Polyphase Induction Motors

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Introduction

Induction Motor is singly-excited a.c. machine. Its stator winding is directly connected to a.c. source, whereas its rotor winding receives its energy from stator by means of induction.

Types of Induction Motor Rotors

1. Squirral - Cage rotor

- Consist of a cylindrical laminated core with slots nearly parallel to the shaft axis or skewed.
- Each slot contains an un-insulated bar conductor of aluminum or copper.
- At each end of the rotor, rotor bar conductors are short circuited by heavy end rings of the same material.

2. Wound rotor or Slip-ring rotor

- Consists of slotted armature.
- Insulated conductors are put in the slots and connected to form a three-phase double layer distributed winding similar to stator winding.
- The open ends of the star circuit are brought outside the rotor and connected to three insulated slip rings.

Remember:

Cage rotor is cheaper, require lesser maintenance and have higher efficiency and higher power factor than wound rotors.

Production of Rotating Magnetic Field in 3ϕ System

- If a 3ϕ balance winding is excited by 3ϕ balance current, a resultant mmf or flux is produced of constant magnitude of $\frac{3}{2}F_m$ or $\frac{3}{2}\phi_m$ in air gap of motor and rotate in space at synchronous speed (N_s).
- The direction of rotation of resultant flux in the air gap depends upon the phase sequence.

Remember:

- Resultant mmf can be given as

$$F_R = \frac{3}{2}F_m \cos(\omega t - \theta)$$

where, F_R = resultant mmf
F_m = maximum mmf

Speep and Slip

Synchronous speed (N_s)

$$N_s = \frac{120f}{P}$$

where, f = Supply frequency
P = Number of poles

Slip (s)

$$s = \frac{N_s - N_r}{N_s}$$

where, N_s = Synchronous speed, rpm
N_r = Rotor speed, rpm

Rotor frequency

$$f_2 = sf_1$$

where, f_1 = Line frequency

Note:

- Speed of stator field w.r.t. stator = N_s.
- Speed of rotor field w.r.t. stator = N_s.
- Speed of rotor w.r.t. stator = N_r.
- Speed of stator field w.r.t. rotor = $(N_s - N_r)$.
- Speed of rotor field w.r.t. rotor = sN_s.
- Relative speed between rotor field and stator field is zero.
- Rotor speed $N_r = (1 - s)N_s$.
- Rotor of induction motor never runs at synchronous speed ($N_r < N_s$) otherwise induced emf in rotor = 0.

Per phase induced emf

In stator winding

$$E_1 = \sqrt{2}\pi N_1 f_1 \phi k_{\omega 1}$$

In rotor winding

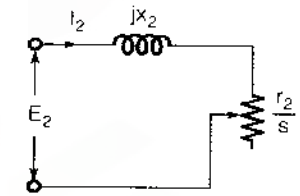
At standstill $$E_2 = \sqrt{2}\pi N_2 f_1 \phi k_{\omega 2}$$

At any slip 's' $$E_2 = \sqrt{2}\pi N_2 (sf_1) \phi k_{\omega 2}$$

where, $k_{\omega 1}, k_{\omega 2}$ = Winding factors of stator and rotor windings respectively
N_1, N_2 = Number of turns of stator and rotor winding respectively

Rotor Emf current and power

- Per phase rotor current at any slip 's'



$$I_2 = \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}}$$

where, r_2 = Rotor resistance at standstill
x_2 = Rotor leakage reactance at standstill
E_2 = Per phase value of induced Emf at standstill

- Rotor power factor
  - Rotor current I_2 lags the rotor voltage E_2 by an angle θ_2.

$$\theta_2 = \tan^{-1}\left(\frac{sx_2}{r_2}\right)$$

  - Rotor power factor = $\cos\theta_2$

- Per phase power input to rotor

$$P_g = E_2 I_2 \cos\theta_2 = I_2^2 \frac{r_2}{s}$$

P_g is the power transferred from stator to rotor across the air gap also called air-gap power.

- Rotor ohmic loss $= I_2^2 r_2 = sP_g$
- Internal mechanical power developed in rotor

$$\boxed{P_m = (1-s)P_g}$$

- Internal (or gross) torque developec per phase

$$\boxed{T_e = \frac{\text{Intermal mechanical power developed in rotor}}{\text{Rotor speed in mechanical rad. per sec.}}}$$

$$\boxed{T_e = \frac{P_g}{\omega_s} = \frac{P_m}{\omega_r} = \frac{\text{Rotor ohmic loss}}{\omega_s \times \text{slip}}}$$

where, ω_s, ω_r = Synchronous speed and rotor speed in mechanical rad. per sec.

- Output or shaft power

$$\boxed{P_{sh} = P_m - \text{Mechanical losses}}$$

$$\boxed{P_{sh} = P_g - \text{Rotor ohmic losses} - \text{Mechanical losses}}$$

Note: ....

Friction and windage losses are the mechanical losses.

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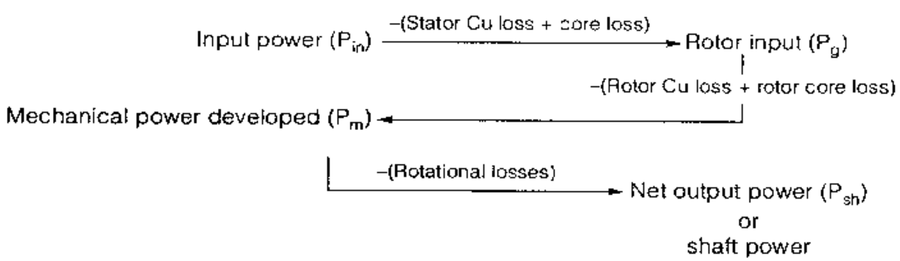
- Output or shaft torque

$$\boxed{T_{sh} = \frac{P_{sh}}{\text{rotor speed}} = \frac{P_{sh}}{(1-s)\omega_s}}$$

- Air gap power

$$\boxed{P_g = \text{Stator power input} - \text{Stator } I^2R \text{ loss} - \text{Stator core loss}}$$

Power Flow Diagram



Efficiency

Efficiency of a 3-phase induction motor (η)

$$\boxed{\eta = \frac{P_{sh}}{P_{sh} + P_f + P_{oh}} \times 100}$$

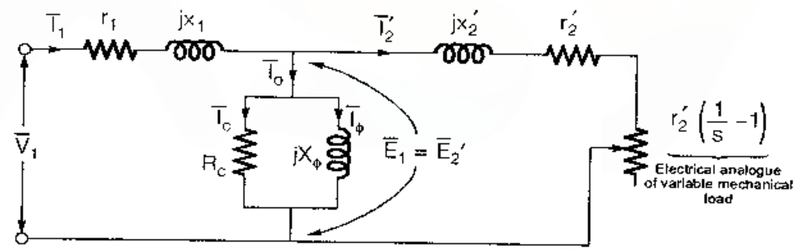
where, P_{sh} = Shaft power

P_f = Fixed losses = core loss + friction and windage losses

P_{oh} = Stator and rotor ohmic losses

Equivalent Circuit

Exact equivalent circuit referred to stator



- Transformation ratio

$$\boxed{a = \frac{E_1}{E_2} = \frac{\left(\begin{array}{c}\text{Effective number of stator-series turns}\\ \text{per phase } (N_1')\end{array}\right)}{\left(\begin{array}{c}\text{Effective number of rotor-series turns}\\ \text{per phase } (N_2')\end{array}\right)}}$$

- Per phase generated Emf in rotor circuit referred to stator side

$$\boxed{E_2' = aE_2}$$

- Per phase rotor current referred to stator side

$$\boxed{I_2' = \frac{I_2}{a}}$$

- Per phase rotor resistance referred to stator side

$$\boxed{r_2' = a^2 r_2}$$

- Per phase rotor leakage reactance referred to stator side

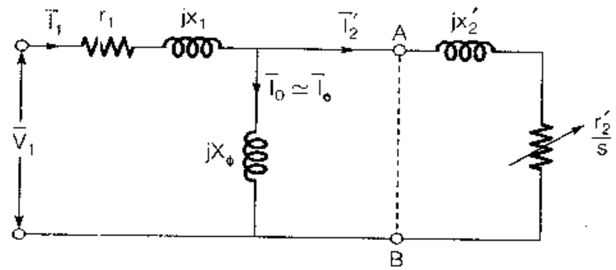
$$\boxed{x_2' = a^2 x_2}$$

where, r_1, r_2 = Per phase stator and rotor resistances.

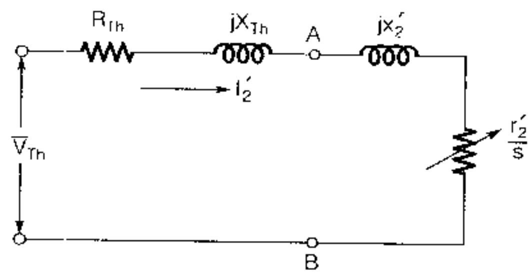
x_1, x_2 = Per phase stator and rotor leakage reactances at standstill.

Analysis of the equivalent circuit

All the rotor quantities have been referred to stator side.



Thevenin equivalent circuit



- Thevenin voltage across points A and B

$$V_{Th} = \frac{V_1}{r_1 + j(x_1 + X_\phi)} \cdot jX_\phi$$

- Thevenin equivalent impedance across points A and B

$$Z_{Th} = \frac{(r_1 + jx_1) \times jX_\phi}{r_1 + j(x_1 + X_\phi)} = R_{Th} + jX_{Th}$$

- Thevenin equivalent resistance and reactance across points A and B *Neglecting r_1 as $r_1 << (x_1 + X_\phi)$*

$$R_{Th} = \frac{r_1 X_\phi}{X} \quad \text{and} \quad X_{Th} = \frac{x_1 X_\phi}{X}$$

where, X_ϕ = Magnetizing reactance

V_1 = Stator applied voltage; $X = x_1 + X_\phi$

- Total torque

$$T_e = \frac{K_t}{\left(R_{Th} + \frac{r_2'}{s}\right)^2 + X_1^2} \cdot \frac{r_2'}{s}, \text{N-m}$$

where, $K_t = \frac{3V_{Th}^2}{\omega_s}$ and $X_1 = x_2' + X_{Th}$

- Maximum internal torque

$$T_{em} = \frac{K_t}{2\left[R_{Th} + \sqrt{R_{Th}^2 + X_1^2}\right]}, \text{N-m}$$

Note: Maximum internal torque is also referred as stalling torque, pull-out torque or breakdown torque.

- Slip at maximum torque

$$s_{mT} = \frac{r_2'}{\sqrt{R_{Th}^2 + X_1^2}}$$

- Starting torque

$$T_{e,st} = \frac{K_t}{(R_{Th} + r_2')^2 + X_1^2} \cdot r_2'$$

- Motor torque in terms of maximum torque

$$T_e = \frac{2\,T_{em}}{\frac{s_{mT}}{s} + \frac{s}{s_{mT}}}$$

- Total mechanical power developed

$$P_m = \frac{3V_{th}^2}{\left[R_{Th} + r_2' + \frac{r_2'(1-s)}{s}\right]^2 + X_1^2} \cdot r_2'\left(\frac{1-s}{s}\right)$$

- Slip at maximum power

$$s_{mP} = \frac{r_2'}{\sqrt{(R_{Th} + r_2')^2 + X_1^2} + r_2'}$$

- Maximum power

$$P_{m,max.} = \frac{3V_{Th}^2}{2\left[R_{Th} + r_2' + \sqrt{(R_{Th} + r_2') + X_1^2}\right]}$$

Remember:

- $P_{m,max.}$ depends on rotor resistance r_2' whereas T_{em} does not.
- At starting, internal mechanical power developed is zero.

- In order to get maximum power output from induction generator The rotor speed (n_r)

$$n_r = n_s\left[1 + \frac{r_2'}{\sqrt{(R_{Th} + r_2')^2 + X_1^2} + r_2'}\right]$$

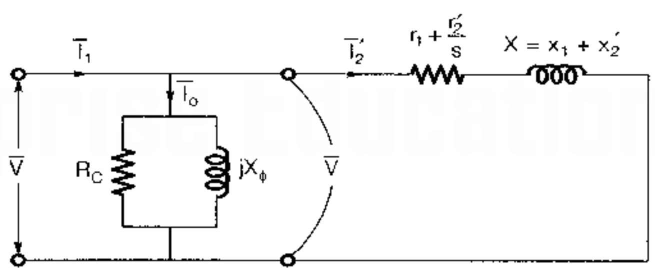
$$\left(\frac{I_{2,st}}{I_2}\right)^2 = \frac{1 + \left(\frac{s_{mT}}{s^2}\right)^2}{1 + s_{mT}^2}$$

$$\left(\frac{I_{2,mT}}{I_2}\right)^2 = \frac{1}{2}\left[1 + \left(\frac{s_{mT}}{s}\right)^2\right]$$

where, $I_{2,st}$ = Stator load component of current at starting

$I_{2,mT}$ = Stator load component of current at maximum torque

Approximate Analysis



$$I_2' = \frac{V}{\sqrt{\left(r_1 + \frac{r_s'}{s}\right)^2 + X^2}}$$

$$T = \frac{3}{\omega_s}\frac{V^2 r_2'/s}{\left(r_1 + \frac{r_2'}{s}\right)^2 + X^2}$$

Starting torque $$T_{st} = \frac{3}{\omega_s}\frac{V^2 r_2'}{(r_1 + r_2')^2 + X^2}$$

Maximum torque $$T_m = \frac{3}{\omega_s}\left(\frac{V^2}{2X_2'}\right)$$

Slip at max. torque $$s_m = \frac{r_2'}{X_2'} = \frac{r_2}{X_2}$$

Remember:

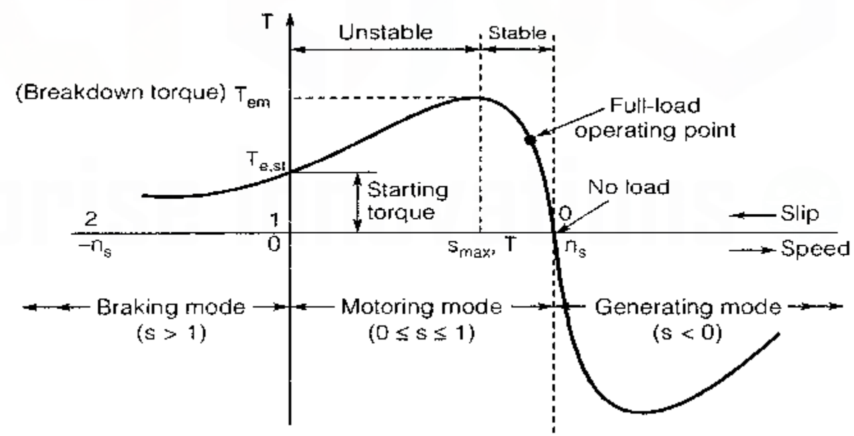
- At high slip (i.e. $s \approx 1$)

$$T \propto \frac{1}{s}$$

- At low slip (i.e. $s \approx 0$)

$$T \propto s$$

Torque Slip Characteristics



Torque-slip characteristics

Remember:

- $T_m \propto \frac{1}{f^2}$
- $T_{st} \propto \frac{1}{f^3}$
- Efficiency of I.M, $\eta \propto \frac{1-s}{1+s}$

where, f = frequency

s = slip

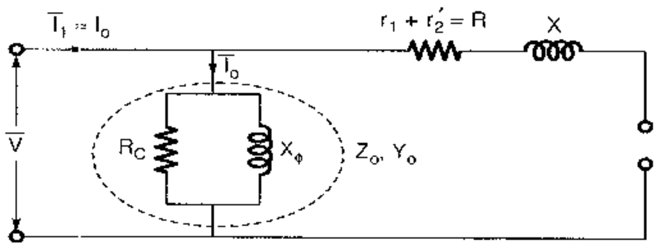
Determination of equivalent circuit parameters

1. No Load Test (O.C. Test):

Test performed at rated voltage to determine

(a) Iron loss at rated voltage i.e. P_i.

(b) Shunt branch parameter i.e. R_C and X_ϕ.



where, I_o = No load current

V = Rated voltage applied to stator

P_i = Fixed losses read by watt meter

$$R_c = \frac{V^2}{P_i}$$

$$G_C = \frac{1}{R_C}$$

$$Z_o = R_C \parallel jX_\phi$$

$$Y_o = \frac{1}{Z_o} = G_C - jB_\phi$$

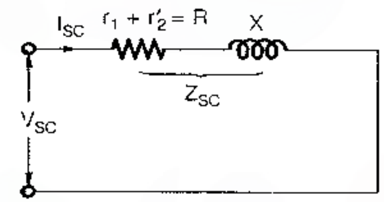
$$Y_o = \frac{I_o}{V}$$

$$B_\phi = \sqrt{Y_0^2 - G_C^2}$$

$$X_\phi = \frac{1}{B_\phi}$$

2. Block Rotor Test

- Supply is given but the rotor is blocked to rotate mechanically.
- This test is also called short circuit test to determine copper loss at rated current. This test is performed at rated current condition.



where, I_{sc} = Rated current

V_{sc} = Voltage that is required to circulate I_{sc} when rotor is blocked

P_{sc} = Total copper loss on full load at standstill, read by wattmeter

$$R = \frac{P_{SC}}{I_{SC}^2}$$

$$Z_{SC} = \frac{V_{SC}}{I_{SC}}$$

$$X = \sqrt{Z_{sc}^2 - R^2}$$

Note:

In block rotor condition, reduced voltage (up to 5% of rated) is required to flow the rated current.

Starting Method of 3ϕ Induction Motor

1. Direct Online Starting (D.O.L.)

To reduce the starting current if voltage is reduce by a fraction x, the starting current is also reduce by a fraction x but the starting torque gets reduce significantly by a factor x^2.

2. Auto Transformer Starting

Both starting current from supply and torque are reduced by same facto x^2.

Remember:

If $x = 1/\sqrt{3}$ results are same as Y-Δ starting.

3. Start-Delta Starting

In this, Δ connected induction motor started as Y connected, and as motor picks up the speed the connection are changed to Δ i.e. it runs as Δ connected.

If delta connected I.M. is started as star connected both the starting current and torque are reduce by a factor of 1/3.

Remember:

By increasing rotor resistance, starting torque increases but maximum torque remains unchanged.

Speed Control of Induction Motor

Rotor speed of induction motor is

$$\boxed{N_r = N_s(1-s)}$$

The rotor speed can be controlled by

(i) slip control technique and

(ii) synchronous speed control technique.

1. Slip Control Technique

$$T = \frac{sV^2}{R} \quad \text{where T remains constant.}$$

(a) **Voltage control technique:** Applicable for both I.M. i.e. slip ring induction motor (SRIM) and squirrel cage I.M.

(b) **Rotor resistance method:** Only in SRIM.

(c) **Rotor emf injection method:** Only in SRIM.

2. Synchronous Speed Control Technique

$$\boxed{N_s = \frac{120f}{P}} \quad \text{and} \quad \boxed{N_r = N_s(1-s)}$$

(a) **Frequency control technique:** Applicable in both induction motor.

(b) **Pole changing technique:** Only in squirrel cage I.M.

(c) **Cascading of 2 induction motor:** In this method one I.M. must be SRIM. while, other can either squirrel cage or SRIM.

Cogging

Number of Slot in stator = S_1

Number of slot in rotor = S_2

If either $S_1 = S_2$ or integral multiple to each other then the stator teeth and rotor teeth which are actually parallel, may develope very strong alignment force at the time of start and therefore the induction motor may fail to start, with rotor teeth blocked against the stator teeth. This phenomenon is known as cogging.

Crawling

The airgap flux in a 3ϕ induction motor contains harmonics of the order of (6n ± 1), then create parasitic torque. Due to these harmonic, motor continue to rotate stably at a speed N much less then full load speed N_{fl}. This phenomenon is called crawling.

Remember:

- Cogging and crawling phenomenon are usually not encountered in S.R.I.M. as their starting torque is high enough to accelerate it.
- **Power factor:** Wide bar rotor < deep bar rotor < semi open slot < closed slot.
- **Leakage flux:** Closed slot > semi closed slot > deep bar > wide bar.

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