)-1+1-(\cos A+\sin A)+\sin A\cos A}{(1-\sin A)(1-\cos A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\sin A\cos A}{(1-\sin A)(1-\cos A)} = \text{RHS}$$

OR

Given,

tan A = n tan B

⇒ tanB = $\frac{1}{n}$tan A

⇒ cotB = $\frac{n}{\tan A}$ ....(1)

Also given,

sin A = m sin B

⇒ sin B = $\frac{1}{m}$sin A

⇒ cosec B = $\frac{m}{\sin A}$ .....(2)

We know that, cosec^2B - $\cot^2$B = 1, hence from (1) & (2) :-

$$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \quad \frac{m^2-n^2\cos^2 A}{\sin^2 A} = 1$$

⇒ m^2 - $n^2\cos^2$A = $\sin^2$A

$\Rightarrow$ m^2 - n^{2}cos^{2}A = 1 - cos^{2}A

$\Rightarrow$ m^2 - 1 = n^{2}cos^{2}A - cos^{2}A

$\Rightarrow$ m^2 - 1 = (n^2 - 1) cos^{2}A

$\Rightarrow \frac{m^2-1}{n^2-1} = \cos^2 A$

31. Area of square ABCD = (side)^2 = $10^2 = 100 \text{ cm}^2$

So Total events n=100

Now , area of the sqaure PQRS = (side)^2 = 5^2 =25 cm^2 [$\because$ side = 5cm, given]

So favorable possibility m = 25

$\therefore$ P(the point will be chosen from the shaded part)=$\frac{m}{n} = \frac{Area(square\ PQRS)}{Area(square\ ABCD)} = \frac{25}{100} = 0.25$

Section D

32. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y-1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2

OR

Let the original average speed of the train be x km/hr.

Time taken to cover 63 km $= \frac{63}{x}$ hours

Time taken to cover 72 km when the speed is increased by 6 km/hr $= \frac{72}{x+6}$ hours

By the question,we have,

$\frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow \frac{21}{x} + \frac{24}{x+6} = 1$

$\Rightarrow \frac{21x+126+24x}{x^2+6x} = 1$

$\Rightarrow$ 45x + 126 = x^2 + 6x

$\Rightarrow$ x^2 - 39x - 126 = 0

$\Rightarrow$ x^2 - 42x + 3x - 126 = 0

$\Rightarrow$ x(x - 42) + 3(x - 42) = 0

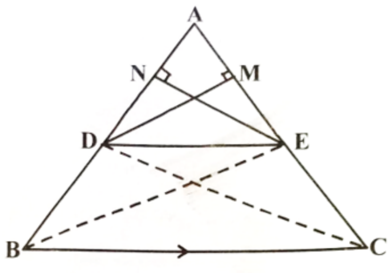
$\Rightarrow$ (x - 42)(x + 3) = 0

$\Rightarrow$ x - 42 = 0 or x + 3 = 0

$\Rightarrow$ x = 42 or x = -3

Since the speed cannot be negative, $x \neq -3$.

Thus, the original average speed of the train is 42 km/hr.

33. 

Given: $\triangle$ABC in which DE||BC

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and DC and Draw EN$\perp$AD and DM $\perp$AE.

Proof: Consider $\triangle$DBE,

Area of $\triangle$DBE=$\frac{1}{2}$DB$\times$EN .....(i)

Again, Consider $\triangle$ADE

Area of $\triangle$ADE= $\frac{1}{2}$AD$\times$EN.....(ii)

Divide eq. (i) by (ii), we get,

$\Rightarrow \frac{Area(\Delta DBE)}{Area(\Delta ADE)} = \frac{\frac{1}{2}\times DB\times EN}{\frac{1}{2}\times AD\times EN}$

$\Rightarrow \frac{Area(\Delta DBE)}{Area(\Delta ADE)} = \frac{DB}{AD}$ .....(iii)

Now Consider $\triangle$ECD,

Area of $\triangle$ECD=$\frac{1}{2}$EC$\times$DM ...(iv)

Again, Consider $\triangle$ADE

Area of $\triangle$ADE= $\frac{1}{2}$AE$\times$DM...(v)

Divide eq. (iv) by (v), we get,

$\Rightarrow \frac{Area(\Delta ECD)}{Area(\Delta ADE)} = \frac{\frac{1}{2}\times EC\times DM}{\frac{1}{2}\times AE\times DM}$

$\Rightarrow \frac{Area(\Delta ECD)}{Area(\Delta ADE)} = \frac{EC}{AE}$ ......(vi)

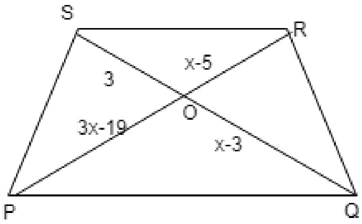
Since $\triangle$DBE and $\triangle$EDC lies in the same base i.e. DE and the same parallels i.e. DE and BC.

$\Rightarrow$ Area ($\triangle$DBE)= Area ($\triangle$EDC)

From (iii) and (vi), we get

$\frac{DB}{AD}=\frac{EC}{AE}$

Hence Proved



Since,SR||PQ, $\Rightarrow \Delta POQ \sim \Delta ROS$ [By AAsimilarty criteria]

$\Rightarrow \frac{PO}{OR} = \frac{OQ}{OS}$

$\Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$

$\Rightarrow$ 3(3x - 19) = (x-5) (x-3)

$\Rightarrow$ 9x - 57 = x^2 - 8x + 15

$\Rightarrow$ x^2 - 8x - 9x + 15 + 57 = 0

$\Rightarrow$ x^2 - 17x + 72 = 0

$\Rightarrow$ x^2 - 8x - 9x + 72 = 0

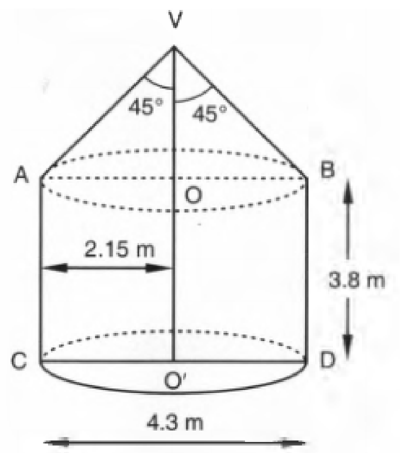
$\Rightarrow$ x(x - 8) -9(x - 8) = 0

$\Rightarrow$ (x - 8) (x - 9) = 0

$\Rightarrow$ x = 8 or x = 9

34. r_1 = Radius of the base of the cylinder = $\frac{4.3}{2}$ m = 2.15 m

$\therefore$ r_2 = Radius of the base of the cone = 2.15 m, h_1 = Height of the cylinder = 3.8 m



In $\triangle$VOA, we have

$$\sin 45^\circ = \frac{OA}{VA} \Rightarrow \frac{1}{\sqrt{2}} = \frac{2.15}{VA} \Rightarrow VA = (\sqrt{2} \times 2.15)\text{m} = (1.414 \times 2.15)\text{m} = 3.04\text{m}$$

Clearly, $\triangle$VOA is an isosceles triangle. Therefore, VO = OA = 2.15 m

Thus, we have

h_2 = Height of the cone = VO = 2.15 m, l_2 = Slant height of the cone = VA = 3.04 m

Let S be the Surface area of the building. Then,

$\Rightarrow$ S = Surface area of the cylinder + Surface area of cone

$\Rightarrow$ S = $(2\pi r_1 h_1 + \pi r_2 l_2)$ m^2

$\Rightarrow$ S = $(2\pi r_1 h_1 + \pi r_1 l_2)$ m^2 [$\because$ $r_1 = r_2$ - 2.15 m]

$\Rightarrow$ S = $\pi r_1(2h_1 + l_2)$ m^2

$\Rightarrow$ S = 3.14 $\times$ 2.15 $\times$ (2 $\times$ 3.8 + 3.04) m^2 = 3.14 $\times$ 2.15 $\times$ 10.64 m^2 = 71.83 m^2

Let U be the volume of the building. Then,

V = Volume of the cylinder + Volume of the cone

$$\Rightarrow \quad V = \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_2^2 h_2\right) \text{m}^3$$

$$\Rightarrow \quad V = \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_1^2 h_2\right) \text{m}^3 \quad [\because r_2 = r_1]$$

$$\Rightarrow \quad V = \pi r_1^2 \left(h_1 + \frac{1}{3}h_2\right) \text{m}^3$$

$$\Rightarrow \quad V = 3.14 \times 2.15 \times 2.15 \times \left(3.8 + \frac{2.15}{3}\right) \text{m}^3$$

$$\Rightarrow \quad V = [3.14 \times 2.15 \times 2.15 \times (3.8 + 0.7166)]\text{m}^3$$

$$\Rightarrow \quad V = (3.14 \times 2.15 \times 2.15 \times 4.5166)\text{m}^3 = 65.55\text{m}^3$$

OR

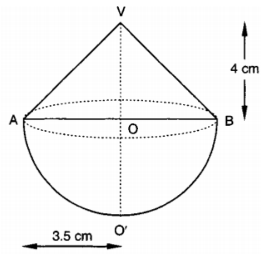
We have, radius of the hemisphere = 3.5 cm

Height of the cone = 4 cm

Radius of the cylinder = 5 cm

|Height of the cylinder = 10.5 cm

We have to find out the volume of water left in the cylindrical tub



$\therefore$ Volume of the solid = Volume of its conical part + Volume of its hemispherical part

$$= \left\{\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3\right\} \text{cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2\{4 + 2 \times 3.5\}\text{cm}^3 = \left\{\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$$

Clearly, when the solid is submerged in the cylindrical tub the volume of water that flows out of the cylinder is equal to the volume of the solid.

Hence,

Volume of water left in the cylinder = Volume of cylinder - Volume of the solid

$$= \left\{\frac{22}{7} \times (5)^2 \times 10.5 - \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$$

$$= \left\{\frac{22}{7} \times 25 \times \frac{21}{2} - \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11\right\} \text{cm}^3$$

$$= \left(11 \times 25 \times 3 - \frac{1}{3} \times 11 \times \frac{7}{2} \times 11\right) \text{cm}^3$$

= (825 - 141.16) cm^3 = 683.83 cm^3

35.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7

200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,

N = Σf_i = 100

⇒ 76 + x + y = 100

⇒ x + y = 24

It is given that the median is 525. Clearly, it lies in the class 500 - 600

∴ l = 500, h = 100, f = 20, F = 36 + x and N = 100

Now, Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h

⇒ 525 = 500 + $\frac{50-(36+x)}{20} \times$ 100

⇒ 525 - 500 = (14 - x)5

⇒ 25 = 70 - 5x

⇒ 5x = 45

⇒ x = 9

Putting x = 9 in x + y = 24, we get y = 15

Hence, x = 9 and y = 15

Section E

36. **Read the text carefully and answer the questions:**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Let 1st year production of TV = x

Production in 6th year = 16000

t_6 = 16000

t_9 = 22,600

t_6 = a + 5d

t_9 = a + 8d

16000 = x + 5d ...(i)
22600 = x + 8d ...(ii)
-　　-　-
-6600 = - 3d

d = 2200

Putting d = 2200 in equation ...(i)

16000 = x + 5 × (2200)

16000 = x + 11000

x = 16000 - 11000

x = 5000

$\therefore$ Production during 1st year = 5000

(ii) Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

OR

Let in n^{th} year production was = 29,200

$t_n = a + (n - 1)d$

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12^{th} year, the production is 29,200

(iii) Production during first 3 year = Production in ($1^{st} + 2^{nd} + 3^{rd}$) year

Production in 1^{st} year = 5000

Production in 2^{nd} year = 5000 + 2200

= 7200

Production in 3^{rd} year = 7200 + 2200

= 9400

$\therefore$ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

37. **Read the text carefully and answer the questions:**

To raise social awareness about the hazards of smoking, a school decided to start a 'No smoking' campaign. 10 students are asked to prepare campaign banners in the shape of a triangle. The vertices of one of the triangles are P(-3, 4), Q(3, 4) and R(-2, -1).

(i) We have, P(-3, 4), Q(3, 4) and R(-2, -1).

$\therefore$ Coordinates of centroid of $\triangle$PQR

$= \left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$

(ii) Coordinates of T = $\left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

OR

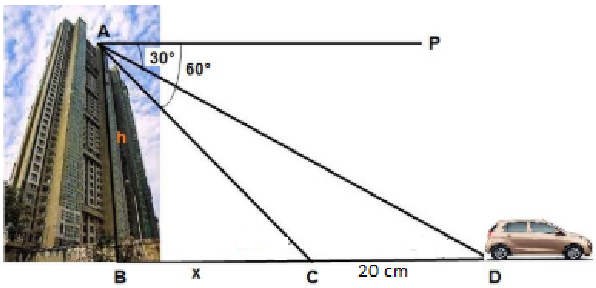
The centroid of the triangle formed by joining the mid-points of sides of a given triangle is the same as that of the given triangle.

So, centroid of $\triangle$STU = $\left(\frac{-2}{3}, \frac{7}{3}\right)$

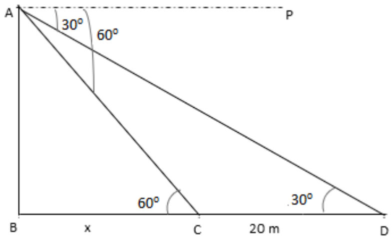
(iii) Coordinates of U = $\left(\frac{-2-3}{2}, \frac{-1+4}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

38. **Read the text carefully and answer the questions:**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^{o}.



(i) The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In $\triangle$ABC,

tan 60 = $\frac{\text{AB}}{\text{BC}} = \frac{\text{h}}{x}$

$\sqrt{3}$ = $\frac{\text{h}}{x}$

h = $\sqrt{3}$x ...(i)

In $\triangle$ABD,

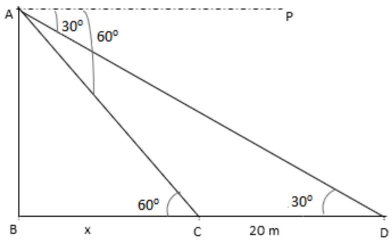
tan 30 = $\frac{\text{AB}}{\text{BD}} = \frac{\text{h}}{x+20}$

$\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

x = 10 m

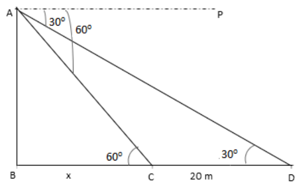
(ii) The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = 10$\sqrt{3}$ = 17.32 m

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

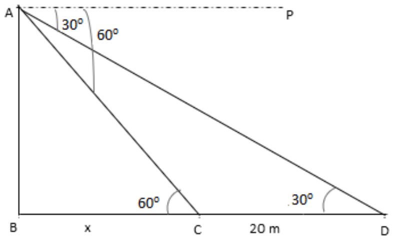
sin 60^{o} = $\frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^{0}}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m

(iii)The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

sin 30^o = $\frac{AB}{AD}$

$\Rightarrow AD = \frac{AB}{\sin 30^0}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$