Chapter 5
Lines and Angles

5.1 You have already learnt about the concept of an angle in the previous class. Now look at the following diagrams and identify the angles (acute angle, right angle, obtuse angle) in these diagrams.

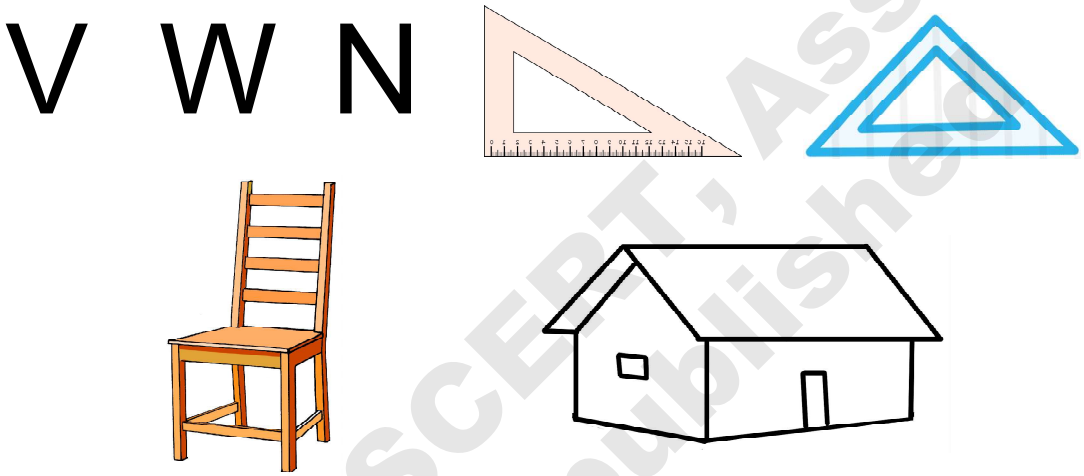


Figure - 5.1

Look at the figures given below and identify (fig 5.2) as right angle, acute angle, obtuse angle, reflex angle, line, ray line segment.

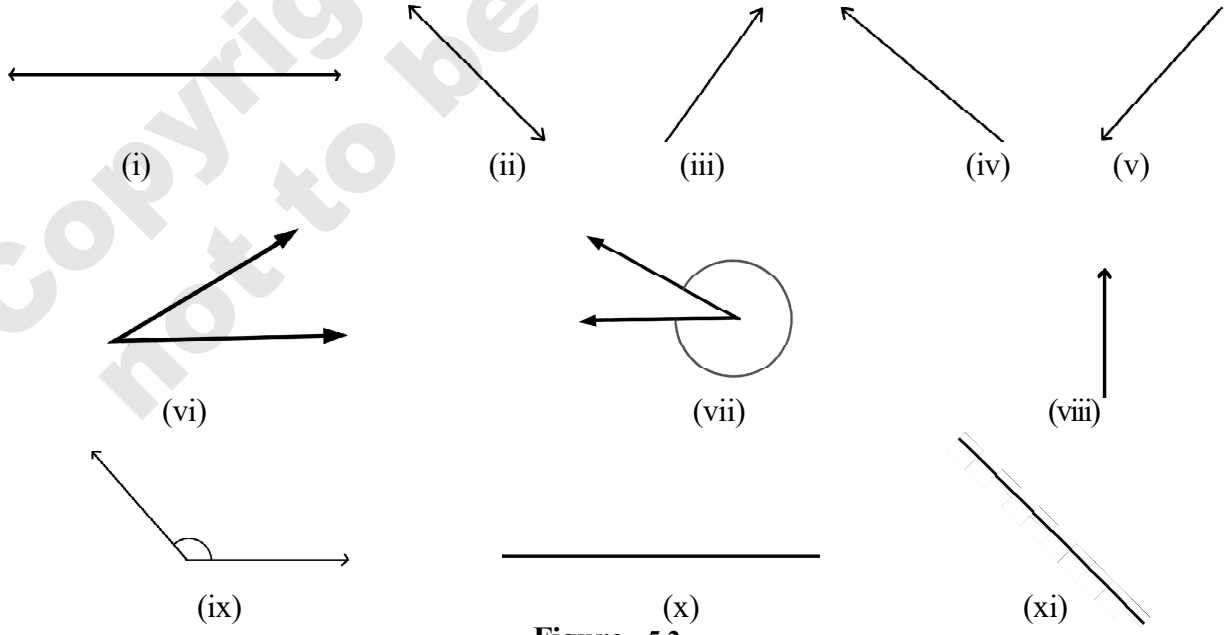


Figure - 5.2

Observe it–

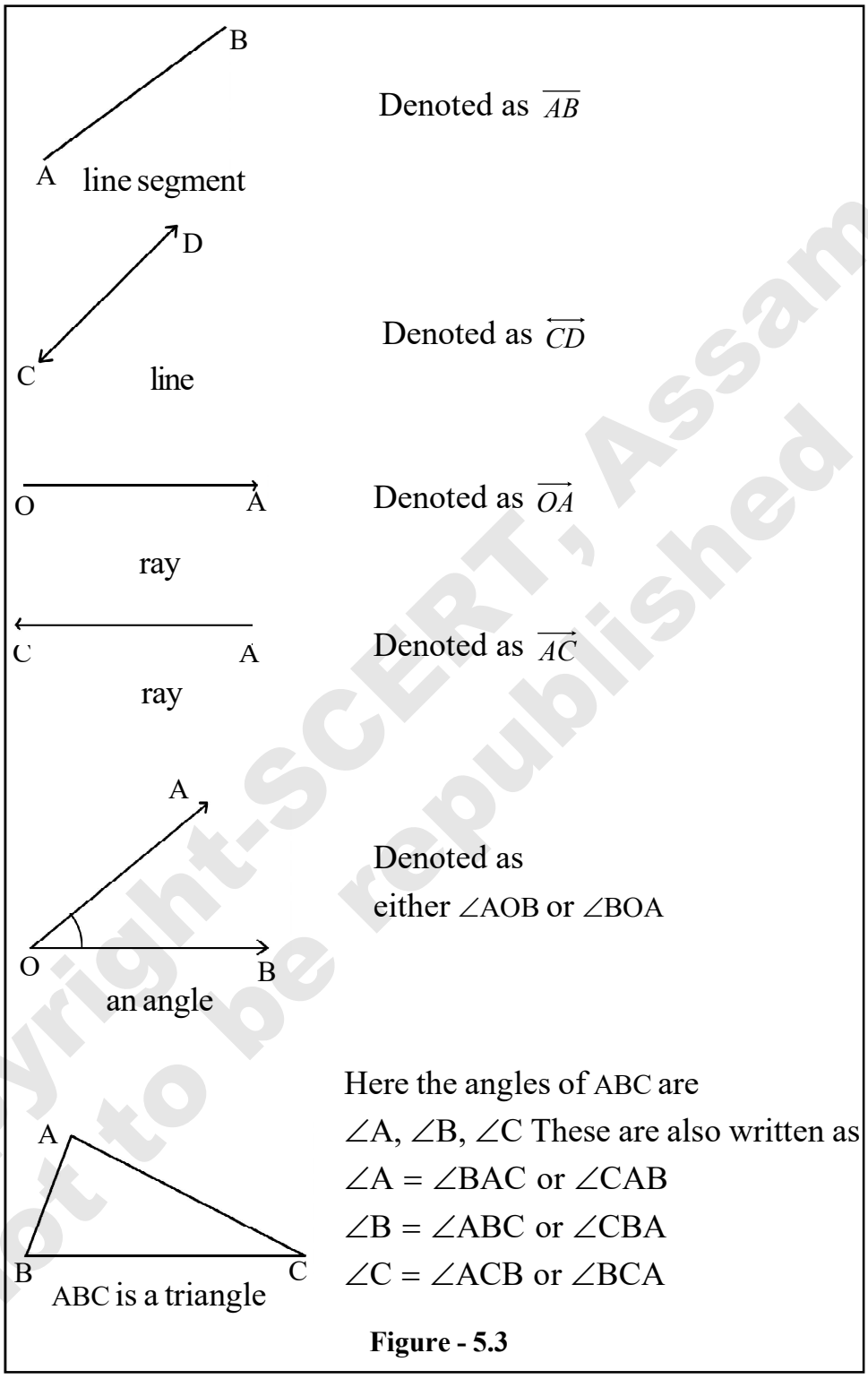


Figure - 5.3

5.2.1 Angle :

Observe the two figures in 5.4. Here 'O' is a point from which two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are produced. AOB is an example of an angle and 'O' is refered to as vertex.

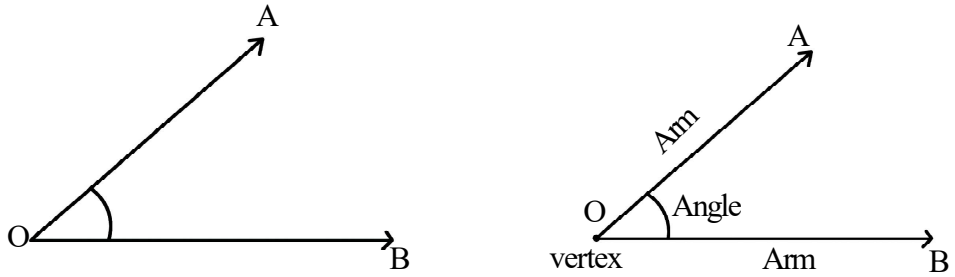


Figure - 5.4

5.2.2 Different parts of an angle :

Arms : The two rays which form an angle are called as the arms of the angle. In the above diagram $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the arms of ∠AOB. While denoting an angle the vertex has to be mentioned in the middle. For example ∠AOB or ∠BOA.

Vertex : The common point where two rays meet to form an angle is called the vertex of the angle. 'O' is the vertex of ∠AOB in fig 5.4.

5.3 Related angles :

5.3.1 Complementary angle

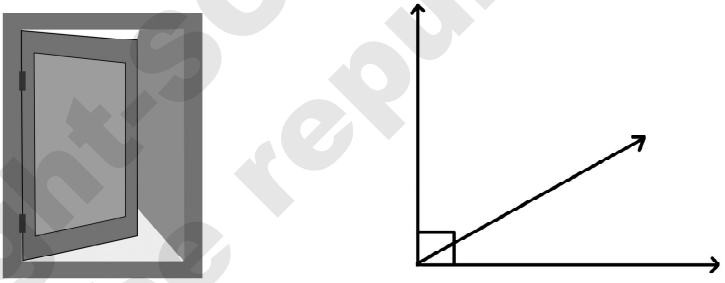
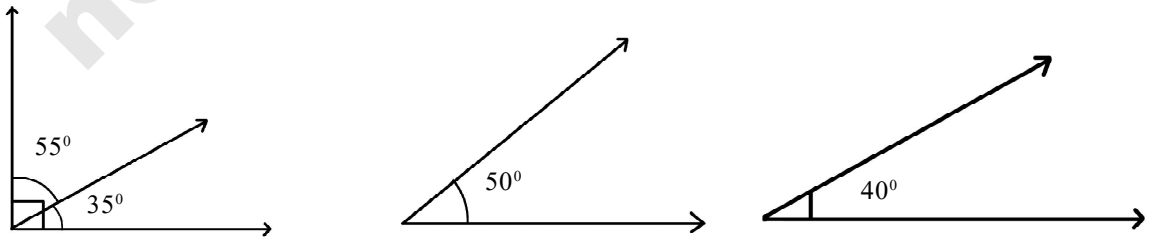
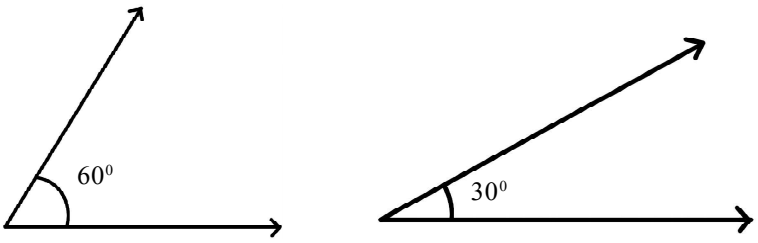


Figure - 5.5

Observe the door in Fig 5.5. On opening a door situated at the corner of a room, it forms two angles. Sum of the two angles is always equal to one right angle. Thus, when the sum of the measures of two angles is equal to 90^0, that is, one right angle then one of the angles is called the complementary angle of the other. Some examples are given below.



(i) (ii)



(iii)

Figure - 5.6

The angles in the figures (i), (ii) and (iii) are three pairs of complementary angles.

Example 1 : An angle is equal to its complementary angle. What is the measure of the angle?

Solution : Let us suppose, the angle = x

So, $x+x = 90$

or, $2x = 90$

or, $x = \left(\frac{90}{2}\right) = 45$

$\therefore$ The required angle = 45^0

Example 2 : The measure of an angle is twice the measure of its complementary angle. What is the measure of the angle ?

Solution :

Let, the angle to be $= x$

The complementary angle will be $= 90 - x$

As per the question $x = 2(90 - x)$

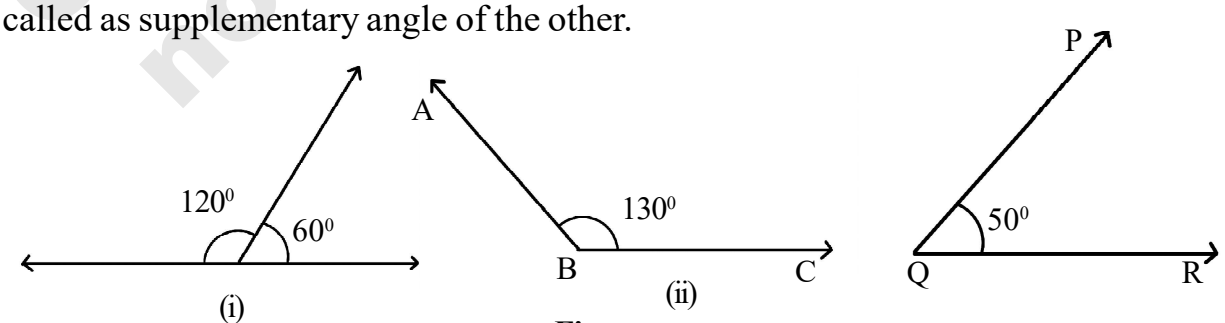
$x = 180 - 2x$

or, $3x = 180$

$\therefore$ The required angle = 60^0

5.3.2 Supplementary Angles :

When the sum of the measures of two angles is equal to 180^0 then one of the angle is called as supplementary angle of the other.



(i) (ii)

Figure - 5.7

Example 3 : Measure of an angle is equal to the measure of its supplementary angle. What is the measure of the angle ?

Solution : Let the angle = x

$\therefore$ supplementary angle $= 180 - x$

According to question, $x = 180 - x$

or, $x + x = 180$

or, $2x = 180$

$x = 180 \div 2 = 90$

$\therefore$ Measure of required angle = 90^0

Example 4 : The measure of an angle is twice of the measure of its supplementary angle. What is the measure of the angle?

Solution : Let the angle be = x

$\therefore$ supplementary angle of $x = 180 - x$

According to question $x = 2(180 - x)$

or $2x + x = 360$

or $3x = 360$

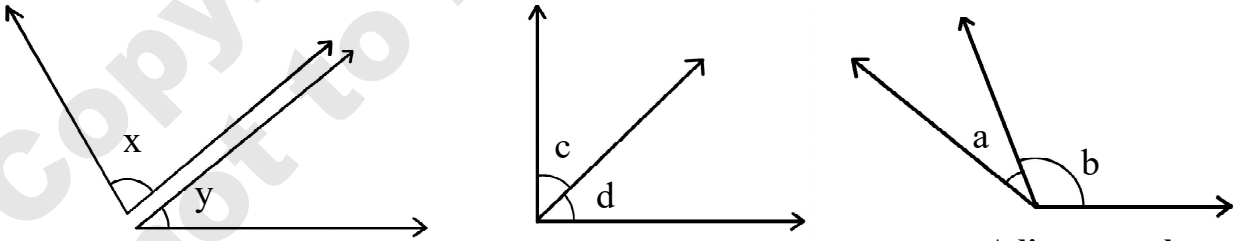
or $x = 120$

$\therefore$ Required angle is $= 120^0$

5.3.3 Adjacent Angles:

Adjacent angles are a pair of angles which are placed next to each other. These angles are such that–

- they have a common vertex ;
- they have a common ray (arm)



Angles are not adjecnt (i) Adjacent angles (ii) Adjacent angles (iii)

Figure - 5.8

5.4 Linear Pair of angles :

A pair of adjacent angles is said to be a linear pair of angles if the two rays excluding the common ray, form a line, that is, they are opposite rays from the same end point.

Remember that the sum of the measures of the angles in a linear pair is 180^0

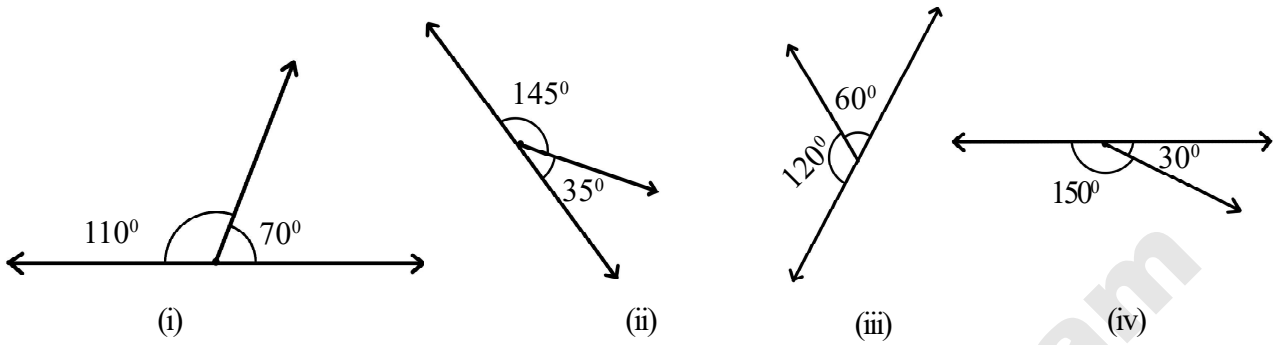


Figure -5.9 (linear pairs)

The pairs of angles shown in figure 5.9 are :

- adjacent, since they have a common ray.
- supplementary since the sum of measures of the angles is 180^0
- linear pair, since sum of the measures of the angles is 180^0 and the two rays excluding the common ray form a line.

Example 5 : Measure of an angle of a linear pair is a right angle; what is the measure of the other angle?

Solution : Measure of an angle of a linear pair = 90^0

Let the measure of other angle = y

$\therefore \quad 90 + y = 180$

or $\quad y = 180 - 90$

$= 90^0$

Thus, when measure of one angle is a right angle of a linear pair angle then the measure of the other angle = 90^0

Example 6 : $\angle PQR$ and $\angle SQR$ is a linear pair. If $\angle PQR = 4x$ and $\angle SQR = 2x$ then find the value of x and also find the measures of the angles.

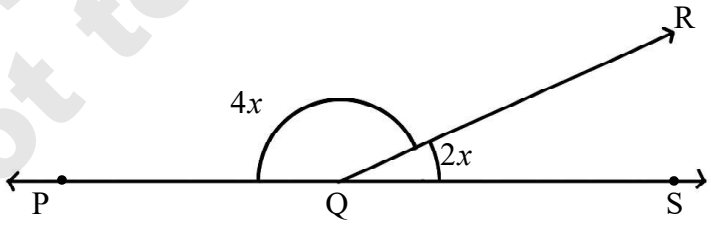


Figure - 5.10

Solution :

Since $\angle PQR$ and $\angle SQR$ is a linear pair

$\therefore \quad \angle PQR + \angle SQR = 180$

or $\quad 4x + 2x = 180$

or, $6x = 180$
or, $x = 180 \div 6$
or, $x = 30$
$\angle PQR = 4x = (4 \times 30)^0 = 120^0$
$\angle SQR = 2x = (2 \times 30)^0 = 60^0$

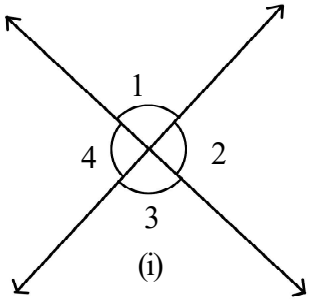
5.5 Vertically Opposite Angles :

When two lines intersect each other, four angles are formed at the point of intersection, of these angles those which are vertically opposite to each other are called as vertically opposite angles.

In the figure 5.11

- $\angle 1$ and $\angle 3$ are a pair of vertically opposite angles.
- $\angle 2$ and $\angle 4$ are vertically opposite angles.
- vertically opposite angles are equal

  that is $\angle 1 = \angle 3$, $\angle 2 = \angle 4$



(i)


(ii) Picture of a pair of scissor


(iii) Picture of Thagaa

Examples of vertically opposite angle found around us

Figure -5.11

Example 7 : Find the values of $\angle x$ and $\angle y$ from the figure 5.12

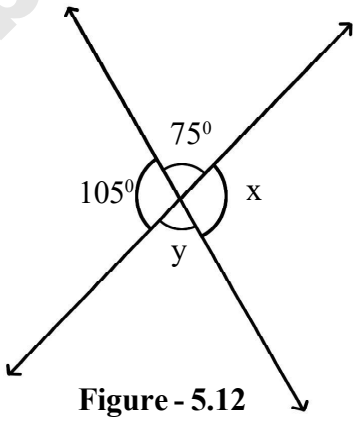


Figure - 5.12

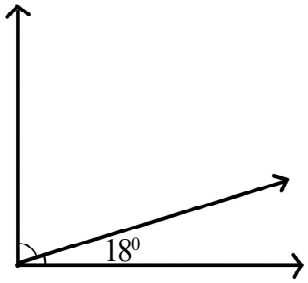
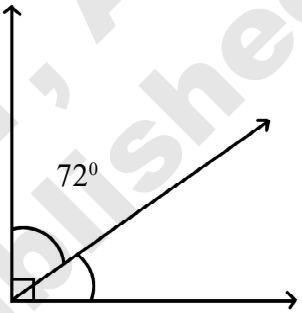
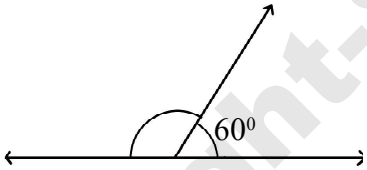
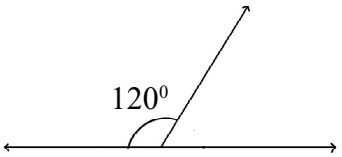
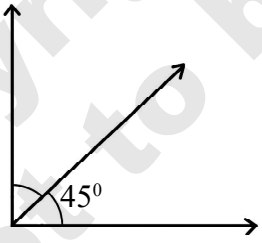
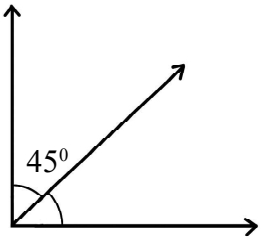
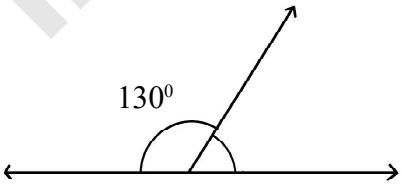
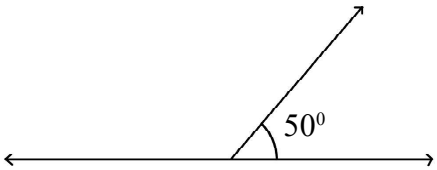
Solution : From fig 5.12

$x = 105^0$ (vertically opposite angle)

Similarlly, $y = 75^0$ (vertically opposite angle)

Activity : Draw angles of different measures in some cards. Angles on two cards will be either complementry or supplementry. Distribute the cards among the students to play a game.

One student will show his card and say "I have ... ^{0} angle" (he will tell the measure of the angle on his card) "who will be my complementary or supplementary?" Other students will try to find complementary or supplementary angles on their own card. The one who has complementary (supplementary) angle on his/her card will stand up and announce " I am your complementry or supplementary". Then another student will mention the angle of his card. In this way the game will be continued till the end.

1st player : Activity starts.  Who has the angle complementry to mine?	It is with me 72^0 
 Who has the angle supplementary to mine?	It is with me 120^0 
 Who has the angle complementry to mine?	It is with me 45^0 
 Who has the angle supplementary to mine?	It is with me 50^0 

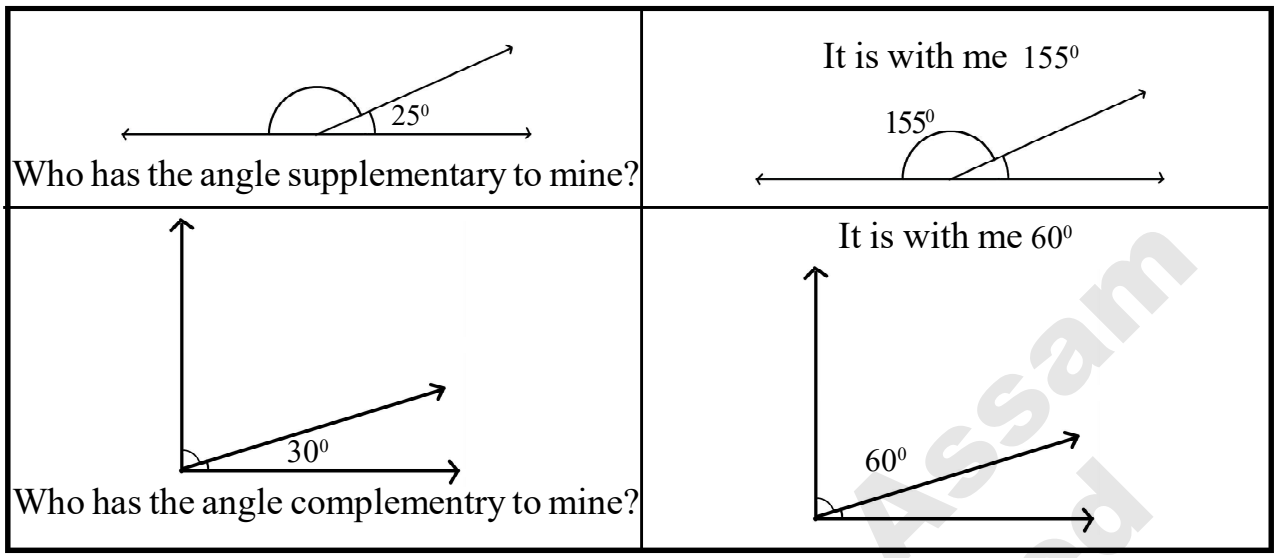
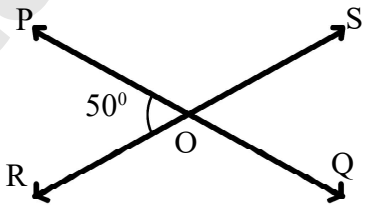


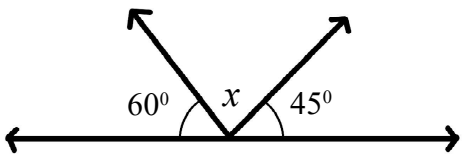
Figure - 5.13

Exercise - 5.1

1. Find out the complementary angles of the following?
   a) 45^0 b) 65^0 c) 41^0 d) 54^0
2. If the difference of measures of a pair of complementary angles is 22^0. Find the angles.
3. Write down the measures of the angles supplementary to each of the following angle.
   a) 100^0 b) 90^0 c) 55^0 d) 125^0
4. A pair of supplementary angles, is such that the larger angle is 44^0 more than the smaller angle. Find the measures of the angles.
5. The lines PQ and RS intersect at O, If $\angle POR = 50^0$ then find the measures of the other angles.

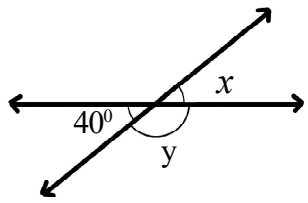


6. Find x from the following figure.

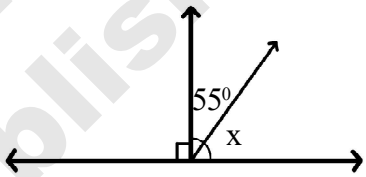


7. Find an angle which is equal to its supplementary angle.
8. The measure of an angle is 24^0 more then its complementary angle. Find the measure of the angle.
9. The measure of an angle is 32^0 less then its complementary angle. Find the measure of the angle.
10. The measure of an angle is five times the measure of its complementary angle. Find the measure of the angle.
11. An angle is five times the measure of its more than its supplementary angle. Find the measure of the angle.
12. The ratio of two supplementary angles is 3 : 2. Find the angles.
13. The ratio of two complementary angles is 4 : 5. Find the angles.
14. Find x and y from the following diagrams.

a)



b)



15. Identify the pairs of complementary angles from the following.
    a) $65^0, 25^0$ b) $63^0, 27^0$ c) $112^0, 68^0$ d) $130^0, 50^0$
16. Identify the pairs of supplementary angles from the following.
    a) $110^0, 70^0$ b) $163^0, 27^0$ c) $112^0, 68^0$ d) $45^0, 45^0$

5.6 Pair of lines

5.6.1 Intersecting Lines

Two lines are said to be intersecting lines if they have only one common point. This common point is called the point of intersection of the two lines.

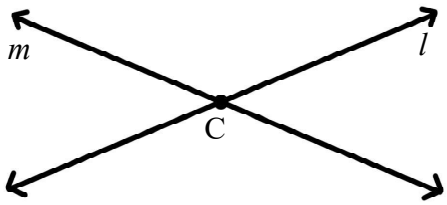


Figure - **5.14**

l and m are two intersecting lines and C is the point of their intersection.

5.6.2 Perpendicular Lines :

Two lines are said to be perpendicular to each other if they form an angle of 90^0 or right angle at the point of intersection.

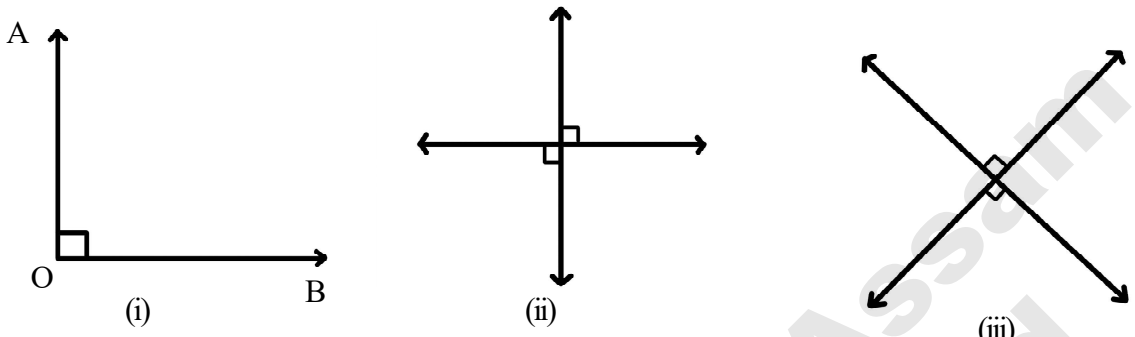


Figure - 5.15 Mutually perpendicular line

Note : Mutually perpendicular lines are always intersecting lines, but interesecting lines may not be perpendicular lines always.

5.6.3 Parallel Lines and Transversal :

What are parallel lines? Do you know?

Look at the figure 5.16. You might have notice the stump of cricket, the rail lines where the train moves, goal post of football or wire of electric lines in the figure. Do you

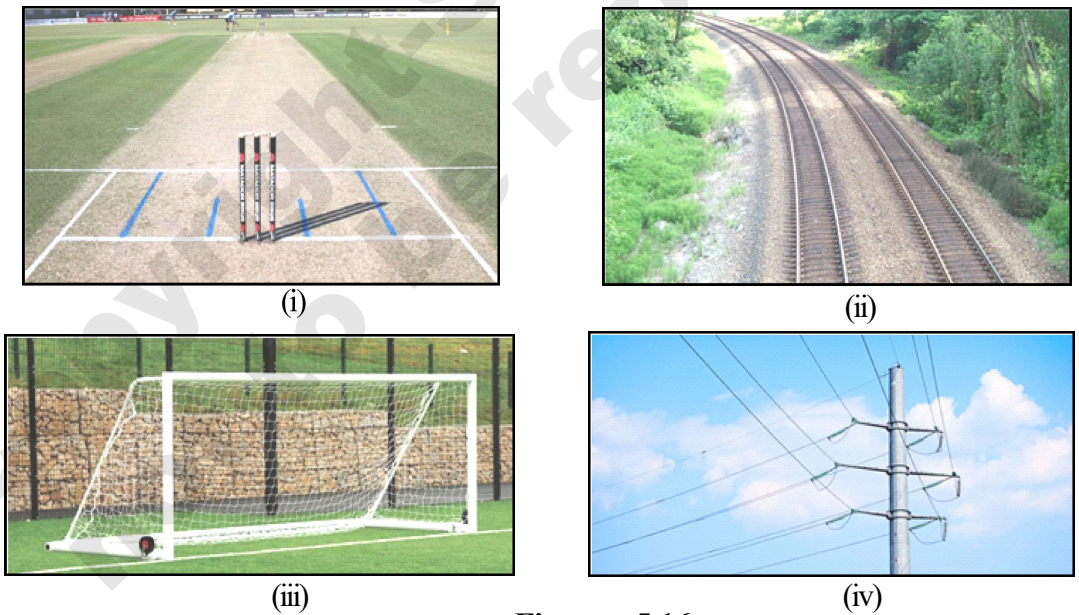


Figure - 5.16

see any similarity among those? The ends of the stumps of cricket or the rail lines do not meet any where. Again, goal posts of football or the wires for electric lines never intersect. Based on these parallel objects, we now discuss about parallel lines, transversal and angles formed by them.

5.6.4 Parallel Lines :

Two lines are said to be parallel if they are always at equal distance in a plane. Two parallel lines never intersect. To represent parallel lines '||' symbol is used.

Some diagrams representating parallel lines are given below.

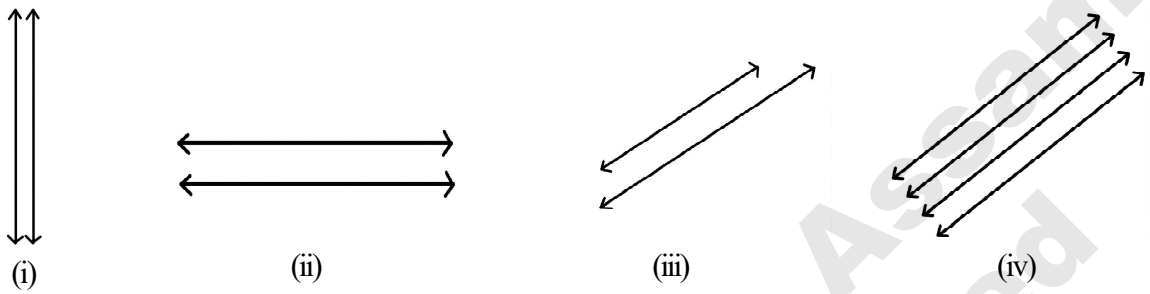


Figure - 5.17

Parallel lines may be two or more in number.

5.6.5 Transversal :

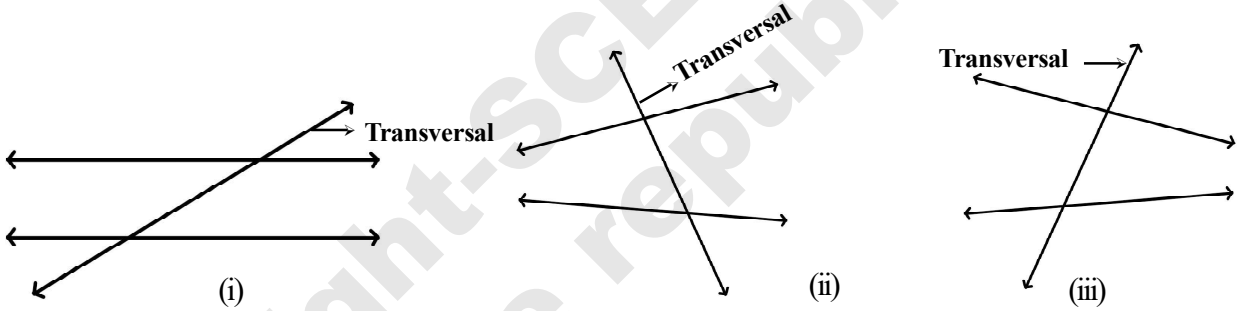


Figure - 5.18

If a line intersects two or more lines at a distinct point then the line is said to be a transversal. Transversals may be found in case of either parallel or non-parallel lines.

Eight angles are formed when a line intersects two lines :–

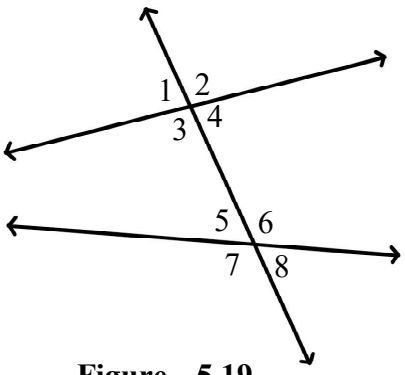


Figure - 5.19

The angles formed by intersection of two lines by a transversal are of the following types.

kinds of angles	see the figure
Interior angles	$\angle 6, \angle 5, \angle 4, \angle 3$
Exterior angles	$\angle 7, \angle 8, \angle 1, \angle 2$
corresponding angles	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ $\angle 4$ and $\angle 8$, $\angle 3$ and $\angle 7$
Alternate interior angles	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Alternate exterior angles	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Interior angles to the same side of transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

5.6.6 Properties of parallel lines and transversal

The following properties can be found to the satisfied when a transversal intersects a pair of parallel lines. (fig 5.20)

a) Each pair of corresponding angles are equal.
$\angle 3 = \angle 7$, $\angle 4 = \angle 8$, $\angle 1 = \angle 5$, $\angle 2 = \angle 6$

b) Each pair of alternate interior angles are equal.
$\angle 3 = \angle 6$, $\angle 4 = \angle 5$

c) Sum of interior angles to the same side is 180^0, that is the angles are supplementary.
$\angle 3 + \angle 5 = 180^0$, $\angle 4 + \angle 6 = 180^0$

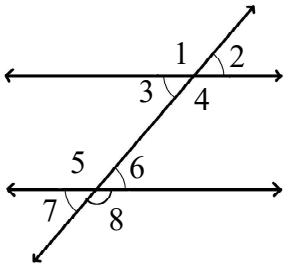


Figure -5.20

5.6.7 Criteria of parallel lines.

When a transveral intersects a pair of lines then the lines will be parallel–

- if the pair of corresponding angle made by the transversal are equal.
- if the pair of alternate angles made by the transversal are equal.
- if the interior angles on the same side of the transversal are supplementary.

Example 8 : l is a line in fig 5.21. P is a point which is not on the line l. Number of lines can be drawn through P and parallel to l is/are

(a) one
(b) two
(c) infinite
(d) none of the above

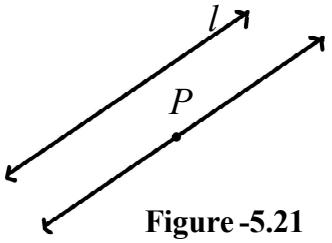


Figure -5.21

Solution : The answer is (a) one, l is a line and P is a point which does not lie on the line. Through P only one line can be drawn which is parallel to l.

Example 9 : The angle $\angle x$ in the figure will be–

(a) Corresponding angles

(b) Interior angles

(c) Alternate angles

(d) Alternate exterior angles

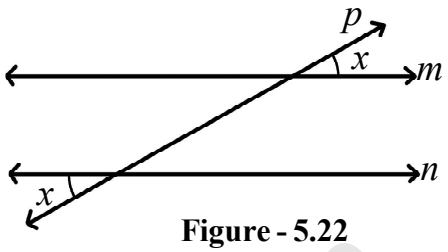


Figure - 5.22

Solution : (d) Alternate exterior angle since the angles are on the opposite sides of the transversal and they are exterior angles.

Example 10 : If XY || QR where $\angle 4 = 50^0$ and $\angle 5 = 45^0$ then find the other three angles.

Solution : Given that XY || QR, $\angle 4 = 50^0$ and $\angle 5 = 45^0$

We have to find out $\angle 1$, $\angle 2$ and $\angle 3$

From the figure, we get.

$\angle 1 + \angle 4 + \angle 5 = 180^0$ [straight angle]

$\Rightarrow \angle 1 = 180^0 - (\angle 4 + \angle 5)$

$\Rightarrow \angle 1 = 180^0 - (50^0 + 45^0)$

$\Rightarrow \angle 1 = 180^0 - 95^0$

$\Rightarrow \angle 1 = 85^0$

XY $\parallel$ QR, PQ is transversal

$\therefore \quad \angle 4 = \angle 2$ [alternate interior angle]

$\Rightarrow \quad \angle 2 = 50^0$

XY $\parallel$ QR, PR is transversal

$\therefore \quad \angle 5 = \angle 3$ [alternate interior angle]

$\Rightarrow \quad \angle 3 = 45^0$

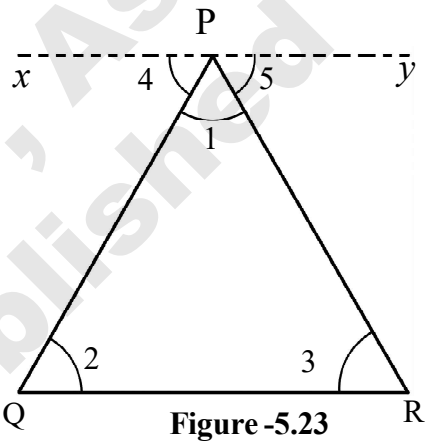


Figure -5.23

Activity 1 : Parallel lines and transversal

How to proceed –

- Form a team of five members.
- Discuss about parallel lines. Discuss the techniques to draw two parallel lines.
- After drawing two parallel lines, draw a third line as transversal which intersect the pair of parallel lines.

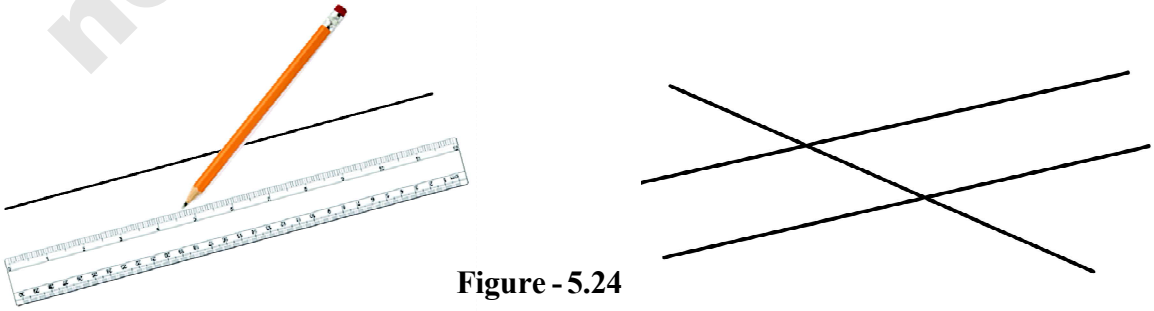


Figure - 5.24

Now (from fig 5.24) answer the following–

a) How many angles are formed by the transversal with the pair of parallel lines?

b) Name the angles. Identify the angles which are equal and give reason.

Activity 2 : Identify the following pairs of lines as parallel, intersecting perpendicularly or intersecting lines.

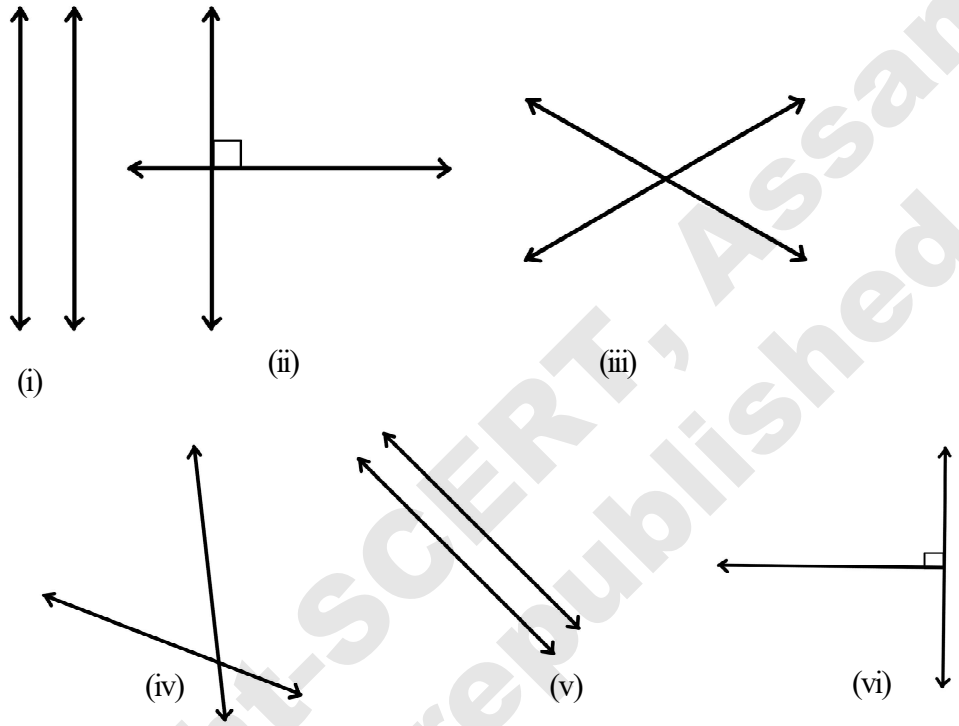
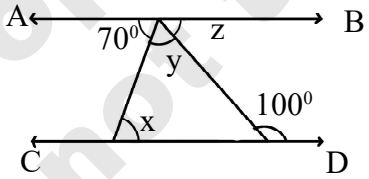


Figure - 5.25

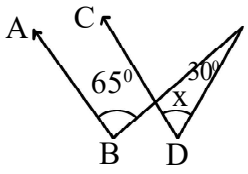
Exercise 5.2

1. In the following figure if AB || CD, find *x*, *y* and *z*

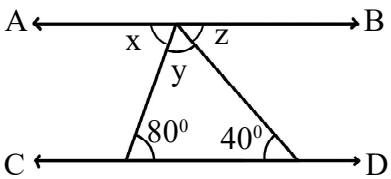
(a)



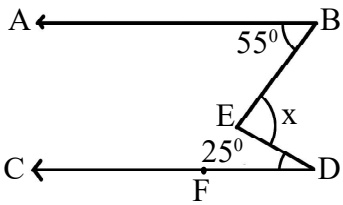
(b)



(c)



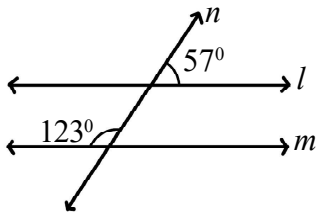
(d)



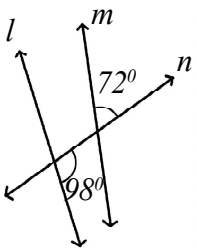
(**Hints :** Produce BE to F)

2. In the following figures, l and m are two lines and n is a transversal. Find the pair of lines which are parallel to each other ?

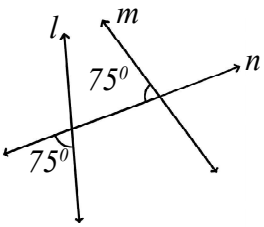
(a)



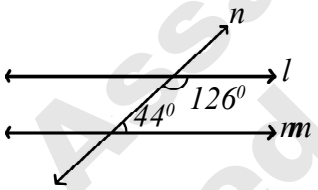
(b)



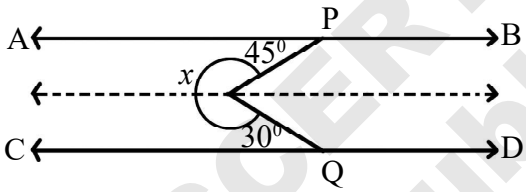
(c)



(d)

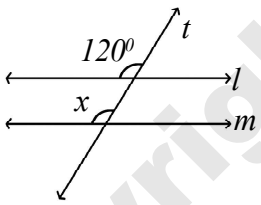


3. From the figure find given below the measure of x.

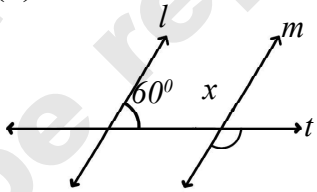


4. In the figure given below $l \parallel m$, t is transversal. Find the value of x.

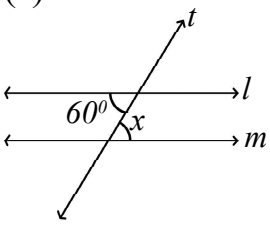
(a)



(b)



(c)



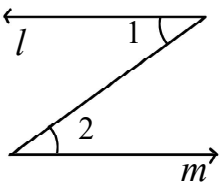
Exercise 5.3

1. If a pair of adjacent angles are supplementary to each other, then they will form
   a) corresponding angles b) vertically opposite angles
   c) linear pair of angles d) a ray

2. If two angles are supplementary their sum will be–
   a) 90^0 b) 180^0 c) 360^0 d) 45^0

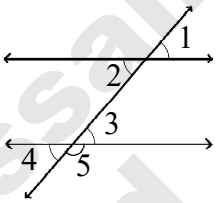
3. If two angles are complementary to each other, their sum will be–
   a) 45^0 b) 180^0 c) 90^0 d) 360^0

4. In the adjoining figure if $l \parallel m$ then $\angle 1 = \angle 2$ since they are



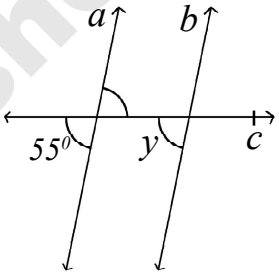
a) corresponding angles
b) vertically opposite angles
c) alternate interior angles
d) supplementary angles

5. In the figure the pair of alternate interior angles will be



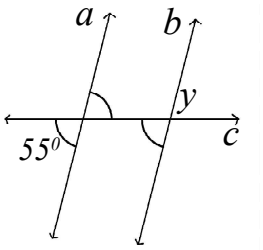
a) $\angle 1, \angle 3$
b) $\angle 2, \angle 3$
c) $\angle 2, \angle 5$
d) $\angle 2, \angle 5$

6. If $a \parallel b$ and c is a transversal, then $\angle y$ is



a) 90^0
b) 125^0
c) 55^0
d) 180^0

7. If $a \parallel b$ and c is a transversal then $\angle y$ is



a) 90^0
b) 25^0
c) 55^0
d) 35^0

8. The measure of the complementary angle of 45^0 is

a) 135^0 b) 25^0 c) 35^0 d) 45^0

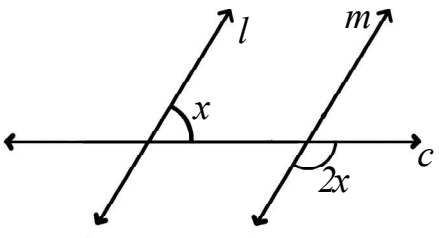
9. Which of the following angles will be equal to its own complementary angle?

a) 30^0 b) 25^0 c) 35^0 d) 45^0

10. Which of the following angles will be equal to its own supplementary angle?

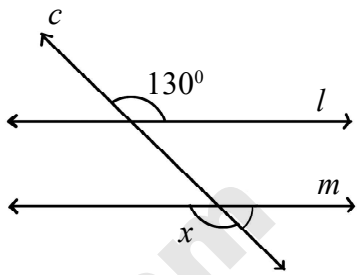
a) 60^0 b) 90^0 c) 180^0 d) None of them

11. If $l \parallel m$ and c is the transversal then x is



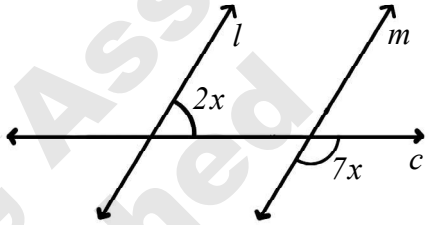
a) 30^0
b) 60^0
c) 90^0
d) 180^0

12. In the given figure $l \parallel m$ and c is transversal. The value of x is



a) 50^0
b) 130^0
c) 120^0
d) 100^0

13. In the given figure $l \parallel m$ and c is transversal. The value of x is



a) 10^0
b) 20^0
c) 30^0
d) 25^0

14. If two lines are equidistant from each other then they are –
a) perpendicular lines
b) non parallel lines
c) Intersecting lines
d) parallel lines.

What we have learnt

1. If the sum of two angles is 90^0 then the angles are complementary to each other.
2. If the sum of two angles is 180^0 then the angles are supplementary to each other.
3. Two angles will be adjacent if they have
   (a) a common vertex
   (b) a common side
   (c) The non-common arms are on either side of the common arm.
4. Two adjcent angles will form a linear pair if the two sides other then the common side are opposite rays.
5. Sum of two adjacent angles will be 180^0 if the two sides other than the common side form a straight line.
6. The sum of angles at a point and on the same side of a line is always 180^0.
7. When two lines intersect each other, then the vertically opposite angles formed are equal.