CHAPTER

13 SURFACE AREAS AND VOLUMES

Syllabus

- *Surface areas and volumes of combinations of any two of the following : cubes, cuboids, spheres, hemi-spheres and right circular cylinders, cones. Frustum of a cone.*
- *Problems involving converting one type of metallic solid into another and other mixed problems. Problems with combination of not more than two different solids.*

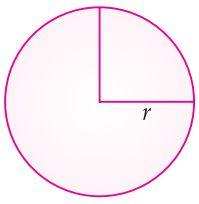
Chapter Analysis

List of Topics	2016		2017		2018
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi & Outside Delhi
Surface Areas and volumes	2 Q (3 Marks)	1 Q (3 M) 1 Q (4 M)	1 Q (1 M)	1 Q (4 M)	1 Q (3 M)
Problems involving converting one type of metallic solid into another	1 Q (3 Marks)	2 Q (3 M)	1 Q (3 M) 1 Q (4 M)	2 Q (3 M)	
Frustum	1 Q (4 Marks)		1 Q (4 M)	1 Q (3 M)	1 Q (4 M)

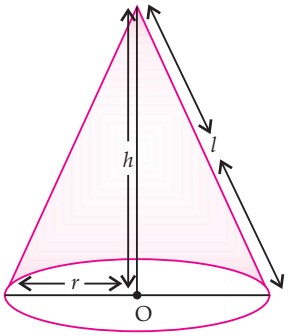
TOPIC-1
Surface Areas and Volumes

Revision Notes

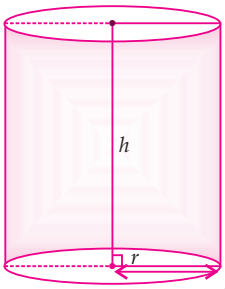
- A sphere is a perfectly round geometrical object in three-dimensional space that is the surface of a completely round ball.



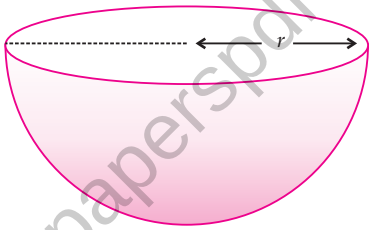
- A Cone is a three dimensional geometric shape tapers smoothly from a flat base to a point called the apex or vertex.



➢ A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.
A right circular cylinder.



➢ A hemisphere is half of a sphere.



Know the Formulae

➢ **Cuboid :**

Lateral surface area or area of four walls $= 2(l + b)h$
Total surface area $= 2(lb + bh + hl)$
Volume $= l \times b \times h$
Diagonal $= \sqrt{l^2 + b^2 + h^2}$

Here, l = length, b = breadth and h = height

➢ **Cube :**

Lateral surface area or area of four walls $= 4 \times (\text{edge})^2$
Total surface area $= 6 \times (\text{edge})^2$
Volume $= (\text{edge})^3$
Diagonal of a cube $= \sqrt{3} \times \text{edge}$.

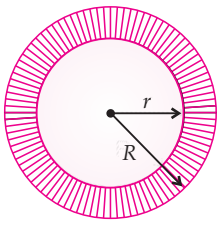
➢ **Right circular cylinder :**

Area of base or top face $= \pi r^2$
Area of curved surface or curved surface area = perimeter of the base × height $= 2\pi rh$
Total surface area (including both ends) $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
Volume = (Area of the base × height) $= \pi r^2 h$

Here, r is the radius of base and h is the height.

➢ **Right circular hollow cylinder :**

Total surface area = (External surface area + internal sufrace area) + (Area of ends)
$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$
$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$
$= [2\pi(R + r)(h + R - r)]$
Curved surface area $= (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$
Volume of the material used = (External volume) – (Internal volume)



$$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$$

Here, R and r are the external and internal radii and h is the height of the hollow clinder.

- **Right circular cone :**

$$\text{Slant height, } l = \sqrt{h^2 + r^2}$$

$$\text{Area of curved surface} = \pi r l = \pi r \sqrt{h^2 + r^2}$$

$$\text{Total surface area} = \text{Area of curved surface} + \text{Area of base}$$
$$= \pi r l + \pi r^2 = \pi r(l + r)$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Here, r, h and l are the radius, vertical height and slant height respectively of the cone.

- **Sphere :**

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Here, r is the radius of the sphere.

- **Spherical shell :**

$$\text{Surface area (outer)} = 4\pi R^2$$

$$\text{Volume of material} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(R^3 - r^3)$$

Here, R and r are the external and internal radii of the spherical shell.

- **Hemisphere :**

$$\text{Area of curved surface} = 2\pi r^2$$

$$\text{Total surface area} = \text{Area of curved surface} + \text{Area of base}$$
$$= 2\pi r^2 + \pi r^2$$
$$= 3\pi r^2$$

$$\text{Volume} = \frac{2}{3}\pi r^3$$

Here, r is the radius of the hemisphere.

Know the Terms

- The platonic solids also called the regular solids or regular polyhedra. 5 such solids are : dodecahedron, icosahedron, octahedron and tetrahedron.
- Greek mathematician Plato equated tetrahedron with the 'element' fire, the cube with earth, the icosahedron with water, the octahedron with air and dodecahedron with the stuff of which the constellations and heavens were made.
- The stone of platonic solids are kept in Ashmolean Museum in Oxford.
- The fonds of Archimedes carried a sculpture consisting of a sphere and cylinder circumscribing it.

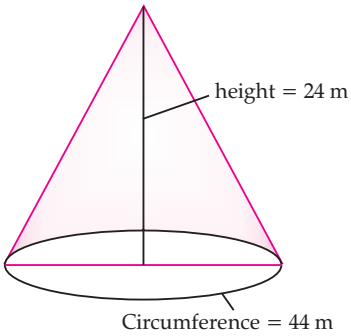
How it is done on GREENBOARD ?

Q. The circumference of the base of a conical tent is 44 m. If the height of tent is 24 m, find the length of the canvas used in making the tent, if the width of the canvas is 2 m. $\left(\text{Use } \pi = \frac{22}{7}\right)$ U

Sol. : Step I. Diagrammatic representation

Given that h = 24 m

Circumference = 44 m



Step II : Calculation of radius

Given that circumference = 44 m

or, $2\pi r = 44$

or, $r = \frac{44}{2\pi} = \frac{44 \times 7}{2 \times 22} = 7$ m

Step III : Calculation of slant height

$l^2 = r^2 + h^2$

$l^2 = (7)^2 + (24)^2$

$= 49 + 576$

$l^2 = 625$

or, $l = 25$ m

Step IV : Calculation for curved surface area

C.S.A $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$ m^2

$= 550$ m^2

Step V : Calculation of length of canvas

We know that area of rectangle = length x breadth

or, $550 = l \times 2$

or, $l = 225$ m

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. A cylindrical pencil sharpened at one edge is the combination of :

(a) a cone and a cylinder
(b) frustum of a cone and a cylinder
(c) a hemisphere and a cylinder
(d) two cylinders U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : The sharpened part of the pencil is cone and unsharpened part is cylinder.

Q. 2. A surahi is the combination of :

(a) a sphere and a cylinder
(b) a hemisphere and a cylinder
(c) two hemispheres
(d) a cylinder and a cone

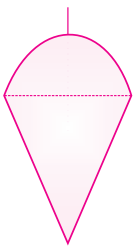
U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : A surahi is the combination of a sphere and a cylinder.

Q. 3. A plumbline (Sahul) is the combination of :

(a) a cone and a cylinder
(b) a hemisphere and a cone
(c) frustum of a cone and a cylinder
(d) sphere and cylinder

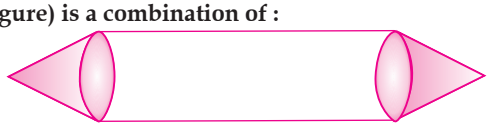


U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Plumbline is an instrument used to check the verticality of an object. It is a combination of a hemisphere and a cone.

Q. 4. The shape of a gilli, in the gilli-danda game (see in Figure) is a combination of :



(a) two cylinders
(b) a cone and a cylinder
(c) two cones and a cylinder
(d) two cylinders and a cone U [NCERT Exemp.]

Sol. Correct option : (c)

*Explanation :*The shape of gilli, in the gilli-danda game is a combination of two cones and a cylinder.

Q. 5. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is :

(a) 142296 **(b) 142396**
(c) 142496 **(d) 142596**

A + U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Let the spherical marble has radius r.

Diameter of the marble = 0.5 cm

$\Rightarrow r = \frac{0.5}{2}$ cm $= 0.25$ cm

Length of side of $l = 22$ cm

Let n marbles can fill the cube.

$\therefore$ Volume of n marbles $= (1 - \frac{1}{8})$

part of volume of cube

$$\Rightarrow n.\frac{4}{3}\pi r^3 = \frac{7}{8}\times l^3$$

$$n = \frac{7l^3}{8}\times\frac{3}{4\pi r^3}$$

$$\Rightarrow \quad = \frac{7\times3\times22\times22\times22\times7}{8\times4\times22\times0.25\times0.25\times0.25}$$

$$\Rightarrow \quad n = 7\times3\times22\times22\times2\times7$$

$$= 42\times484\times7$$

$$n = 142296$$

So, cube can accommodate up 142296 marbles.

Q. 6. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each to its ends. The length of entire capsule is 2 cm. The capacity of the capsule is :

(a) 0.36 cm^3 **(b) 0.35 cm^3**

(c) 0.34 cm^3 **(d) 0.33 cm^3**

A [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Capsule consists of 2 hemispheres and a cylinder.

$$r = \frac{0.5}{2}\text{ cm} = 0.25\text{ cm}$$

$\Rightarrow r = 0.25$ cm

Total length of capsule $= r + h + r$

$\Rightarrow 2\text{ cm} = 2r + h$

$\Rightarrow \quad 2 = 2\times0.25 + h$

$\Rightarrow \quad h = 2 - 0.5 = 1.5$ cm

Volume of capsule = Volume of two hemispheres + Volume of cylinder

$$= 2\times\left(\frac{4}{3}\pi r^3\times\frac{1}{2}\right)+\pi r^2h$$

$$= \frac{4}{3}\pi r^3 + \pi r^2h$$

$$= \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= \frac{22}{7}\times0.25\times0.25\left(\frac{4}{3}\times0.25+\frac{15}{10}\right)$$

$$= \frac{22}{7}\times0.25\times0.25\left(\frac{1}{3}+\frac{3}{2}\right)$$

$$= \frac{22}{7}\times\frac{25}{100}\times\frac{25}{100}\times\frac{11}{6} = \frac{121}{336}$$

$\therefore$ Volume of capsule $= 0.3601\text{ cm}^3 = 0.36\text{ cm}^3$.

Q. 7. If two solid hemispheres of same base radius 'r' are joined together along their bases, then curved surface area of this new solid is :

(a) $4\pi r^2$ **(b) $6\pi r^2$**

(c) $3\pi r^2$ **(d) $8\pi r^2$**

A [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : When two hemispheres of equal radii are joined base to base, new solid becomes sphere and curved surface area of sphere is $4\pi r^2$.

Q. 8. A right circular cylinder of radius r cm and height h cm (where $h > 2r$) just encloses of sphere of diameter :

(a) r cm **(b) $2r$ cm**

(c) h cm **(d) $2h$ cm**

A [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : As the cylinder just enclosed the sphere so the radius or diameter of cylinder and sphere are equal, *i.e.*, $2r$ and height $h > 2r$.

Q. 9. In a right circular cone, the cross-section made by a plane parallel to the base is a :

(a) circle **(b) frustum of a cone**

(c) sphere **(d) hemisphere**

A [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : In a right circular cone, if any cut is made parallel to its base, we get a circle.

Q. 10. Volumes of two spheres are in the ratio 64 :27. The ratio of their surface areas is :

(a) 3 : 4 **(b) 4 : 3**

(c) 9 : 16 **(d) 16 : 9**

A [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : $\frac{V_1}{V_2} = \frac{64}{27}$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$ [r_1 and r_2 are the radii of two spheres.]

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

Now, the ratio of their surface areas,

$$\frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Q. 11. The surface areas of two spheres are in the ratio 16 : 9. The ratio of their volumes is :

(a) 64 : 27 **(b) 16 : 9**

(c) 4 : 3 **(d) 16^3 : 9^3**

A [Board Term-2, Set-I, 2013]

Sol. Correct option : (a)

Explanation :

Given, $\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{16}{9}$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{16}{9}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Now, volumes of two spheres,

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^2}{\frac{4}{3}\pi r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27} = 64 : 27.$$

[B] Very Short Answer Type Questions :

Q. 1. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. Find the ratio of its height to its diameter. A [Board Term-2, 2014]

Sol. Curved Surface area of cylinder $= 2\pi rh$

Volume of cylinder $= \pi r^2 h$

$$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{7}{2}$$

$\therefore \quad r = 7$ m

$2\pi rh = 264$

or, $2 \times \frac{22}{7} \times 7 \times h = 264$

or, $h = 6$ m

$\therefore \quad \frac{h}{2r} = \frac{6}{14} = \frac{3}{7}$ **1**

Hence, $h : r = 3 : 7$

Q. 2. A rectangular sheet of paper 40cm × 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder. A [Foreign Set I, II, III, 2014]

Sol. Here, $h = 40$ cm, circumference $= 22$ cm

$2\pi r = 22$

or, $r = \frac{22 \times 7}{2 \times 22}$

or, $r = \frac{7}{2}$

$= 3.5$ cm **1**

Q. 3. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes. U [Delhi CBSE, 2014]

Sol. Volume of cylinder : Volume of cone : Volume of hemisphere

$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$

$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 \times h$ $(\because h = r)$

$= 1 : \frac{1}{3} : \frac{2}{3}$

or, $3 : 1 : 2$ **1**

Q. 4. What is the ratio of the total surface area of the solid hemisphere to the square of its radius. U [Board Term-2, 2012 Set (21, 22)]

Sol. $\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$

$\therefore$ Total surface area of hemisphere : Square of radius $= 3\pi : 1$ [CBSE Marking Scheme, 2012] **1**

Q. 5. Two cubes each of volume 8 cm^3 are joined end to end, then what is the surface area of resulting cuboid. A [Board Term II, 2012 Set (23)]

Sol. Side of the cube, $a = \sqrt[3]{8} = 2$ cm

Now the length of cuboid

$l = 4$ cm

breadth, $b = 2$ cm

height, $h = 2$ cm

Surface area of cuboid $= 2(l \times b + b \times h + h \times l)$ ½

$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$

$= 2 \times 20 = 40\ cm^2$ ½

[CBSE Marking Scheme, 2012]

Q. 6. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts. A [Board Term-2, 2012, Set (26)]

Sol. $\because$ Whole surface of each part

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$ ½

$\therefore$ Total surface of two parts

$= 3\pi r^2 + 3\pi r^2 = 6\pi r^2$ ½

[CBSE Marking Scheme, 2012]

Q. 7. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? $\left(\text{Use } \pi = \frac{22}{7}\right)$ R [Board Term-2, 2012 Set (59)]

Sol. Here $r = 7$ cm, $h = 10$ cm,

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10$

$= 1540\ cm^3$ **1**

[CBSE Marking Scheme, 2012]

Q. 8. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder. U [Board Term-2, 2012 Set (40)]

Sol.

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h}$$

$= \frac{1}{4} = 1 : 4$ **1**

[CBSE Marking Scheme, 2012]

Q. 9. If the areas of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid. A [Board Term-2, 2012, Set (5)]

Sol. Let the length, breadth and height of the cuboid is l, b and h respectively.

$$X = l \times b$$
$$Y = b \times h$$
$$Z = l \times h$$
$$XYZ = l^2 b^2 h^2$$
$$\text{Volume of cuboid} = l \times b \times h$$
$$\therefore \quad l^2 b^2 h^2 = XYZ$$
$$\text{or,} \quad lbh = \sqrt{XYZ} \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 10. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes. U **[Board Term-2, 2012, Set (44)]**

Sol.
$$\frac{\text{Voume of } 1^{st} \text{ cylinder}}{\text{Voume of } 2^{nd} \text{ cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$
$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$
$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$
$$= 20 : 27 \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 11. Volumes of two spheres are in the ratio 64 : 27, find the ratio of their surface areas.

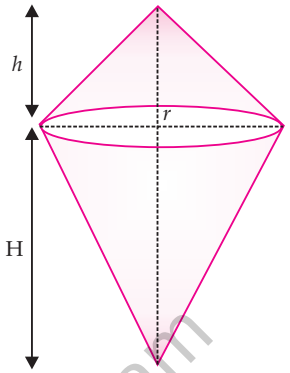
A **[KVS 2014][Board Term-2, 2012, Set (22)]**

Sol.
$$\frac{\text{Volume of } 1^{st} \text{ sphere}}{\text{Volume of } 2^{nd} \text{ sphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$
$$\therefore \quad = \frac{r_1^3}{r_2^3} = \frac{64}{27}$$
$$= \frac{r_1}{r_2} = \frac{4}{3} \qquad \frac{1}{2}$$

Ratio of their surface areas
$$= \frac{\text{Surface area of } 1^{st} \text{ sphere}}{\text{Surface area of } 2^{nd} \text{ sphere}}$$
$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$
$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$
$$= 16 : 9 \qquad \frac{1}{2}$$

[CBSE Marking Scheme, 2012]

Q. 12. A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace. A **[Board Term-2, 2012, Set (34, 50)]**



Sol. Volume of the upper cone $= \frac{1}{3}\pi r^2 h$

Volume of the lower cone $= \frac{1}{3}\pi r^2 H$

Total volume of both the cones $= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$
$$= \frac{1}{3}\pi r^2 (h + H)$$

Thus, the quantity of water displaced will be $\frac{1}{3}\pi r^2 (h + H)$ units^3. **1**

[CBSE Marking Scheme, 2012]

Q. 13. Find the volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm. A **[Board Term-2, 2012, Set (22)]**

Sol. Edge of the cube $= 4.2$ cm.
Height of the cone $= 4.2$ cm.
$$\text{Radius of the cone} = \frac{4.2}{2} = 2.1 \text{ cm.}$$
$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$$
$$= 19.4 \text{ cm}^3 \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 14. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm^3.

A **[Board Term-2, 2012, Set (1)]**

Sol. Let r be the radius of bowl.
$$\therefore \quad 2\pi r = 132$$
$$\text{or,} \quad r = \frac{132 \times 7}{2 \times 22} = 21 \text{cm}$$
$$\text{Capacity of the bowl} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19404 \text{ cm}^3 \quad \textbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 15. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ? A **[Delhi Set-I 2017]**

Sol. Let radius of sphere be r.

Given, volume of hemisphere = Surface area of hemisphere

or, $\frac{2}{3}\pi r^3 = 3\pi r^2$

or, $r = \frac{9}{2}$ units

$\therefore$ Diameter $= \frac{9}{2} \times 2 = 9$ units **1**

[CBSE Marking Scheme, 2012]

Q. 16. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

R **[O.D. Compt. Set I, II, III-2018]**

Sol. Given, $\frac{a^3}{A^3} = \frac{1}{27}$ ½

$\Rightarrow \frac{a}{A} = \frac{1}{3}$

Ratio of sufrace areas $= \frac{6a^2}{6A^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$. ½

[CBSE Marking Scheme, 2018]

Short Answer Type Questions-I (2 marks each)

Q. 1. A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

A **[Board Term-2, 2012 Set (59)]**

Sol. Volume of cylinder $= \pi(5)^2 \times 4 \text{ cm}^3$

$= 100\pi \text{ cm}^3$. **1**

Volume of cone $= \frac{1}{3}\pi \times 3^2 \times 8$ ½

$= 24\pi \text{ cm}^3$

$\therefore$ Required ratio $= 100\pi : 24\pi$

$= 25 : 6$. ½

[CBSE Marking Scheme, 2012]

Q. 2. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

A **[Board Term-2, 2012 Set (5)]**

Sol. Diameter of sphere = Radius of hemisphere

$= 6$ cm

or, radius of sphere $= 3$ cm ½

$V = \frac{4}{3}\pi r^3$ ½

$= \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3$. ½

$= 113.14 \text{ cm}^3$. ½

[CBSE Marking Scheme, 2012]

Q. 3. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter that the hemisphere can have ? Find the surface area of the solid. A **[Board Term-2, 2012 Set (17)]**

Sol. Diameter of hemisphere = Side of cubical block

$2R = 7$

or, $R = \frac{7}{2}$

Surface area of solid = Surface area of the cube – Area of base of hemisphere + curved surface area of hemisphere

$= 6l^2 - \pi R^2 + 2\pi R^2$ **1**

$= 6 \times 49 - 11 \times \frac{7}{2} + 77$ ½

$= 332.5 \text{ cm}^2$ ½

[CBSE Marking Scheme, 2012]

Q. 4. A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$ A **[Board Term-2, 2012 Set (34)]**

OR

A cylindrical glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed completely. By how much the water level will rise in the glass tube ?

[Board Term-2, 2014, 2015]

Sol. Let the height of water raised measured be h cm. ½

$\therefore$ Volume of water displaced in cylinder $= \pi(10)^2 h$ ½

Volume of cube $= 8 \times 8 \times 8 \text{ cm}^3$ ½

$\therefore \pi(10)^2 h = 8 \times 8 \times 8$

$\therefore h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$ ½

$= 1.629$ cm.

[CBSE Marking Scheme, 2012]

Q. 5. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

A **[Board Term-2, 2015]**

Sol. Length of the cuboid so formed be l cm

$\therefore\ l = 5 + 5 = 10$ cm, $b = 5$ cm ; $h = 5$ cm.

Total surface area $= 2(l \times b + b \times h + h \times l)$ **1**

$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)$

$= 2(50 + 25 + 50)$

$= 2 \times 125$

$= 250$ cm^2. **1**

[CBSE Marking Scheme, 2015]

Q. 6. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [Take $\pi = \frac{22}{7}$]

U [CBSE O.D. 2014]

Sol. Given, TSA of hemisphere $= 462$ cm^2.

Then, $3\pi r^2 = 462$ ½

or, $\frac{22}{7} \times r^2 = \frac{462}{3}$

or, $r^2 = \frac{462 \times 7}{22 \times 3} = 49$

or, $r = 7$ cm. ½

$\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3$ ½

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{2156}{3}$

$= 718.67$ cm^3. ½

Q. 7. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of ₹25 per metre.

A [Delhi CBSE, Term-2, 2014]
[Foreign Set I, II, III, 2014]

Sol. Given, radius $(r) = 7$ m and height $(h) = 24$ m

Slant height of tent $(l) = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$

$= \sqrt{625} = 25$ m.

C.S.A. $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = 550$ m^2. **1**

Let x m of cloth is required

CSA of tent = area of cloth.

or, $5x = 550$ or, $x = \frac{550}{5} = 110$ m.

$\therefore$ 110 m of cloth is required.

Cost of cloth $= 25 \times 110 = ₹\ 2750$. **1**

Q. 8. Find the number of plates. 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular cylinder of height 10 cm and diameter 4.5 cm. A [Board Term-2, 2014]

Sol. Each one of the circular plate is also a cylinder.

Its volume is,

$V = \pi r^2 h = \pi \times (0.75)^2 (0.2)$

$= \frac{9\pi}{80}$ cm^3. **1**

The volume of right circular cylinder

$V = \pi(2{\cdot}25)^2(10) = 405\frac{\pi}{8}$ cm^3.

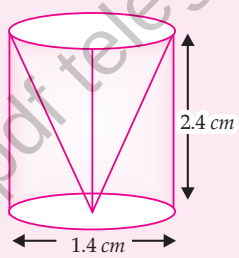
$\therefore$ Number of plates $= \frac{405\pi}{\frac{9\pi}{80}} = \frac{405\pi}{9\pi} \times \frac{80}{8}$

$= 450$ plates. **1**

Q. 9. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3.

[Use $\pi = \frac{22}{7}$] U [Board Term-2, 2012 Set (44)]

Sol. Volume of remaining solid



= Volume of cylinder – Volume of cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^2 h$ **1**

$= \frac{2}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$ ½

$= 44 \times 0.1 \times 0.7 \times 0.8$

$= 4.4 \times .56 = 2.464$ cm^3. ½

[CBSE Marking Scheme, 2012]

Q. 10. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the ratio between radius of their bases to their height is 3 : 4.

U [O.D. Compt. Set I, II, III-2018]

Sol. Let r be the radii of bases of cylinder and cone and h be the height

Slant height of cone $= \sqrt{r^2 + h^2}$ ½

$\therefore\ \frac{2\pi r h}{\pi r \sqrt{r^2 + h^2}} = \frac{8}{5}$ ½

$\frac{h}{\sqrt{r^2 + h^2}} = \frac{4}{5}$

$\Rightarrow\ \frac{h^2}{r^2 + h^2} = \frac{16}{25}$

$\Rightarrow\ 25h^2 = 16r^2 + 16h^2$

$\Rightarrow\ 9h^2 = 16r^2$ ½

$\Rightarrow\ \frac{r^2}{h^2} = \frac{9}{16} \Rightarrow \frac{r}{h} = \frac{3}{4}$ ½

[CBSE Marking Scheme, 2018]

Commonly Made Error

- In such types of problems, mostly students write incorrect formulas of surface area of cylinder and cone and also they do errors in calculation.
- Students write the formula of cylinder in place of cones and vice-versa.

Answering Tip

- Adequate practice and remembering of formulae is necessary.

Q. 11. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4m with the conical upper part of same diameter but of height 2.8 m and the canvas to be used cost ₹ 100 per sq.m, find the amount, the associations will have to pay. $\left[\text{Use } \pi = \frac{22}{7}\right]$

AE [OD Set I, II, III, 2015]

Sol. Here, height of upper conical part

$h = 2.8$ m

and radius $r = \frac{4.2}{2} = 2.1$ m

Slant height $l = \sqrt{h^2 + r^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41} = 3.5$ m ½

Surface area of tent $= 2\pi rh + \pi rl.$

Area of canvas for 1 tent

= S. area of cylinder + S. ar of cone

$= 2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times 2.1 \times 4 + \frac{22}{7} \times 2.1 \times 3.5$

$= 6.6\ (8 + 3.5)$

$= 6.6 \times 11.5$ m^2 ½

Area for 100 tents $= 6.6 \times 11.5 \times 100$

$= 66 \times 115$ m^2

$= 7590$ m^2

Cost of 100 tents = ₹ 7590 × 100 ½

50% cost $= \frac{50}{100} \times 7590 \times 100$

= ₹ 379500 ½

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II (3 marks each)

Q. 1. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm^2. Find the volume of the cone. (Use $\pi = 3.14$) A [Delhi Set II, 2016]

Sol. Given, $r = 3$, $\pi rl = 47.1$

$\therefore$ $l = \frac{47.1}{3 \times 3.14} = 5$ cm **1**

$h = \sqrt{5^2 - 3^2} = 4$ cm ½

Volume of cone $= \frac{1}{3}\pi r^2 h$ ½

$= \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$

$= 37.68$ cm^3. **1**

[CBSE Marking Scheme, 2016]

Q. 2. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$ U [Delhi Set I, 2016]

Sol. Here $r + h = 37$ and $2\pi r(r + h) = 1628$ ½

or, $2\pi r \times 37 = 1628$ ½

or, $2\pi r = \frac{1628}{37}$

or, $r = 7$ cm ½

and $h = 30$ cm. ½

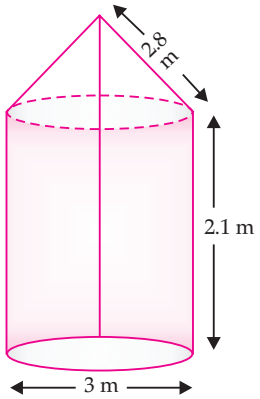
Hence, volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30$

$= 4620$ cm^3. **1**

[CBSE Marking Scheme, 2016]

Q. 3. In the given figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500 per sq. metre. $\left[\text{Use } \pi = \frac{22}{7}\right]$

C + A [O.D. Set I, II, III, 2016]



Sol. Height of cylinder = 2.1 m

Radius of cylinder = radius of cone = $\frac{3}{2}$ m **1**

Slant height of cone = 2.8 m

Then, area of canvas required = Surface area of tent

= C.S.A of cone + C.S.A. of cylinder.

$= \pi rl + 2\pi rh$ ½

$CSA = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1) = \frac{33}{7} \times 7$

$= 33 \text{ m}^2$ **1**

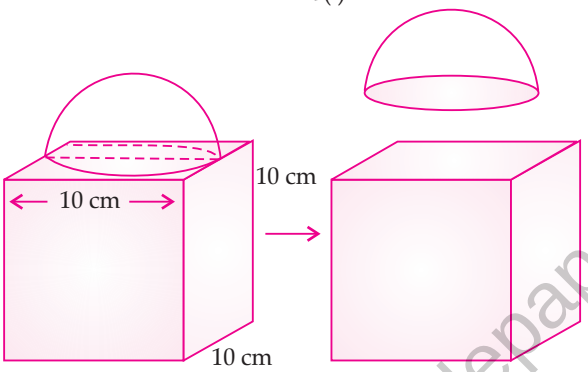
Total cost = 500 × ₹ 33

= ₹ 16,500 ½

Q. 4. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹ 5 per 100 sq. cm. [Use $\pi = 3.14$] C + A

[Outside Delhi CBSE Board 2015, Set I, II, III]

Sol. Side of the cubical block (l) = 10 cm.



The hemisphere is surmounted on it.

The largest diameter the hemisphere can have = side of the cubical block

Diameter of the hemisphere = 10 cm

Radius of the hemisphere (r) = 5 cm

Total surface area of the solid formed

$$= \begin{pmatrix} \text{TSA of the cubical} \\ + \text{CSA of the hemisphere} \\ - \text{Area of the base of the hemisphere} \end{pmatrix}$$

$= 6l^2 + 2\pi r^2 - \pi r^2$

$= 6l^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times (5)^2$

$= 6 \times 100 + 3.14 \times 25$

$= 600 + 78.50$

$= 678.5 \text{ cm}^2$

Rate of paintaing = ₹ 5 per 100 cm^2

Cost pf painting the solid formed = $₹ \frac{5}{100} \times 678.5$

= ₹ 33.925

= ₹ 33.93 (approx)

Q. 5. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find. the height of the each bottle, if 10% liquid is wasted in this transfer. C + A

[Outside Delhi CBSE Board, 2015, Set I, II, III]

Sol. Volume of bowl = $\frac{2}{3}\pi R^3$

Volume of liquid in bowl = $\frac{2}{3}\pi \times (18)^3 \text{ cm}^3$ ½

Volume of liquid after wastage

$= \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100} \text{ cm}^3$ ½

Volume of one bottle = $\pi r^2 h$

Volume of liquid in 72 bottles

$= \pi \times (3)^2 \times h \times 72 \text{ cm}^3$ ½

Volume of bottles = volume of liquid after wastage

$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}$

or, $h = \frac{\frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$

Hence, the height of bottle = 5.4 cm. ½ + **1**

[CBSE Marking Scheme, 2015]

Q. 6. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

C + A **[Foreign Set I, II, III, 2015]**

Sol. Volume of cylinder $= \pi r^2 h = \pi(3)^2 \times 5$

$= 45\pi \text{ cm}^3$ ½

Volume of conical hole $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9}$

$= \frac{2}{3}\pi \text{ cm}^3$ **1**

Metal left in cylinder $= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3} \text{ cm}^3$

1

Again, the required ratio

$$= \frac{\text{Volume of metal left}}{\text{Volume of metal taken out}}$$

$= \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$ ½

Hence, Volume of metal left : Volume of metal taken out = 133 : 2

[CBSE Marking Scheme, 2015]

Q. 7. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. [use $\pi = \frac{22}{7}$].

C + A **[Foreign Set I, II, III, 2015]**

Sol. Volume of water in cylinder = Volume of cylinder

$= \pi r^2 h$

$= \pi \times (60)^2 \times 180$

$= 648000\pi \text{ cm}^3$ **1**

Water displaced on dropping cone = Volume of solid cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (30)^2 \times 60$

$= 18000\pi \text{ cm}^3$ **1**

Volume of water left in cylinder

= Volume of cylinder – volume of cone

$= 648000\pi - 18000\pi$

$= 630000\pi \text{ cm}^3$

$= \frac{630000 \times 22}{1000000 \times 7} \text{m}^3$

$= 1.98 \text{ m}^3$. **1**

[CBSE Marking Scheme, 2015]

Q. 8. The rain water from 22 m × 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4}{5}^{\text{th}}$ of cylindrical vessel then find the rainfall in cm. C + A **[Foreign Set I, II, III, 2015]**

Sol. Volume of water collected in cylindrical vessel

$= \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{m}^3$ **1**

$= \frac{44}{5} \text{ m}^3$ **1**

Let the rainfall is h m.

Volume of rain water from roof $= 22 \times 20 \times h \text{ m}^3$

or, $22 \times 20 \times h = \frac{44}{5}$

or, $h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50} \text{m}$

$= \frac{1}{50} \times 100 = 2 \text{ cm}$ **1**

[CBSE Marking Scheme, 2015]

Q. 9. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe. C + A **[Board Term-2, 2015]**

Sol. Height of cylindrical pipe h = 21 dm

= 210 cm

External radius $R = \frac{10}{2} = 5$ cm

Internal radius $r = \frac{6}{2} = 3$ cm **1**

Volume of copper used in making the pipe = (Volume of external cylinder) – (Volume of internal cylinder)

$= \pi R^2 h - \pi r^2 h$ **1**

$= \pi h (R^2 - h^2)$

$= \frac{22}{7} \times 210(5^2 - 3^2) = \frac{22}{7} \times 16 \times 210$

$= 10560 \text{ cm}^3$. **1**

[CBSE Marking Scheme, 2015]

Q. 10. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[Board Term-2, 2015]**

Sol. Radius of the glass = 7 cm

Height of the glass = 10 cm

Volume of 1 glass $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540 \text{ cm}^3$ **1**

∴ Volume of juice to fill 6 glasses

$= 6 \times 1540 = 9240 \text{ cm}^3$ **1**

∴ Volume in litre $= \frac{9240}{1000} = 9.240$ litre. **1**

[CBSE Marking Scheme, 2015]

Q. 11. The largest possible sphere is cut out from a wooden solid cube of side 7 cm. Find the volume of the wood left. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[CBSE O.D. 2014]**

Sol. Given, the side of cube a = 7 cm

Since, the diameter of the largest possible sphere

= Side of the cube

Hence, the radius of sphere $= \frac{7}{2}$ cm. **1**

Volume of the wood left = Volume of cube – Volume of sphere

$= a^3 - \frac{4}{3}\pi r^3$ **1**

$= 7 \times 7 \times 7 - \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= 7 \times 7 \times 7\left(1 - \frac{11}{21}\right) = 7 \times 7 \times 7 \times \frac{10}{21} = \frac{490}{3}$

Hence, Volume of wood left = 163.3 cm^3. **1**

Q. 12. A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

C + A **[Foreign Set I, II, III, 2014]**

Sol. Let r_1 and r_2 be the radii of the cylinder and cone respectively.

Volume of cone = Volume of Cylinder

$$\Rightarrow \quad \frac{1}{3}\pi r_2^2 h = \pi r_1^2 h \qquad \textbf{1}$$

$$\frac{1}{3}\times\pi\times r_2^2\times 24 = \pi\times 18\times 18\times 32$$

or, $r_2^2 = 1296$

Hence, the radius of cone = 36 cm

Now,slant height of cone

$$l = \sqrt{h^2+r^2} \qquad \textbf{1}$$

$$= \sqrt{24^2+36^2}$$

$$= \sqrt{576+1296}$$

$$= \sqrt{1872}$$

$$= 43{\cdot}2 \text{ cm. approx.} \qquad \textbf{1}$$

Q. 13. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

A [Delhi 2013]

Sol. Radius of toy = radius of hemisphere = radius of cylinder = 3.5 cm

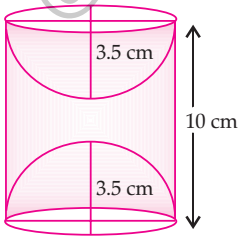
Volume of toy = Volume of cylinder – 2 × Volume of hemisphere

$$= \pi r^2 h - 2\times\frac{2}{3}\pi r^3 \qquad \textbf{1}$$

$$= \pi r^2\left[h-\frac{4}{3}r\right]$$

$$= \frac{22}{7}\times(3.5)^2\left[10-\frac{4}{3}\times 3.5\right] \qquad \textbf{1}$$

$$= 22\times 0.5\times 3.5\times 5.3$$

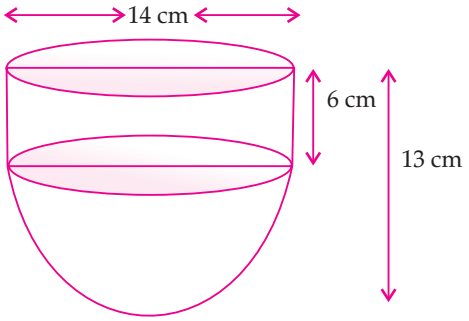


$$= 204.05 \text{ cm}^3. \text{ approx.} \qquad \textbf{1}$$

Q. 14. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

C + A [Delhi 2013]

Sol.



Radius of hemisphere = $\frac{14}{2}$ = 7 cm

Height of cylinder = 13 – 7 = 6 cm

T.S.A. of vessel = 2(CSA of hemisphere + CSA of cylinder)

$$= (2\pi r^2 + 2\pi rh) \qquad \textbf{1}$$

$$= 4\pi r(r+h)$$

$$= 4\times\frac{22}{7}\times 7(7+6) \qquad \textbf{1}$$

$$= 88\times 13$$

$$= 1144 \text{ cm}^2. \qquad \textbf{1}$$

Q. 15. The radii of two right circular cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 4. Calculate the ratio of their curved surface areas and ratio of their volumes.

A [Board Term-2, 2012 Set (22)]

Sol. Let the radii of two cylinders be $2x$ and $3x$ and their heights be $5y$ and $4y$ respectively. ½

Again, ratio of their curved surface areas

$$= \frac{2\pi\times 2x\times 5y}{2\pi\times 3x\times 4y} = \frac{5}{6} \qquad \textbf{1}$$

$\because$ Hence, their curved surface areas are in the ratio of 5 : 6.

$$\therefore \text{ Ratio of their volumes} = \frac{\pi\times(2x)^2\times 5y}{\pi\times(3x)^2\times 4y} \qquad \textbf{1}$$

$$= \frac{5\times 4}{4\times 9}$$

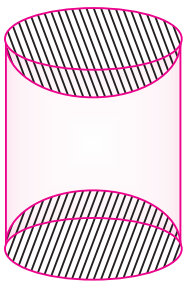
$$= \frac{5}{9} \qquad ½$$

Hence, their volumes are in the ratio of 5 : 9.

and their CSAs are in the ratio of 5 : 6

[CBSE Marking Scheme, 2012]

Q. 16. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in fig. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



C + A **[Delhi/O.D. Set- 2018]**

Sol. Total surface Area of articles = CSA of cylinder + CSA of 2 hemispheres

CSA of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 10$

$= 220 \text{ cm}^2$ **1**

Surface area of two hemispherical scoops

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154 \text{ cm}^2$ **1**

$\therefore$ Total surface area of article $= 220 + 154$

$= 374 \text{ cm}^2$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Mostly students are unable to find the radius of hemisphere also they use the value of π, 3.14 in place of $\frac{22}{7}$ and they do errors in calculation, they subtract the area of hemisphere from T.S.A of cylinder in place of adding these.

Answering Tip

- They should read the question clearly and use right formula and correct calculation for which good practice is necessary.

Q. 17. Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

A **[Board Term-2, 2012 Set (40)]**

Sol. Volume of water in 30 minutes

$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$ **1**

Let height of water in tank be h cm

and radius = 40 cm

Volume of water in the tank = Volume of water flowed through pipe in 30 minutes

$\pi(40)^2 \times h = 700 \times 60 \times 30 \times 4 \times \pi$

or, $h = \frac{700 \times 60 \times 30 \times 4}{40 \times 40}$ **1**

$= \frac{6300}{2} \text{ cm} = \frac{63}{2} \text{ m}$

Hence, water level increased = 31.5 m. **1**

[CBSE Marking Scheme, 2012]

Q. 18. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

A **[CBSE Delhi/O.D. Set- 2018]**

Sol. Radius of conical heap = 12 m ½

Volume of rice $= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ m}^3$

$= 528 \text{ m}^3$ **1**

Area of canvas cloth required $= \pi rl$

$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m}$ ½

$\therefore$ Area of canvas required $= \frac{22}{7} \times 12 \times 12.5$

$= 471.4 \text{ m}^2$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Sometimes the students find TSA of the canvas in place of C.S.A.

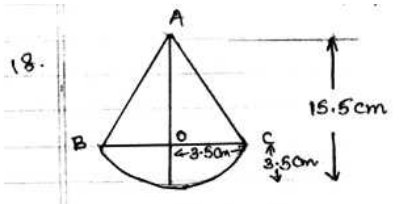
Answering Tip

- They should have clear idea about C.S.A and T.S.A and volume.

Q. 19. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

A **[Delhi/OD Set 2017, Board Term-2, 2012 Set (21, 22)]**

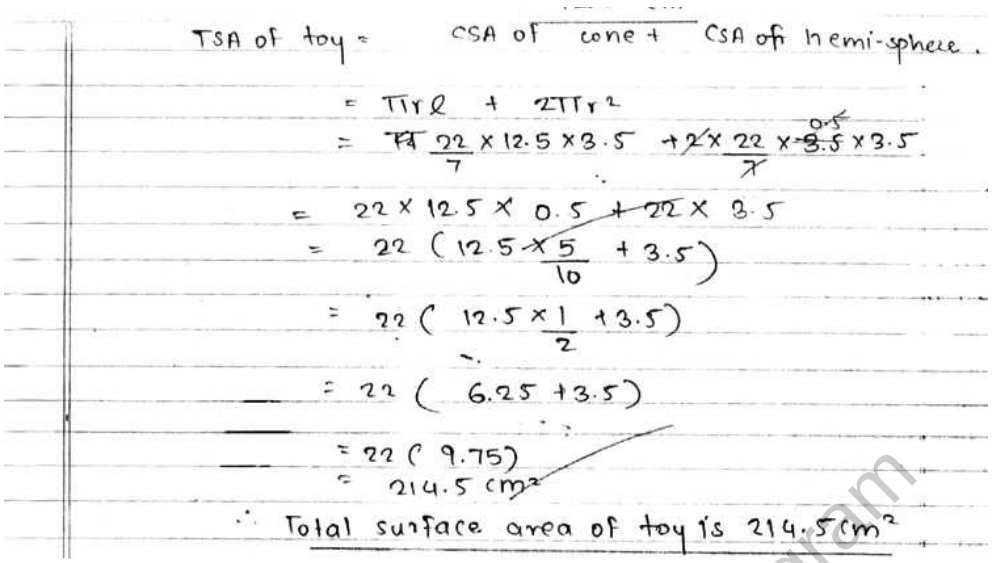
Sol. 18.



Height of hemisphere = r
= 3.5cm

height of cone = 15.5cm − 3.5cm
= 12cm = h.

Slant height of cone = $\sqrt{r^2 + h^2}$
= $\sqrt{12.25 + 144}$
= $\sqrt{156.25}$
= 12.5 cm



[Topper Answer, 2017] 3

Q. 20. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment. U [Delhi Set I, II, III, 2016]

Sol. Diameter of earth dug out = 4 m

Radius of earth dug out, $r = 2$ m

Depth of the earth, $h = 21$ m,

Volume of earth dug out = $\pi r^2 h$

$$= \frac{22}{7} \times 2 \times 2 \times 21$$

$$= 264 \text{ m}^2 \quad \mathbf{1}$$

Width of embankment = 3 m

Outer radius of ring = 2 + 3 = 5 m

Let the height of embankment be h

∴ Volume of embankment = Volume of earth dug out

$$\pi(R^2 - r^2)h = 264$$

$$\frac{22}{7} \times (25 - 4) \times h = 264 \quad \mathbf{1}$$

$$h = \frac{264 \times 7}{22 \times 21} = 4$$

∴ Height of embankment = 4 m. **1**

[CBSE Marking Scheme, 2016]

Q. 21. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cones.

C + A [Foreign Set I, II, III, 2016]

Sol. For cylindrical tub,

Diameter = 12 cm

or, Radius $R = 6$ cm

Height $H = 15$ cm.

∴ Volume = $\pi R^2 H$

$= \pi(6)^2 \times 15$

$= 540\ \pi \text{ cm}^3$

Each child will get the ice-cream = $\frac{540\pi}{10}$ cm^3

$= 54\pi \text{ cm}^3$ **1**

For cone, height = 2 × Diameter

$h = 2(2r)$

or, $h = 4r$

∴ Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^2 \times 4r = \frac{4}{3}\pi r^3$$

Volume of hemisphere = $\frac{2}{3}\pi r^3$.

Total volume of cone and hemisphere

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \frac{6}{3}\pi r^3$$

$$= 2\pi r^3 \quad \mathbf{1}$$

According to question,

$2\pi r^3 = 54\pi$

or, $r^3 = 27$

$r = 3$

Hence, diameter = $2r = 2 \times 3 = 6$ cm. **1**

Q. 22. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty ? $\left[\text{Use } \pi = \frac{22}{7}\right]$

A [Foreign Set I, II, III, 2016]

Sol. Diameter of tank = 3 m

∴ Radius $r = \frac{3}{2}$ m

Volume of hemispherical tank $= \frac{2}{3}\pi r^3$

or, $V = \frac{2}{3}\pi\left(\frac{3}{2}\right)^3 \text{m}^3$

$= \frac{2}{3}\times\frac{22}{7}\times\frac{27}{8}\ \text{m}^3$

$= \frac{11}{7}\times\frac{9}{2} = \frac{99}{14}\ \text{m}^3$

$V = \frac{99}{14} \times 1000$ litre [Since 1 m^3 = 1000 litre] **1**

$\therefore$ Half the volume of hemisphere $= \frac{V}{2}$

$= \frac{1}{2}\times\frac{99}{14} \times 1000$ Litres

Let time taken for this volume to flow out be t sec. Then according to question,

$t \times 3\frac{4}{7} = \frac{1}{2}\times\frac{99}{14} \times 1000$ **1**

$t \times \frac{25}{7} = \frac{1}{2}\times\frac{99}{14} \times 1000$

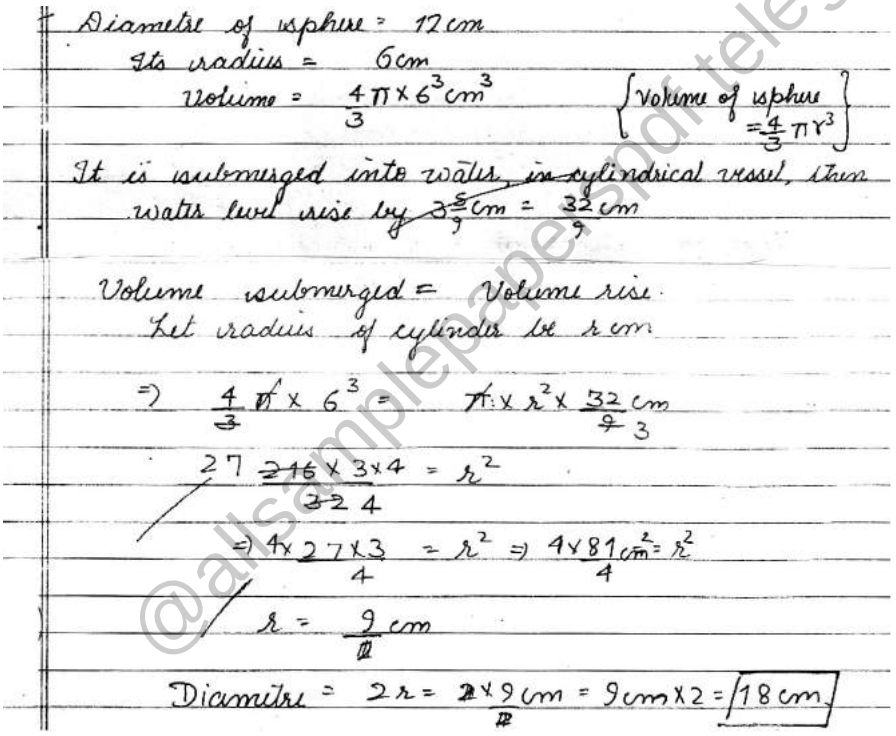
or, $t = \frac{7}{25}\times\frac{1}{2}\times\frac{99}{14} \times 1000$

$= 990$ sec

$= 16$ minutes 30 sec. **1**

Q. 23. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel. A **[Outside Delhi Set–II, 2016]**

Sol.



[Topper Answer, 2016] 3

Q. 24. The $\frac{3}{4}$ th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel. A **[Delhi Set-I 2017]**

Sol. Radius of conical vessel = 5 cm

and its height = 24 cm

Volume of this vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\pi\times5\times5\times24$

$= 200\pi\ \text{cm}^3$. **1**

Internal radius of cylindrical vessel = 10 cm

Let the height of emptied water be h.

$\therefore$ Volume of water in cylinder

$= \frac{3}{4}\times$ Volume of cone

$\Rightarrow \pi r^2 h = \frac{3}{4}\times$ Volume of cone **1**

$\Rightarrow \pi \times 10 \times 10 \times h = 150\pi$

$\Rightarrow h = 1.5$ cm

Hence the height of water = 1.5 cm **1**

[CBSE Marking Scheme, 2017]

Q. 25. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is ₹ 40 per meter, find the amount by which Rampal helped the centre.

C + A **[Outside Delhi Compt. Set-I, II III 2017]**

Sol. Diameter of tent = 14 m and height = 24 m

$\therefore$ radius of tent = 7 m

Slant height $= \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2}$

$= \sqrt{576 + 49} = 25$ m **1**

Surface area of the tent $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550 \text{ m}^2$ **1**

Surface area of 10 tents $= 550 \times 10$

$= 5500 \text{ m}^2$

Total cost $= 5500 \times \frac{40}{2} = ₹ 110000$

Hence, the amount by which Rampal helped the centre = ₹ 110000 **1**

[CBSE Marking Scheme, 2017]

Q. 26. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ? A **[OD Set II, 2017]**

Sol.

speed of water in canal = 25 km/hr.

in 40 min = $\frac{40}{60} = \frac{2}{3}$ hr,

length of water = $25 \times \frac{2}{3} = \frac{50}{3}$ km = $\frac{50000}{3}$ m

volume of water in canal in 40 minutes = volume of water for irrigation.

$\frac{54}{10} \times \frac{18}{10} \times \frac{50000}{3} \text{ m}^3 = \frac{10}{100} \times l \times b \text{ m}^3$

$324 \times 5000 = l \times b$

$1620000 = l \times b$

area irrigated in 40 minutes is

1620000 m^2

$= \frac{1620000}{1000000}$

$= 1.62 \text{ km}^2$ or 162 hectares.

[Topper Answer, 2017]

Q. 27. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of spheres respectively, then find the $(R - r) : r$. U **[Board Term-2, 2012, Set (22)]**

Sol. Ratio of volumes

$$\frac{\text{Volume of 1}^{\text{st}}\text{ sphere}}{\text{Volume of 2}^{\text{nd}}\text{ sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} \quad \textbf{1}$$

$$= \frac{8}{27}$$

or, $\frac{r}{R} = \frac{2}{3}$

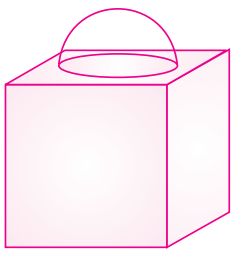
$\therefore$ $R = \frac{3}{2}r$ **1**

$\therefore$ $(R - r) : r = \left(\frac{3}{2}r - r\right) : r$

$= \frac{r}{2} : r = 1 : 2$ **1**

[CBSE Marking Scheme, 2012]

Q. 28. The given figure is a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$.



A **[Delhi Set I, II, III, 2016]**

Sol. Surface area of block

$= 216 - \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} + 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$ 1+½+½

$= 225.625\ cm^2$. **[CBSE Marking Scheme, 2016]** 1

Detailed Answer :

Given, side of cube = 6 cm

diameter of hemisphere = 3.5 cm

radius of hemisphere = $\frac{3.5}{2}$

Total surface area of cube = $6a^2$

$= 6 \times (6)^2 = 216\ cm^2$

Total surface area of solid = TSA of cube – Area of circle + TSA of hemisphere.

$= 216\ cm^2 - \pi.\left(\frac{3.5}{2}\right)^2 cm^2 + 2.\pi.\left(\frac{3.5}{2}\right)^2 cm^2$

$= \left(216 - \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} + 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\right) cm^2$

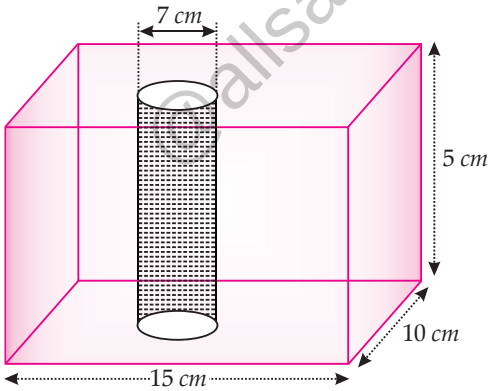
$= \left(216 - \frac{77}{8} + \frac{77}{4}\right) cm^2$

$= \left(\frac{1728 - 77 + 154}{8}\right) cm^2$

$= \frac{1805}{8} = 225.625\ cm^2$

Q. 29. In fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[Delhi CBSE Board, 2015 Set-I, II, III]**



Sol. Total surface area = $2(lb + bh + hl) + 2\pi rh - 2\pi r^2$

Here, l = 15 cm, b = 10 cm, h = 5 cm, $r = \frac{7}{2}$ cm

TSA of cuboidal block = 2 (15 × 10 + 10 × 5 +5 × 15)

$= 550\ cm^2$. **1**

C.S.A. of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 5$

$= 110\ cm^2$ **1**

Area of two circular bases = $2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$

$= 77\ cm^2$ ½

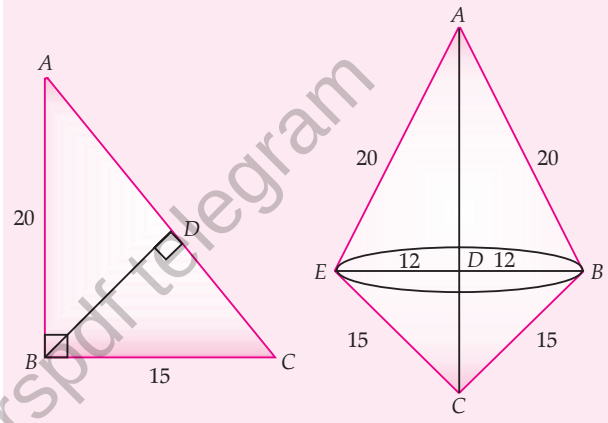
Required area = 550 + 110 – 77 = $583\ cm^2$ ½

[CBSE Marking Scheme, 2015]

Q. 30. A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use π = 3.14) [Board Sample Paper, 2016]

A **[Board Term-2, 2012 Set (28)]**

Sol.



(i) $AC^2 = 20^2 + 15^2 = 625$

or, $AC = 25$ cm

(ii) $ar\ (\Delta ABC) = ar\ (\Delta ABC)$

$\frac{1}{2} \times BC \times AB = \frac{1}{2} \times AC \times BD$

or, $15 \times 20 = 25 \times BD$

or, $BD = 12$ cm **1**

Volume of double cone

= Volume of upper cone + Volume of lower cone

$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$

$= \frac{1}{3}\pi(BD)^2\{AD + CD\} = \frac{1}{3}\pi(BD)^2(AC)$

$= \frac{1}{3}\times 3.14 \times 144 \times 25 = 3768\ cm^3$ **1**

Surface area = C.S.A. of upper cone + C.S.A. of lower cone

$= \pi(12)(20) + \pi(12)(15)$

$= 12\pi\{20 + 15\}$

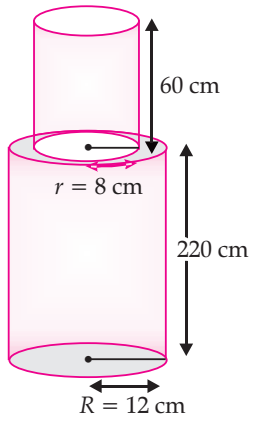
$= 12 \times 3.14 \times 35$

$= 1318.8\ cm^2$. **1**

[CBSE Marking Scheme, 2016, 2012]

Q. 31. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use π = 3.14)

C + A **[Board Term-2, 2012 Set (31)]**



Sol. Radius of lower cylinder $= R = 12$ cm
Radius of upper cylinder $= r = 8$ cm
Height of upper cylinder $= h = 60$ cm
Height of lower cylinder $= H = 220$ cm
Volume of solid iron pole $= \pi R^2 H + \pi r^2 h$ **1**

$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$

$= 111532.8 \text{ cm}^3$ **1**

Mass of the pole $= 111532.8 \times 8$ g

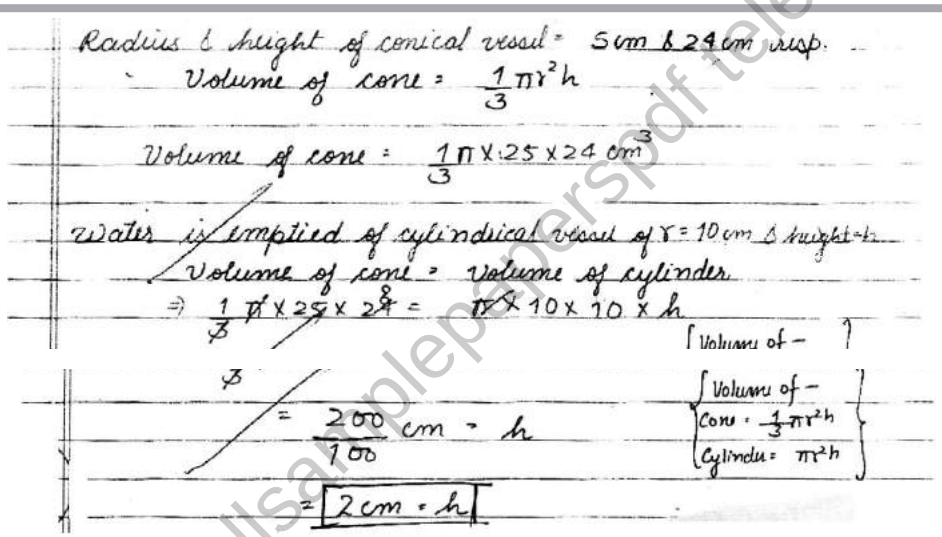
$= 892.2624$ kg. **1**

[CBSE Marking Scheme, 2012]

Q. 32. A conical vessel, with base radius 5 cm height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[Outside Delhi, Set–II 2016]**

Sol.



[Topper Answer, 2016] **3**

Q. 33. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ **cm^3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of ₹ 10 per cm^2.** $\left[\text{Use } \pi = \frac{22}{7}\right]$

C + A **[Delhi CBSE Board 2015 set I, II, III]**

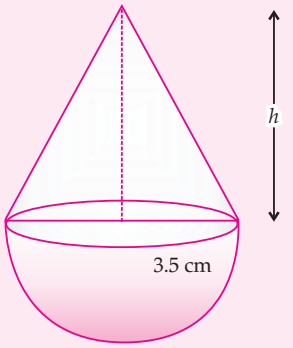
Sol. Given, radius of cone = radius of hemisphere = r

$r = 3.5$ cm.

Total volume, $V = 166\frac{5}{6}\text{ cm}^3 = \frac{1001}{6}\text{ cm}^3$ ½

Let the height of cone be h.

Total volume = Volume of cone + Volume of hemisphere



$$\frac{1001}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

or, $$\frac{1001}{6} = \frac{1}{3}\pi(3.5)^2 h + \frac{2}{3}\pi(3.5)^3$$

or, $$\frac{1001}{6} = \frac{1}{3}\pi[12.25\,h + 2 \times 42.875]$$

or, $\frac{1001\times3\times7}{6\times22} = 12.25h + 85.75$

or, $\frac{21021}{132} = 12.25h + 85.75$ ½

or, $12.25\,h = 159.25 - 85.75$

or, $h = \frac{73.5}{12.25} = 6$ cm ½

Height of the toy = 6 + 3.5 = 9.5 cm. ½

Curved surface area of hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7}\times3.5\times3.5$

$= 77$ cm^2 ½

Cost of painting = ₹ 10 × 77

= ₹ 770 ½

[CBSE Marking Scheme, 2015]

Q. 34. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, if the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

C + A **[Delhi CBSE Board 2015 Set I, II, III]**

Sol. Let the internal diameter of the pipe be r m.

Water flows in 1 hour = 2.52 km.

Water flows in $\frac{1}{2}$ hour = $\frac{2.52}{2}$ = 1.26 km

= 1260 m **1**

Volume of water flows in $\frac{1}{2}$ hour = $\pi r^2 h$.

$= \pi r^2 \times 1260$

Volume of the water in cylindrical tank

$= \pi \times \left(\frac{40}{100}\right)^2 \times 3.15$ **1**

Volume of water flow = Volume of increased water

$\pi r^2 \times 1260 = \pi\left(\frac{2}{5}\right)^2 \times 3.15$

or, $1260r^2 = \frac{2}{5} \times \frac{2}{5} \times 3.15$

or, $r^2 = \frac{4}{25}\times\frac{315}{100}\times\frac{1}{1260} = \frac{1}{2500}$ ½

or, $r = \frac{1}{50}\text{ m} = 2\text{ cm}$

Internal diameter of pipe = 4 cm. ½

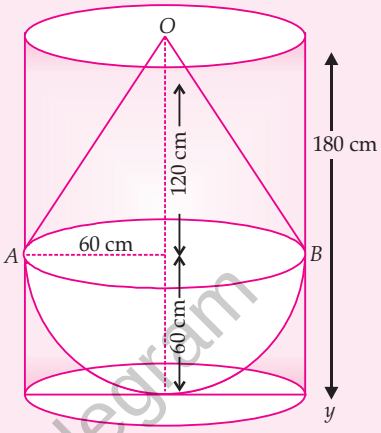
[CBSE Marking Scheme, 2015]

Q. 35. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. A **[Board Term-2, 2015]**

Sol. Given, height of cone, h = 120 cm, radius of cone r = 60 cm.

Radius of hemisphere = 60 cm.

Volume of cone = $\frac{1}{3}\pi r^2 h$



$= \frac{1}{3}\times 3.14 \times 60 \times 60 \times 120$

$= 3.14 \times 60 \times 60 \times 40$

$= 452160$ cm^3 **1**

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\times 3.14 \times 60 \times 60 \times 60$

$= 452160$ cm^3 ½

Total volume = Volume of cone + Volume of hemisphere

$= 452160 + 452160$

$= 904320$ cm^3 ½

Height of cylinder = 180 cm,

radius = 60 cm.

Volume of water in the cylinder

= Volume of cylinder

$= \pi r^2 h$

$= 3.14 \times 60 \times 60 \times 180$

$= 2034720$ cm^3 ½

Water left in the cylinder = Volume of water in cylinder – Volume of (cone + sphere)

$= 2034720 - 904320$

$= 1130400$ cm^3 ½

[CBSE Marking Scheme, 2015]

Q. 36. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter If there common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of canvas used in the tent.

A **[Delhi Compt. Set-I, II, III 2017]**

Sol. Total height of tent = 27 m

Height of cylindrical part = 6 m

∴ Height of conical part = 27 – 6 = 21 m

radius of cone $= \frac{56}{2} = 28$ m

Slant height of cone $= \sqrt{r^2 + h^2}$

$= \sqrt{28^2 + 21^2}$

$= \sqrt{784 + 441} = \sqrt{1225}$

= 35 m **1**

Area of canvas used $= 2\pi rh + \pi rl$ **1**

$= \pi r(2h + l)$

$= \frac{22}{7} \times 28(2 \times 6 + 35)$

$= 22 \times 4 \times 47$

$= 4136 \text{ m}^2$ **1**

[CBSE Marking Scheme, 2017]

Q. 37. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cutout. Find the total surface area of the remaining solid. U **[Outside Delhi Set-II, III 2017]**

Sol. Radius $r = 0.7$ cm

and height $h = 2.4$ cm

∴ Slant height $l = \sqrt{h^2 + r^2}$

$= \sqrt{(2.4)^2 + (0.7)^2}$

= 2.5 m **1**

Total surface area of remaining solid

= C.S.A. of cylinder + C.S.A. of cone + Area of top

$= 2\pi rh + \pi rl + \pi r^2$ ½

$= \pi r(2h + l + r)$

$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$

$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$

Hence total surface area = 17.6 cm^2 **1½**

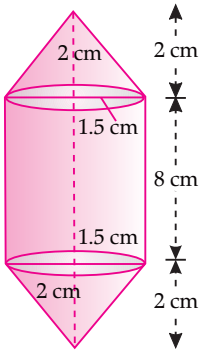
[CBSE Marking Scheme, 2017]

Q. 38. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm.

(i) Find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).

(ii) Which mathematical concept is used in the above problem ? AE

Sol.



(i) Here, radius of two cones and cylinder $= \frac{3}{2}$ cm

= 1.5 cm

Height of each cone = 2 cm

∴ Height of cylindrical portion = 12 – 2 – 2 = 8 cm

∴ Volume of the air in model

= Volume of cylindrical part + 2 × Volumes of conical part

$= \pi(1.5)^2 \times 8 + 2 \frac{1}{3}\pi(1.5)^2 \times 2$ **1**

$= \frac{22}{7} \times (1.5)^2 \left[8 + \frac{4}{3}\right]$

$= \frac{22}{7} \times 2.25 \times \frac{28}{3}$ **1**

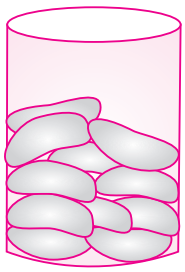
$= 66 \text{ cm}^3$.

(ii) Volume (Mensuration). **1**

Q. 39. A *gulab jamun*, contains sugar syrup up to about 30% of its volume.

(i) Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm (see Figure).

(ii) Which mathematical concept is used in the above problem ?



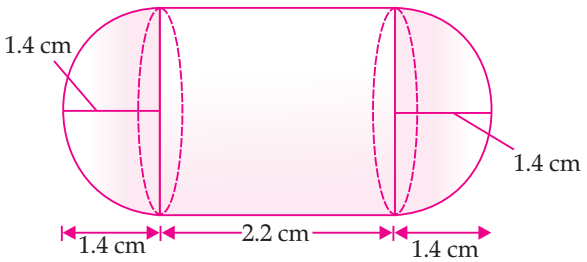
[NCERT Ex.]

Ans. (i) Radius of cylindrical portion and hemispherical portion of a gulab jamun

$= \frac{2.8}{2} = 1.4$ cm

Length of cylindrical portion

= 5 – 1.4 – 1.4 = 2.2 cm



Now, Volume of one gulab jamun = Volume of cylinder part + 2 × Volume of hemispherical part

$= \pi(1.4)^2 \times 2.2 + 2 \times \frac{2}{3}\pi(1.4)^3$

$= \frac{22}{7} \times (1.4)^2 \left[2.2 + \frac{4}{3} \times 1.4\right]$

$$= \frac{22}{7} \times 1.96 \times \frac{12.2}{3} = \frac{75.152}{3} \text{ cm}^3$$ 1

Volume of 45 gulab jamun

$$= 45 \times \frac{75.152}{3} = 1127.28 \text{ cm}^3$$

Volume of syrup in 45 gulab jamun

= 30% of 1127.28

$$= \frac{30}{100} \times 1127.28 = 338.18 \text{ cm}^3$$

$$= 338 \text{ cm}^3 \text{ (approx.)}$$

(ii) Volume (Mensuration)

Long Answer Type Questions (4 marks each)

Q. 1. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

A **[Delhi CBSE Board, 2015 Set I, II]**

Sol. Given, Depth of well = 14 m, radius = 2 m.

Volume of earth taken out = $\pi r^2 h$

$$= \frac{22}{7} \times 2 \times 2 \times 14$$

$$= 176 \text{ m}^3$$ **1**

Let r be the width of embankment

The radius of outer circle of embankment

$$= 2 + r$$

Area of upper surface of embankment

$$= \pi[(2 + r)^2 - (2)^2]$$

Volume of embankment = Volume of earth taken out **1½**

or, $\pi[(2 + r)^2 - (2)^2] \times 0.4 = 176$

or, $\pi[4 + r^2 + 4r - 4] \times 0.4 = 176$

or, $r^2 + 4r = \frac{176 \times 7}{0.4 \times 22}$

or, $r^2 + 4r = 140$

or, $r^2 + 4r - 140 = 0$

or, $(r + 14)(r - 10) = 0$

or, $r = 10$ m **1½**

Hence, width of embankment = 10 m.

[CBSE Marking Scheme, 2015]

Q. 2. A hemispherical depression is cut from one face of a cubical block, such that diameter 'l' of hemisphere is equal to the edge of cube. Find the surface area of the remaining solid.

A **[Foreign Set I, II, III, 2014]**

Sol. Let the radius of hemisphere = r

$\therefore$ Therefore, $r = \frac{l}{2}$ **1**

Now, the required surface area

= Surface area of cubical block – Area of base of hemisphere + Curved surface area of hemisphere. **1**

$$= 6(\text{side})^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2} l^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$ **1**

Required urface area $= \frac{1}{4}(24 + \pi) l^2$ unit^2.

$$= \frac{1}{4}\left(24 + \frac{22}{7}\right)^2 l^2$$

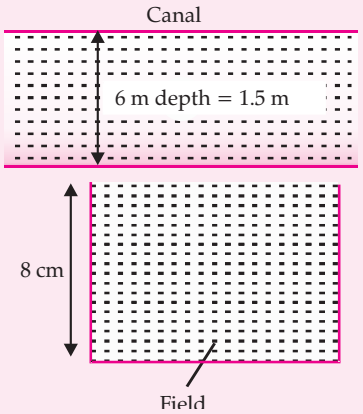
$$= \frac{1}{4} \times \frac{190}{7} \times \frac{190}{7} l^2$$

$$= 184.18\, l^2 \text{ unit}^2$$ **1**

[CBSE Marking Scheme, 2014]

Q. 3. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area in hectare will it irrigate in 30 minutes if 8 cm of standing water is needed ? A **[KVS 2014] [Delhi Set, 2014] [Board Term-2, 2012 (13)]**

Sol.



½

Water flows in 1 hr = 10 km

Water flows in $\frac{1}{2}$ hr $= \frac{10}{2}$

$$= 5 \text{ km}$$

$$= 5000 \text{ m}$$

Now volume of water flows in $\frac{1}{2}$ hr

$$= lbh$$ **1**

$$= 5000 \times 6 \times 1.5 \text{ m}^3$$

$$= 45000 \text{ m}^3.$$ ½

According to the question,

Volume of water in $\frac{1}{2}$ hr

$= \text{area of irrigated field} \times \frac{8}{100}\text{ m}$ **1**

or, $45000 = \text{Area} \times \frac{8}{100}$

$\therefore \quad \text{Area} = \frac{45000 \times 100}{8} = 562500 \text{ m}^2$

$= 56.25$ hectare. **1**

[CBSE Marking Scheme, 2012]

Q. 4. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled ?

A **[Delhi Set, 2014]**

[Board Term-2, 2012 (31)]

Sol. Diameter of pipe $= 20$ cm.

$\therefore \quad$ Radius of pipe $= \frac{20}{2} = 10\text{cm}$

$= 0.10$ m ½

Diameter of tank $= 10$ m

$\therefore$ radius of the tank $= \frac{20}{2} = 5$ m **1**

Depth of tank, $d = 2$ m

Volume of tank $= \pi r^2 d$

$= \pi \times 5 \times 5 \times 2$

$= 50\pi$

Speed of the water 3 km/ hr.

$= \frac{3000}{60} = 50 \text{ m/min}$ **1**

Volume of water supplied in one minute

$= \pi r^2 h$

$= \pi \times 0.10 \times 0.10 \times 50$

Let time taken by tank to fill

$= \frac{50\pi}{\pi \times .10 \times 0.10 \times 50} = 100$ **1**

Hence, time taken to fill the tank

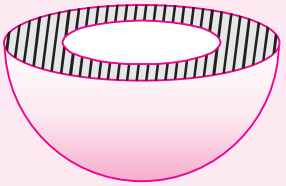
$= 100$ minutes. ½

[CBSE Marking Scheme 2012]

Q. 5. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is ₹ 5·00, find the total cost of painting the vessel all over. (Use $\pi = 3{\cdot}14$)

A **[Board Term-2, 2012 (40)]**

Sol. R $= 8$ cm, $r = 6$ cm



½

Total Surface area $= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$ **1**

$= \pi[8^2 \times 2 + 6^2 \times 2 + (8^2 - 6^2)]$

$= \pi[64 \times 2 + 36 \times 2 + (64 - 36)]$

$= \pi[128 + 72 + 28]$

$= 228 \times 3.14$ **2**

$= 715.92 \text{ cm}^2$

$\therefore \quad$ Total cost $= 715.92 \times ₹\ 5 = ₹\ 3579{\cdot}60$. ½

[CBSE Marking Scheme, 2012]

Q. 6. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour. A **[Delhi 2013]**

Sol. Volume of water flowing through pipe in 1 sec

$= \pi R^2 H$

$= \pi \times (1)^2 \times 0.4 \times 100 \text{ cm}^3$ **1**

Volume of water flowing in 30 min (30 × 60 sec)

$= \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$ **1**

Volume of water in cylindrical tank in 30 min

$= \pi r^2 h$

$= \pi \times (40)^2 \times h$ ½

$\pi \times (40)^2 \times h = \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$ ½

Rise in water level

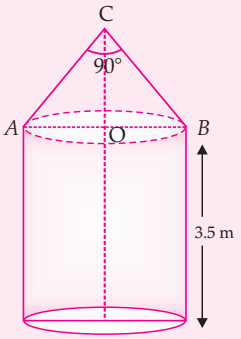
$h = \frac{\pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60}{\pi \times 40 \times 40}$

$= 45$ cm.

$\therefore$ Rise in level of water in the tank is 45 cm. **1**

Q. 7. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3·5 m surmounted by a cone whose vertical angle is 90°. Find total surface area of the toy. U **[Board Term-2, 2012 (44)]**

Sol.



Since $\angle C = 90°$

$AC = BC = x$ (say) **2**

$\therefore \quad AB^2 = AC^2 + BC^2$

$AB^2 = x^2 + x^2$

$\therefore \quad 2x^2 = (2\sqrt{2})^2$

or, $x = 2$ and $r = \sqrt{2}$ m

$\therefore$ Slant height of conical portion $= 2$ m **1**

Total surface area of toy $= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r[7 + \sqrt{2} + 2] \text{ m}^2$

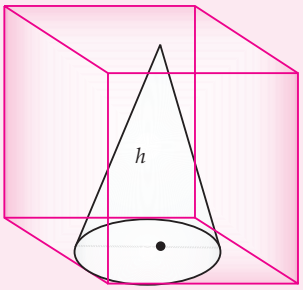
$= \pi\sqrt{2}[9+\sqrt{2}]\,m^2$

$= \pi[2+9\sqrt{2}]\,m^2$ **1**

[CBSE Marking Scheme, 2012]

Q. 8. Find the volume of the largest solid right circular cone that can be cut out off a solid cube of side 14 cm. A **[Board Term-2, 2012 (1)]**

Sol.



The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube. **2**

Radius of cone $= \frac{14}{2} = 7$ cm

Height of cone $= 14$ cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 14$

$= \frac{2156}{3} = 718.67\ cm^2.$ **2**

[CBSE Marking Scheme, 2012]

Q. 9. Water is flowing at the rate of 15 km/hr through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

U **[Board Term-2, 2012 Set (5)]**

[SQP 2018]

Sol. Speed of water flowing through the pipe

$= 15$ km/hr $= 15000$ m/hr **1**

Volume of water flowing in 1 hr $= \pi R^2 H$

$= \frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 15000\ m^3$

$= 231\ m^3$ **1**

Volume of water in the tank when the depth is 21 cm

$= lbh = 50\times 44\times\frac{21}{100}\ m^3$ **1**

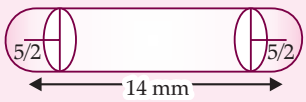
$= 462\ m^3$

$\therefore$ Time taken to fill $462\ m^3 = \frac{462}{231} = 2$ hrs. **1**

[CBSE Marking Scheme, 2018]

Q. 10. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule. A **[Board Term-2, 2012 Set (12)]**

Sol.



Total height $= 14$ mm

Height of cylinder $= 14 - 2\times 2.5$

$= 14 - 5 = 9$ mm **1**

Radius of cylinder $= 2.5$ mm

Radius of hemisphere $= 2.5$ mm

Volume of capsule = Volume of two hemispheres + Volume of cylinder **1**

$= 2\times\frac{2}{3}\pi r^3 + \pi r^2 h$

$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2\times 9$ **1**

$= \left(\frac{5}{2}\right)^2\times\pi\left[\frac{4}{3}\times\frac{5}{2}+9\right]$

$= \frac{25}{4}\pi\left[\frac{10}{3}+9\right]$

$= \frac{25}{4}\pi\left[\frac{10+27}{3}\right]$

$= \frac{25}{4}\pi\left[\frac{37}{3}\right]$

$= \frac{25}{4}\times\frac{22}{7}\times\frac{37}{3}$

$= \frac{10175}{42}\ mm^3$

or $= 242.26\ mm^3.$ **1**

[CBSE Marking Scheme, 2012]

Q. 11. A milk tanker cylindrical in shape having diameter 2 m and length 4·2 m supplies milk to the two booths in the ratio 3 : 2. One of the milk booths has cuboidal vessel having base area 3·96 sq. m. and the other has a cylindrical vessel having radius 1 m. Find the level of milk in each of the vessels. $\left[\text{Use } \pi = \frac{22}{7}\right]$ A **[Board Term-2, 2012 (28)]**

Sol. Volume of milk $= \frac{22}{7}\times 1\times 1\times 4{\cdot}2 = 13{\cdot}2\ m^3$ **1**

Milk to booth I $= 13{\cdot}2\times\frac{3}{5} = 2{\cdot}64\times 3$

$= 7.92\ m^3$ ½

Milk to booth II $= 13{\cdot}2\times\frac{2}{5} = 2{\cdot}64\times 2$

$= 5.28\ m^3$ ½

$\therefore$ Height in 1st vessel $= \frac{7{\cdot}92}{3{\cdot}96} = 2$ m 1

Height in 2nd vessel $= \frac{5{\cdot}28}{\frac{22}{7}\times 1} = \frac{5{\cdot}28\times 7}{22}$

$= 1.68$ m. 1

[CBSE Marking Scheme, 2012]

Q. 12. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel. A [CBSE O.D. 2014]

Ans. Diameter of spherical marble = 1.4 cm

Radius $r_1 = \frac{1.4}{2} = 0.7 = \frac{7}{10}$ cm

Diameter of cylindrical vessel = 7 cm

Radius $R = \frac{7}{2}$ cm 1

Let h be the rise in water level then

Then, the volume of 150 spherical marbles

= volume of water rises

or, $150\times\frac{4}{3}\times\pi\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10} = \pi\times\frac{7}{2}\times\frac{7}{2}\times h$ 1½

or, $h = \frac{4\times 7}{5}$

or, $\frac{28}{5} = h$ ½

or, $h = 5.6$ cm 1

Thus, the rise in the level of water, $h = 5.6$ cm.

Q. 13. A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment. A [Foreign Set-I, II, III 2017]

Sol. Try yourself similar to Q. 20. SATQ-II.

Q. 14. Water is following at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m × 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

A [Delhi Compt. Set-I, II, III 2017]

Sol. Speed of water in pipe = 5 km/hour

In an hour length of water = 5000 m

Let time taken to fill the tank be t.

$\therefore$ Total length of water = $t \times 5000$ m ½

Volume of water flown = Volume of water in tank ½

$\Rightarrow \pi r^2 h = l \times b \times h$

$\Rightarrow \frac{22}{7}\times\left(\frac{7}{100}\right)^2\times 5000t = 50\times 44\times\frac{7}{100}$ 1

$\Rightarrow \frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 5000t = 50\times 44\times\frac{7}{100}$ 1

$\Rightarrow t = \frac{50\times 44}{22\times 50} = 2$

Hence, time taken to fill the tank = 2 hours. 1

[CBSE Marking Scheme, 2017]

Q. 15. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball. [Foreign Set I, II, III, 2015]

Sol. Volume of water in cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\times(5)^2\times 8$

$= \frac{200}{3}\pi$ cm^3 ½

Volume of water flown out

$= \frac{1}{4}\times\frac{200}{3}\pi = \frac{50}{3}\pi$ cm^3 1

Let the radius of one spherical ball be r cm 1½

$\therefore \frac{4}{3}\pi r^3\times 100 = \frac{50}{3}\pi$

$r^3 = \frac{50}{4\times 100} = \frac{1}{8}$

or, $r = \frac{1}{2} = 0.5$ cm 1

[CBSE Marking Scheme, 2015]

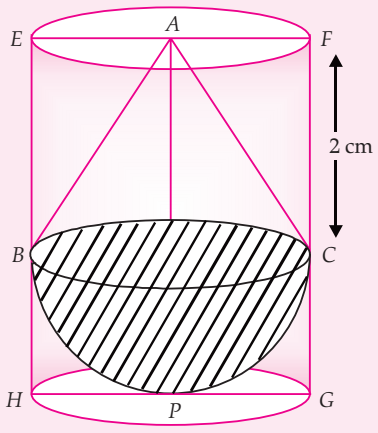
Q. 16. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the longest side. Find the surface area of figure obtained. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[Board Term-2, 2012 (44)]

Ans. Try yourself, similar to Q. No. 30 in SATQ- II.

Q. 17. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Take $\pi = 3.14$) C [Board Term-2, 2012 Set (34)]

Sol.



½

Let BPC is a hemisphere and ABC is a cone.

Radius of hemisphere = Radius of cone

$= \frac{4}{2} = 2$ cm

h = Height of cone = 2 cm

Volume of toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi r^2(2r+h) = \frac{1}{3}\times 3.14\times 2\times 2(2\times 2+2)$

$= \frac{1}{3}\times 3.14\times 4\times 6$

$= 25.12 \text{ cm}^3$ ...(i)

Let right circular cylinder $EFGH$ circumscribe the given solid toy. **1½**

Radius of cylinder = 2 cm,
Height of cylinder = 4 cm

Volume of right circular cylinder $= \pi r^2 h$

$= 3.14 \times (2)^2 \times 4 \text{ cm}^3$ ...(ii)

$= 50.24 \text{ cm}^3$ **1**

$\therefore$ Difference of two volumes = Volume of cylinder – Volume of toy

$= 50.24 - 25.12$

$= 25.12 \text{ cm}^3$. **1**

[CBSE Marking Scheme, 2012]

TOPIC-2

Problems involving converting one type of metallic solid into another

Revision Notes

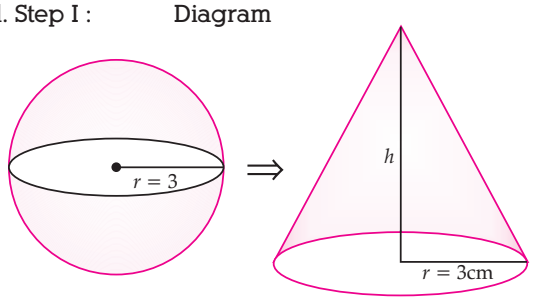
- While converting one metallic object into another, the volume will remain same by assuming no wastage of metal.
- Total surface area always be different from the original.
- Total surface area of the solid formed by the combination of solids remains the sum of the curved surface areas of each of the individual pants.
- The solids having the same curved surface do not necessarily have the same volume.

Table for the converting of the solids :

Name of solids	Volume	Total surface Area	Lateral surface Area
Cube	$V = a^3$	$TSA = 6a^2$	$LSA = 4a^2$
Cuboid	$V = l \times b \times h$	$TSA = 2(lb + bh + hl)$	$LSA = 2h(l + b)$
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi r(h + r)$	$CSA = 2\pi rh$
Hollow cylinder ($R > r$)	$V = \pi(R^2 - r^2)h$	$TSA = 2\pi(R + r)(h + R - r)$	
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi r(l + r)$	$CSA = \pi rl$
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$	$CSA = 4\pi r^2$
Hemisphere	$V = \frac{2}{3}\pi r^3$	$TSA = 3\pi r^2$	$CSA = 2\pi r^2$

Q. A spherical ball of radius 3 cm is melted and recast into a cone of same radius. Calculate the height of cone A

Sol. Step I : Diagram



Step II : Volume of spherical ball

$$V = \frac{4}{3}\pi(3)^3 \text{ cm}^3$$

Volume of cone $V = \frac{1}{3}\pi(3)^2 h$

Step III : According to question

$$\frac{4}{3}\pi \times 27 = \frac{\pi}{3} \times 9 \times h$$

$$\frac{4 \times 27}{9} = h$$

$$h = 12 \text{ cm}$$

$\therefore$ Height of cone = 12 cm.

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is :

(a) 12 cm **(b) 14 cm**

(c) 15 cm **(d) 18 cm**

A [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : During recasting a shape into another shape it's volume does not change.

For Spherical shell

$r_1 = \frac{4}{2} = 2$ cm

$r_2 = \frac{8}{2} = 4$ cm

For Cone

$r = \frac{8}{2} = 4$ cm

$h = ?$

During recasting volume remains same so,

Volume of cone = Volume of hollow spherical shell

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi(r_2^3 - r_1^3)$$

$$\Rightarrow r^2 h = 4(r_2^3 - r_1^3)$$

$$\Rightarrow 4 \times 4h = 4[(4)^3 - (2)^3]$$

$$\Rightarrow 4h = 64 - 8$$

$$\Rightarrow h = \frac{56}{4}$$

$$\Rightarrow h = 14 \text{ cm}$$

Q. 2. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is :

(a) 21 cm **(b) 23 cm**

(c) 25 cm **(d) 19 cm**

A [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Solid cuboid of iron is moulded into solid sphere.

For hence, volume of cuboid and sphere are equal.

For sphere

$r = ?$

cuboid

$l = 49$ cm

$b = 33$ cm

$h = 24$ cm

$\therefore$ Volume of sphere (solid) = Volume of cuboid

$$\Rightarrow \frac{4}{3\pi r^3} = l \times b \times h$$

$$\Rightarrow r^3 = \frac{(l \times b \times h \times 3)}{(4 \times \pi)}$$

$$= \frac{(49 \times 33 \times 24 \times 3 \times 7)}{(4 \times 22)}$$

$$\Rightarrow r^3 = 7 \times 7 \times 7 \times 3 \times 3 \times 3$$

$$\Rightarrow r = 21 \text{ cm}$$

Q. 3. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that 1/8 space is covered by the mortar. Then the number of bricks used to construct the wall is :

(a) 11100 **(b) 11200**

(c) 11000 **(d) 11300**

A [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : The volume of the wall covered by mortar = $\frac{1}{8}$ part

So, the volume covered by bricks of wall = $\left(1 - \frac{1}{8}\right)$

volume of wall = $\frac{7}{8}$ volume of wall

Bricks (Cuboid)	Wall (Cuboid)
l_1 = 22.5 cm	l = 270 cm
b_1 = 11.25 cm	b = 300 cm
h_1 = 8.75 cm	h = 350 cm

Let n be the number of bricks.

According to the question, we have

Volume of n bricks = $\frac{7}{8}$ Volume of wall (Cuboid)

$$\Rightarrow n \times l_1 \times b_1 \times h_1 = \frac{7}{8} \times l \times b \times h$$

$$\Rightarrow n = \frac{(7 \times l \times b \times h)}{(8 \times l_1 \times b_1 \times h_1)} = \frac{(7 \times 270 \times 300 \times 350)}{(8 \times 22.5 \times 11.25 \times 8.75)}$$

$$\Rightarrow n = \frac{(7 \times 270 \times 300 \times 350 \times 100 \times 10 \times 100)}{(8 \times 225 \times 1125 \times 875)}$$

$\Rightarrow n = 2 \times 4 \times 350 \times 4 = 32 \times 350 = 11{,}200$ bricks.

Q. 4. Twelve solid sphere of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is :

(a) 4 cm **(b) 3 cm**
(c) 2 cm **(d) 6 cm**

A [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : Solid cylinder is recasted into 12 spheres.

So, the volume of 12 spheres will be equal to the volume of the cylinder.

12 For spheres

R = ?

For cylinder

$r = \frac{2}{2} = 1$ cm

$h = 16$ cm

$\therefore$ Volume of 12 spheres = Volume of cylinder

$$\Rightarrow \frac{4}{3\pi r^3} = \pi r^2 h$$

$$\Rightarrow R^3 = \frac{(3r^2h)}{(4 \times 12)} = \frac{(3 \times 1 \times 1 \times 16)}{(4 \times 12)} = 1$$

$\Rightarrow R = 1$ cm

Hence, diameter $= 2R = 2 \times 1 = 2$ cm.

Q. 5. During conversion of a solid from one shape to another, the volume of new shape will :

(a) increase **(b) decrease**
(c) remains unaltered **(d) be doubled**

A [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : During reshaping a solid, the volume of new solid will be equal to old one or remains unaltered.

Q. 6. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is :

(a) 3.5 **(b) 7**
(c) $\frac{80}{7}$ **(d) 5**

A [Board, Term-2, Outside Delhi Set-I, II, III, 2014]

Sol. Correct option : (a)

Explanation : Circumference = 22 cm

$$2\pi r = 22$$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = 3.5 \text{ cm}$$

Q. 7. The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm, is :

(a) 3 **(b) 5**
(c) 4 **(d) 6**

A [Board, Term-2, Foreign Set-I, II, III, 2014]

Sol. Correct option : (b)

Explanation :

$$\text{No. of solid spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{\pi R^2 h}{\frac{4}{3}\pi r^3}$$

$$= \frac{\pi(2)^2 \times 45 \times 3}{4 \times \pi \times (3)^3}$$

$$= 5.$$

[B] Very Short Answer Type Questions :

Q. 1. Find the number of solid spheres of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm.

A [Delhi CBSE Term-2, 2014]

Sol. Let the number of spheres be n.

Radius of sphere $r_1 = 3$ cm, radius of cylinder $r_2 = 2$ cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r^3 = \pi r_1^2 h$$

$$\text{or, } n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$

$$\text{or, } 36n = 180$$

$$\text{or, } n = \frac{180}{36} = 5$$

Thus, the number of solid spheres = 5. **1**

Q. 2. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius. A [Board Term-2, 2014]

Sol. Let the radius of spherical ball be R.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi [(3)^3 + (4)^3 + (5)^3]$$

$$\text{or, } R^3 = 27 + 64 + 125$$

$$\text{or, } R^3 = 216$$

$$\text{or, } R = 6 \text{ cm}$$ **1**

Q. 3. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere. A [Board Term-2, 2014]

Sol. No. of spheres = 12
Radius of cone, r = 1 cm
Height of the cone = 48 cm
$\therefore$ Volume of 12 spheres = Volume of cone
Let the radius of sphere be R cm

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

or, $$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$$

$$16R^3 = 16$$

or, $$R^3 = 1$$

or, R = 1 cm **1**

[CBSE Marking Scheme, 2014]

Q. 4. Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

U **[Delhi CBSE Term-2, 2012 Set (13)]**

Sol. Let the edge of single cube be x cm
Volume of single cube = Volume of three cubes

$$x^3 = (3)^3 + (4)^3 + (5)^3$$
$$= 27 + 64 + 125$$
$$= 216$$

or, x = 6 cm **1**

[CBSE Marking Scheme, 2012]

Q. 5. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

U **[Delhi Board Term-2, 2012, Set (22)]**

Sol. Volume of sphere = Volume of cone
Let the radius of cone be R cm.

$\therefore$ $$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$

or, $$4r^3 = R^2 r$$

or, $$R^2 = 4r^2$$

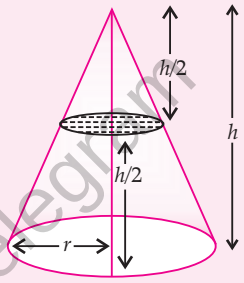
or, $R = 2r$ **1**

[CBSE Marking Scheme, 2012]

Q. 6. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, find the ratio of the volume of the upper part and the cone. **[Board Term-2, 2011, Set A1]**

Sol. Volume of upper cone $= \frac{1}{3}\pi \left(\frac{r}{2}\right)^2 \times \frac{h}{2}$

$$= \frac{1}{3}\pi \frac{r^2}{4} \times \frac{h}{2}$$



$$= \frac{1}{3}\pi \frac{r^2 h}{8}$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$\frac{\text{Volume of upper part of cone}}{\text{Volume of cone}} = \frac{\frac{1}{3}\pi \times \frac{r^2 h}{8}}{\frac{1}{3}\pi r^2 h}$$

$$= \frac{1}{8}$$

$$= 1 : 8$$ **1**

[CBSE Marking Scheme, 2011]

Short Answer Type Questions-I (2 marks each)

Q. 1. A solid metallic cylinder of radius 3.5 cm and height 14 cm is melted and recast into a number of small solid metallic balls, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

A **[CBSE S.A.2 2016 Set–HODM4OL]**

Sol. Let the number of recast balls be N
radius of cylinder R = 3.5 cm
height of cylinder h = 14 cm
radius of recast balls $r = \frac{7}{12}$ cm

$\therefore$ Volume of n balls = Volume of cylinder

$\Rightarrow$ $$n\frac{4}{3}\pi r^3 = \pi R^2 h$$ **1**

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$

$$\Rightarrow n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$

$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$
$$= 648$$

Hence, number of recast balls = 648 **1**

[CBSE Marking Scheme, 2016]

Q. 2. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ? A **[Board Sample Paper, 2016]**

Sol. Diameter of sphere = 6 cm
Diameter of cylindrical vessel = 12 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\times\pi\times3\times3\times3$$

$$= 36\pi \text{ cm}^3 \quad \mathbf{1}$$

$\therefore$ Volume of sphere = Increase volume in cylinder

$$36\pi = \pi(6)^2 \times h$$

$$h = 1 \text{ cm}$$

$\therefore$ Level of water rise in vessel = 1 cm. **1**

[CBSE Marking Scheme, 2016]

Q. 3. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

A **[Board Sample Paper 2016]**

Sol. Volume of coin $= \pi r^2 h$

$$= \frac{22}{7} \times (0.75)^2 \times 0.2 \text{ cm}^3 \quad ½$$

$$\text{Volume of cylinder} = \frac{22}{7} \times (2.25)^2 \times 10 \text{ cm}^3 \quad ½$$

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}} \quad ½$$

$$= \frac{\left(\frac{22}{7}\times(2.25)^2\times10\right)}{\left(\frac{22}{7}\times(0.75)^2\times0.2\right)}$$

$$= 450 \quad ½$$

[CBSE Marking Scheme, 2016]

Q. 4. A solid metallic cuboid of dimensions 9 m × 8 m × 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

A **[Foreign Set-I, II 2017]**

Sol. Volume of cuboid $= 9 \times 8 \times 2 \text{ cm}^3$ ½

Volume of cube $= 2 \times 2 \times 2 \text{ cm}^3$ ½

Let number of recast cubes be n.

$\therefore$ Volume of n cubes = Volume of cuboid

$$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$$

$$n = \frac{9\times8\times2}{2\times2\times2} = 18 \quad \mathbf{1}$$

Hence, number of cubes recast = 18.

[CBSE Marking Scheme, 2017]

Q. 5. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0·5 cm. What is the height of cylinder ?

A **[Board Term-2, 2012 (22)]**

Sol. Volume of cylinder = Volume of sphere, ½

$$\pi r^2 h = \pi$$

where r and h are radius of base and height of cylinder ½

$$(0.5)^2 h = 1$$

$$\left(\frac{1}{2}\right)^2 h = 1$$

$$h = 4 \text{ cm.} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 6. A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

A **[Board Term-2, 2012 (1)]**

Sol. Try Similar to Q. 5. in SATQ-I.

Short Answer Type Questions-II

(3 marks each)

Q. 1. A Solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of balls.

A **[Sample Q. Paper- 2018]**

Sol. Number of balls $= \frac{\text{Volume of solid sphere}}{\text{Volume of 1 spherical ball}}$ **1**

$$= \frac{\frac{4}{3}\times\pi\times3\times3\times3}{\frac{4}{3}\times\pi\times0.3\times0.3\times0.3} \quad \mathbf{1}$$

$$= 1000 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2018]

Q. 2. A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones each of radius 3.5 cm and height 3 cm. How many cones will be made ?

A **[Delhi Set-II 2017]**

Sol. Radius of given sphere = 10.5 cm

$$\therefore \text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\times10.5\times10.5\times10.5$$

$$= 4\pi \times 3.5 \times 10.5 \times 10.5 \text{ cm}^3 \quad \mathbf{1}$$

Radius of one recast cone = 3.5 cm

and height = 3 cm

$$\therefore \text{Volume} = \frac{1}{3}\pi\times3.5\times3.5\times3$$

$$= \pi \times 3.5 \times 3.5 \text{ cm}^3 \quad \mathbf{1}$$

Let the number of recast cones be n.

$\therefore n \times \pi \times 3.5 \times 3.5 \text{ cm}^3 = 4 \times \pi \times 3.5 \times 10.5 \times 10.5 \text{ cm}^3$

$$\Rightarrow \quad n = \frac{4\times3.5\times10.5\times10.5}{3.5\times3.5} = 126$$

Hence, number of recast cones = 126. **1**

[CBSE Marking Scheme, 2017]

Q. 3. A solid metallic sphere of diameter 16 cm is melted and recast into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed. A [Delhi Set-III 2017]

Sol. Diameter of sphere = 16 cm

$\therefore$ radius $= \frac{16}{2} = 8$ cm

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times8\times8\times8 \text{ cm}^3$$

1

Radius and height of recast cones = 4 cm and 8 cm respectively.

$\therefore$ Volume of each cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times4\times4\times8 \text{ cm}^3$$

1

Let number of cones recasted be n

$$\therefore \quad n = \frac{\text{Volume of Sphere}}{\text{Volume of each Cone}}$$

$$= \frac{\frac{4}{3}\times\pi\times8\times8\times8}{\frac{1}{3}\times\pi\times4\times4\times8} = 16$$

Hence number of recast cones = 16. **1**

Q. 4. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

U [Outside Delhi CBSE Board, 2015, Set I, II, III]

Sol. Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of metal in 504 cones

$$= 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\times3 \quad ½$$

Volume of Sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times r^3$ **1**

Since, Volume of sphere = Volume of 504 cones

$$\frac{4}{3}\times\frac{22}{7}\times r^3 = 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times3 \quad ½$$

or, $r^3 = \left(\frac{21}{2}\right)^3$

or, $r = 10.5$ cm

$\therefore$ Diameter $= 21$ cm ½

and surface area $= 4\pi r^2$

$$= 4\times\frac{22}{7}\times10.5\times10.5$$

$= 1386$ cm^2 ½

[CBSE Marking Scheme, 2015]

Q. 5. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere. U [Board Term-2, 2014]

Sol. Let the radius of sphere be R cm.

Volume of sphere = Volume of cone **1**

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

or, $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi\times2\times2\times8$ **1**

or, $R^3 = \frac{2\times2\times8}{4}$

or, $R^3 = 8$

or, $R = 2$ cm **1**

[CBSE Marking Scheme, 2014]

Q. 6. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

U [Sample Question Paper 2017]

Sol. Side of cube = 14 cm.

Cone of maximum size is curved out

$\therefore$ Diameter of cone = 14 cm

radius of cone = 7 cm

Slant height $l = \sqrt{h^2+r^2} = \sqrt{14^2+7^2}$

$= \sqrt{196+49} = \sqrt{245}$

$= 15.65$ cm. **1**

Total surface area = Total surface area of cube + Curved surface area of cone – Circular area of base of cone

$= 6a^2 + \pi rl - \pi r^2$ **1**

$$= 6\times14\times14 + \frac{22}{7}\times7\times15.65 - \frac{22}{7}\times7\times7$$

$= 1176 + [22(15.65 - 7)]$

$= 1176 + 22 \times 8.65$

$= 1176 + 190.3 = 1366.3$ cm^2 **1**

Q. 7. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

U [Outside Delhi Compt. Set-I, II III 2017]

Sol. Height of cylinder = height of cone = 8 cm

radius of cylinder = radius of cone = 6 cm

$\therefore$ Slant height of cone $= \sqrt{8^2+6^2} = \sqrt{64+36}$

$= 10$ cm ½

Total surface area of remaining solid = Curved surface area of cylinder + Surface area of cone + Area of top cylinder $= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + l + r)$ **1**

$$= \frac{22}{7}\times6(2\times8+10+6)$$

$$= \frac{22}{7}\times6\times32$$

$= 603.43$ **1**

Hence total surface area of remaining solid $= 603.43$ cm^2 [CBSE Marking Scheme, 2017] ½

Q. 8. From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

A [Delhi Compt. Set-I, II III 2017]

Sol. Try yourself similar to Q. 7. in SATQ-II.

Long Answer Type Questions (4 marks each)

Q. 1. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. $\left[\text{Use } \pi = \frac{22}{7}\right]$ A **[Outside Delhi Set I, II, III, 2015]**

Sol. Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times 10$ ½

$= 554.40 \text{ cm}^3$ ½

Volume of metal scooped out = 2 × volume of hemisphere

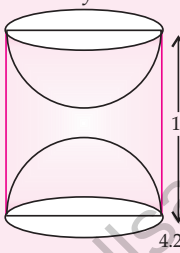
$= 2 \times \frac{2}{3} \times \pi r^3$

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{42}{10}\right)^3$ ½

$= 310.46 \text{ cm}^3$ ½

Volume of rest of cylinder = 554.40 – 310.46



$= 243.94 \text{ m}^3$ ½

Let the length of wire be l cm

$\therefore$ Volume of wire $= \pi r^2 l$

and $\pi r^2 l = 243.94 \text{ cm}^3$ ½

$\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times l = 243.94 \text{ cm}^3$ ½

or, $l = \frac{243.94 \times 10 \times 10}{22 \times 7}$

$l = 158.4 \text{ cm}$ ½

[CBSE Marking Scheme, 2015]

Q. 2. From a rectangular block of wood, having dimensions 15 cm × 10 cm × 3.5 cm, a pen stand is made by making four conical depressions. The radius of each one of the depression is 0.5 cm and the depth 2.1 cm. Find the volume of wood left in the pen stand. A **[Delhi Compt. Set-I, II, III 2017]**

Sol. Volume of cuboidal block $= l \times b \times h$

$= 15 \times 10 \times 3.5 = 525 \text{ cm}^3$ **1**

Volume of one cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \text{ cm}^3$

$= 0.55 \text{ cm}^3$ **1**

Volume of 4 cones $= 0.55 \times 4 = 2.2 \text{ cm}^3$ **1**

Volume of wood remaining in pen stand

$= 525 - 2.2$

$= 522.80 \text{ cm}^3$. **1**

[CBSE Marking Scheme, 2017]

Q. 3. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys so formed.

A **[Outside Delhi Compt. Set-II, III 2017]**

Sol. Given, height of cylinder = 15 cm

and its diameter = 12 cm

$\therefore$ radius = 6 cm

radius of cone = 3 cm ½

and height = 9 cm

Let the number of toys recast be n. ½

$\therefore$ Volume of n conical toys = Volume of cylinder **1**

$n \times \frac{1}{3}\pi \times 3 \times 3 \times 9 = \pi \times 6 \times 6 \times 15$

$n = \frac{6 \times 6 \times 15}{3 \times 9}$

$n = 20$

Hence the number of toys = 20. **2**

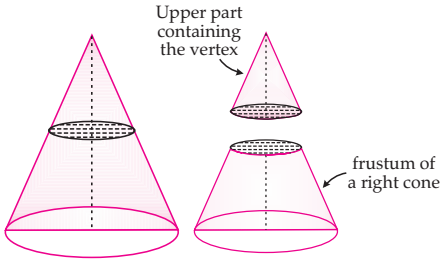
[CBSE Marking Scheme, 2015]

TOPIC-3
Frustum of Cone

Revision Notes

When the smaller conical portion of a given a right circular cone, which is sliced through by a plane parallel to its base, is removed, the resulting solid is called a Frustum of Right Circular Cone.

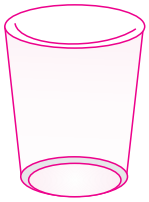
- Volume of a frustum of a cone = $\frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$
- Curved surface area of a frustum = $\pi l\,(r_1 + r_2)$
- Total surface area = $\pi l\,(r_1 + r_2) + \pi\,(r^2_1 + r_2^2)$
  where r_1 and r_2 area the radii of two ends and h is the height.



Objective Type Questions (1 mark each)

[A] Multiple Choice Questions :

Q. 1. The shape of a glass (tumbler) (see Figure) is usually in the form of a :



(a) cone **(b) frustum of a cone**
(c) cylinder **(d) sphere**

U [NCERT Exemp.]

Sol. Correct option : (b)
Explanation : The radius of the lower circul ar part is smaller than the upper part. So, it is frustum of a cone.

Q. 2. A shuttle cock used for playing badminton has the shape of the combination of :
(a) a cylinder and a sphere
(b) a cylinder and a hemisphere
(c) a sphere and a cone
(d) frustum of a cone and a hemisphere

U [NCERT Exemp.]

Sol. Correct option : (d)
Explanation : A shuttle cock used for playing badminton has the shape of the combination of frustum of a cone and a hemisphere.

Q. 3. A cone is cut through a plane parallel to its base and then the cone is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called :
(a) a frustum of cone **(b) cone**
(c) cylinder **(d) sphere**

U [NCERT Exemp.]

Sol. Correct option : (a)
Explanation : The new part that is left over on the other sides of the plane is called a frustum of a cone.

Q. 4. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm respectively. The curved surface area of the bucket is :
(a) 4950 cm^2 **(b) 4951 cm^2**
(c) 4952 cm^2 **(d) 4953 cm^2**

A [NCERT Exemp.]

Sol. Correct option : (a)
Explanation : Here, $r_1 = 7$ cm, $r_2 = 28$ cm, $l = 45$ cm
Curved surface area of bucket = $\pi l(r_1 + r_2)$

$= \frac{22}{7} \times 45(7 + 28)$

$= \frac{22}{7} \times 45 \times 35$

$\Rightarrow$ Curved surface area of bucket = $22 \times 45 \times 5$ cm^2
$= 4{,}950$ cm^2

Q. 5. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of bucket is 35 cm. The capacity of bucket is :
(a) 32.7 *L* **(b) 33.7 *L***
(c) 34.7 *L* **(d) 37.7 *L***

A [NCERT Exemp.]

Sol. Correct option : (a)
Explanation : Bucket is in the shape of a frustum of a cone.

Here, $r_1 = 24/2 = 12$ cm, $r_2 = \frac{44}{2} = 22$ cm, $h = 35$ cm

The volume of the bucket is given by,

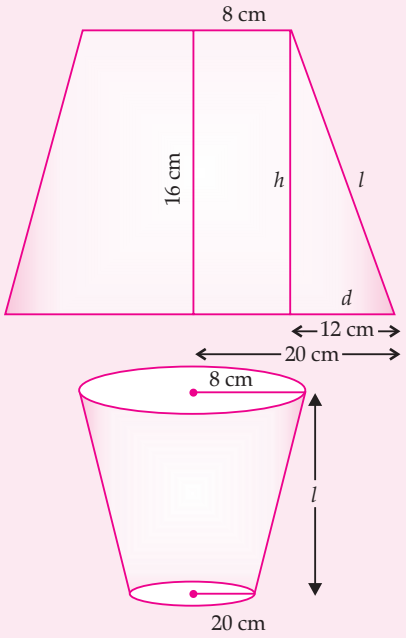
$$V = \frac{1}{2}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 35(12^2 + 22^2 + 12 \times 22)$$
$$= \frac{1 \times 22 \times 35}{3 \times 7}(144 + 484 + 264)$$
$$= \frac{22 \times 35 \times 892}{3 \times 7}\text{cm}^3 = \frac{110 \times 892}{3 \times 1{,}000} \text{ litres}$$
$$= \frac{9{,}812}{300} \text{ litres} = 32.706 \text{ litres}$$

[B] Very Short Answer Type Questions :

Q. 1. What is the frustum of a right circular cone of height 16 cm with radii of its circular ends as 8 cm and 20 cm has slant height equal to ?

A [Board Term-2, 2014 A1]

Sol.



Slant height of the frustum

$$l = \sqrt{h^2 + d^2} = \sqrt{16^2 + (20-8)^2}$$

$$= \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400}$$

$= 20$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 2. The slant height of a bucket is 26 cm. The diameter of upper and lower circular ends are 36 cm and 16 cm. Find the height of the bucket.

A **[Board Term-2, 2012 31]**

Sol. Here, $l = 26$ cm, upper radius $= 18$ cm,

lower radius $= 8$ cm

d = difference in radius $= 18 - 8 = 10$ cm.

Let h be the height of bucket

$$\therefore \quad h = \sqrt{l^2 - d^2} = \sqrt{26^2 - (18-8)^2}$$

$$= \sqrt{(26)^2 - (10)^2}$$

$$= \sqrt{676 - 100}$$

$$= \sqrt{576} = 24 \text{ cm.}$$ **1**

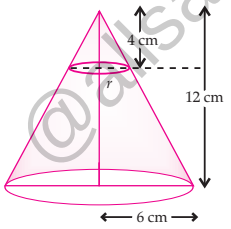
[CBSE Marking Scheme, 2012]

Short Answer Type Questions-I

(2 marks each)

Q. 1. In fig from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.

$$\left(\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{5} = 2.236\right)$$



A **[Delhi CBSE Board, 2015 set I, II, III]**

Sol. Let r be the radius of the top., $h = 12 - 4 = 8$ cm

$$\frac{4}{r} = \frac{12}{6}$$

$$\therefore \quad r = 2 \text{ cm} \qquad \frac{1}{2}$$

$$l = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{(8)^2 + (6-2)^2}$$

$$= \sqrt{64 + 16} = \sqrt{80}$$

$$= 4\sqrt{5} = 4 \times 2.236$$

$= 8.944$ cm $\frac{1}{2}$

Total surface area of frustum

$$= \pi[R^2 + r^2 + l(R + r)]$$

$$= \frac{22}{7}[(6)^2 + (2)^2 + 8.944(6 + 2)]$$

$$= \frac{22}{7}[36 + 4 + 71.552]$$

$$= \frac{22}{7} \times 111.552$$

$= 350.59$ cm^2. **1**

[CBSE Marking Scheme, 2015]

Q. 2. Milk in a container, which is in the form of frustum of a cone of height 30 cm and the radii of whose lower and upper circular ends are 20 cm and 40 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of ₹ 35 per litre and 880 litre of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and that cost will it put on the donor agency for this.

AE **[Foreign Set I, II, III, 2015]**

Sol. Volume of the milk container = Volume of frustum

$$= \frac{1}{3}\pi h[R^2 + r^2 + Rr]$$

$$= \frac{1}{3}\pi \times 30(40^2 + 20^2 + 40 \times 20)$$

$$= 10\pi\,(1600 + 400 + 800)$$

$$= 10 \times \frac{22}{7} \times 2800$$

$$= 88000 \text{ cm}^3$$

$$= 88 \text{ litre} \quad \mathbf{1}$$

$$\text{Number of containers needed} = \frac{880}{88} = 10$$

$$\text{Cost of milk} = ₹\ 880 \times 35$$

$$= ₹\ 30800 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II

(3 marks each)

Q. 1. The perimeters of the ends of the frustum of a cone are 207.24 cm and 169.56 cm. If the height of the frustum be 8 cm, find the whole surface area of the frustum. (Use $\pi = 3.14$)

A **[Board Sample Paper, 2016]**

Sol. Let R and r be the radii of the circular ends of the frustum. $(R > r)$

$$2\pi R = 207.24$$

$$R = 207.24/(2 \times 3.14)$$

$$R = 33 \text{ cm} \quad ½$$

$$2\pi r = 169.56 \text{ cm}$$

$$r = 169.56/(2 \times 3.14)$$

$$r = 27 \text{ cm} \quad ½$$

$$l^2 = h^2 + (R - r)^2$$

$$= 8^2 + (33 - 27)^2 \quad ½$$

$$l = 10 \text{ cm} \quad ½$$

Whole surface area of the frustum

$$= \pi(R^2 + r^2 + (R + r)l)$$

$$= 3.14\,[(33)^2 + (27)^2 + (33 + 27)10]$$

$$= 3.14\,(1089 + 729 + 600)$$

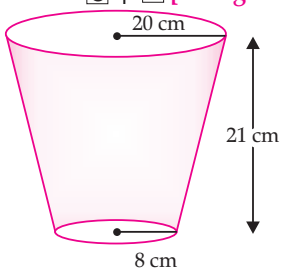
$$= 3.14 \times 2418 \text{ cm}^2$$

$$= 7592.52 \text{ cm}^2. \quad \mathbf{1}$$

[CBSE Marking Scheme, 2016]

Q. 2. A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹ 35 per litre. $\left[\text{Use } \pi = \frac{22}{7}\right]$

C + A **[Foreign Set I, II, III, 2016]**



Sol. If r_1 and r_2 be the radii of two circular ends and h be the height of frustum, then volume

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$$

Given that $r_1 = 8$ cm

$r_2 = 20$ cm

and $h = 21$ cm **1**

$$\therefore \text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 21\,[(8)^2 + (20)^2 + 8 \times 20]$$

$$= 22\,[64 + 400 + 160]$$

$$= 22 \times 624$$

$$= 13728 \text{ cm}^3$$

$$= \frac{13728}{1000} \text{lit} \quad (\because 1000 \text{ cm}^3 = 1 \text{ lit.})$$

$V = 13.728$ litres **1**

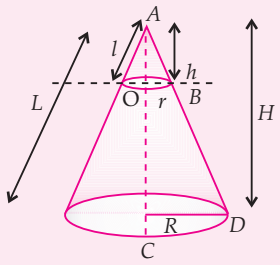
$\therefore$ Total Cost of milk $= 13.728 \times ₹\ 35$

$= ₹\ 480.48$ **1**

Q. 3. A cone is cut by a plane parallel to the base and upper part is removed. If the curved surface area of upper cone is $\frac{1}{9}$ times the curved surface of original cone. Find the ratio of line segment to which the cone's height is divided by the plane.

A **[Board Term-2, 2014]**

Sol.



$$\frac{\text{Curved surface of upper cone}}{\text{Curved surface of original cone}} = \frac{1}{9}$$

or, $$\frac{\pi rl}{\pi RL} = \frac{1}{9}$$

or, $$\frac{rl}{RL} = \frac{1}{9} \quad \text{...(i)}\ \mathbf{1}$$

Now

$\Delta AOB \sim \Delta ACD$ (by AA similarity)

$\therefore \quad \frac{r}{R} = \frac{h}{H} = \frac{l}{L}$ ...(ii) **1**

Substituting (ii) in (i),

or, $\frac{h}{H} \times \frac{h}{H} = \frac{1}{9}$

or, $\frac{h^2}{H^2} = \frac{1}{9}$

or, $\frac{h}{H} = \frac{1}{3}$

Hence $\frac{\text{Height of upper cone}}{\text{Height of lower frustum}} = \frac{1}{3-1} = \frac{1}{2}$

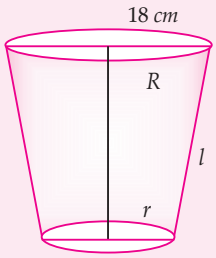
$\therefore$ Ratio of the line segments $OA : OC = 1 : 2$. **1**

[CBSE Marking Scheme, 2014]

Q. 4. The slant height of a frustum of a cone is 4 cm and the perimeter (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

U **[Board Term-2, 2012 Set (12)]**

Sol.



$l = 4$ cm

$2\pi R = 18$

or, $R = \frac{18}{2\pi} \Rightarrow R = \frac{9}{\pi}$ **1**

$2\pi r = 6$ **1**

or, $r = \frac{6}{2\pi}$

$= \frac{3}{\pi}$ cm

Curved surface area of frustum

$= \pi l(R + r)$

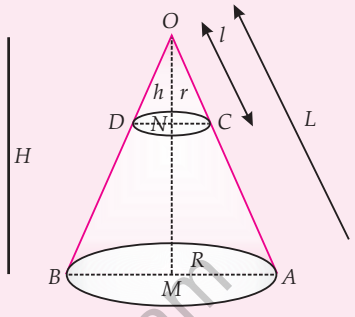
$= \pi \times 4\left(\frac{9}{\pi} + \frac{3}{\pi}\right)$

$= 4\pi \times \frac{12}{\pi} = 48 \text{ cm}^2$. **1**

[CBSE Marking Scheme, 2012]

Q. 5. A cone is cut by a plane parallel to the base and upper part is removed. If the C.S.A. of the remainder is $\frac{15}{16}$ of the C.S.A. of whole cone, find the ratio of the line segments to which the cone's height is divided by the plane. **[Board Term-2, 2014]**

Sol.



Let the height of larger cone be H

Let height of smaller cone be h

and radius of larger & smaller cones are R and r

Now, $\Delta ONC \sim \Delta OMA$

$\therefore \quad \frac{h}{H} = \frac{r}{R} = \frac{l}{L}$ **1**

or C.S.A. of the frustum $= \frac{15}{16}$ of (C.S.A. of cone OAB)

and C.S.A. of cone $OCD = 1 - \frac{15}{16} = \frac{1}{16}$ of (C.S.A. of cone OAB)

or, $\frac{\text{C.S.A. of cone } OCD}{\text{C.S.A. of cone } OAB} = \frac{1}{16}$

or, $\frac{\pi rl}{\pi RL} = \frac{1}{16}$ **1**

or, $\left(\frac{r}{R}\right)\left(\frac{l}{L}\right) = \frac{1}{16}$

or, $\left(\frac{h}{H}\right)\left(\frac{h}{H}\right) = \frac{1}{16} \quad \left(\because \frac{l}{L} = \frac{h}{H}\right)$

or, $\frac{h}{H} = \frac{1}{4}$

or, $h = \frac{1}{4}H$ **1**

$\therefore \quad ON = \frac{1}{4}H$

and $MN = \frac{3}{4}H$

or, $ON : MN = 1 : 3$ **1**

[CBSE Marking Scheme, 2014]

Long Answer Type Questions **(4 mark each)**

Q. 1. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi = 3.14$)

C + A **[Delhi Set I, II, III, 2016]**

Sol. Here, $R = 20, r = 12, V = 12308.8$

$$V = \frac{1}{3}\pi\left(R^2 + r^2 + Rr\right)h$$

$$12308.8 = \frac{1}{3} \times 3.14\,(400 + 240 + 144)\,h \quad \mathbf{1}$$

$$12308.8 = \frac{1}{3} \times 3.14 \times 784 \times h$$

or, $h = 15$ cm ½

$$l = \sqrt{(20-12)^2 + 15^2} = 17 \text{ cm} \quad ½$$

Total area of metal sheet used = CSA + base area

$= \pi\,[(20 + 12) \times 17 + 12 \times 12]$ **1**

$= 2160.32$ cm^2 **1**

[CBSE Marking Scheme, 2016]

Q. 2. The radii of the circular ends of a frustum of cone of height 6 cm are 14 cm and 6 cm respectively. Find the lateral surface area and total surface area of the frustum. A **[Board Term-2, 2012 Set (59)]**

Sol. $r_1 = 14$ cm, $r_2 = 6$ cm, $h = 6$ cm

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{6^2 + (14-6)^2} = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64} = 10 \text{ cm} \quad \mathbf{1}$$

$\therefore$ Lateral surface area $= \pi(r_1 + r_2)l$

$$= \frac{22}{7} \times (14+6) \times 10 \text{ cm}^2$$

$$= 628{\cdot}57 \text{ cm}^2 \quad \mathbf{1½}$$

Total surface area $= \pi[r_1^2 + r_2^2 + l\,(r_1 + r_2)]$

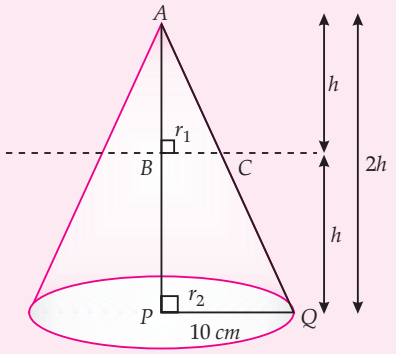
$$= \frac{22}{7} \times [(196 + 36) + 20 \times 10]\,\text{cm}^2$$

$$= \frac{22}{7} \times 432 = 1357.71 \text{ cm}^2. \quad \mathbf{1½}$$

[CBSE Marking Scheme, 2012]

Q. 3. A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the mid-point of its height. Compare the volume of the two parts. A **[Delhi Set-III 2017]**

Sol.



$\Delta ABC \sim \Delta APQ$

$$\Rightarrow \quad \frac{h}{2h} = \frac{r_1}{10}$$

$$\Rightarrow \quad r_1 = 5 \text{ cm} \quad \mathbf{1}$$

Volume of smaller cone $= \frac{1}{3}\pi(5)^2 \times h$ **1**

Volume of frustum $= \frac{1}{3}\pi \times h\left(5^2 + 10^2 + 5 \times 10\right)$

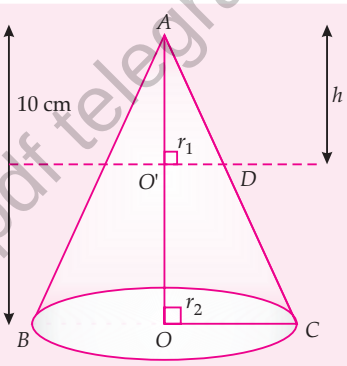
$$= \frac{1}{3}\pi \times h \times 175 \quad \mathbf{1½}$$

Required ratio $= \dfrac{\frac{1}{3} \times \pi \times 25 \times h}{\frac{1}{3} \times \pi \times h \times 175} = \frac{1}{7}$ ½

[CBSE Marking Scheme, 2017]

Q. 4. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of volume the two parts. A **[Delhi Set-I, II, III 2017]**

Sol.



Let the radius of cone be r_2 and cut of cone r_1

Height of the cone = 10 cm **1**

And the height the cone cut off = 5 cm

$\Delta AOC \sim \Delta AO'D$

$$\therefore \quad \frac{AO}{AO'} = \frac{r_2}{r_1} = \frac{10}{5}$$

$$\Rightarrow \quad r_2 = 2r_1 \quad \mathbf{1}$$

Volume of cut off cone $= \frac{1}{3}\pi r_1^2 \times 5$

$$= \frac{5}{3}\pi r_1^2 \text{ sq. units}$$

Volume of original cone $= \frac{1}{3}\pi(2r_1)^2 \times 10$

$$= \frac{40}{3}\pi r_1^2 \text{ sq. units}$$

Volume of frustum = Volume of original cone − Volume of cut of cone

$$= \frac{40}{3}\pi r_1^2 - \frac{5}{3}\pi r_1^2$$

$$= \frac{35}{3}\pi r_1^2 \text{ sq. units} \quad \mathbf{1}$$

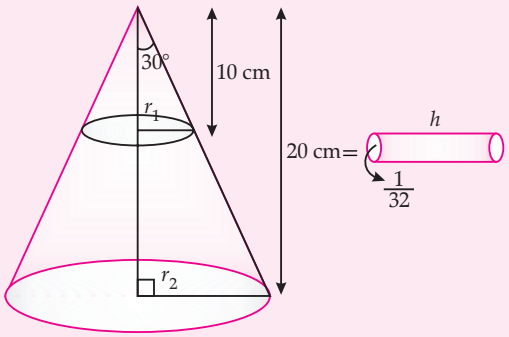
Ratio of two parts $= \dfrac{35\pi r_1^2}{5\pi r_1^2} = \frac{7}{1}$

Hence the ratio of two parts = 7 : 1 **1**

[CBSE Marking Scheme, 2017]

Q. 5. A metallic right circular cone 20 cm high and whose vertical angel is 60° is cut into two parts at the middle of its height by a plane parallel to its base if the frustum so obtained be drawn into a wire of uniform diameter $\frac{1}{16}$ cm, find the length of the wire. **[Foreign Set-I 2017]**

Sol.



Total height of cone = 20 cm

and vertex angle = 30°

Let the radius of cone be r_2

$$\therefore \quad \frac{r_2}{20} = \tan 30° \Rightarrow \frac{1}{\sqrt{3}}$$

$$r_2 = \frac{20}{\sqrt{3}} \text{ cm} \qquad \mathbf{1}$$

The height of the cone cut off = 10 cm

Let its radius be r_1

$$\Rightarrow \quad \frac{r_1}{10} = \tan 30° \Rightarrow r_1 = \frac{10}{\sqrt{3}} \text{ cm.}$$

1

Let the length of wire be l

$$\text{Its radius} = \frac{1}{32} \text{cm}$$

$\because$ Volume of frustum = Volume of wire

$$\Rightarrow \quad \frac{1}{3}\pi \times h\left[(r_1)^2 + (r_2)^2 + (r_1 r_2)\right] = \pi r^2 l$$

$$\Rightarrow \frac{1}{3}\times 10 \times \pi\left[\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2 + \frac{10}{\sqrt{3}} \times \frac{20}{\sqrt{3}}\right]$$

$$= \pi\left(\frac{1}{32}\right)^2 \times l$$

$$\Rightarrow \frac{1}{3}\times 10\left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right] = \frac{1}{32\times 32} \times l \qquad \mathbf{1}$$

$$\Rightarrow \quad \frac{1}{3}\times 10 \times \frac{700}{3} = \frac{1}{32} \times \frac{1}{32} \times l$$

$$\Rightarrow \quad l = \frac{32\times 32\times 700\times 10}{3\times 3}$$

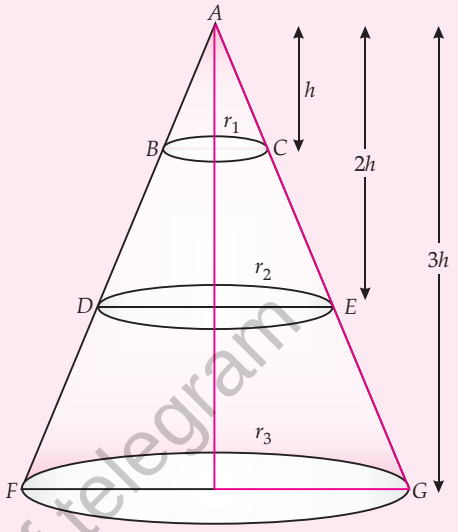
$$= 796444.44 \text{ cm.}$$

Hence, the length of wire = 7964.44 m. **1**

[CBSE Marking Scheme, 2017]

Q. 6. A right circular cone is divided into three parts trisecting its height by two planes drawn parallel to the base. Show that volumes of the three portions starting from the top are in the ratio 1 : 7 : 19. A **[Foreign Set-III 2017]**

Sol.



Let the radii of three cones from top be r_1, r_2 and r_3 respectively.

Let the height of given cone be 3 h.

So, the height of cone ADE = 2h.

and height of cone ABC = h

$$\therefore \Delta ABC \sim ADE, \quad \frac{r_1}{r_2} = \frac{h}{2h}$$

$$\Rightarrow \quad 2r_1 = r_2 \qquad ½$$

$$\Delta ABC \sim AFG \quad \frac{r_1}{r_3} = \frac{h}{3h}$$

$$3r_1 = r_3 \qquad ½$$

$$\text{Volume of cone } ABC = \frac{1}{3}\pi r_1^2 h$$

$$\text{Volume of cone } ADE = \frac{1}{3}\pi (r_2)^2 2h \qquad ½$$

$$= \frac{1}{3}\pi (2r_1)^2 2h$$

$$\text{Volume of frustum } BCED = \frac{1}{3}\pi 4r_1^2 2h - \frac{1}{3}\pi r_1^2 h$$

$$= \frac{7}{3}\pi r_1^2 h \qquad \mathbf{1}$$

Volume of frustum $DEGF$

$$= \frac{1}{3}\pi r_3^2 .3h - \frac{1}{3} r_2^2 .2h$$

$$= \frac{1}{3}\pi (3r_1)^2 3h - \frac{1}{3}(2r_1)^2 .2h$$

$$= \frac{1}{3}\pi r_1^2 h(27 - 8)$$

$$= \frac{19}{3}\pi r_1^2 h$$

$$\text{Ratio} = \frac{1}{3}\pi r_1^2 h : \frac{7}{3}\pi r_1^2 h : \frac{19}{3}\pi r_1^2 h \quad 1$$

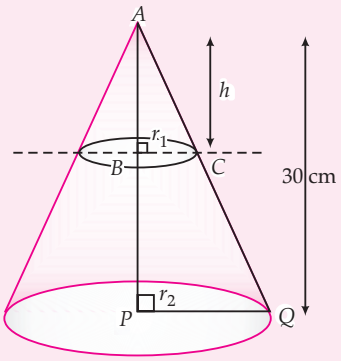
Hence, required ratio = 1 : 7 : 19. ½

[CBSE Marking Scheme, 2017]

Q. 7. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the cone then at what height it is cut from the base ?

A [Delhi Set-II, 2017]

Sol.



Let the radii of smaller cone and original cone be r_1 and r_2 respectively and the height of smaller cone be h.

$$\Delta ABC \sim \Delta APQ$$

$$\Rightarrow \quad \frac{h}{30} = \frac{r_1}{r_2} \quad \text{...(i)} \quad 1$$

Volume of smaller cone

$$= \frac{1}{27} \times \text{Volume of original cone}$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 \times h = \frac{1}{27} \times \frac{1}{3}\pi r_2^2 \times 30 \quad 1$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

$$\Rightarrow \quad \left(\frac{h}{30}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

$$\left(\text{Using } \frac{h}{30} = \frac{r_1}{r_2} \text{ From (i)}\right)$$

$$\Rightarrow \quad \left(\frac{h}{30}\right)^3 = \frac{1}{27}$$

$$\Rightarrow \quad h^3 = \frac{30 \times 30 \times 30}{27} \quad 1$$

$$h = 10 \text{ cm} \quad ½$$

Hence, required height = (30 – 10) = 20 cm. ½

[CBSE Marking Scheme, 2017]

Q. 8. The diameters of the lower and upper ends of a bucket in the form of a frustum of the cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$] A [Delhi/O.D. Set- 2018]

Sol. Here $r_1 = 15$ cm, $r_2 = 5$ cm and $h = 24$ cm

(i) Area of metal sheet = CSA of the bucket + Area of lower circle

$$= \pi l(r_1 + r_2) + \pi r_2^2 \quad 1$$

where $l = \sqrt{24^2 + (15-5)^2} = 26$ cm 1

$\therefore$ Surface area of metal sheet = $3.14\,(26 \times 20 + 25)$ cm^2

$$= 1711.3 \text{ cm}^2 \quad 1$$

(ii) We should avoid use of plastic because it is non-degradable or similar value. 1

[CBSE Marking Scheme, 2018]

Commonly Made Error

- In problems related to frustum, students write incorrect formula and do wrong calculation.

Answering Tip

- Students should learn the formula clearly by adequate practice and do the correct calculation.

Q. 9. A man donates 10 aluminium buckets to a orphanage. A bucket made of aluminium is of height 20 cm and has its upper and lower ends of radius 36 cm and 21 cm respectively. Find the cost of preparing 10 buckets if the cost of aluminium sheet is ₹ 42 per 100 cm^2. Write your comments on the act of the man.

A [O.D. Compt. Set I, II, III- 2018]

Sol. Surface area of bucket = $\pi(r_1 + r_2)\, l + \pi r_1^2$

$$l = \sqrt{h^2 + (r_2 - r_1)^2}$$

$$= \sqrt{20^2 + (36-21)^2}$$

$$= \sqrt{625} = 25 \text{ cm} \quad ½$$

$\therefore$ Surface area of 1 bucket

$$= \frac{22}{7}\,[(36 + 21) \times 25 + 21^2]$$

$$= \frac{22}{7} \times 1866 \text{ cm}^2 \quad 1$$

Surface are of 10 buckets

$$= \frac{22}{7} \times 18660 \text{ cm}^2 \quad ½$$

Cost of aluminium sheet = ₹ $\frac{22}{7} \times \frac{18660 \times 42}{100}$ 1

= ₹ 24631.20

Any relevant comment 1

[CBSE Marking Scheme, 2018]

Q. 10. The radii of circular ends of a bucket of height 24 cm are 15 cm and 5 cm. Find the area of its curved surface. A [S.Q.P- 2018]

Sol. $r_1 = 15$ cm, $r_2 = 5$ cm

$h = 24$ cm

$l = \sqrt{h^2 + (r_1 - r_2)^2}$ **1**

$= \sqrt{24^2 + 10^2} = 26$ cm

Curved surface area of bucket $= \pi(r_1 + r_2)l$ **1**

$= \frac{22}{7} \times (15 + 5) \times 26$

$= \frac{22 \times 20 \times 26}{7}$ **1**

$= \frac{11440}{7}$ cm^2 or 1634.3 cm^2 **1**

[CBSE Marking Scheme, 2018]

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