Constructions

Exercise-12.1

Construct the following with the help of straight-edge and compass only :

Question 1:

Draw $\overline{AB}$ of length 7.4 cm and divide it in the ratio 5 : 7.

Solution :

Given : $\overline{AB}$ of length 7.4cm.

To construct: $\overline{AB}$ is to be divided in the ratio 5 : 7.

Steps of construction:

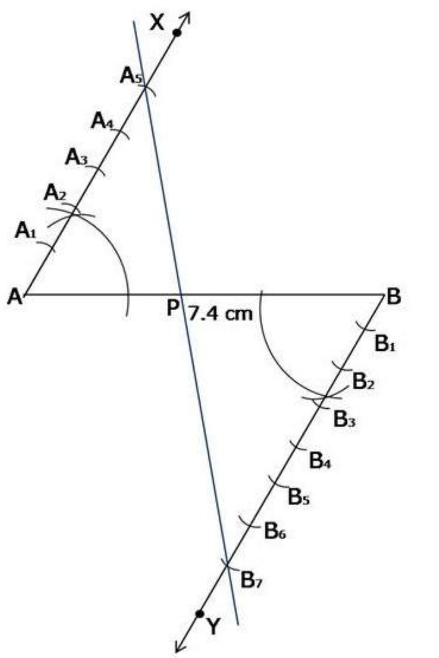
1) For $\overleftrightarrow{AB}$ containing $\overline{AB}$, draw $\overrightarrow{AX}$ and $\overrightarrow{BY}$ in different half-planes such that $\angle XAB$ and $\angle YBA$ are congruent acute angles.
2) With some appropriate radius and centre A, draw and arc to intersect $\overrightarrow{AX}$ at A_1. Similarly, with centre A_1 and the same radius, draw an arc to intersect $\overrightarrow{AX}$ at A_2 such that $A - A_1 - A_2$. Similarly, draw an arc with the same radius and centre A_k to intersect $\overrightarrow{AX}$ at A_k+1 such that $A_{k-1} - A_k - A_k+1$, where k = 2, 3, 4. Thus, we obtain five points A_1, A_2, A_3, A_4 and A_5 on $\overrightarrow{AX}$ such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3) Now, with the same radius and beginging with point B as center, obtain seven points B_1, B_2, B_3, B_4, B_5, B_6 and B_7.
4) Draw $\overline{A_5B_7}$ to intersect $\overline{AB}$ at P.

Thus, $P \in \overline{AB}$ is the point which divides $\overline{AB}$ in the ratio 5 : 7.

Justification: Here $\frac{AA_5}{BB_7} = \frac{5}{7}$.

Since $\overrightarrow{AX} \parallel \overrightarrow{BY}$, the correspondence $AA_5P \leftrightarrow BB_7P$ between ΔAA_5P and ΔBB_7P is similarity.

$\therefore \frac{AP}{BP} = \frac{AA_5}{BB_7} = \frac{5}{7}$.



Question 2:

Divide a line-segment into three parts in the ratio 2 : 3 : 4 in the same order.

Solution :

Given : $\overline{AB}$

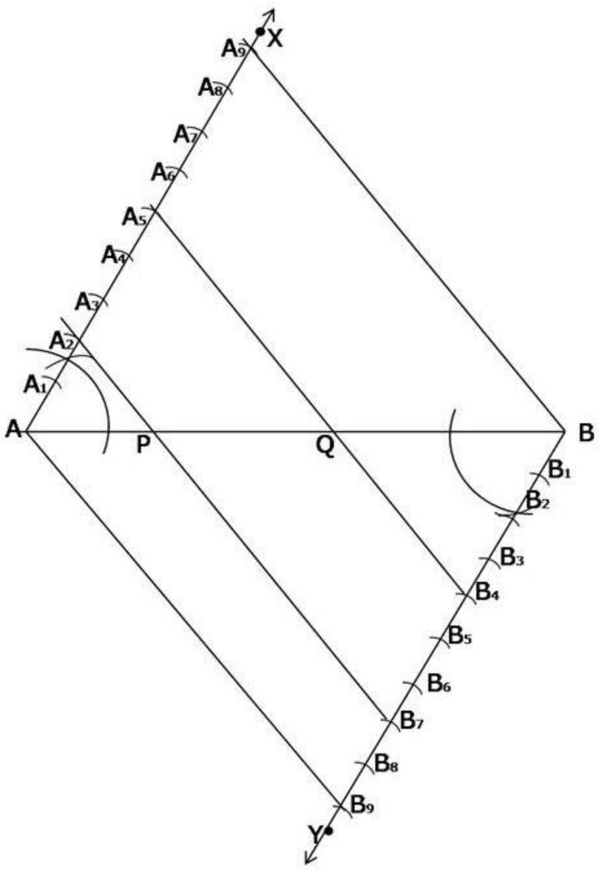
To Construct: $\overline{AB}$ is to divided into three parts in the ratio 2 : 3 : 4 from A.

Steps of construction:

1) For $\overleftrightarrow{AB}$ containing $\overline{AB}$, draw $\overrightarrow{AX}$ and $\overrightarrow{BY}$ in different half planes such that $\angle XAB$ and $\angle YBA$ are congruent acute angles.
2) With some appropriate radius and centre A, draw an arc to intersect $\overrightarrow{AX}$ at A_1.
   Similarly, with centre A_1 and the same radius, draw an arc to intersect $\overrightarrow{AX}$ at A_2 such that $A - A_1 - A_2$.
   Similarly, draw an arc with the same radius and centre A_k to intersect $\overrightarrow{AX}$ at A_{k+1} such that $A_{k-1} - A_k - A_{k+1}$, where $k = 2,3,4,\dots,8$.
   Thus, we obtain nine points A_1, A_2, ... A_9 on $\overrightarrow{AX}$ such that $AA_1 = A_1A_2 = A_8A_9$.
3) Now, with the same radius and begining with point B as center, obtain nine points B_1, B_2, ... B_9 on $\overrightarrow{BY}$ such that $BB_1 = B_1B_2 = \dots . B_8B_9$.
4) Draw $\overline{AB_9}$, $\overline{A_2B_7}$, $\overline{A_5B_4}$ and $\overline{A_9B}$ so that $\overline{A_2B_7}$ intersects $\overline{AB}$ at P and $\overline{A_5B_4}$ intersects $\overline{AB}$ at Q.
   Thus, we obtain points P and Q which divide $\overline{AB}$ in the ratio 2 : 3 : 4 from A, i.e., AP : PQ : QB = 2 : 3 : 4.

Justification: Here, the intercepts made on transversals $\overleftrightarrow{AX}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{BY}$ by $\overline{AB_9} \parallel \overline{A_2B_7} \parallel \overline{A_5B_4} \parallel \overline{A_9B}$ are in proportion with each other.

Hence, $AP : PQ : QB = AA_2 : A_3A_5 : A_5A_9 = B_9B_7 : B_7B_4 : B_4B = 2 : 3 : 4$.



Question 3:

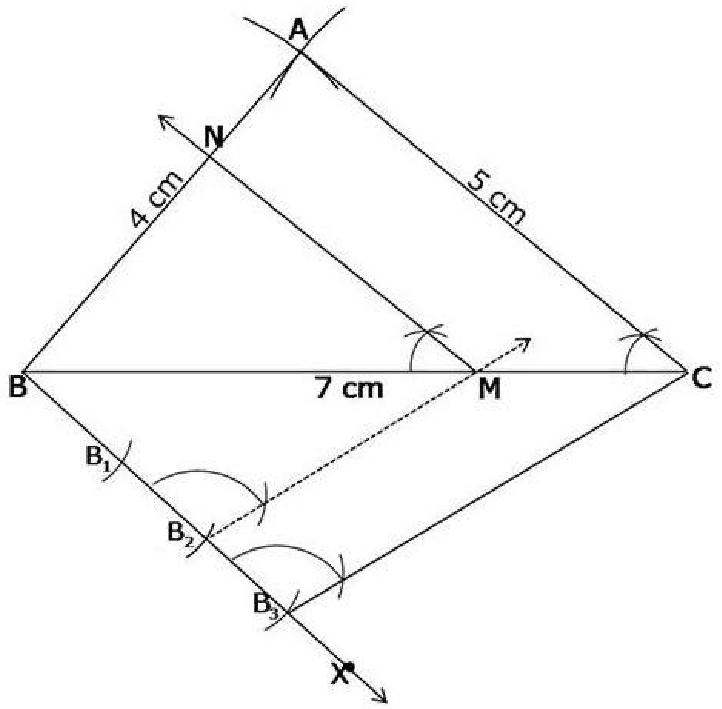
Construct a triangle with sides 4 cm, 5 cm, 7 cm and then construct a triangle similar to it whose sides have lengths in the ratio 2 : 3 to the lengths of the corresponding sides of the first triangle.

Solution :

Data: Construct ΔABC in which AB = 4 cm, BC = 7 cm and AC = 5 cm.
To construct: Construct ΔBMQ similar to ΔABC such that ratio of their corresponding sides is 2 : 3.

Steps of construction:
1) Construct ΔABC in which AB = 4 cm, BC = 7 cm and AC = 5 cm.
2) In the half plane of $\overline{BC}$ that does not contain A, draw $\overrightarrow{BX}$ such that $\angle CBX$ is an acute angle.
3) With some approprriate radius and centre B, draw an arc which intersects $\overrightarrow{BX}$ at B_1. Similarly, with center B_1 and the same radius, draw an arc that intersects $\overrightarrow{BX}$ at B_3 such that $B_1 - B_2 - B_3$.
4) Draw $\overline{B_3C}$.
5) Through B_2 draw a ray parallel to $\overline{B_3C}$ that intersect $\overline{BC}$ at M.
6) Through P draw a ray parallel to $\overline{CA}$ to intersect $\overline{AB}$ at N.
Thus, ΔBPQ is the required triangle.

Justification: $\overleftrightarrow{BX}$ and $\overleftrightarrow{BC}$ are transversals to $\overleftrightarrow{B_3C} \parallel \overleftrightarrow{B_2M}$
$\therefore BM : BC = BB_2 : BB_3 = 2 : 3$
Similarly, $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ are transversals to $\overleftrightarrow{CA} \parallel \overleftrightarrow{MN}$.
$\therefore BN : BA = BM : BN = 2 : 3$



Question 4:

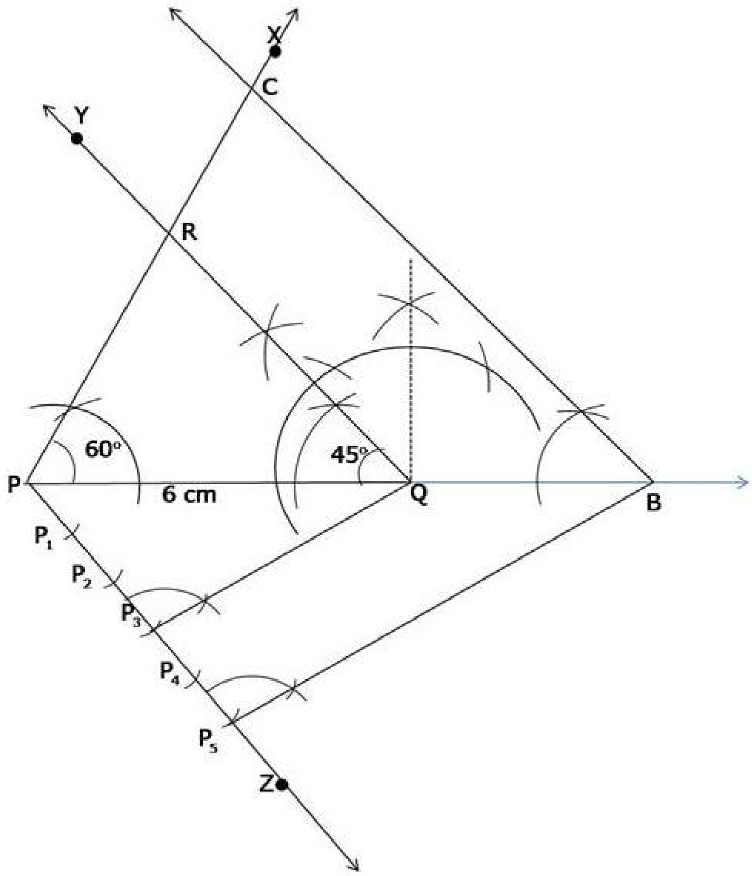
Draw ΔPQR with $m\angle P = 60$, $m\angle Q = 45$ and PQ = 6 cm. Then construct ΔPBC whose sides have lengths $\frac{5}{3}$ times the lengths of the corresponding sides of ΔPQR.

Solution :

Data : Construct ΔPQR with $m\angle P = 60^\circ$, $m\angle Q = 45^\circ$ and $PQ = 6$ cm.
To construct: Construct ΔPBC similar to ΔPQR such that the ratio of their corresponding sides is 5 : 3.

Steps of construction :
1) Construct ΔPQR with $m\angle P = 60^\circ$, $m\angle Q = 45^\circ$ and $PQ = 6$ cm.
2) In the half plane of $\overleftrightarrow{PQ}$ that does not contain R, draw $\overrightarrow{PZ}$ such that $\angle QPZ$ is an acute angle.
3) With some appropriate radius and centre P, draw an arc to intersect $\overrightarrow{PZ}$ at P_1. With the same radius and centre P_1, draw an arc to intersect $\overrightarrow{PZ}$ at P_2. Similarly, with the same radius and center P_k, draw an arc to intersect $\overrightarrow{PZ}$ at P_{k+1} such that $P_{k-1} - P_k - P_{k+1}$ where k = 2, 3, 4.
4) Draw $\overline{P_3Q}$.
5) Through P_5, draw a ray parallel to $\overline{P_3Q}$ to intersect $\overrightarrow{PQ}$ at B.
6) Through B, draw a ray parallel to $\overline{QR}$ to intersect $\overrightarrow{PR}$ at C.
Thus, ΔPBC is the required triangle.

Justification: In ΔPP_5B, $P - P_3 - P_5$, $P - Q - B$ and $\overline{P_3Q} \parallel \overline{P_5B}$.
$\therefore \dfrac{PB}{PQ} = \dfrac{PP_5}{PP_3} = \dfrac{5}{3}$
Similary, in ΔPBC, $P - Q - B$, $P - R - C$ and $\overline{QR} \parallel \overline{BC}$.
$\therefore \dfrac{PC}{PR} = \dfrac{PB}{PQ} = \dfrac{5}{3}$



Question 5:

Draw ΔABC having $m\angle ABC = 90$, BC = 4 cm and AC = 5 cm. Then construct ΔBXY, where scale factor is $\frac{4}{3}$.

Solution :

Data : Construct ΔABC with $m\angle ABC = 90°$, BC = 4 cm and AC = 5 cm.

To construct:Construct ΔBXY similar to ΔABC where the scale factor is $\frac{4}{3}$.

Steps of construction:

1) Construct ΔABC with $m\angle ABC = 90°$, BC = 4 cm and AC = 5 cm.

2) In the half plane of $\overleftrightarrow{BC}$ not containing A, draw $\overrightarrow{BQ}$ such that $\angle CBQ$ is an angle.

3) With some appropriate radius and centre B, draw an arc to intersect $\overrightarrow{BQ}$ at B_1.With the same radius and centre B_1, draw an arc to intersect $\overrightarrow{BQ}$ at B_2 such that $B - B_1 - B_2$.
Similarly, with the same radius and centre B_k draw an arc to intersect $\overrightarrow{BQ}$ at B_{k+1} such that $B_{k-1} - B_k - B_{k+1}$, where k=2, 3.

4) Draw $\overline{B_3C}$.

5) Through B_4, draw a ray parallel to $\overline{B_3C}$ to intersect $\overrightarrow{BC}$ at X.

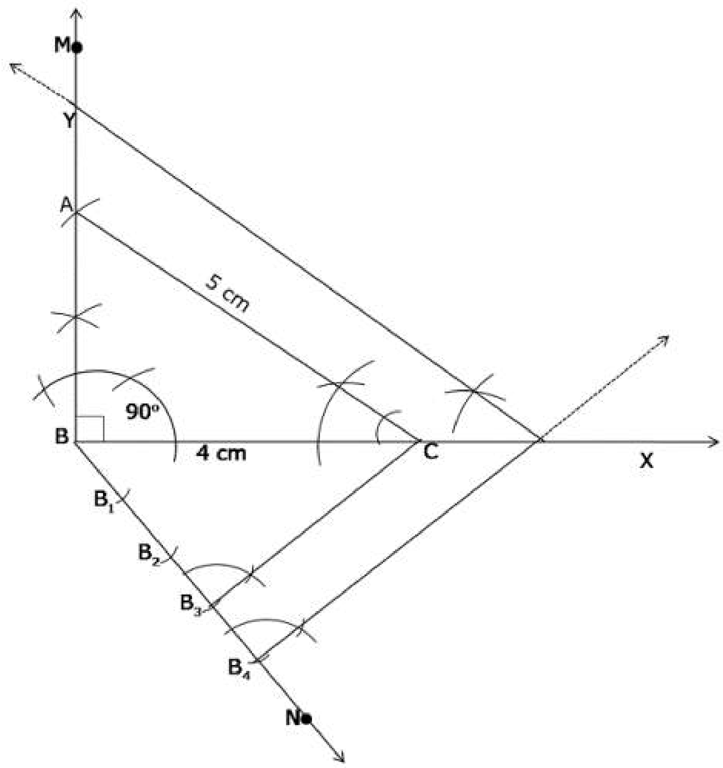
6) Through X, draw a ray parallel to $\overline{CA}$ to intersect $\overrightarrow{BA}$ at Y.
Thus, ΔBXY is the required triangle.

Justification: In ΔBB_4X, $B - B_3 - B_4$, $B - C - X$ and $\overline{B_3C} \parallel \overline{B_4X}$.

$\therefore \frac{BX}{BC} = \frac{BB_4}{BB_3} = \frac{4}{3}$

Similarly, in ΔBXY, $B - C - X$, $B - A - Y$ and $\overline{AC} \parallel \overline{XY}$.

$\therefore \frac{BY}{BA} = \frac{BX}{BC} = \frac{4}{3}$



Question 6:

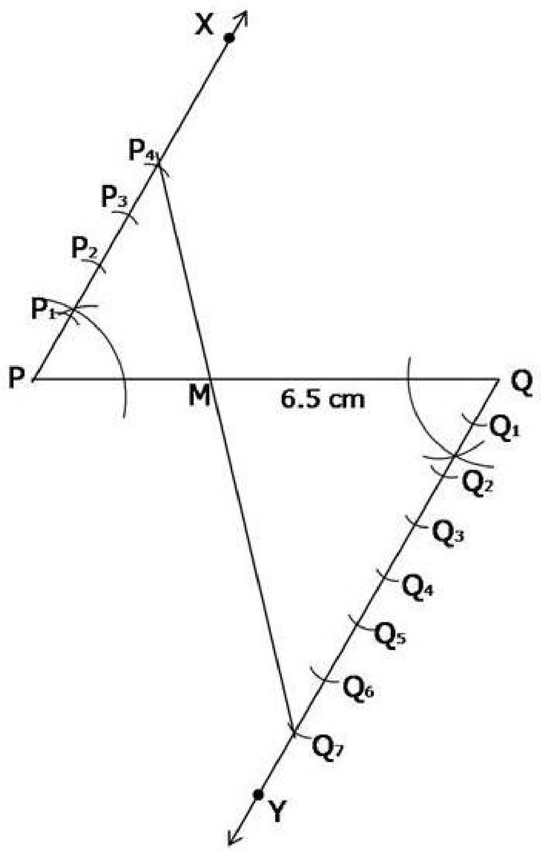
Draw $\overline{PQ}$ of length 6.5 cm and divide it in the ratio 4 : 7. Measure the two parts.

Solution :

Data: $\overline{PQ}$ of length 6.5 cm is given.
To construct: $\overline{PQ}$ is to be divided in the ratio 4 :7.

Steps of construction:
1) For the $\overleftrightarrow{PQ}$ containing $\overline{PQ}$, draw $\overrightarrow{PX}$ and $\overrightarrow{PY}$ in different half planes, such that $\angle XPQ$ and $\angle YQP$ are congruent acute angles.
2) With some appropriate radius and centre P, draw an arc to intersect $\overrightarrow{PX}$ at P_1 with the same radius and centre P_1, draw an arc that intersect $\overrightarrow{PX}$ at P_2, such that $P - P_1 - P_2$.
Similarly, with the same radius and centre P_k, draw an arc to intersect $\overrightarrow{PX}$ at P_{k+1}, such that $P_{k-1} - P_k - P_{k+1}$, where k= 2, 3. Now, with the same radius and beginning with Q obtain seven points Q_1, Q_2, ...,Q_7 on $\overrightarrow{QY}$ such that $QQ_1 = Q_1Q_2 = ... = Q_6Q_7$.
3) Draw $\overline{P_4Q_7}$ to intersect $\overline{PQ}$ at M.
Thus, point M divides $\overline{PQ}$ in the ratio 4:7.
Here, PM = 2.4cm and QM = 4.1cm.

Justification: The correspondence $PMP_4 \leftrightarrow QMQ_7$ between ΔPMP_4 and ΔQMQ_7 is a similarity.
$\therefore \frac{PM}{QM} = \frac{PP_4}{QQ_7} = \frac{4}{7}$



Exercise-12

Question 1:

Draw a circle of radius 5 cm. From a point 8 cm away from the centre, construct two tangents to the circle from this point. Measure them.

Solution :

Data: Draw $\odot(A, 5\text{ cm})$ and point B is its exterior such that AB = 8 cm.

To construct: Through B, tangents are to be drawn to $\odot(A, 5\text{ cm})$.

Steps of constrction:

1) Draw $\odot(A, 5\text{ cm})$ and take point B in its exterior such that AB = 8 cm.

2) Draw $\overline{AB}$.

3) Obtain the midpoint P of $\overline{AB}$ by constructing its perpendicular bisector.

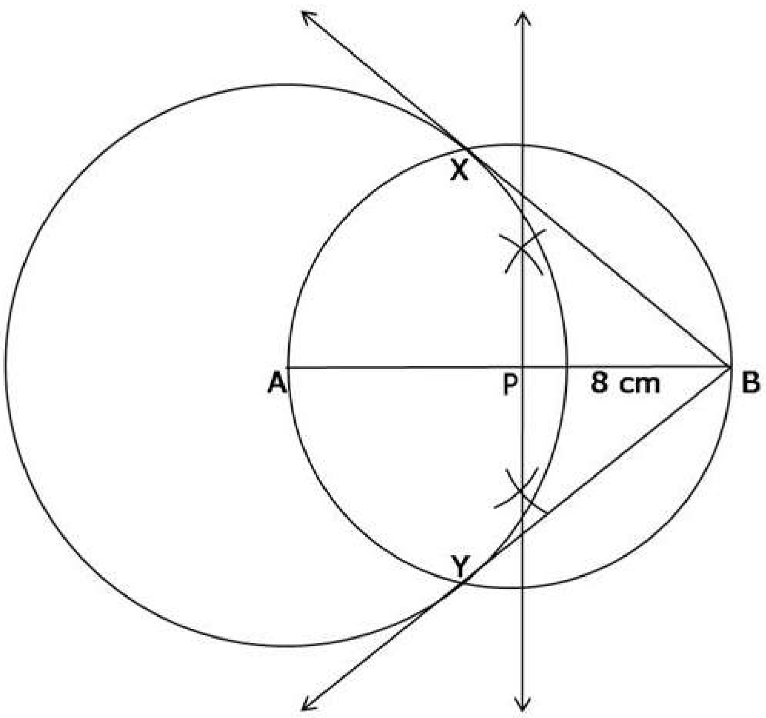
4) Draw $\odot(P, PA)$ to intersect $\odot(A, 5\text{cm})$ at X and Y.

5) Draw $\overline{BX}$ and $\overline{BY}$.

Thus, $\overleftrightarrow{BX}$ and $\overleftrightarrow{BY}$ are the required tangents.

The length of tangents $\overleftrightarrow{BX}$ and $\overleftrightarrow{BY}$ is 6.2 cm (approx.),

i.e., BX = BY = 6.2 cm (approx.)



Question 2:

Draw ⊙(O, 4). Construct a pair of tangents from A where OA = 10 units.

Solution :

Data: Draw ⊙(O, 4 cm) and take point A in the exterior of ⊙(O, 4 cm) such that OA = 10cm.

To construct: Through A, tangents are to be drawn to ⊙(O, 4 cm).

Steps of construction:

1) Draw ⊙(O, 4 cm) and take point A in its exterior such that OA = 10cm.

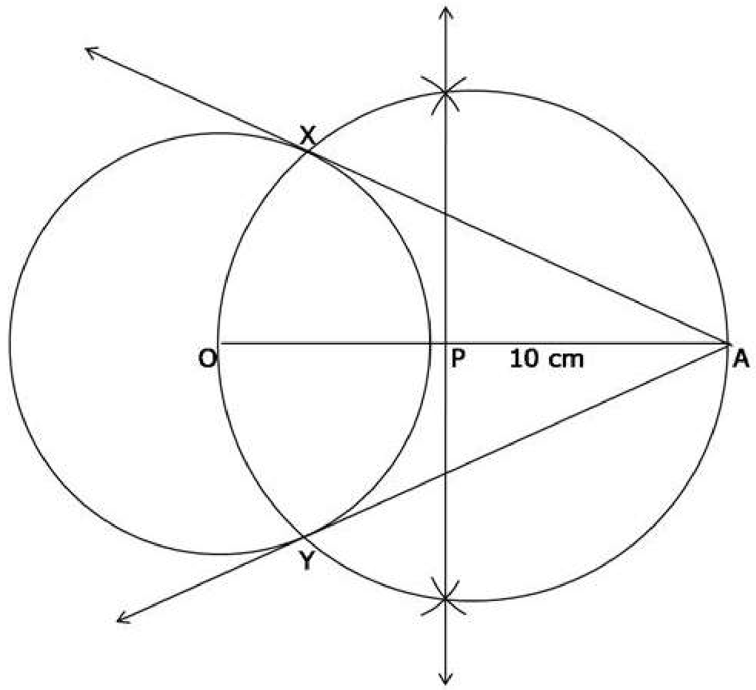
2) Draw $\overline{OA}$.

3) Obtain the midpoint P of $\overline{OA}$ by constructing its perpendicular bisector.

4) Draw ⊙(P, PA) to intersect ⊙(O, 4 cm) at X and Y.

5) Draw $\overrightarrow{AX}$ and $\overrightarrow{AY}$.

Thus, $\overleftrightarrow{AX}$ and $\overleftrightarrow{AY}$ are the required tangents.



Question 3:

Draw a circle with the help of a circular bangle. Construct two tangets to this circle through a point in the exterior of the circle.

Solution :

Data: Draw a circle with the help of a circular bangle and take point A in the exterior of the circle .

To construct: Through A, tangents are to be drawn to the circle.

Steps of construction:

1) Draw a circle with the help of a bangle and take point A in the exterior of the circle.

2) Draw two non-parallel chords $\overline{PQ}$ and $\overline{RS}$ in this circle.

3) Draw the perpendicular bisectors of $\overline{PQ}$ and $\overline{RS}$ to intersect each other at O. Then, O is the centre of the circle drawn with the help of the circular bangle.

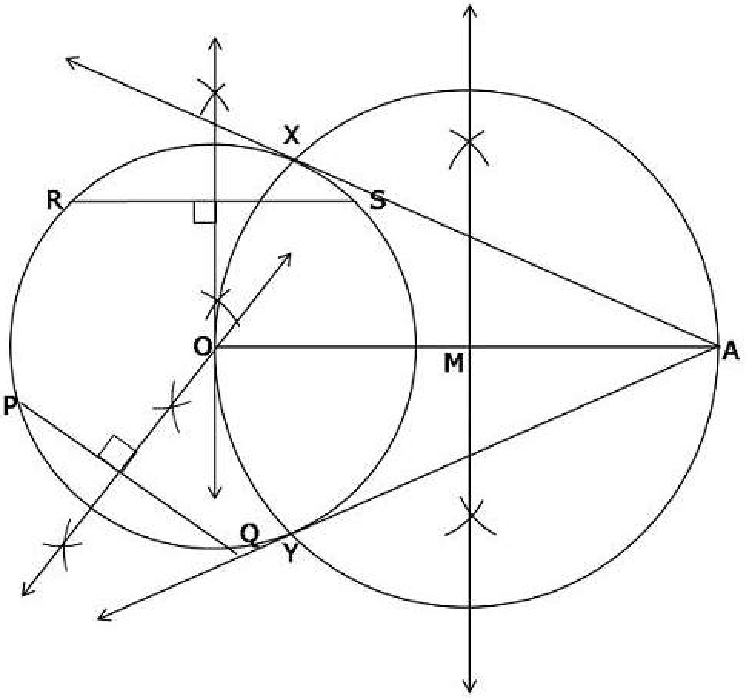
4) Draw $\overline{OA}$.

5) Obtain the midpoint M of $\overline{OA}$ by constructing the perpendicular bisector of $\overline{OA}$.

6) Draw $\odot(M, MA)$ to intersect the first circle at X and Y.

7) Draw $\overleftrightarrow{AX}$ and $\overleftrightarrow{AY}$.

Thus, $\overleftrightarrow{AX}$ and $\overleftrightarrow{AY}$ are the required tangents.



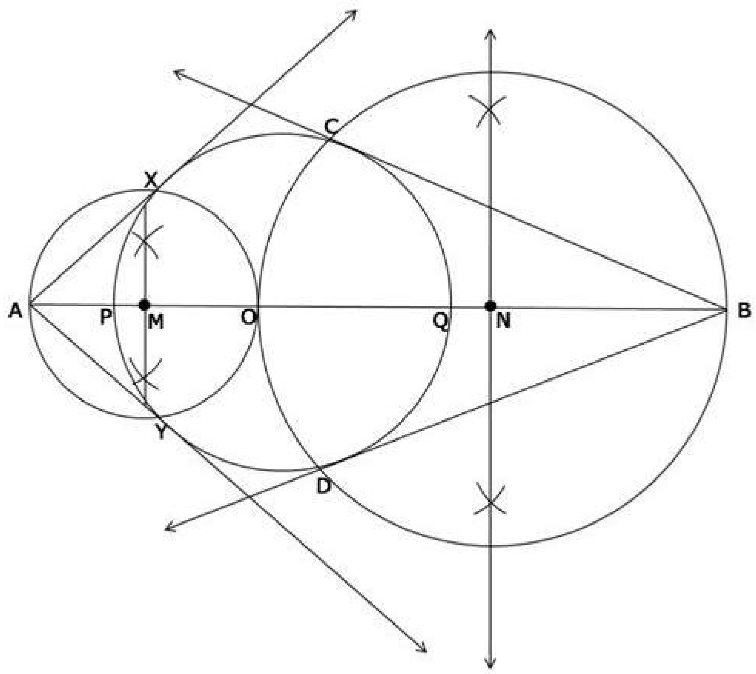
Question 4:

Draw ⊙(O, r). $\overline{PQ}$ is a diameter of ⊙(O, r). Points A and B are on the $\overleftrightarrow{PQ}$ such that A – P – Q and P – Q – B. Construct tangents through A and B to ⊙(O, r).

Solution :

Data : With some appropriate radius, draw ⊙(O, r) and diameter $\overline{PQ}$ in it. Take points A and B in the exterior of the circle such that A – P – Q and P – Q – B.
To construct: Tangents to ⊙(O, r) are to be drawn from points A and B.

Steps of construction:
1) With some appropriate radius, draw ⊙(O, r) and diameter $\overline{PQ}$ in it. Take points A and B in the exterior of the circle such that A – P – Q and P – Q – B.
2) Obtain the midpoint M of $\overline{OA}$ by constructing its perpedicular bisector.
3) Construct ⊙(M, MA) to intersect ⊙(O, r) at X and Y.
4) Obtain the midpoint N of $\overline{OB}$ by constructing its perpendicular bisector.
5) Construct ⊙(N, NB) to intersect ⊙(O, r) at C and D.
6) Draw $\overrightarrow{AX}$, $\overrightarrow{AY}$, $\overrightarrow{BC}$, and $\overrightarrow{BD}$.
Thus, $\overrightarrow{AX}$, $\overrightarrow{AY}$, $\overrightarrow{BC}$, and $\overrightarrow{BD}$ are the required tangents.



Question 5:

Draw $\overline{AB}$ such that AB = 10 cm. Draw ⊙(A, 3) and ⊙(B, 4). Construct tangents to each circle through the centre of the other circle.

Solution :

Data : Draw $\overline{AB}$ such that AB = 10cm. Draw ⊙(A, 3 cm) and ⊙(B, 4 cm).

To construct: From the center of each circle tangents are to be drawn to the other circle.

Steps of construction:

1) Draw $\overline{AB}$ such that AB = 10 cm. Draw ⊙(A, 3 cm) and ⊙(B, 4 cm).

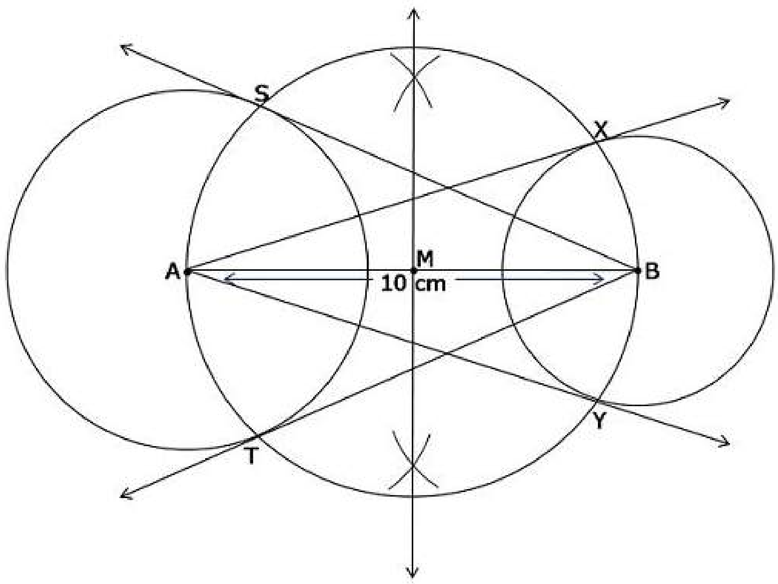
2) Obtain the midpoint M of $\overline{AB}$ by constructing its perpendicular bisector.

3) Draw ⊙(M, MA) to intersect ⊙(A, 3 cm) at C and D and ⊙(B, 4 cm) at X and Y.

4) Draw $\overrightarrow{AX}$ and $\overrightarrow{AY}$ as well as $\overrightarrow{BC}$ and $\overrightarrow{BD}$.

Thus, AX and AY are the tangent from A to ⊙(B, 4 cm).

and BC and BD are the tangents from B to ⊙(A, 3 cm).



Question 6:

⊙(P, 4) is given. Draw a pair of tangents through A which is in the exterior of ⊙(P, 4) such that measure of an angle between the tangents is 60.

Solution :

Data : Draw ⊙(P, 4 cm)

To construct: Draw a pair of tangents to ⊙(P, 4 cm) such that the measure of the angle between the tangents at their point of intersection A is 60.

Steps of construction :

1) Draw ⊙(P, 4 cm) and two radii $\overline{PN}$ and $\overline{PM}$ such that m∠NPM = 120.

2) Through N, draw a line perpendicular to $\overline{PN}$.

3) Through M, draw a line perpendicular to $\overline{PM}$

4)Let the lines drawn in step (3) and (4) intersect at A,

Thus, AM and AN are the required tangents such that the measure of the angle between them is 60.