DAY THIRTY EIGHT

Mock Test 1

(Based on Complete Syllabus)

Instructions

1. This question paper contains of 30 Questions of Mathematic, divided into two Sections : **Section A** Objective Type Questions and **Section B** Numerical Type Questions.
2. Section A contains 20 Objective questions and all Questions are compulsory (**Marking Scheme** : Correct +4, Incorrect −1).
3. Section B contains 10 Numerical value questions out of which only 5 questions are to be attempted (**Marking Scheme** : Correct +4, Incorrect 0).

Section A : Objective Type Questions

1 Let $f(x)$ satisfies the requirements of Lagrange's mean value theorem in [0, 2]. If $f(0)=0$ and $|f'(x)| \le \frac{1}{2}$ for all x in [0, 2], then

(a) $f(x) \le 2$ (b) $|f(x)| \le 1$
(c) $f(x) = 2x$ (d) $f(x) = 3$ for atleast one x in [0, 2]

2 A woman purchases 1 kg of onions from each of the 4 places at the rate of 1kg, 2kg, 3kg, 4kg per rupee respectively. On the average she has purchased x kg of onions per rupee, then the value of x is

(a) 2 (b) 2.5
(c) 1.92 (d) None of these

3 The statement $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

(a) p (b) $\sim p$ (c) q (d) $\sim q$

4 If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and $B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}$, then $A \cap B$ is equal to

(a) $\left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$ (b) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{2}\right\}$
(c) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6} \text{ or } \pi \le \theta \le \frac{3\pi}{2}\right\}$
(d) None of the above

5 A square $OABC$ is formed by line pairs $xy=0$ and $xy+1=x+y$, where O is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy=0$ and another circle with centre C_2 and radius twice that of C_1, is drawn to touch the circle C_1 and the other line pair. The radius of the circle with centre C_1 is

(a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ (b) $\frac{2\sqrt{2}}{\sqrt{2}(\sqrt{2}+1)}$
(c) $\frac{\sqrt{2}}{3(\sqrt{2}+1)}$ (d) $\frac{\sqrt{2}+1}{3\sqrt{2}}$

6 A function $y=f(x)$ has a second order derivative $f''(x)=6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y=3x-5$, then the function is

(a) $(x-1)^3$ (b) $(x+1)^3$
(c) $(x+1)^2$ (d) $(x-1)^2$

7 If z is non-real and $i=\sqrt{-1}$, then $\sin^{-1}\left(\frac{1}{i}(z-1)\right)$ be the angle of a triangle, if

(a) Re $(z)=1$, Im $(z)=2$
(b) Re$(z)=1$, $-1 \le$ Im $(z) \le 1$
(c) Re (z) + Im $(z) = 0$
(d) None of these

8 The ends A and B of a rod of length $\sqrt{5}$ are sliding along the curve $y = 2x^2$. Let x_A and x_B be the x-coordinate of the ends. At the moment when A is at (0, 0) and B is at (1, 2) the derivative $\frac{dx_B}{dx_A}$ has the value equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{8}$ (d) $\frac{1}{9}$

9 Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n$; $(n \in N)$ satisfies the equation $\int_{-1}^{1} (px+q) g_n(x)\, dx = 0$ for all linear functions $(px+q)$, then

(a) $a_n = b_n = 0$ (b) $b_n = 0, a_n = -\frac{3}{2n+3}$

(c) $a_n = 0, b_n = -\frac{3}{2n+3}$ (d) $a_n = \frac{3}{2n+3}, b_n = -\frac{3}{2n+3}$

10 The function $f(x) = \max.\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$, is

(a) continuous at all points except at $x = 1$ and $x = -1$
(b) differentiable at all points except at $x = 1$ and $x = -1$
(c) differentiable at all points
(d) None of the above

11 The two vertices of a triangle are $(4, -3)$ and $(-2, 5)$. If the orthocentre of the triangle is at $(1, 2)$, then the third vertex is

(a) (– 33, – 26) (b) (33, 26)
(c) (26, 33) (d) None of these

12 The unit vectors **a** and **b** are perpendicular and the unit vector **c** is inclined at an angle θ to both **a** and **b**. If $c = \alpha a + \beta b + \gamma (a \times b)$, then which is not true ?

(a) $\gamma^2 = 1 - 2\alpha^2$ (b) $\alpha = 2\beta$

(c) $\gamma^2 = -\cos 2\theta$ (d) $\beta^2 = \frac{1+\cos 2\theta}{2}$

13 If $0° \le \theta \le 180°$ and $81^{\sin^2\theta} + 81^{\cos^2\theta} = 30$, then θ is

(a) 30° (b) 45° (c) 120° (d) 150°

14 The sum of n terms of the following series

` $1 + (1+x) + (1+x+x^2) + \ldots$ will be

(a) $\frac{1-x^n}{1-x}$ (b) $\frac{x(1-x^n)}{1-x}$

(c) $\frac{n(1-x) - x(1-x^n)}{(1-x)^2}$ (d) None of these

15 Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then f is

(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) Neither one-one nor onto

16 For the arithmetic progression $a, a+d, a+2d, a+3d, \ldots, a+2nd$, the mean deviation from mean is

(a) $\frac{n(n+1)\,d}{2n-1}$ (b) $\frac{n\,(n+1)\,d}{2n+1}$

(c) $\frac{n(n-1)\,d}{2n+1}$ (d) None of these

17 At a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, tangent PQ is drawn. If the point Q be at a distance $\frac{1}{p}$ from the point P, where p is distance of the tangent from the origin, then the locus of the point Q is

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{1}{a^2 b^2}$ (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 - \frac{1}{a^2 b^2}$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{a^2 b^2}$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{1}{a^2 b^2}$

18 The lines $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} \times \mathbf{c})$ and $\mathbf{r} = \mathbf{b} + \mu(\mathbf{c} \times \mathbf{a})$ will intersect if

(a) $\mathbf{a} \times \mathbf{c} = \mathbf{b} \times \mathbf{c}$ (b) $\mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c}$
(c) $\mathbf{b} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$ (d) None of these

19 The volume of the parallelopiped formed by vectors $\hat{\mathbf{i}} + a\hat{\mathbf{j}}$, $a\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ is maximum, when a equals to

(a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}$

20 Solution of the differential equation $(3xy^2 + x\sin(xy))dy + (y^3 + y\sin(xy))dx = 0$ is

(a) $xy^3 - \cos xy = C$ (b) $xy^3 + \cos xy = C$
(c) $xy^2 - \cos xy = C$ (d) $xy^2 + \sin xy = C$

Section B : Numerical Type Questions

21 The number of ordered pairs (α, β), where $\alpha, \beta \in (-\pi, \pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ is

22 The coefficient of x^{50} in $(1+x)^{41}(1-x+x^2)^{40}$ is

23 The value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ is $k + m\log_7 n$, then value of $|k| + |m| + |n|$ is equal to

24 The integral $\int_{-1/2}^{1/2} \left[[x] + \log\left(\frac{1+x}{1-x}\right) \right] dx$ is equal to $\frac{p}{q}$, then value of $p^2 + q^2$ is equal to

25 A natural number x is chosen at random from the first one hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{(x-30)} < 0$ is $7/a$, then a is equal to

26 If $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, where r is a natural number, then $|A_1| + |A_2| + \ldots + |A_{2018}|$ must be equal to k^2, then value of k is equal to

27 $\lim_{\theta \to 0} \frac{4\theta(\tan\theta - 2\theta\tan\theta)}{(1-\cos 2\theta)}$ is equal to

28 Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If ΔOA_1A_2 is equilateral, then possible values of h is

29 A dictionary is printed consisting of 7 lettered words only that can be made with a letter of the word CRICKET. If the words are printed in alphabetical order, as in an ordinary dictionary, then the number of words before the word CRICKET is

30 If $a, b, c > 0$, $a^2 = bc$ and $a + b + c = abc$, then the least value of $a^4 + a^2 + 7$ must be equal to

Hints and Explanations

1 *(b)* Since, $\mathbf{f(x)}$ satisfies the Lagrange's mean value theorem.

$\therefore \quad f'(c) = \dfrac{f(x) - f(0)}{x - 0}$

where, $\mathbf{0 < c < x < 2}$ i.e. $\mathbf{0 < c < 2}$

$\Rightarrow \quad f(x) = x f'(c)$

$\Rightarrow \quad |f(x)| = |x f'(c)|$

$\quad = |x||f'(c)| \le 2 \cdot \dfrac{1}{2} = 1$

$\Rightarrow \quad |f(x)| \le 1$

2 *(c)* Cost of 1kg onion, purchased from place 1 = ₹ 1

Cost of 1kg onion, purchased from place **2** = ₹ $\dfrac{1}{2}$

Cost of 1kg onion, purchased from place **3** = ₹ $\dfrac{1}{3}$

Cost of 1kg onion, purchased from place **4** = ₹ $\dfrac{1}{4}$

Now, average rate of 1kg onion

$= ₹\left(\dfrac{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{4}\right) = ₹\dfrac{25}{48}$

Thus, in ₹ $\dfrac{25}{48}$, we get 1 kg onoin.

$\therefore$ In ₹1, we get

$\dfrac{48}{25}$ kg onion = $\mathbf{1.92}$ kg

Alternate Method

Harmonic mean will give the correct answer, here

$\text{HM} = \dfrac{4}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \dfrac{4 \times 12}{12+6+4+3}$

$= \dfrac{48}{25} = 1.92$ kg

3 *(b)* $\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q)$

$\equiv \sim p \wedge t \equiv \sim p$

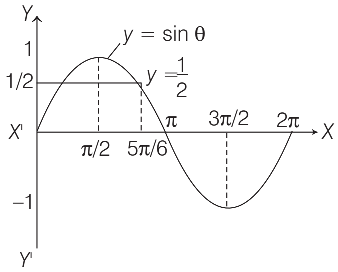
4 *(c)* Consider, $2\cos^2\theta + \sin\theta \le 2$ and

$\dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2} \Rightarrow 2 - 2\sin^2\theta + \sin\theta \le 2$

$\Rightarrow \sin\theta\,(2\sin\theta - 1) \ge 0$

Case I $\quad \sin\theta \ge 0$ and $2\sin\theta - 1 \ge 0$

$\therefore \quad \sin\theta \ge 0$ and $\sin\theta \ge \dfrac{1}{2}$



$\Rightarrow \quad \sin\theta \ge \dfrac{1}{2} \Rightarrow \dfrac{\pi}{2} \le \theta \le \dfrac{5\pi}{6} \quad$ ...(i)

Case II $\quad \sin\theta \le 0$ and $2\sin\theta - 1 \le 0$

$\therefore \quad \sin\theta \le 0$ and $\sin\theta \le \dfrac{1}{2}$

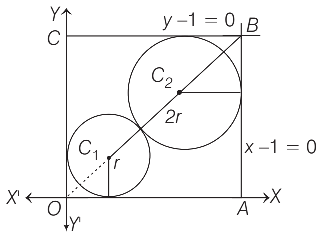
$\Rightarrow \quad \sin\theta \le 0$

$\Rightarrow \quad \pi \le \theta \le \dfrac{3\pi}{2} \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$A \cap B = \left\{\theta : \dfrac{\pi}{2} \le \theta \le \dfrac{5\pi}{6} \text{ or } \pi \le \theta \le \dfrac{3\pi}{2}\right\}$

5 *(a)* Diagonal of the square = $\sqrt{2}$



Also, $\quad d = r\sqrt{2} + 3r + 2\sqrt{2}r$

$\Rightarrow \quad \sqrt{2} = 3\sqrt{2}r + 3r \Rightarrow r = \dfrac{\sqrt{2}}{3(\sqrt{2}+1)}$

6 *(a)* Given that, $f''(x) = 6(x - 1)$

$f'(x) = 3(x-1)^2 + C_1 \quad$ ...(i)

But at point (2, 1) the line $y = 3x - 5$ is tangent to the graph $\mathbf{y = f(x)}$.

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{(x=2)} = 3$ or $f'(2) = 3$

From Eq. (i), $f'(2) = 3(2-1)^2 + C_1$

$\Rightarrow \quad 3 = 3 + C_1 \Rightarrow C_1 = 0$

$\therefore \quad f'(x) = 3(x-1)^2$

$\Rightarrow \quad f(x) = (x-1)^3 + C_2$

$\because \quad f(2) = 1$

$\therefore \quad 1 + C_2 = 1 \Rightarrow C_2 = 0$

Hence, $\quad f(x) = (x-1)^3$

7 *(b)* By the properties of inverse trigonometric function $\dfrac{z-1}{i}$ = **real**

$\Rightarrow \dfrac{x - 1 + iy}{i} = \text{real} \Rightarrow \dfrac{x-1}{i} + y = \text{real}$

$\Rightarrow \quad x - 1 = 0 \quad \Rightarrow \quad x = 1$

$\therefore \quad \sin^{-1}\left(\dfrac{z-1}{i}\right) = \sin^{-1}(y)$

So, $\quad -1 \le y \le 1$

$\therefore \text{Re}(z) = x = 1, \; -1 \le \text{Im}(z) \le 1$

8 *(d)* We have, $y = 2x^2$

$(AB)^2 = (x_B - x_A)^2 + (2x_B^2 - 2x_A^2)^2 = 5$

$\Rightarrow \quad (x_B - x_A)^2 + 4(x_B^2 - x_A^2)^2 = 5$

On differentiating w.r.t. x_A and denoting $\dfrac{dx_B}{dx_A} = D$, we get

$2(x_B - x_A)(D-1) + 8(x_B^2 - x_A^2)$
$\quad (2x_B D - 2x_A) = 0$

On putting $x_A = 0$; $x_B = 1$, then

$2(1-0)(D-1) + 8(1-0)(2D-0) = 0$

$\Rightarrow \quad 2D - 2 + 16D = 0$

$\Rightarrow \quad D = \dfrac{1}{9}$

9 *(b)* We have,

$\int_{-1}^{1} (px + q)(x^{2n+1} + a_n x + b_n)\,dx = 0$

Equating the odd component to be zero and integrating, we get

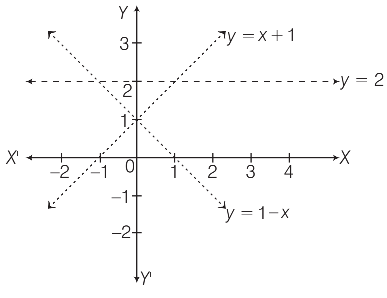
$\dfrac{2p}{2n+3} + \dfrac{2a_n p}{3} + 2b_n q = 0$ for all *p*, *q*

Hence, $\quad b_n = 0$

and $\quad a_n = -\dfrac{3}{2n+3}$

10 *(c)* We have, $f(x) = \max \cdot \{1 - x, x + 1, 2\}$

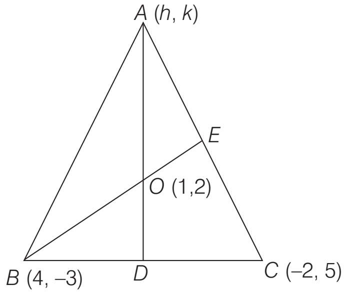
Let us draw the graph of $\mathbf{y = f(x)}$, as shown below



From the graph it is clear that, $\mathbf{f(x)}$ is continuous everywhere but not differentiable at $\mathbf{x = -1, 1}$.

11 *(b)* Let the third vertex be $\mathbf{(h, k)}$.

Now, the slope of *AO* or **AD** is $\dfrac{k-2}{h-1}$.



Slope of **BC** is $\dfrac{5+3}{-2-4} = -\dfrac{4}{3}$

Slope of **BE** is $\dfrac{-3-2}{4-1} = -\dfrac{5}{3}$

and slope of **AC** is $\dfrac{k-5}{h+2}$.

Since, $AD \perp BC$, $\dfrac{k-2}{h-1} \times \left(-\dfrac{4}{3}\right) = -1$

$\Rightarrow \quad 3h = 4k - 5 \qquad \ldots(i)$

Again, $BE \perp AC$,

$-\dfrac{5}{3} \times \dfrac{k-5}{h+2} = -1$

$\Rightarrow \quad 3h = 5k - 31 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$h = 33$ and $k = 26$

12 *(b)* Here, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1, \mathbf{a} \cdot \mathbf{b} = 0$

and $\cos\theta = \mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c}$

Now, $\mathbf{c} = \alpha\, \mathbf{a} + \beta\, \mathbf{b} + \gamma\, (\mathbf{a} \times \mathbf{b}) \quad \ldots(i)$

$\Rightarrow \mathbf{a} \cdot \mathbf{c} = \alpha(\mathbf{a} \cdot \mathbf{a}) + \beta\, (\mathbf{a} \cdot \mathbf{b}) + \gamma\{\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})\}$

$\Rightarrow \cos\theta = \alpha\, |\mathbf{a}|^2 \Rightarrow \cos\theta = \alpha$

Similarly, by taking dot product on both sides of Eq. (i) by **b**, we get

$\beta = \cos\theta$

$\therefore \quad \alpha = \beta$

$\therefore$ From Eq. (i), we get

$|\mathbf{c}|^2 = |\alpha\, \mathbf{a} + \beta \mathbf{b} + \gamma(\mathbf{a} \times \mathbf{b})|^2$

$= \alpha^2\, |\mathbf{a}|^2 + \beta^2\, |\mathbf{b}|^2 + \gamma^2\, |\mathbf{a} \times \mathbf{b}|^2 + 2\alpha\beta\, (\mathbf{a} \cdot \mathbf{b}) + 2\alpha\gamma\, \{\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})\} + 2\beta\gamma\, \{\mathbf{b} \cdot (\mathbf{a} \times \mathbf{b})\}$

$\Rightarrow \quad 1 = \alpha^2 + \beta^2 + \gamma^2\, |\mathbf{a} \times \mathbf{b}|^2$

$\Rightarrow \quad 1 = 2\alpha^2 + \gamma^2 \left\{|\mathbf{a}|^2 \cdot |\mathbf{b}|^2 \sin^2 \dfrac{\pi}{2}\right\}$

$\Rightarrow \quad 1 = 2\alpha^2 + \gamma^2$

$\Rightarrow \quad \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

$\Rightarrow \quad \alpha^2 = \beta^2 = \dfrac{1-\gamma^2}{2} = \dfrac{1+\cos 2\theta}{2}$

13 *(a)* Let $81^{\sin^2\theta} = t$

Given, $81^{\sin^2\theta} + 81^{\cos^2\theta} = 30$

$\therefore \quad t + \dfrac{81}{t} = 30 \Rightarrow t^2 - 30\,t + 81 = 0$

$\Rightarrow \quad (t-27)(t-3) = 0 \Rightarrow t = 27, 3$

$\Rightarrow \quad 81^{\sin^2\theta} = 3^{4\sin^2\theta} = 3^3, 3^1$

$\Rightarrow \quad 4\sin^2\theta = 3, 4\sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = \pm\dfrac{\sqrt{3}}{2}, \pm\dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{\pi}{3}$

14 *(c)* Let S_n

$= 1 + (1+x) + (1+x+x^2) + \ldots + (1+x+x^2+x^3+\ldots+x^{n-1})$.

Then, $S_n = \dfrac{1}{(1-x)}\{(1-x) + (1-x^2) + (1-x^3) + (1-x^4) + \ldots +$ upto **n** terms$\}$

$= \dfrac{1}{(1-x)}[n - (x + x^2 + x^3 + \ldots + \text{upto } n \text{ terms})]$

$= \dfrac{1}{(1-x)}\left[n - \dfrac{x(1-x^n)}{1-x}\right]$

$= \dfrac{n(1-x) - x(1-x^n)}{(1-x)^2}$

15 *(a)* $\because \quad f(x) = 2x + \sin x$

$\therefore \quad f'(x) = 2 + \cos x > 0$ for all **x**

Since, $f(x)$ is strictly increasing. So, **f** is one-one.

Here, $\lim\limits_{x \to -\infty} f(x) = -\infty$ and $\lim\limits_{x \to \infty} f(x) = \infty$

Hence, **f** is onto.

16 *(b)* Clearly, mean

$\bar{x} = \dfrac{1}{(2n+1)}[a + (a+d) + (a+2d) + \ldots + (a+2nd)]$

$= \dfrac{1}{(2n+1)}\left[\dfrac{2n+1}{2}(a + a + 2nd)\right]$

$= a + nd$

Now, mean deviation from mean

$= \dfrac{1}{(2n+1)}\sum_{r=0}^{2n}|(a+rd) - (a+nd)|$

$= \dfrac{1}{(2n+1)}\sum_{r=0}^{2n}|(r-n)d|$

$= \dfrac{1}{(2n+1)} \times 2d\,(1 + 2 + \ldots + n)$

$= \dfrac{n(n+1)}{2n+1}d$

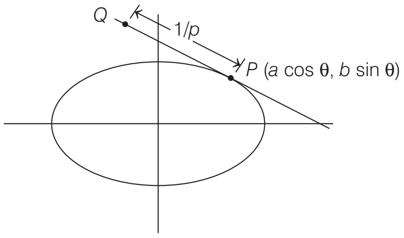
17 *(a)* Equation of the tangent at **P** is

$\dfrac{x - a\cos\theta}{a\sin\theta} = \dfrac{y - b\sin\theta}{-b\cos\theta}$

$\Rightarrow \quad xb\cos\theta + ay\sin\theta = ab$

The distance of the tangent from the origin is $p = \left|\dfrac{ab}{\sqrt{b^2\cos\theta + a^2\sin^2\theta}}\right|$

$\Rightarrow \quad \dfrac{1}{p} = \dfrac{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}{ab}$



Now, the coordinates of the point *Q* are given as follows

$\dfrac{x - a\cos\theta}{-a\sin\theta} \over \sqrt{b^2\cos^2\theta + a^2\sin^2\theta}$

$= \dfrac{\dfrac{y - b\sin\theta}{b\cos\theta}}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$

$= \dfrac{1}{p} = \dfrac{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}{ab}$

$x = a\cos\theta - \dfrac{a\sin\theta}{ab}$

and $y = b\sin\theta + \dfrac{b\cos\theta}{ab}$

$\Rightarrow \left(\dfrac{x}{a}\right)^2 + \left(\dfrac{y}{b}\right)^2 = 1 + \dfrac{1}{a^2b^2}$ is the required locus.

18 *(b)* Clearly, the lines $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} \times \mathbf{c})$ and $\mathbf{r} = \mathbf{b} + \mu(\mathbf{c} \times \mathbf{a})$ will intersect, if the shortest distance between them is zero.

i.e. $(\mathbf{a} - \mathbf{b}) \cdot \{(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})\} = 0$

$\Rightarrow \quad (\mathbf{a} - \mathbf{b}) \cdot \{[\mathbf{b}\,\mathbf{c}\,\mathbf{a}]\mathbf{c} - [\mathbf{b}\,\mathbf{c}\,\mathbf{c}]\mathbf{a}\} = 0$

$\Rightarrow \quad ((\mathbf{a} - \mathbf{b}) \cdot \mathbf{c})[\mathbf{b}\,\mathbf{c}\,\mathbf{a}] = 0$

$\Rightarrow \quad \mathbf{a} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} = 0$

$\Rightarrow \quad \mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c}$

19 *(d)* Let $\quad V = \begin{vmatrix} 1 & a & 0 \\ a & 1 & 1 \\ 0 & 1 & a \end{vmatrix} = a - 1 - a^3$

$\therefore \quad \dfrac{dV}{da} = 1 - 3a^2 = 0$

$\Rightarrow \quad a = \pm\dfrac{1}{\sqrt{3}}$

Now, $\quad \dfrac{d^2V}{da^2} = -6a$

$\Rightarrow \quad \left(\dfrac{d^2V}{da^2}\right)_{\left(a = \frac{1}{\sqrt{3}}\right)} = -\dfrac{6}{\sqrt{3}}$

Hence, it is maximum at

$a = \dfrac{1}{\sqrt{3}}$.

20 *(a)* We have, $(3xy^2 + x\sin(xy))dy + (y^3 + y\sin(xy))dx = 0$

$\Rightarrow \quad (3xy^2dy + y^3dx) + \sin(xy)(xdy + ydx) = 0$

$\Rightarrow \quad d(xy^3) + \sin(xy)d(xy) = 0$

On integrating, we get

$xy^3 - \cos(xy) = C$

21 *(4)* Since, $\cos(\alpha - \beta) = 1, \; \alpha - \beta = 2n\pi$

But $\quad -2\pi < \alpha - \beta < 2\pi$

$[\because \alpha, \beta \in (-\pi, \pi)]$

$\therefore \quad \alpha - \beta = 0$

Now, $\cos(\alpha + \beta) = \frac{1}{e}$

$\Rightarrow \cos 2\alpha = \frac{1}{e} < 1$, which is true for four values of α, as $-2\pi < 2\alpha < 2\pi$.

22. *(0)* Coefficient of x^{50} in $(1 + x)^{41}(1 - x + x^2)^{40}$

$= $ Coefficient of x^{50} in $(1 + x)(1 + x^3)^{40}$

$= $ Coefficient of x^{50} in

$(1 + x)(1 + {}^{40}C_1 x^3 + \ldots + {}^{40}C_{16}(x^3)^{16} + {}^{40}C_{17}(x^3)^{17} + \ldots)$

$= 0$

23 *(6)* $\log_7 \log_7 7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}$

$= \log_7\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$

$= \log_7\left(\frac{7}{8}\right) = 1 - \log_7 2^3 = 1 - 3\log_7 2$

$\Rightarrow k = 1$, $m = -3$ and $n = 2$

$\therefore |k| + |m| + |n|$

$= 1 + 3 + 2 = 6$

24 *(5)* Let $I = \int_{-1/2}^{1/2}\left[[x] + \log\left(\frac{1 + x}{1 - x}\right)\right]dx$

$= \int_{-1/2}^{0}[x]\,dx + \int_{0}^{1/2}[x]\,dx + 0$

$\left[\because \log\left(\frac{1 + x}{1 - x}\right) \text{ is an odd function}\right]$

$= \int_{-1/2}^{0} -1\,dx + \int_{0}^{1/2} 0\,dx$

$= -(x)_{-1/2}^{0} + 0 = -\frac{1}{2}$

$\therefore p^2 + q^2 = 1 + 4 = 5$

25 *(25)* Since, $\frac{(x - 20)(x - 40)}{(x - 30)} < 0$

$\Rightarrow x \in (-\infty, 20) \cup (30, 40)$

Let $E = \{1, 2, 3, \ldots, 19, 31, 32, \ldots, 39\}$, then $n(E) = 28$

Now, required probability

$P(E) = \frac{28}{100} = \frac{7}{25}$

$\therefore \quad a = 25$

26 *(2018)* $|A_r| = r^2 - (r - 1)^2$

$\therefore |A_1| + |A_2| + \ldots + |A_{2018}|$

$= \sum_{r=1}^{2018}\{r^2 - (r - 1)^2\}$

$= (2018)^2 - (0)^2 = (2018)^2$

$\therefore \quad k = 2018$

27 *(2)* $\lim_{\theta \to 0} \frac{4(\theta \tan\theta - 2\theta^2 \tan\theta)}{1 - \cos 2\theta}$

$= \frac{4(\theta\sec^2\theta + \tan\theta - 4\theta\tan\theta - 2\theta^2\sec^2\theta)}{2\sin 2\theta}$

[using L'Hospital's rule]

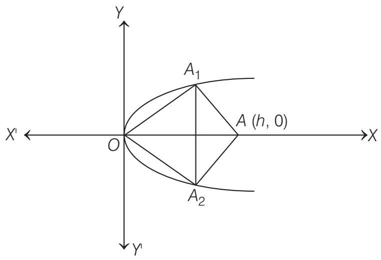
$= \lim_{\theta \to 0} \frac{\begin{bmatrix} 4(\sec^2\theta + 2\theta\sec^2\theta\tan\theta \\ + \sec^2\theta - 4\tan\theta - 4\theta\sec^2\theta \\ -4\theta\sec^2\theta - 4\theta^2\sec^2\theta\tan\theta) \end{bmatrix}}{4\cos 2\theta}$

[using L'Hospital's rule]

$= \frac{4(1 + 0 + 1)}{4} = 2$

28 *(28)* Let

$A_1 = (2t_1^2, 4t_1)$, $A_2 = (2t_1^2, -4t_1)$



Clearly, $\angle A_1OA = \frac{\pi}{6} \Rightarrow \frac{2}{t_1} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad t_1 = 2\sqrt{3}$

Equation of normal at A_1 is

$y = -t_1x + 4t_1 + 2t_1^3$

Since, $A(h, 0)$ lies on it,

$\Rightarrow \quad h = 4 + 2t_1^2 = 4 + 2 \cdot 12 = 28$

29 *(530)* Number of words starting with CC is 5!

Number of words starting with CE is 5!

Number of words starting with CI is 5!

Number of words starting with CK is 5!

Number of words starting with CRC is 4!

Number of words starting with CRE is 4!

Now, the first word starting with CRI is CRICEKT and next of it is CRICETK and next of it is CRICKET.

Hence, number of words before the word CRICKET

$= 4 \times 5! + 2 \times 4! + 2$

$= 480 + 48 + 2 = 530$

30 *(19)* $\because bc = a^2$

and $b + c = abc - a$

$= a(a^2) - a$

$= a^3 - a$

$\therefore$ **b and c** are the roots of the equation

$x^2 - (b + c)x + bc = 0$

ie, $x^2 - (a^3 - a)x + a^2 = 0$

$\because$ Roots (b, c) are real.

$\therefore \quad (a^3 - a)^2 - 4 \cdot 1 \cdot a^2 \geq 0$

$\Rightarrow (a^3 - a + 2a)(a^3 - a - 2a) \geq 0$

or $\quad a^2(a^2 + 1)(a^2 - 3) \geq 0$

$\Rightarrow \quad a^2 \geq 3$

or $\quad a^4 + a^2 + 7 \geq 3^2 + 3 + 7 = 19$

or $\quad a^4 + a^2 + 7 \geq 19$