Three Dimensional Geometry

Case Study Based Questions

Case Study 1

Two motorcycles X and Y are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.

Based on the above information, solve the following questions:

Q1. The cartesian equation of the line along which motorcycle X is running, is:

a. $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z-1}{-1}$ b. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

c. $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ d. None of these

Q2. The direction cosines of line along which motorcycle X is running, are:

a. $<1, -2, 1>$ b. $<1, 2, -1>$

c. $< \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} >$ d. $< \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} >$

Q3. The direction ratios of the line along which motorcycle Y is running, are:

a. $<1, 0, 2>$ b. $<2, 1, 0>$

c. $<1, 1, 2>$ d. $<2, 1, 1>$

Q4. The shortest distance between the given lines is:

a. 4 units b. $2\sqrt{3}$ units

c. $3\sqrt{2}$ units d. zero

Q5. The motorcycles will meet with an accident at the point:

a. $(-1, 1, 2)$ b. $(2, 1, -1)$

c. $(1, 2, -1)$ d. None of these

Solutions

1. The line along which motorcycle X is running, is $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can rewrite as:

$$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$$

$$\Rightarrow \quad x = \lambda, \quad y = 2\lambda, \quad z = -\lambda$$

$$\Rightarrow \quad \frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = \lambda$$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$.

So, option (b) is correct.

2. Clearly, direction ratios of the required line are

$$<1, 2, -1>.$$

$\therefore$ Direction cosines are:

$$< \frac{1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}}, \frac{2}{\sqrt{(1)^2 + (2)^2 + (-1)^2}}, \frac{-1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}} >$$

i.e., $$< \frac{1}{\sqrt{1+4+1}}, \frac{2}{\sqrt{1+4+1}}, \frac{-1}{\sqrt{1+4+1}} >$$

i.e., $$< \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} >$$

So, option (d) is correct.

3. The line along which motorcycle B is running, is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$, which is parallel to the vector $2\hat{i} + \hat{j} + \hat{k}$.

$\therefore$ D.R.'s of the required line are $<2, 1, 1>$.

So, option (d) is correct.

4. Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$

and $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore \quad \vec{a}_2 - \vec{a}_1 = (3\hat{i} + 3\hat{j}) - (0\hat{i} + 0\hat{j} + 0\hat{k}) = 3\hat{i} + 3\hat{j}$

and $\quad \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$

$$= (2+1)\hat{i} - (1+2)\hat{j} + (1-4)\hat{k}$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k})$

$$= (3)(3) + (3)(-3) + (0)(-3) = 9 - 9 = 0$$

Hence, shortest distance between the given lines is 0.

So, option (d) is correct.

5. Cartesian equation of Motorcycle x is

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} \quad \ldots(1)$$

and cartesian equation of Motorcycle y is

$$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} \quad \ldots(2)$$

Take a point $(x, y, z) = (1, 2, -1)$

from eq. (1), $\frac{1}{1} = \frac{2}{2} = \frac{-1}{-1} \Rightarrow 1 = 1 = 1$ (true)

from eq. (2), $\frac{1-3}{2} = \frac{2-3}{1} = \frac{-1}{1} \Rightarrow \frac{-2}{2} = \frac{-1}{1} = \frac{-1}{1}$

$\Rightarrow 1 = 1 = 1$ (true)

Since, the point (1, 2, -1) satisfy both the equations of lines, therefore point of intersection of given lines is (1, 2, -1).

So, the Motorcycles will meet with an accident at the point (1, 2, -1).

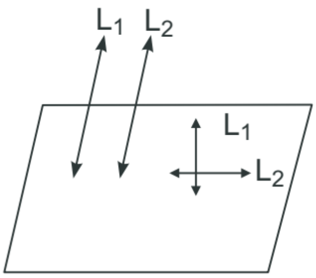
So, option (c) is correct.

Case Study 2

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively.

Then $L_1 || L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

and $L_1 \perp L_2$ iff $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.



Based on the above information, solve the following questions:

Q 1. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of L_1 and L_2 respectively, then L_1 will be perpendicular to L_2, iff:

a. $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

b. $l_1 m_2 + m_1 l_2 + n_1 n_2 = 0$

c. $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

d. None of the above

Q 2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of L_1 and L_2 respectively, then L_1 will be parallel to L_2 iff:

a. $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

b. $l_1 m_2 + m_1 l_2 + n_1 n_2 = 0$

c. $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

d. None of the above

Q 3. The coordinates of the foot of the perpendicular drawn from the point A (2, 1, 2) to the line $\frac{x-3}{1} = \frac{y-2}{1} = \frac{z-5}{1}$ are:

a. $\left(\frac{4}{3}, \frac{1}{3}, \frac{10}{3}\right)$

b. (2, 4, 5)

c. (3, 4, 5)

d. (4, 3, 5)

Q 4. The direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (–1 , 3, 2) and (4, 0, –3), are:

a. $< -9, 5, -12 >$ b. $< 1, 2, 1 >$

c. $< 2, -1, 2 >$ d. $< -1, 2, 2 >$

Q 5. The lines $\frac{-x+2}{-3} = \frac{-y+1}{2} = \frac{z-2}{0}$

and $\frac{-x+1}{-1} = \frac{2y+3}{3} = \frac{z+5}{2}$ **are:**

a. parallel b. perpendicular

c. skew-lines d. non-intersecting

Solutions

1. Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be perpendicular to L_2 iff

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

So, option (a) is correct.

2. Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be parallel to L_2, iff

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

So, option (c) is correct.

3. Equation of line is:

$$\frac{x-3}{1} = \frac{y-2}{1} = \frac{z-5}{1}$$

Let coordinates of foot of perpendicular be D (x, y, z).

$\therefore$ Direction ratios of AD are $< x-2, y-1, z-2 >$

Also AD is perpendicular to the given line

$$\therefore \quad 1(x-2) + 1(y-1) + 1(z-2) = 0$$

$$\Rightarrow \quad x-2+y-1+z-2 = 0$$

$$\Rightarrow \quad x+y+z = 5 \qquad \ldots(1)$$

Any point on the line is $(\lambda+3,\ \lambda+2,\ \lambda+5)$.

This will satisfy the eq. (1).

$\lambda+3+\lambda+2+\lambda+5=5 \Rightarrow 3\lambda=5-10$

$\Rightarrow \qquad \lambda=-\frac{5}{3}$

$\therefore$ Required coordinate of foot of perpendicular is

$$\left(-\frac{5}{3}+3,\ -\frac{5}{3}+2,\ -\frac{5}{3}+5\right)$$

i.e., $\left(\frac{4}{3}, \frac{1}{3}, \frac{10}{3}\right)$

So, option (a) is correct.

4. Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are $(-1, 3, 2)$ and $(4, 0, -3)$ respectively.

$\therefore \qquad -a+3b+2c=0 \qquad \ldots(1)$

and $\qquad 4a+0\cdot b-3c=0 \qquad \ldots(2)$

On solving eqs. (1) and (2) by cross-multiplication, we get

$$\frac{a}{-9+0}=\frac{b}{8-3}=\frac{c}{0-12} \Rightarrow \frac{a}{-9}=\frac{b}{5}=\frac{c}{-12}$$

Thus, the direction ratios of the required line are

$$<-9, 5, -12>.$$

So, option (a) is correct.

5. Given lines are

$$\frac{-x+2}{-3}=\frac{-y+1}{2}=\frac{z-2}{0}$$

or $\qquad \frac{x-2}{3}=\frac{y-1}{-2}=\frac{z-2}{0}$

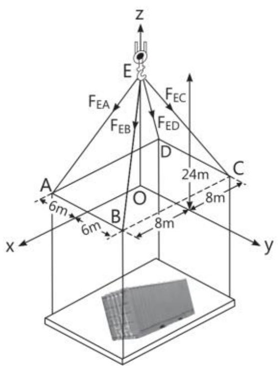
and $\qquad \frac{-x+1}{-1}=\frac{2y+3}{3}=\frac{z+5}{2}$

or $\qquad \frac{x-1}{1}=\frac{y-(-3/2)}{3/2}=\frac{z-(-5)}{2}$

$\therefore$ Direction ratios of given lines are $< 3, -2, 0 >$
and $< 1, 3/2, 2 >$.
Now, as $(3)(1) + (-2)(3/2) + (0)(2) = 3 - 3 + 0 = 0$
Thus, given lines are perpendicular to each other.
So, option (b) is correct.

Case Study 3

Consider the following diagram, where the forces in the cable are given:



Based on the above information, solve the following questions:

Q 1. The vector $\overrightarrow{ED}$ is:

a. $8\hat{i} - 6\hat{j} + 24\hat{k}$ b. $-8\hat{i} - 6\hat{j} + 24\hat{k}$

c. $-8\hat{i} - 6\hat{j} - 24\hat{k}$ d. $8\hat{i} + 6\hat{j} + 24\hat{k}$

Q 2. The length of the cable EB is:

a. 24 units b. 26 units

c. 27 units d. 25 units

Q 3. The length of cable EC is equal to the length of:

a. EA b. EB

c. ED d. All of these

Q 4. The sum of all vectors along the cables is:

a. $96\hat{i}$ b. $96\hat{j}$

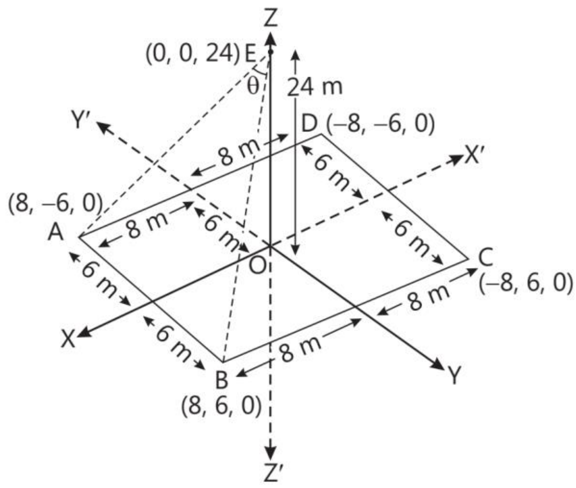
c. $-96\hat{k}$ d. $96\hat{k}$

Q 5. The angle between $\overrightarrow{EA}$ and $\overrightarrow{EB}$ is:

a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$

c. $\cos^{-1}\left(\frac{57}{71}\right)$ d. $\cos^{-1}\left(\frac{151}{169}\right)$

Solutions



1. Clearly, the coordinates of D are $(-8, -6, 0)$ and that of E are $(0, 0, 24)$.

$\therefore$ Vector $\overrightarrow{ED}$ is $(-8-0)\hat{i}+(-6-0)\hat{j}+(0-24)\hat{k}$,

i.e., $-8\hat{i}-6\hat{j}-24\hat{k}$.

So, option (c) is correct.

2. Since, the coordinates of B are $(8, 6, 0)$ and that of E are $(0, 0, 24)$, therefore length of cable

$$EB = \sqrt{(8-0)^2+(6-0)^2+(0-24)^2}$$
$$= \sqrt{64+36+576} = \sqrt{676} = 26 \text{ units.}$$

So, option (b) is correct.

3. Since, the coordinates of C are $(-8, 6, 0)$, therefore length of cable $EC = \sqrt{(-8-0)^2+(6-0)^2+(0-24)^2}$

$$= \sqrt{64+36+576} = \sqrt{676} = 26 \text{ units.}$$

Similarly, length of cable $EA = ED = EB = 26$ units.

So, option (d) is correct.

4. Sum of all vectors along the cables

$$= \overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$$
$$= (8\hat{i}-6\hat{j}-24\hat{k})+(8\hat{i}+6\hat{j}-24\hat{k})$$
$$+(-8\hat{i}+6\hat{j}-24\hat{k})+(-8\hat{i}-6\hat{j}-24\hat{k})$$
$$= -96\hat{k}$$

So, option (c) is correct.

5. Let the angle between $\overrightarrow{EA}$ and $\overrightarrow{EB}$ be θ.

$\overrightarrow{EA} = 8\hat{i} - 6\hat{j} - 24\hat{k}$ and $\overrightarrow{EB} = 8\hat{i} + 6\hat{j} - 24\hat{k}$

$$\therefore \cos\theta = \frac{|\overrightarrow{EA}\cdot\overrightarrow{EB}|}{|\overrightarrow{EA}||\overrightarrow{EB}|} = \frac{|(8\hat{i}-6\hat{j}-24\hat{k})\cdot(8\hat{i}+6\hat{j}-24\hat{k})|}{|8\hat{i}-6\hat{j}-24\hat{k}||8\hat{i}+6\hat{j}-24\hat{k}|}$$

$$= \frac{|(8)(8)+(-6)(6)+(-24)(-24)|}{\sqrt{(8)^2+(-6)^2+(-24)^2}\sqrt{(8)^2+(6)^2+(-24)^2}}$$

$$= \frac{|64-36+576|}{\sqrt{64+36+576}\sqrt{64+36+576}} = \frac{604}{\sqrt{676}\sqrt{676}}$$

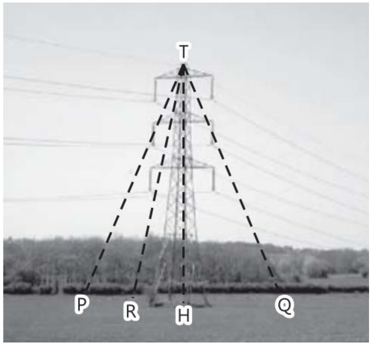
$$= \frac{604}{676} = \frac{151}{169}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{151}{169}\right)$$

So, option (d) is correct.

Case Study 4

An electricity tower stands on an agricultural horizontal field, where agricultural activities are not obstructed in any manner. Consider the surface on which the electricity tower stands as a plane having points P (2,-1,3), Q (0,4,1)R (2,1,-1) and H (0, -1, 2) on it. The electricity tower is tied with three cables from the points P, Q and R such that it stands vertically on the field. The top of the electricity tower is at the point T (4, 1, 3) as shown in the following figure:



Based on the above information, solve the following questions:

Q 1. Find the equation of the perpendicular line drawn from the top of the electricity tower to the horizontal field.

Q 2. Find the distance between the points P and Q.

Q 3. If the points P, Q and R connect each other with a wire and form a triangle, then find the area of ΔPQR.

Or

Check the points P, Q and R collinear or not.

Solutions

1. The equation of the perpendicular line TH drawn from the top T (4, 1, 3) of the electricity tower to the horizontal field at point H is

$$\frac{x-4}{4}=\frac{y-1}{2}=\frac{z-3}{1}$$

where, $<4, 2, 1>$ are direction ratios of the line TH.

2. The distance between the points, P and Q is

$$PQ=\sqrt{(0-2)^2+(4+1)^2+(1-3)^2}$$

$$=\sqrt{4+25+4}=\sqrt{33}$$

3. Since, P (2, –1, 3), Q (0, 4, 1) and R (2, 1, –1)

$\therefore$
$$\Delta_x=\frac{1}{2}\begin{vmatrix}-1 & 3 & 1\\ 4 & 1 & 1\\ 1 & -1 & 1\end{vmatrix}$$

$$=\frac{1}{2}[-1(1+1)-3(4-1)+1(-4-1)]$$

$$=\frac{1}{2}[-2-9-5]$$

$$=\frac{-16}{2}=-8$$

$$\Delta_y=\frac{1}{2}\begin{vmatrix}2 & 3 & 1\\ 0 & 1 & 1\\ 2 & -1 & 1\end{vmatrix}$$

$$=\frac{1}{2}[2(1+1)-3(0-2)+1(0-2)]$$

$$=\frac{1}{2}[4+6-2]=\frac{8}{2}=4$$

and
$$\Delta_z=\frac{1}{2}\begin{vmatrix}2 & -1 & 1\\ 0 & 4 & 1\\ 2 & 1 & 1\end{vmatrix}$$

$$= \frac{1}{2}[2(4-1)+1(0-2)+1(0-8)]$$

$$= \frac{1}{2}[6-2-8] = -\frac{4}{2} = -2$$

$\therefore$ Area of $\Delta PQR = \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$

$$= \sqrt{(-8)^2 + (4)^2 + (-2)^2}$$

$$= \sqrt{64+16+4} = \sqrt{84}$$

$$= 2\sqrt{21} \text{ sq. units.}$$

Or

Given, points are

$P(2,-1,3) \equiv (x_1, y_1, z_1)$,

$Q(0,4,1) \equiv (x_2, y_2, z_2)$

and $R(2,1,-1) \equiv (x_3, y_3, z_3)$

If the points are collinear then

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{Z_3 - Z_1}{Z_2 - Z_1}$$

$$\Rightarrow \quad \frac{2-2}{0-2} = \frac{1+1}{4+1} = \frac{-1-3}{1-3}$$

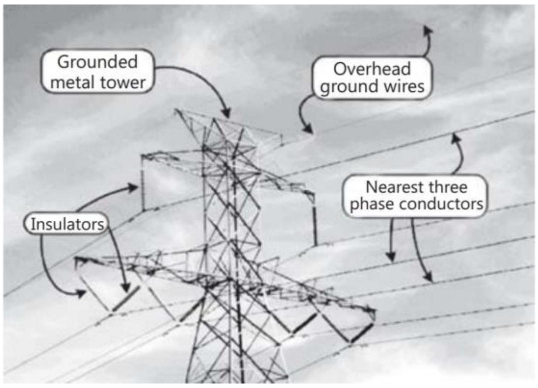
$$\Rightarrow \quad \frac{0}{-2} = \frac{2}{5} = \frac{-4}{-2}$$

$$\Rightarrow \quad 0 \neq \frac{2}{5} \neq 2$$

Thus, the points P, Q and R are not collinear.

Case Study 5

Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$$

Based on the given information, solve the following questions:

Q 1. Find the angle between the lines l_1 and l_2.

Q 2. Find the point of intersection of the lines l_1 and l_2.

Solutions

1. The direction ratios of given lines are 3, -2, -1 and −1, 3, −2 respectively.

$$\therefore \cos\theta = \left| \frac{(3)(-1)+(-2)(3)+(-1)(-2)}{\sqrt{(3)^2+(-2)^2+(-1)^2}\,\sqrt{(-1)^2+(3)^2+(-2)^2}} \right|$$

$$= \left| \frac{-3-6+2}{\sqrt{9+4+1}\sqrt{1+9+4}} \right| = \left| \frac{-7}{\sqrt{14}\sqrt{14}} \right|$$

$$= \frac{7}{14} = \frac{1}{2} = \cos 60^\circ$$

$$\Rightarrow \quad \theta = 60^\circ = \frac{\pi}{3}.$$

2. From line (1),

$$\frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} = \lambda \qquad \text{(say)}$$

$\Rightarrow x = 3\lambda - 1,\ y = -2\lambda + 3$ and $z = -\lambda - 2$.

So, the coordinates of a general point on this line are $(3\lambda - 1, -2\lambda + 3, -\lambda - 2)$.

From line (2),

$$\frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} = \mu \qquad \text{(say)}$$

So, the coordinates of a general point on this line are $(-\mu, 3\mu + 7, -2\mu - 7)$.

If the line intersects, then they have a common point.

So, for some values of λ and μ, we must have

$$3\lambda - 1 = -\mu,\ -2\lambda + 3 = 3\mu + 7,\ -\lambda - 2 = -2\mu - 7$$

$$\Rightarrow \quad 3\lambda + \mu = 1, 3\mu + 2\lambda = -4, \lambda - 2\mu = 5$$

On solving first two equations, we get $\lambda = 1$ and $\mu = -2$
Since, $\lambda = 1$ and $\mu = -2$ satisfy the third equation. So the given lines intersect. On putting $\lambda = 1$ in $(3\lambda - 1, -2\lambda + 3, -\lambda - 2)$, the coordinates of the required point of intersection are $(2, 1, -3)$.

Case Study 6

The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were no way looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are A (4, 5, 2) and B (1, 2, 3) respectively.

Based on the above information, solve the following questions:

Q1. If the soldier decides to shoot the boat at given instant of time, where the distance measured in metres, then what is the distance that bullet has to travel?

Q2. If the speed of bullet is 45 m/s, then how much time will the bullet take to hit the boat after the shot is fired?

Q3. Find the direction cosines of line passing through the positions of helicopter and boat.

Or

At the given instant of time, find the equation of line passing through the positions of helicopter and boat.

Solutions

1. Required distance = Distance between A and B

$$= \sqrt{(1-4)^2 + (2-5)^2 + (3-2)^2}$$

$$= \sqrt{9+9+1} = \sqrt{19} \text{ m}$$

2. We know that,

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\therefore \text{ Required time } = \frac{\text{Distance}}{\text{Speed}} = \frac{\sqrt{19}}{45} \text{ sec}$$

3. Direction ratios of line AB are $< 1-4, 2-5, 3-2 >$; i.e; $< -3, -3, 1 >$ From part (1), length of AB is $\sqrt{19}$ m.

$\therefore$ Direction cosines of line AB are $< \frac{-3}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{1}{\sqrt{19}} >$

Or

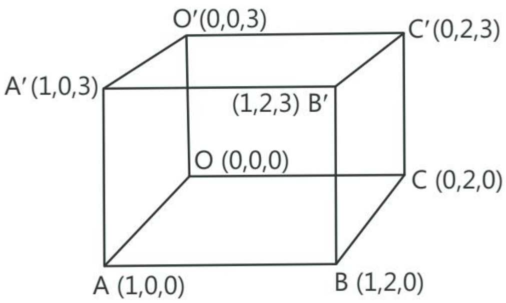
$\therefore$ Required equation of line AB is

$$\frac{x-4}{-3} = \frac{y-5}{-3} = \frac{z-2}{1}$$

$[\because \text{Dr's of line AB are } < -3, -3, 1 >]$

Case Study 7

In a diamond exhibition, a diamond is covered in cuboidal glass box having coordinates O (0, 0, 0), A (1, 0, 0), B (1, 2, 0), C (0, 2, 0), O′ (0, 0, 3), A′ (1, 0, 3), B′ (1, 2, 3) and C′ (0, 2, 3).



Based on the above information, solve the following questions:

Q 1. Find the direction ratios of OA′.

Q 2. Find the equation of diagonal OB′.

Or

Find the length of the longest rod place in cuboidal glass box.

Q 3. Find the angle between OB and OB′.

Solutions

1. Direction ratios of OA′ are

$$< 1-0, 0-0, 3-0 > \textit{ i.e., } < 1, 0, 3 >$$

2. Since Dr's of OB′ are $<1-0, 2-0, 3-0>$ i.e., $<1, 2, 3>$

$\therefore$ Equation of diagonal OB′ is

$$\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3} \textit{ i.e., } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

Or

Length of the longest rod

$= \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14}$ units

3. Direction ratios of OB are

$< a_1, b_1, c_1 > \equiv < 1-0, 2-0, 0-0 > \equiv < 1, 2, 0 >$

and direction ratios of OB′ are

$< a_2, b_2, c_2 > \equiv < 1-0, 2-0, 3-0 > \equiv < 1, 2, 3 >$.

Let θ be the angle between OB and OB′, then

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \quad \cos\theta = \frac{|(1)(1) + (2)(2) + (0)(3)|}{\sqrt{1^2 + 2^2 + 0^2}\,\sqrt{1^2 + 2^2 + 3^2}} = \frac{1+4+0}{\sqrt{5}\,\sqrt{14}}$$

$$\Rightarrow \quad \cos\theta = \frac{5}{\sqrt{5}\,\sqrt{14}} = \sqrt{\frac{5}{14}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$$

Solutions for Questions 8 to 17 are Given Below

Case Study 8

The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are (2, 3, 5) and (1, 4, 2) respectively.

Based on the above information, answer the following questions.

(i) If the line joining the positions of the helicopter and boat is perpendicular to the plane in which boat moves, then equation of plane is

(a) $x - y + 3z = 2$ (b) $x + y + 3z = 2$

(c) $x - y + 3z = 3$ (d) $x + y + 3z = 3$

(ii) If the soldier decides to shoot the boat at given instant of time, where the distance measured in metres, then what is the distance that bullet has to travel?

(a) $\sqrt{5}$ m (b) $\sqrt{8}$ m

(c) $\sqrt{10}$ m (d) $\sqrt{11}$ m

(iii) If the speed of bullet is 30 m/sec, then how much time will the bullet take to hit the boat after the shot is fired?

(a) 30 seconds (b) 1 second

(c) $\frac{1}{2}$ second (d) $\frac{\sqrt{11}}{30}$ seconds

(iv) At the given instant of time, the equation of line passing through the positions of helicopter and boat is

(a) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{3}$ (b) $\frac{x-1}{1}=\frac{y-4}{-1}=\frac{z-2}{3}$

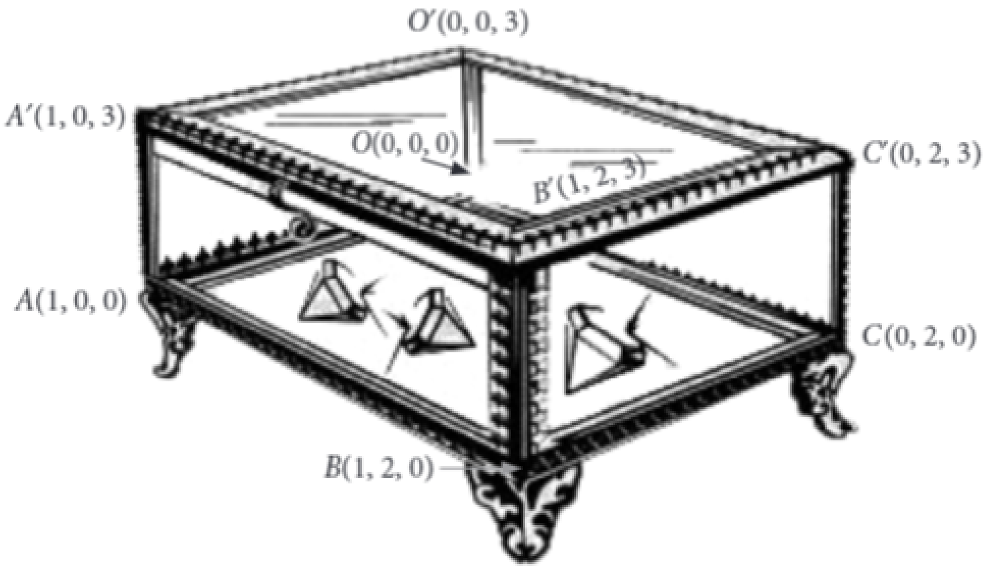
(c) $\frac{x}{1}=\frac{y}{1}=\frac{z}{-3}$ (d) $\frac{x-1}{1}=\frac{y-4}{1}=\frac{z-2}{-3}$

(v) At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat for the bullet to hit the boat if soldier shoots the bullet along the line whose equation is $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$?

(a) $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{3}{4},\frac{3}{2},\frac{5}{4}\right)$ (c) $\left(\frac{1}{3},\frac{1}{4},\frac{1}{5}\right)$ (d) none of these

Case Study 9

In a diamond exhibition, a diamond is covered in cubical glass box having coordinates $O(0, 0, 0)$, $A(1, 0, 0)$, $B(1, 2, 0)$, $C(0, 2, 0)$, $O'(0, 0, 3)$, $A'(1, 0, 3)$, $B'(1, 2, 3)$ and $C'(0, 2, 3)$.



Based on the above information, answer the following questions.

(i) Direction ratios of OA are

(a) $< 0, 1, 0 >$ (b) $< 1, 0, 0 >$ (c) $< 0, 0, 1 >$ (d) none of these

(ii) Equation of diagonal OB' is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (b) $\frac{x}{0}=\frac{y}{1}=\frac{z}{2}$ (c) $\frac{x}{1}=\frac{y}{0}=\frac{z}{2}$ (d) none of these

(iii) Equation of plane $OABC$ is

(a) $x=0$ (b) $y=0$ (c) $z=0$ (d) none of these

(iv) Equation of plane $O'A'B'C'$ is

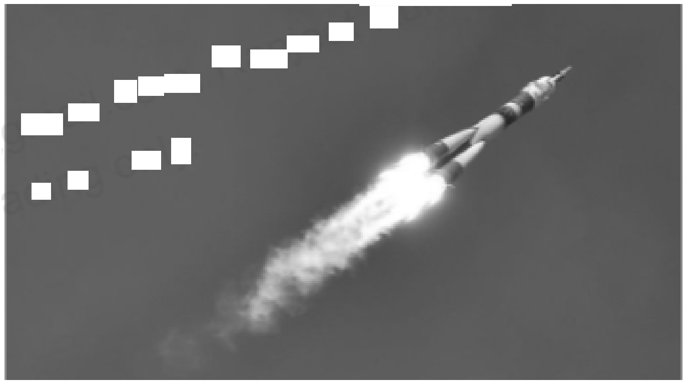
(a) $x=3$ (b) $y=3$ (c) $z=3$ (d) $z=2$

(v) Equation of plane $ABB'A'$ is

(a) $x=1$ (b) $y=1$ (c) $z=2$ (d) $x=3$

Case Study 10

The equation of motion of a rocket are : $x = 2t$, $y = -4t$, $z = 4t$, where the time 't' is given in seconds, and the distance measured is in kilometres.



Based on the above information, answer the following questions.

(i) What is the path of the rocket?

(a) Straight line (b) Circle (c) Parabola (d) none of these

(ii) Which of the following points lie on the path of the rocket?

(a) (0, 1, 2) (b) (1, –2, 2) (c) (2, –2, 2) (d) none of these

(iii) At what distance will the rocket be from the starting point (0, 0, 0) in 10 seconds?

(a) 40 km (b) 60 km (c) 30 km (d) 80 km

(iv) If the position of rocket at certain instant of time is (3, –6, 6), then what will be the height of the rocket from the ground, which is along the xy-plane?

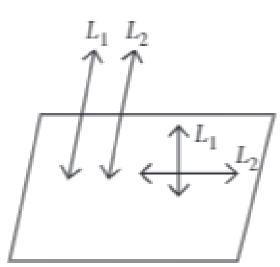
(a) 3 km (b) 2 km (c) 4 km (d) 6 km

(v) At certain instant of time, if the rocket is above sea level, where equation of surface of sea is given by $3x - y + 4z = 2$ and position of rocket at that instant of time is (1, –2, 2), then the image of position of rocket in the sea is

(a) $\left(\frac{20}{13}, \frac{15}{13}, \frac{18}{13}\right)$ (b) $\left(\frac{-20}{13}, \frac{-15}{13}, \frac{-18}{13}\right)$ (c) $\left(\frac{20}{13}, \frac{-15}{13}, \frac{18}{13}\right)$ (d) none of these

Case Study 11

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



Based on the above information, answer the following questions.

(i) If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of L_1 and L_2 respectively, then L_1 will be perpendicular to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(b) $l_1m_2 + m_1l_2 + n_1n_2 = 0$

(c) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(d) none of these

(ii) If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, then L_1 will be parallel to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(b) $l_1m_2 + m_1l_2 + n_1n_2 = 0$

(c) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(d) $m_1n_2 + m_2n_2 + l_1l_2 = 0$

(iii) The coordinates of the foot of the perpendicular drawn from the point $A(1, 2, 1)$ to the line joining $B(1, 4, 6)$ and $C(5, 4, 4)$, are

(a) $(1, 2, 1)$ (b) $(2, 4, 5)$ (c) $(3, 4, 5)$ (d) $(4, 3, 5)$

(iv) The direction ratios of the line which is perpendicular to the lines with direction ratios proportional to $(1, -2, -2)$ and $(0, 2, 1)$ are

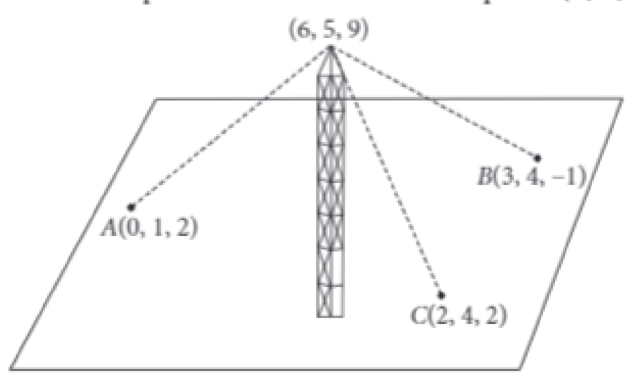
(a) $< 1, 2, 1 >$ (b) $< 2, -1, 2 >$ (c) $< -1, 2, 2 >$ (d) none of these

(v) The lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$ are

(a) parallel (b) perpendicular (c) skew lines (d) non-intersecting

Case Study 12

A mobile tower stands at the top of a hill. Consider the surface on which tower stand as a plane having points $A(0, 1, 2)$, $B(3, 4, -1)$ and $C(2, 4, 2)$ on it. The mobile tower is tied with 3 cables from the point A, B and C such that it stand vertically on the ground. The peak of the tower is at the point (6, 5, 9), as shown in the figure.



Based on the above information, answer the following questions.

(i) The equation of plane passing through the points A, B and C is

(a) $3x - 4y + z = 0$ (b) $3x - 2y + z = 0$ (c) $4x - 3y + z = 0$ (d) $4x - 3y + 3z = 0$

(ii) The height of the tower from the ground is

(a) 6 units (b) 5 units (c) $\frac{17}{\sqrt{14}}$ units (d) $\frac{5}{\sqrt{14}}$ units

(iii) The equation of line of perpendicular drawn from the peak of tower to the ground is

(a) $\frac{x-6}{3}=\frac{y-4}{-2}=\frac{z-9}{1}$ (b) $\frac{x-6}{3}=\frac{y-5}{-2}=\frac{z-9}{1}$

(c) $\frac{x-6}{3}=\frac{y-4}{2}=\frac{z-9}{1}$ (d) $\frac{x-6}{3}=\frac{y-5}{2}=\frac{z-9}{1}$

(iv) The coordinates of foot of perpendicular drawn from the peak of tower to the ground are

(a) $\left(\frac{33}{14},\frac{104}{14},\frac{109}{14}\right)$ (b) $\left(\frac{33}{14},\frac{109}{14},\frac{104}{14}\right)$ (c) $\left(\frac{33}{14},\frac{105}{14},\frac{109}{14}\right)$ (d) none of these

(v) The area of ΔABC is

(a) $\frac{1}{2}\sqrt{14}$ sq. units (b) $\frac{3}{2}\sqrt{14}$ sq. units (c) $\sqrt{14}$ sq. units (d) $2\sqrt{14}$ sq. units

Case Study 13

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r}=\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}+\mu(2\hat{i}+\hat{j}+\hat{k})$, respectively.

Based on the above information, answer the following questions.

(i) The cartesian equation of the line along which motorcycle A is running, is

(a) $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z-1}{-1}$ (b) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ (c) $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ (d) None of these

(ii) The direction cosines of line along which motorcycle A is running, are

(a) $<1, -2, 1>$ (b) $<1, 2, -1>$ (c) $<\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}>$ (d) $<\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}}>$

(iii) The direction ratios of line along which motorcycle B is running, are

(a) $<1, 0, 2>$ (b) $<2, 1, 0>$ (c) $<1, 1, 2>$ (d) $<2, 1, 1>$

(iv) The shortest distance between the gives lines is

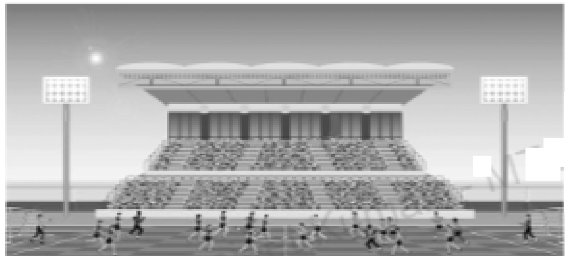
(a) 4 units (b) $2\sqrt{3}$ units (c) $3\sqrt{2}$ units (d) 0 units

(v) The motorcycles will meet with an accident at the point

(a) $(-1, 1, 2)$ (b) $(2, 1, -1)$ (c) $(1, 2, -1)$ (d) does not exist

Case Study 14

A football match is organised between students of class XII of two schools, say school A and school B. For which a team from each school is chosen. Remaining students of class XII of school A and B are respectively sitting on the plane represented by the equation $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=6$, to cheer up the team of their respective schools.



Based on the above information, answer the following questions.

(i) The cartesian equation of the plane on which students of school A are seated is

(a) $2x - y + z = 8$ (b) $2x + y + z = 8$ (c) $x + y + 2z = 5$ (d) $x + y + z = 5$

(ii) The magnitude of the normal to the plane on which students of school B are seated, is

(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

(iii) The intercept form of the equation of the plane on which students of school B are seated, is

(a) $\frac{x}{6}+\frac{y}{6}+\frac{z}{6}=1$ (b) $\frac{x}{3}+\frac{y}{(-6)}+\frac{z}{6}=1$ (c) $\frac{x}{3}+\frac{y}{6}+\frac{z}{6}=1$ (d) $\frac{x}{3}+\frac{y}{6}+\frac{z}{3}=1$

(iv) Which of the following is a student of school B?

(a) Mohit sitting at (1, 2, 1) (b) Ravi sitting at (0, 1, 2)

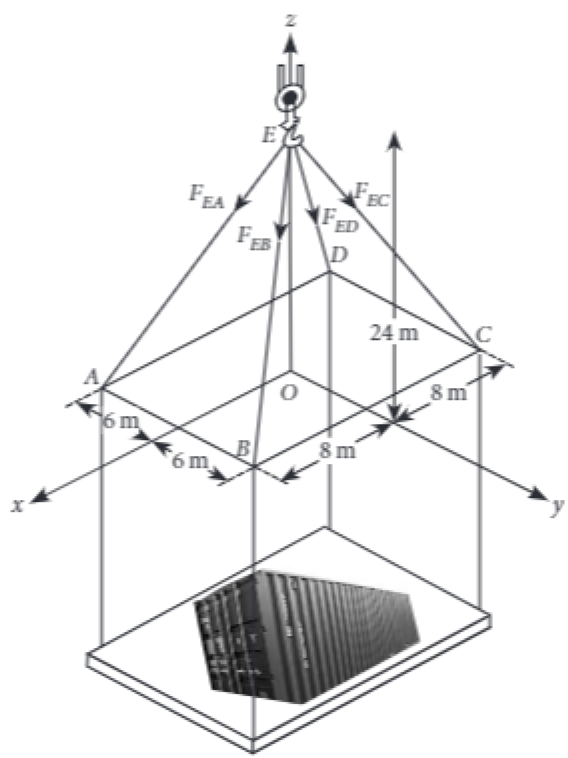
(c) Khushi sitting at (3, 1, 1) (d) Shewta sitting at (2, –1, 2)

(v) The distance of the plane, on which students of school B are seated, from the origin is

(a) 6 units (b) $\frac{1}{\sqrt{6}}$ units (c) $\frac{5}{\sqrt{6}}$ units (d) $\sqrt{6}$ units

Case Study 15

Consider the following diagram, where the forces in the cable are given.



Based on the above information, answer the following questions.

(i) The cartesian equation of line along EA is

(a) $\frac{x}{-4}=\frac{y}{3}=\frac{z}{12}$ (b) $\frac{x}{-4}=\frac{y}{3}=\frac{z-24}{12}$ (c) $\frac{x}{-3}=\frac{y}{4}=\frac{z-12}{12}$ (d) $\frac{x}{3}=\frac{y}{4}=\frac{z-24}{12}$

(ii) The vector $\overline{ED}$ is

(a) $8\hat{i}-6\hat{j}+24\hat{k}$ (b) $-8\hat{i}-6\hat{j}+24\hat{k}$ (c) $-8\hat{i}-6\hat{j}-24\hat{k}$ (d) $8\hat{i}+6\hat{j}+24\hat{k}$

(iii) The length of the cable EB is

(a) 24 units (b) 26 units (c) 27 units (d) 25 units

(iv) The length of cable EC is equal to the length of

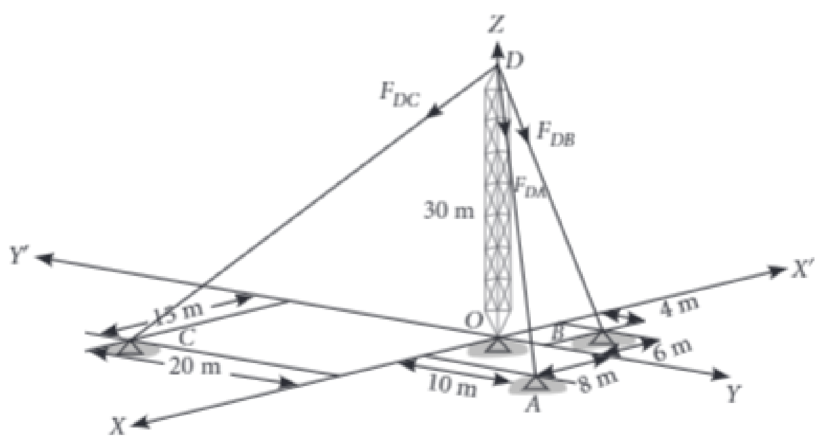
(a) EA (b) EB (c) ED (d) All of these

(v) The sum of all vectors along the cables is

(a) $96\hat{i}$ (b) $96\hat{j}$ (c) $-96\hat{k}$ (d) $96\hat{k}$

Case Study 16

Consider the following diagram, where the forces in the cable are given.



Based on the above information, answer the following questions.

(i) The equation of line along the cable AD is

(a) $\frac{x}{5}=\frac{y}{4}=\frac{z-30}{15}$ (b) $\frac{x}{4}=\frac{y}{5}=\frac{z-30}{15}$

(c) $\frac{x}{5}=\frac{y}{4}=\frac{30-z}{15}$ (d) $\frac{x}{4}=\frac{y}{5}=\frac{30-z}{15}$

(ii) The length of cable DC is

(a) $4\sqrt{61}$ m (b) $5\sqrt{61}$ m (c) $6\sqrt{61}$ m (d) $7\sqrt{61}$ m

(iii) The vector DB is

(a) $-6\hat{i}+4\hat{j}-30\hat{k}$ (b) $6\hat{i}-4\hat{j}+30\hat{k}$ (c) $6\hat{i}+4\hat{j}+30\hat{k}$ (d) none of these

(iv) The sum of vectors along the cables, is

(a) $17\hat{i}+6\hat{j}+90\hat{k}$ (b) $17\hat{i}-6\hat{j}-90\hat{k}$ (c) $17\hat{i}+6\hat{j}-90\hat{k}$ (d) none of these

(v) The sum of distances of points A, B and C from the origin, *i.e.*, $OA + OB + OC$, is

(a) $\sqrt{164}+\sqrt{52}+\sqrt{625}$ (b) $\sqrt{52}+\sqrt{625}+\sqrt{48}$ (c) $\sqrt{164}+\sqrt{625}+\sqrt{49}$ (d) none of these

Case Study 17

Suppose the floor of a hotel is made up of mirror polished Kota stone. Also, there is a large crystal chandelier attached at the ceiling of the hotel. Consider the floor of the hotel as a plane having equation $x - 2y + 2z = 3$ and crystal chandelier at the point $(3, -2, 1)$.

Based on the above information, answer the following questions.

(i) The d.r.'s of the perpendicular from the point $(3, -2, 1)$ to the plane $x - 2y + 2z = 3$, is

(a) <1, 2, 2 > (b) <1, – 2, 2 > (c) <2, 1, 2> (d) <2, –1, 2>

(ii) The length of the perpendicular from the point $(3, -2, 1)$ to the plane $x - 2y + 2z = 3$, is

(a) $\frac{2}{3}$ units (b) 3 units (c) 2 units (d) none of these

(iii) The equation of the perpendicular from the point $(3, -2, 1)$ to the plane $x - 2y + 2z = 3$, is

(a) $\frac{x-3}{1}=\frac{y-2}{-2}=\frac{z-1}{2}$ (b) $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-1}{2}$

(c) $\frac{x+3}{1}=\frac{y+2}{-2}=\frac{z-1}{2}$ (d) none of these

(iv) The equation of plane parallel to the plane $x - 2y + 2z = 3$, which is at a unit distance from the point $(3, -2, 1)$ is

(a) $x - 2y + 2z = 0$ (b) $x - 2y + 2z = 6$ (c) $x - 2y + 2z = 12$ (d) Both (b) and (c)

(v) The image of the point $(3, -2, 1)$ in the given plane is

(a) $\left(\frac{5}{3},\frac{2}{3},\frac{-5}{3}\right)$ (b) $\left(\frac{-5}{3},\frac{-2}{3},\frac{5}{3}\right)$ (c) $\left(\frac{-5}{3},\frac{2}{3},\frac{5}{3}\right)$ (d) none of these

HINTS & EXPLANATIONS

8. **(i) (c) :** Let $P(2, 3, 5)$ and $Q(1, 4, 2)$ be the positions of helicopter and boat respectively.
Now, direction ratios of PQ are proportional to $1-2, 4-3, 2-5$, *i.e.*, $-1, 1, -3$.
So, equation of plane passing through $Q(1, 4, 2)$ and perpendicular to PQ is
$-(x-1) + (y-4) + (-3)(z-2) = 0 \Rightarrow x - y + 3z = 3$

(ii) (d) : Required distance = Distance between P and Q

$= \sqrt{(1-2)^2+(4-3)^2+(2-5)^2} = \sqrt{1+1+9} = \sqrt{11}$ m

(iii) (d) : We know, Distance = Speed × Time

$\therefore$ Required time $= \frac{\sqrt{11}}{30}$ seconds

(iv) (b) : Equation of line PQ is $\frac{x-1}{1}=\frac{y-4}{-1}=\frac{z-2}{3}$.

(v) (b) : Any point on the line $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$ is given by $(\lambda+1, -2\lambda+1, 3\lambda+2)$.

Now, on substituting this point in the equation of plane $x-y+3z=3$, we get

$(\lambda+1)-(-2\lambda+1)+3(3\lambda+2)=3$

$\Rightarrow \lambda+1+2\lambda-1+9\lambda+6=3 \Rightarrow 12\lambda=-3$

$\Rightarrow \lambda=\frac{-1}{4}$

Thus, the required point is $\left(\frac{-1}{4}+1, \frac{1}{2}+1, \frac{-3}{4}+2\right)$

i.e., $\left(\frac{3}{4}, \frac{3}{2}, \frac{5}{4}\right)$.

9. **(i) (b) :** D.R.'s of OA are $<1-0, 0-0, 0-0>$, *i.e.*, $<1, 0, 0>$.

(ii) (a) : Equation of diagonal OB' is

$\frac{x-0}{1}=\frac{y-0}{2}=\frac{z-0}{3}$ *i.e.*, $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(iii) (c) : $OABC$ is xy-plane, therefore its equation is $z=0$.

(iv) (c) : Plane $O'A'B'C'$ is parallel to xy-plane passing through $(0, 0, 3)$, therefore its equation is $z=3$.

(v) (a) : Plane $ABB'A'$ is parallel to yz-plane passing through $(1, 0, 0)$, therefore its equation is $x=1$.

10. **(i) (a) :** Eliminating 't' from the given equations, we get equation of path as, $\frac{x}{2}=\frac{y}{-4}=\frac{z}{4}$ or $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$.

Thus, the path of the rocket represents a straight line.

(ii) (b) : Since, only $(1, -2, 2)$ satisfy the equation of path of rocket therefore $(1, -2, 2)$ lie on the path of rocket.

(iii) (b) : For $t=10$ sec, we have $x=20, y=-40, z=40$

Now, required distance $=\sqrt{x^2+y^2+z^2}$

$=\sqrt{20^2+(-40)^2+(40)^2}=\sqrt{400+1600+1600}$

$=\sqrt{3600}=60$ km

(iv) (d) : Clearly, height of rocket from the ground $=z$-coordinate of given position $=6$ km

(v) (b) : Let Q be the image of point $P(1, -2, 2)$ in the plane $3x-y+4z=2$. Then, equation of PQ is

$\frac{x-1}{3}=\frac{y+2}{-1}=\frac{z-2}{4}$

Let the coordinates of Q be $(3r+1, -r-2, 4r+2)$.

Let R be the mid-point of PQ. Then, coordinates of R are

$\left(\frac{3r+2}{2}, \frac{-r-4}{2}, \frac{4r+4}{2}\right)$ or $\left(\frac{3}{2}r+1, \frac{-r}{2}-2, 2r+2\right)$

Since, R lies on $3x-y+4z=2$.

$\therefore\ 3\left(\frac{3}{2}r+1\right)-\left(\frac{-r}{2}-2\right)+4(2r+2)=2$

$\Rightarrow \frac{9r}{2}+3+\frac{r}{2}+2+8r+8=2$

$\Rightarrow 13r+13=2 \Rightarrow r=\frac{-11}{13}$

Hence, the coordinates of Q are

$\left(\frac{-33}{13}+1, \frac{11}{13}-2, \frac{-44}{13}+2\right)$ *i.e.*, $\left(\frac{-20}{13}, \frac{-15}{13}, \frac{-18}{13}\right)$.

11. **(i) (a) :** Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be perpendicular to L_2 iff

$l_1l_2+m_1m_2+n_1n_2=0$

(ii) (c) : Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be parallel to L_2, iff

$\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$

(iii) (c) : Equation of line joining B and C is

$\frac{x-1}{4}=\frac{y-4}{0}=\frac{z-6}{-2}$

Let coordinates of foot of perpendicular be $D(x, y, z)$.

$\therefore$ D.R.'s of AD are $<x-1, y-2, z-1>$.

Now, $4(x-1)+0(y-2)-2(z-1)=0 \Rightarrow 4x-2z=2$

Also, (x, y, z) will satisfy equation of line BC.

Here, $(3, 4, 5)$ satisfy both the conditions.

$\therefore$ Required coordinates are $(3, 4, 5)$.

(iv) (b) : Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are $(1, -2, -2)$ and $(0, 2, 1)$ respectively.

$\therefore\ a-2b-2c=0$ ...(i)

$0\cdot a+2b+c=0$ ...(ii)

On solving (i) and (ii) by cross-multiplication, we get

$\frac{a}{-2+4}=\frac{b}{0-1}=\frac{c}{2} \Rightarrow \frac{a}{2}=\frac{b}{-1}=\frac{c}{2}$

Thus, the direction ratios of the required line are $<2, -1, 2>$.

(v) (b) : D.R.'s of given lines are $<3, -2, 0>$ and $<1, \frac{3}{2}, 2>$

Now, as $3\cdot1+(-2)\cdot\left(\frac{3}{2}\right)+0\cdot2=3-3+0=0$

$\therefore$ Given lines are perpendicular to each other.

12. **(i) (b) :** The equation of plane passing through three non-collinear points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x & y-1 & z-2 \\ 3-0 & 4-1 & -1-2 \\ 2-0 & 4-1 & 2-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y-1 & z-2 \\ 3 & 3 & -3 \\ 2 & 3 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ $x(0+9) - (y-1)(0+6) + (z-2)(9-6) = 0$

$\Rightarrow$ $9x - 6y + 6 + 3z - 6 = 0 \Rightarrow 3x - 2y + z = 0$

(ii) (c) : Height of tower = Perpendicular distance from the point (6, 5, 9) to the plane $3x - 2y + z = 0$

$$= \left| \frac{18-10+9}{\sqrt{3^2+(-2)^2+1^2}} \right| = \frac{17}{\sqrt{14}} \text{ units}$$

(iii) (b) : D.R.'s of perpendicular are $< 3, -2, 1 >$

[∵ Perpendicular is parallel to the normal to the plane]

Since, perpendicular is passing through the point (6, 5, 9), therefore its equation is

$$\frac{x-6}{3} = \frac{y-5}{-2} = \frac{z-9}{1}$$

(iv) (a) : Let the coordinates of foot of perpendicular are $(3\lambda + 6, -2\lambda + 5, \lambda + 9)$

Since, this point lie on the plane $3x - 2y + z = 0$, therefore we get

$3(3\lambda + 6) - 2(-2\lambda + 5) + (\lambda + 9) = 0$

$\Rightarrow$ $9\lambda + 4\lambda + \lambda + 18 - 10 + 9 = 0$

$\Rightarrow$ $14\lambda = -17 \Rightarrow \lambda = \frac{-17}{14}$

Thus, the coordinates of foot of perpendicular are

$$\left(\frac{-51}{14}+6, \frac{34}{14}+5, \frac{-17}{14}+9\right) \text{ i.e., } \left(\frac{33}{14}, \frac{104}{14}, \frac{109}{14}\right)$$

(v) (b) : Clearly, area of $ABC = \frac{1}{2}|\overline{AB} \times \overline{AC}|$

$$= \frac{1}{2}\left|(3\hat{i}+3\hat{j}-3\hat{k}) \times (2\hat{i}+3\hat{j})\right|$$

$$= \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -3 \\ 2 & 3 & 0 \end{matrix} \right\| = \frac{1}{2}|9\hat{i}-6\hat{j}+3\hat{k}|$$

$$= \frac{1}{2}\sqrt{9^2+6^2+3^2} = \frac{1}{2}\sqrt{126} = \frac{3}{2}\sqrt{14} \text{ sq. units}$$

13. **(i) (b)** : The line along which motorcycle A is running, is $\vec{r} = \lambda(\hat{i}+2\hat{j}-\hat{k})$, which can be rewritten as

$(x\hat{i}+y\hat{j}+z\hat{k}) = \lambda\hat{i}+2\lambda\hat{j}-\lambda\hat{k}$

$\Rightarrow$ $x = \lambda, y = 2\lambda, z = -\lambda \Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

(ii) (d) : Clearly, D.R.'s of the required line are $< 1, 2, -1 >$

∴ D.C.'s are

$$< \frac{1}{\sqrt{1^2+2^2+(-1)^2}}, \frac{2}{\sqrt{1^2+2^2+(-1)^2}}, \frac{-1}{\sqrt{1^2+2^2+(-1)^2}} >$$

i.e., $< \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} >$

(iii) (d) : The line along which motorcycle B is running, is $\vec{r} = (3\hat{i}+3\hat{j}) + \mu(2\hat{i}+\hat{j}+\hat{k})$, which is parallel to the vector $2\hat{i}+\hat{j}+\hat{k}$.

∴ D.R.'s of the required line are <2, 1, 1>.

(iv) (d): Here, $\vec{a}_1 = 0\hat{i}+0\hat{j}+0\hat{k}, \vec{a}_2 = 3\hat{i}+3\hat{j}, \vec{b}_1 = \hat{i}+2\hat{j}-\hat{k},$

$\vec{b}_2 = 2\hat{i}+\hat{j}+\hat{k}$ ∴ $\vec{a}_2 - \vec{a}_1 = 3\hat{i}+3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i}-3\hat{j}-3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i}+3\hat{j})\cdot(3\hat{i}-3\hat{j}-3\hat{k})$

$= 9 - 9 = 0$

Hence, shortest distance between the given lines is 0.

(v) (c) : Since, the point (1, 2, –1) satisfy both the equations of lines, therefore point of intersection of given lines is (1, 2, –1).

So, the motorcycles will meet with an accident at the point (1, 2, –1).

14. **(i) (c)** : Clearly, the plane for students of school A is $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 5$, which can be rewritten as

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}+2\hat{k}) = 5$

$\Rightarrow$ $x + y + 2z = 5$, which is the required cartesian equation.

(ii) (b) : Clearly, the equation of plane for students of school B is $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 6$, which is of the form $\vec{r}\cdot\vec{n} = d$

∴ Normal vector to the plane is, $\vec{n} = 2\hat{i}-\hat{j}+\hat{k}$ and its magnitude is $|\vec{n}| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$

(iii) (b) : The cartesian form is $2x - y + z = 6$, which can be rewritten as

$$\frac{2x}{6} - \frac{y}{6} + \frac{z}{6} = 1 \Rightarrow \frac{x}{3} + \frac{y}{(-6)} + \frac{z}{6} = 1$$

(iv) (c) : Since, only the point (3, 1, 1) satisfy the equation of plane representing seating position of students of school B, therefore Khushi is the student of school B.

(v) (d) : Equation of plane representing students of school B is $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 6$, which is not in normal form, as $|\vec{n}| \neq 1$.

On dividing both sides by $\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}$, we get

$$\vec{r}\cdot\left(\frac{2}{\sqrt{6}}\hat{i}-\frac{1}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}\right)=\frac{6}{\sqrt{6}},$$

which is of the form $\vec{r}\cdot\hat{n}=d$

Thus, the required distance is $\sqrt{6}$ units.

15. **(i) (b)** : Clearly, the coordinates of A are (8, –6, 0) and that of E are (0, 0, 24).

Also, cartesian equation of line along EA is given by

$$\frac{x-0}{8-0}=\frac{y-0}{-6-0}=\frac{z-24}{0-24}$$

$$\Rightarrow \frac{x}{8}=\frac{y}{-6}=\frac{z-24}{-24} \Rightarrow \frac{x}{-4}=\frac{y}{3}=\frac{z-24}{12}$$

(ii) (c) : Clearly, the coordinates of D are (–8, –6, 0) and that of E are (0, 0, 24)

$\therefore$ Vector $\overline{ED}$ is $(-8-0)\hat{i}+(-6-0)\hat{j}+(0-24)\hat{k}$,

i.e., $-8\hat{i}-6\hat{j}-24\hat{k}$.

(iii) (b) : Since, the coordinates of B are (8, 6, 0) and that of E are (0, 0, 24), therefore length of cable

$EB=\sqrt{(8-0)^2+(6-0)^2+(0-24)^2}$

$=\sqrt{64+36+576}=\sqrt{676}=26$ units

(iv) (d) : Since, the coordinates of C are (–8, 6, 0) therefore length of cable $EC=\sqrt{(-8-0)^2+(6-0)^2+(0-24)^2}$

$=\sqrt{64+36+576}=\sqrt{676}=26$ units

Similarly, length of cable $EA=ED=26$ units

(v) (c) : Sum of all vectors along the cables

$=\overline{EA}+\overline{EB}+\overline{EC}+\overline{ED}$

$=(8\hat{i}-6\hat{j}-24\hat{k})+(8\hat{i}+6\hat{j}-24\hat{k})+(-8\hat{i}+6\hat{j}-24\hat{k})$
$+(-8\hat{i}-6\hat{j}-24\hat{k})$

$=-96\hat{k}$

16. **(i) (d)** : Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$$\frac{x-0}{8-0}=\frac{y-0}{10-0}=\frac{z-30}{-30}$$

$$\Rightarrow \frac{x}{4}=\frac{y}{5}=\frac{30-z}{15}$$

(ii) (b) : The coordinates of point C are (15, –20, 0) and D are (0, 0, 30)

$\therefore$ Length of the cable DC

$=\sqrt{(0-15)^2+(0+20)^2+(30-0)^2}$

$=\sqrt{225+400+900}=\sqrt{1525}=5\sqrt{61}$ m

(iii) (a) : Since, the coordinates of point B are (–6, 4, 0) and D are (0, 0, 30), therefore vector DB is

$(-6-0)\hat{i}+(4-0)\hat{j}+(0-30)\hat{k}$, *i.e.*, $-6\hat{i}+4\hat{j}-30\hat{k}$

(iv) (b) : Required sum

$=(8\hat{i}+10\hat{j}-30\hat{k})+(-6\hat{i}+4\hat{j}-30\hat{k})+(15\hat{i}-20\hat{j}-30\hat{k})$

$=17\hat{i}-6\hat{j}-90\hat{k}$

(v) (a) : Clearly, $OA=\sqrt{8^2+10^2}=\sqrt{164}$

$OB=\sqrt{6^2+4^2}=\sqrt{36+16}=\sqrt{52}$

and $OC=\sqrt{15^2+20^2}=\sqrt{225+400}=\sqrt{625}$

17. **(i) (b)** : Equation of plane is $x-2y+2z=3$

$\therefore$ D.R.'s of normal to the plane are $\langle 1, -2, 2\rangle$, which is also the D.R.'s of perpendicular from the point (3, –2, 1) to the given plane.

(ii) (c) : Required length = Perpendicular distance from (3, –2, 1) to the plane $x-2y+2z=3$

$$=\left|\frac{3-2(-2)+2(1)-3}{\sqrt{1^2+(-2)^2+2^2}}\right|=\frac{6}{3}=2 \text{ units}$$

(iii) (b) : The equation of perpendicular from the point (x_1, y_1, z_1) to the plane $ax+by+cz=d$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Here, $x_1=3, y_1=-2, z_1=1$ and $a=1, b=-2, c=2$

$\therefore$ Required equation is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-1}{2}$.

(iv) (d) : The equation of the plane parallel to the plane $x-2y+2z-3=0$ is $x-2y+2z+\lambda=0$

Now, distance of this plane from the point (3, –2, 1) is

$$\left|\frac{3+4+2+\lambda}{\sqrt{1^2+(-2)^2+2^2}}\right|=\left|\frac{9+\lambda}{3}\right|$$

But, this distance is given to be unity

$\therefore$ $|9+\lambda|=3 \Rightarrow \lambda+9=\pm 3 \Rightarrow \lambda=-6$ or -12

Thus, required equation of planes are

$x-2y+2z-6=0$ or $x-2y+2z-12=0$

(v) (a) : Let the coordinate of image of (3, –2, 1) be $Q(r+3, -2r-2, 2r+1)$

Let R be the mid-point of PQ, then coordinate of R be

$$\left(\frac{r+6}{2}, \frac{-2r-4}{2}, r+1\right)$$

Since, R lies on the plane $x-2y+2z=3$

$$\therefore \left(\frac{r+6}{2}\right)-2\left(\frac{-2r-4}{2}\right)+2(r+1)=3$$

$$\Rightarrow 9r=-12 \Rightarrow r=-\frac{4}{3}$$

Thus, the coordinates of Q be $\left(\frac{5}{3}, \frac{2}{3}, \frac{-5}{3}\right)$.