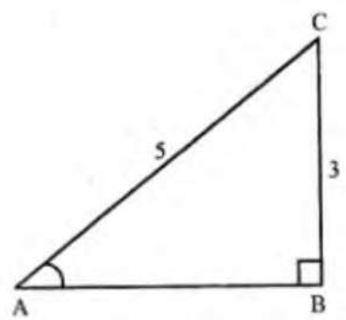
Chapter 18
Trignometric identities

Exercise 18.1

1. If A is an acute angle and sin A = $\frac{3}{5}$, find all other trignomometric ratios of angle A (using trignometric identites).

Solution:



Given,

Sin A = $\frac{3}{5}$ and A is an acute angle

So, in ΔABC we have $\angle B = 90^\circ$

And,

AC = 5 and BC = 3

By Pythagoras theorem,

AB = $\sqrt{(AC)^2 - (BC)^2}$

= $\sqrt{(5^2 - 3^2)}$

= $\sqrt{25 - 9}$

= $\sqrt{16}$

= 4

Now,

Cos A = $\frac{AB}{AC} = \frac{4}{5}$

tan A = $\frac{BC}{AB} = \frac{3}{4}$

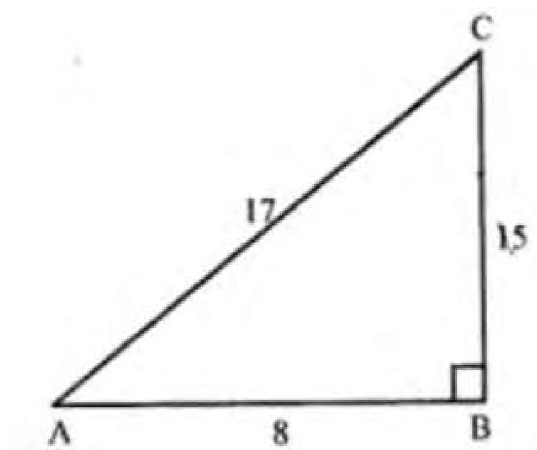
cot A = $\frac{1}{tan\theta} = \frac{4}{3}$

sec A = $\frac{1}{\cos\theta} = \frac{5}{4}$

cosec A = $\frac{1}{sin\theta} = \frac{5}{3}$

2. If A is an acute angle and secA= $\frac{17}{8}$, find all other trignometric ratios of angle A (using trignometric identities).

Solution:



Given,

Sec A = $\frac{17}{8}$ and A is an acute angle

So, in ΔABC we have $\angle B = 90^\circ$

And,

AC = 17 and AB = 8

By Pythagoras theorem,

BC = $\sqrt{AC^2 - AB^2}$

= $\sqrt{17^2 - 8^2}$

= $\sqrt{289 - 64}$

= $\sqrt{225}$

= 15

Now,

Sin A = $\frac{BC}{AC} = \frac{15}{17}$

cos A $= \frac{1}{secA} = \frac{8}{17}$

tan A = $\frac{BC}{AB} = \frac{15}{8}$

cot A = $\frac{1}{\tan A} = \frac{8}{15}$

cosec A = $\frac{1}{\sin A} = \frac{17}{15}$

3. Express the ratios cosA, tan A and sec A in terms of sin A.

Solution:

We know that,

$sin^2A + cos^2A = 1$

So,

Cos A = $\sqrt{1 - sin^2A}$

tan A = $\frac{\sin A}{cosA}$

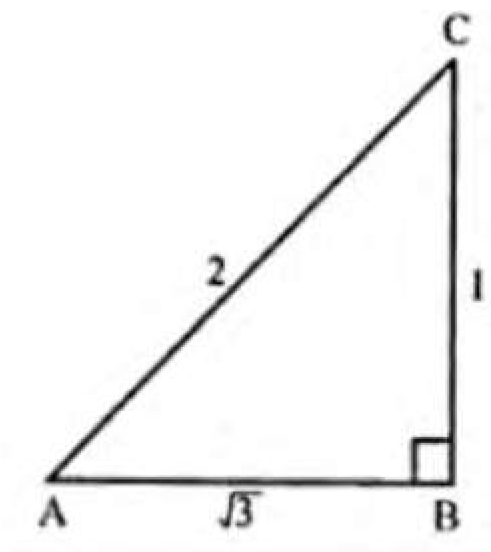
= $\frac{\sin A}{\sqrt{1-sin^2A}}$

sec A = $\frac{1}{\cos A}$

= $\frac{1}{(\sqrt{1-sin^2 A})}$

4. If tan A = $\frac{1}{\sqrt{3}}$, find all other trigonometric ratios of angle A.

Solution:



Given, tan A = $\frac{1}{\sqrt{3}}$

In right ΔABC,

tan A = $\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

So,

BC = 1 and AB = $\sqrt{3}$

By Pythagoras theorem,

$AC = \sqrt{AB^2 + BC^2}$

$= \left[\sqrt{\left(\sqrt{3}\right)^2 + (1)^2}\right]$

$= \left(\sqrt{3} + 1\right) = \sqrt{4} = 2$

Hence,

Sin A $= \frac{BC}{AC} = \frac{1}{2}$

cos A $= \frac{1}{secA} = \frac{\sqrt{3}}{2}$

tan A $= \frac{BC}{AB} = \sqrt{3}$

CotA $= \frac{1}{tanA} = \frac{2}{\sqrt{3}}$

Cosec A $= \frac{1}{sinA} = \frac{2}{1} = 2$

5. If $12\ cosec\theta = 13$, find the value of $\frac{(2\ sin\theta - 3cos\theta)}{(4\ sin\theta - 9\cos\theta)}$

Solution:

Given,

$12\ cosec\theta = 13$

$\Rightarrow\ cosec\theta = \frac{13}{12}$

in right ΔABC,

$\angle A = \theta$

So, cosec $\theta = \frac{AC}{BC} = \frac{13}{12}$

AC = 13 and BC = 12

By Pythagoras therorem,

$$AB = \sqrt{(AC^2 - BC^2)}$$

$$= \sqrt{((13)^2 - (12)^2)}$$

$$= \sqrt{169 - 144}$$

$$= \sqrt{25}$$

$$= 5$$

Now,

$$sin\theta = \frac{BC}{AC} = \frac{12}{13}$$

$$cos\theta = \frac{AB}{AC} = \frac{5}{13}$$

Hence,

$$\frac{2sin\theta - 3cos\theta}{4sin\theta - 9cos\theta}$$

$$= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}}$$

$$= \frac{9}{13} \times \frac{13}{3} = 3$$

6. Without using trignometric tables, evaluate the following :

(i) $Cos^2\ 26^\circ + Cos^2\ 64^\circ sin^2\ 26^\circ + \frac{(\boldsymbol{tan36^\circ})}{(\boldsymbol{cot\ 54^\circ})}$

(ii) $\frac{(\boldsymbol{Sec\ 17^\circ})}{(\boldsymbol{cosec\ 73^\circ})} + \frac{(\boldsymbol{tan\ 68^\circ})}{(\boldsymbol{cot\ 22^\circ})} + cos^2 44^\circ + cos^2 46^\circ$

Solution:

Given,

(i) $Cos^2\ 26^\circ + Cos^2\ 64^\circ sin\ 26^\circ + \frac{(\boldsymbol{tan36^\circ})}{(\boldsymbol{cot\ 54^\circ})}$

$= Cos^2\ 26^\circ + \boldsymbol{cos}\ (90^\circ - 16^\circ)\boldsymbol{sin}\ 26^\circ + \left[\frac{\boldsymbol{tan36^\circ}}{\boldsymbol{cos}(90^\circ - 54^\circ)}\right]$

$= [Cos^2\ 26^\circ + Sin^2\ 26^\circ] + \frac{(\boldsymbol{tan36^\circ})}{(\boldsymbol{tan\ 36^\circ})}$

= 1 + 1

= 2

(ii) $\frac{(\boldsymbol{Sec\ 17^\circ})}{(\boldsymbol{cosec\ 73^\circ})} + \frac{(\boldsymbol{tan\ 68^\circ})}{(\boldsymbol{cot\ 22^\circ})} + cos^2 44^\circ + cos^2 46^\circ$

$= \left[\frac{Sec\ 17^\circ}{cosec\ (90^\circ - 73^\circ)}\right] + \left[\frac{tan\ 90^\circ - 22^\circ}{cot\ 22^\circ}\right] + \cos^2(90^\circ - 44^\circ) + \cos^2 46^\circ$

$= \left[\frac{Sec\ 17^\circ}{sec17^\circ}\right] + \left[\frac{cot\ 22^\circ}{cot\ 22^\circ}\right] + [\sin^2 46^\circ + \cos^2 46^\circ]$

= 1 + 1 + 1

= 3

7. (i) $\left(\frac{Sin\ 65^\circ}{cos\ 25^\circ}\right) + \left(\frac{cos\ 32^\circ}{Sin\ 58^\circ}\right)$ - $sin 28^\circ sec 62^\circ + cosec^2 30^\circ$

(ii) $\left[\frac{\sin 29^\circ}{Cosec 61^\circ}\right] + 2\ Cot\ 8^\circ Cot\ 17^\circ Cot\ 45^\circ Cot\ 73^\circ Cot\ 82^\circ - 3(sin^2 38^\circ + sin^2 52^\circ)$

Solution:

(i) $\left(\frac{Sin\ 65^\circ}{cos\ 25^\circ}\right) + \left(\frac{cos\ 32^\circ}{Sin\ 58^\circ}\right)$ - $sin 28^\circ sec 62^\circ + cosec^2 30^\circ$

$= \left(\frac{Sin\ 65^\circ}{cos\ (90^\circ - 65^\circ)}\right) + \left(\frac{cos\ 32^\circ}{sin\ (90^\circ - 32^\circ)}\right)$ - $sin 28^\circ sec(90^\circ - 28^\circ) + 2^2$

$= \left(\frac{Sin\ 65^\circ}{Sin\ 65^\circ}\right) + \left(\frac{cos\ 32^\circ}{cos\ 32^\circ}\right)$ - $[sin 28^\circ\ x\ cosec 28^\circ] + 4$

= 1 + 1 – 1 + 4

= 5

(ii) $\left[\frac{\sin 29^\circ}{Cosec 61^\circ}\right] + 2\ Cot\ 8^\circ\ Cot\ 17^\circ Cot\ 45^\circ\ Cot\ 73^\circ\ Cot\ 82^\circ -$
$3\ (sin^2 38^\circ + sin^2 52^\circ)$

$= \left[\frac{\sin 29^\circ}{Cosec(90^\circ - 29^\circ)}\right] + [2\ Cot\ 8^\circ\ Cot\ 17^\circ Cot\ 45^\circ cot(90^\circ - 17^\circ)\ cot\ (90^\circ - 8^\circ)]$ -3 $\left(sin^2 38^\circ + sin^2(90^\circ - 38^\circ)\right)$

$= \left[\frac{\sin 29^\circ}{\sin 29^\circ}\right] + [2\ Cot\ 8^\circ\ Cot\ 17^\circ Cot\ 45^\circ tan 17^\circ tan 8^\circ]$- 3 $(sin^2 38^\circ + Cos^2 38^\circ)$

$= 1 + 2[(Cot\ 8^\circ\ tan\ 8^\circ)(cot 17^\circ\ tan 17^\circ)cot 45^\circ] - 3(1)$

$= 1 + 2[\ 1\times 1 \times 1] - 3$

$= 1 + 2 - 3$

$= 0$

8. (i) $\frac{(\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ)}{(Cosec^2 10^\circ - \tan^3 80^\circ)}$

(ii) $\sin^2 34^\circ + \sin^2 35^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

Solution:

(i) $\frac{(\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ)}{(Cosec^2 10^\circ - \tan^3 80^\circ)}$

$= \frac{\sin 35^\circ \cos(90^\circ - 35^\circ) + \cos 35^\circ \sin(90^\circ - 35^\circ)}{Cosec^2 10^\circ - \tan^2(90^\circ - 10^\circ)}$

$= \frac{\sin 35^\circ \sin 35^\circ + \cos 35^\circ + \cos 35^\circ}{Cosec^2 10^\circ - Cot^2 \quad 10^\circ}$

$= \frac{1}{1}$

$= 1$

(ii) $\sin^2 34^\circ + \sin^2 35^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

$= [\sin^2 34^\circ + \sin^2(90^\circ - 34^\circ)] + 2 \tan 18^\circ \tan(90^\circ - 18^\circ) - \cot^2 30^\circ$

$= \left[1 + 2 \times 1 - (\sqrt{3})^2\right]$

$= 1 + 2 - 3$

$= 0$

9. (i) $\left(\frac{\tan 25^\circ}{Cosec\, 65^\circ}\right)^2 + \left(\frac{\cot 25^\circ}{sec\, 65^\circ}\right)^2 + 2tan18^\circ tan45^\circ\ tan75^\circ$

(ii) $(cos^2 25^\circ + cos^2 65^\circ) + cosec\theta sec\,(90^\circ - \theta) - cot\theta tan(90^\circ - \theta)$

Solution:

(i) $\left(\frac{\tan 25^\circ}{Cosec\, 65^\circ}\right)^2 + \left(\frac{\cot 25^\circ}{sec\, 65^\circ}\right)^2 + 2tan18^\circ tan45^\circ\ tan75^\circ$

$= \left(\frac{\tan 25^\circ}{Cosec\,(90^\circ - 25^\circ)}\right)^2 - \left(\frac{\cot 25^\circ}{sec\,(90^\circ - 25^\circ)}\right)^2 + 2tan18^\circ\ tan(90^\circ - 18^\circ)\tan 45^\circ.$

$= \left(\frac{\tan 25^\circ}{sec\, 25^\circ}\right)^2 + \left(\frac{\cot 25^\circ}{Cosec\, 25^\circ}\right)^2 + 2tan18^\circ\ cot18^\circ \tan 45^\circ.$

$= \left(\frac{\text{Sin } 25^\circ \times \cos 25^\circ}{\cos 25^\circ \times 1}\right) + \left[\frac{Cos25^\circ \times \text{Sin } 25^\circ}{\text{Sin } 25^\circ \times 1}\right] + 2 \times 1 \times 1$

$= Sin^2\ 25^\circ + Cos^2\ 25^\circ + 2$

$= 1 + 2$

$= 3$

(ii) $(cos^2 25^\circ + cos^2 65^\circ) + cosec\theta sec\,(90^\circ - \theta) - cot\theta tan(90^\circ - \theta)$

$= cos^2 25^\circ + cos^2(90^\circ - 25^\circ) + cosec\theta sec(90^\circ - \theta) - cot\theta . cot\theta$

$= (cos^2 25^\circ + sin^2 25^\circ) + (cosec^2\theta + cot^2\theta)$

$= 1 + 1$

$= 2$

10. (i) $\mathbf{2(sec^2 35^\circ - cot^2 55^\circ)}$ **-** $\mathbf{\frac{Cos28^\circ Cosec62^\circ}{tan18^\circ\ tan36^\circ\ tan30^\circ tan54^\circ tan72^\circ}}$

(ii) $\mathbf{\frac{Cosec^2\ (90-\theta) - tan^2\theta}{2(Cos^2 48^\circ + Cos^2 42^\circ)}}$ **-** $\mathbf{\frac{2tan^2 30^\circ\ sec^2\ 52^\circ\ sin^2\ 38^\circ}{Cosec^2 70^\circ - tan^2 20^\circ}}$

Solution:

(i) $2(sec^2\ 35^\circ - cot^2\ 55^\circ) - \frac{Cos28^\circ Cosec62^\circ}{tan18^\circ\ tan36^\circ\ tan30^\circ tan54^\circ tan72^\circ}$

$= 2\left(sec^2\ 35^\circ - cot^2\ (90^\circ - 35^\circ)\right) - \left[\frac{Cos28^\circ Cosec(90^\circ - 28^\circ)}{tan18^\circ\ tan(90^\circ - 18^\circ)\ tan36^\circ tan(90^\circ - 36^\circ) tan30^\circ}\right]$

$= \quad 2[sec^2\ 35^\circ - tan^2 35^\circ\ ] - \frac{Cos28^\circ\ Sec\ 28^\circ}{tan18^\circ\ cot18^\circ tan36^\circ\ cot36^\circ\ tan30^\circ}$

$$\left\{\begin{matrix} \because\ sec^2\theta - tan^2\ \theta = 1 \\ tan\theta cot\theta = 1 \\ cos\theta sec\theta = 1 \end{matrix}\right\}$$

$= 2(1) - \frac{1}{1\times1\times\frac{1}{\sqrt{3}}}$

$= 2 - \frac{\sqrt{3}}{1}$

$= 2 - \sqrt{3}$

(ii) $\mathbf{\frac{Cosec^2\ (90-\theta) - tan^2\theta}{2(Cos^2 48^\circ + Cos^2 42^\circ)}}$ **-** $\mathbf{\frac{2tan^2 30^\circ\ sec^2\ 52^\circ\ sin^2\ 38^\circ}{Cosec^2 70^\circ - tan^2 20^\circ}}$

$= \frac{sec^2\theta - tan^2\theta}{2\left(Cos^2 48^\circ + Cos^2(90^\circ - 48^\circ)\right)} - \frac{2[tan^2 30^\circ sec^2 52^\circ sin^2(90^\circ - 52^\circ)]}{cosec^2 70^\circ - tan^2(90^\circ - 70^\circ)}$

$$\left\{\begin{matrix} \because\ sin^2\theta + Cos^2\,\theta = 1 \\ sec^2\theta - tan^2\,\theta = 1 \\ cosec^2\theta - \ Cot^2\theta = 1 \end{matrix}\right\}$$

$= \frac{1}{2[Cos^2 48^\circ + sin^2 48^\circ]} - \frac{2\left[\left(\frac{1}{\sqrt{3}}\right)^2 sec^2 52^\circ sec^2 52^\circ\right]}{cosec^2 70^\circ - Cot^2 70}$

$= \frac{1}{2\times1} - \frac{2\left[\frac{1}{3}\times1\right]}{1}$

$= \frac{1}{2} - \frac{2}{3}$

$= \frac{3-4}{6}$

$= \frac{-1}{6}$

11. Prove that following :

(i) $cos\theta sin(90^\circ - \theta) + sin\theta\ cos(90^\circ - \theta) = 1$

(ii) $\frac{tan\theta}{tan(90^\circ - \theta)} + \frac{Sin(90^\circ - \theta)}{cos\theta} = sec^2\theta$

(iii) $\frac{(cos\ (90^\circ - \theta)cos\theta)}{tan\theta + cos^2(90^\circ - \theta)} = 1$

(iv) $sin(90^\circ - \theta)cos(90^\circ - \theta) = \frac{tan\theta}{(1 + tan^2\theta)}$

Solution:

(i) $\boldsymbol{cos\theta sin(90^{\circ} - \theta) + sin\theta\ cos(90^{\circ} - \theta) = 1}$

L.H.S. = $cos\theta sin(90^{\circ} - \theta) + sin\theta\ cos(90^{\circ} - \theta)$

$= cos\theta \times cos\theta + sin\theta \times sin\theta$

$= cos^2\theta + sin^2\theta$

= 1

= R.H.S.

(ii) L.H.S. = $\frac{\boldsymbol{tan\theta}}{\boldsymbol{tan(90° - \theta)}} + \frac{\boldsymbol{Sin(90° - \theta)}}{\boldsymbol{cos\theta}}$

$= \frac{tan\theta}{Cot\theta} + \frac{cos\theta}{cos\theta}$

$= \frac{tan\theta}{\left(\frac{1}{tan\theta}\right)} + 1$

$= tan^2\theta + 1$

$= Sec^2\theta$ = R.H.S.

(iii) L.H.S. = $\frac{\boldsymbol{(cos\ (90° - \theta)cos\theta)}}{\boldsymbol{tan\theta + cos^2(90° - \theta)}}$

$= \frac{(sin\theta cos\theta)}{tan\theta + sin^2\theta}$

$= \frac{(sin\theta cos\theta)}{\left(\frac{sin\theta}{cos\theta}\right)} + sin^2\theta$

$= \cos^2\theta + \sin^2\theta$

$=1$

= R.H.S.

(iv) $sin(90^{\circ} - \theta)cos(90^{\circ} - \theta) = \frac{tan\theta}{(1+tan^2\theta)}$

L.H.S. = $sin(90^{\circ} - \theta)\, cos(90^{\circ} - \theta)$

$= \cos\theta sin\theta$

$$\left\{\begin{matrix} \because sin(90^{\circ} - \theta) - \cos\theta \\ sin^2\theta + cos^2\theta = 1 \end{matrix}\right\}$$

R.H.S. $= \frac{tan\theta}{1+tan^2\theta} = \frac{\left(\frac{sin\theta}{cos\theta}\right)}{1+\frac{sin^2\theta}{cos^2\theta}}$

$= \frac{\frac{sin\theta}{cos\theta}}{\frac{cos^2\theta+sin^2\theta}{cos^2\theta}}$

$= \frac{\frac{sin\theta}{cos\theta}}{\frac{1}{cos^2\theta}} = \frac{sin\theta}{cos\theta} \times cos^2\theta = sin\theta cos\theta$

$\therefore L.H.S. = R.H.S.$

Prove that following (12 to 30) identities, where the angles involved are acute angles for which the trignometric ratios as defined :

12. (i) $(\sec A + \tan A)\,(1 - \sin A) = cosA$

(ii) $(1 + tan^2 A)(1 - sinA)(1 + sinA) = 1.$

Solution:

Given .

(i) L.H.S. = $(\sec A + \tan A)\,(1 - \sin A)$

$= \left(\frac{1}{CosA} + \frac{sinA}{cosA}\right)(1 - sinA)$

$= \frac{(1+sinA)\times(1-sinA)}{cosA} = \frac{1-sin^2 A}{cosA}$

$= \frac{Cos^2 A}{cosA} = CosA$

= R.H.S.

$(1 - sin^2 A = cos^2 A)$

1

(ii) L.H.S. = $(1 + tan^2 A)(1 - sinA)(1 + sinA)$

= $\left(1 + \frac{sin^2 A}{Cos^2 A}\right)(1 - sin^2 A)$

= $\frac{Cos^2 A + sin^2 A}{Cos^2 A} \times Cos^2 A$

= $\frac{1}{Cos^2 A} \times Cos^2 A$

= 1

= R.H.S.

$$\begin{Bmatrix} \because 1 - sin^2 A = cos^2 A \\ sin^2 A + cos^2 A = 1 \end{Bmatrix}$$

13. (i) $\boldsymbol{tanA + cotA = secA\ cosecA}$

(ii) $\boldsymbol{(1 - cosA)(1 + secA) = tanAsinA}$

Solution:

(i) L.H.S. = $tanA + cotA$

= $\frac{sinA}{cosA} + \frac{cosA}{sinA}$

= $\frac{(Sin^2 A + cos^2 A)}{(sinAcosA)}$

= $\frac{1}{(sinAcosecA)}$

= secAcosecA

= R.H.S.

(ii) L.H.S. = $(1 - cosA)\ (1 + secA)$

$= \ (1 - cosA)\left(1 + \frac{1}{cosA}\right)$

$= (1 - cosA)\frac{(cosA+1)}{cosA}$

$= \frac{(1-cosA)(1+cosA)}{cosA} = \frac{1-cos^2A}{cosA} = \frac{sin^2A}{cosA}$

$= \frac{sin^2A}{cosA} = sinA \times \frac{sinA}{cosA}$

$\{1 - cos^2A = sin^2A\}$

$= \ tanAsinA = R.H.S.$

14. (i) $\mathbf{\frac{1}{(1+cosA)} + \frac{1}{(1-cosA)} = \ 2cosec^2A}$

(ii) $\mathbf{\frac{1}{(secA+tanA)} + \frac{1}{(secA-tanA)} = 2secA}$

Solution:

(i) $\frac{1}{(1+cosA)} + \frac{1}{(1-cosA)} = \ 2cosec^2A$

$= \ \frac{1-cosA+1+cosA}{(1+cosA)(1-cosA)}$

$= \frac{2}{1-cos^2A}$

$= \frac{2}{sin^2A}$

($\because$ 1-$cos^2A = sin^2A$)

$= \ 2cosec^2A$

= R.H.S.

(ii) $\frac{\mathbf{1}}{(\boldsymbol{secA+tanA})} + \frac{\mathbf{1}}{(\boldsymbol{secA-tanA})} = \boldsymbol{2secA}$

L.H.S. =

$$\frac{1}{(secA + tanA)} + \frac{1}{(secA - tanA)}$$

$= \frac{secA-tanA+SecA+tanA}{(secA+tanA)(SecA-tanA)}$

$= \frac{2\ secA}{sec^2A-tan^2A}$

$= \frac{2\ secA}{1}$

$(\because sec^2A - tan^2A = 1)$

= 2 secA = R.H.S.

15. (i) $\frac{sinA}{(1+cosA)} = \frac{(1-cosA)}{sinA}$

(ii) $\frac{(1-tan^2A)}{(cot^2A-1)} = tan^2A$

(iii) $\frac{sinA}{(1+cosA)} = cosecA - cotA$

Solution:

(i) L.H.S. $= \frac{sinA}{(1+cosA)}$

On multiplying and dividing by (1-cosA), we have

$= \frac{\boldsymbol{sinA}\ (1 - \cos A)}{(\mathbf{1}-\boldsymbol{cos^2A})}$

$= \frac{\boldsymbol{sinA}\ (1 - \cos A)}{\boldsymbol{sin^2A}}$

$$(\because\ \mathbf{1 - cos^2 A = sin^2 A})$$

$= \frac{(1 - \cos A)}{sinA}$ = R.H.S.

(ii) $\frac{\mathbf{1-tan^2 A}}{\mathbf{Cot^2 A-1}} = \frac{\mathbf{1-\frac{sin^2 A}{cos^2 A}}}{\mathbf{\frac{cos^2 A}{sin^2 A}-1}}$

$= \frac{\frac{\mathbf{cos^2 A-sin^2 A}}{\mathbf{cos^2 A}}}{\frac{\mathbf{cos^2 A-sin^2 A}}{\mathbf{sin^2 A}}}$

$= \frac{\mathbf{cos^2 A-sin^2 A}}{\mathbf{cos^2 A}} \times \frac{\mathbf{sin^2 A}}{\mathbf{cos^2 A-sin^2 A}}$

$= \frac{\mathbf{sin^2 A}}{\mathbf{cos^2 A}}$

$= tan^2 A$

= R.H.S.

(iii) R.H.S. = $cosecA - cotA$

$= \frac{1}{sinA} - \frac{cosA}{sinA} = \frac{1-cosA}{sinA}$

$= \frac{(1-cosA)(1+cosA)}{sinA(1+cosA)}$

{ Multiplying and dividing by 1 + cosA}

$= \frac{1-cos^2 A}{sinA(1+cosA)} = \frac{sin^2 A}{sinA(1+cosA)}$

$$\{\because\ 1 - cos^2 A = sin^2 A\}$$

$= \frac{sinA}{1+cosA}$ =L.H.S.

16. (i) $\frac{(\boldsymbol{secA-1})}{(\boldsymbol{secA+1})} = \frac{(\boldsymbol{1-cosA})}{(\boldsymbol{1+cosA})}$

(ii) $\frac{\boldsymbol{tan^2\theta}}{(\boldsymbol{sec\theta-1})^2} = \frac{(\boldsymbol{1+cos\theta})}{(\boldsymbol{1-cos\theta})}$

(iii) $(1 + tanA)^2 + (1 - tanA)^2 = 2sec^2A$

(iv) $sec^2A + cosec^2A = sec^2A.cosec^2A$

Solution:

(i) $\frac{(\boldsymbol{secA-1})}{(\boldsymbol{secA+1})} = \frac{(\boldsymbol{1-cosA})}{(\boldsymbol{1+cosA})}$

$= \frac{\frac{1-cosA}{cosA}}{\frac{1+cosA}{cosA}} = \frac{1-cosA}{cosA} \times \frac{cosA}{1+cosA}$

$= \frac{1-cosA}{1+cosA}$ = R.H.S.

(ii) $$\boldsymbol{\frac{tan^2\theta}{(sec\theta-1)^2} = \frac{(1+cos\theta)}{(1-cos\theta)}}$$

$$= \frac{\frac{sin^2\theta}{Cos^2\theta}}{\frac{1}{Cos^2\theta}+1-\frac{2}{cos\theta}}$$

$$= \frac{sin^2\theta}{Cos^2\theta} \times \frac{Cos^2\theta}{1+Cos^2\theta-2cos\theta}$$

$$= \frac{sin^2\theta}{Cos^2\theta} \times \frac{Cos^2\theta}{1+Cos^2\theta-2cos\theta}$$

$$= \frac{sin^2\theta}{(1-cos\theta)^2} = \frac{(1-Cos^2\theta)}{(1-cos\theta)^2}$$

$$= \frac{\boldsymbol{(1+cos\theta)(1-cos\theta)}}{(1-cos\theta)^2}$$

$$= \frac{1+cos\theta}{1-cos\theta}$$

R.H.S. Proved

(iii) $(1 + tanA)^2 + (1 - tanA)^2 = 2sec^2A$

$= 1 + 2tanA + tan^2A + 1 - 2tanA + tan^2A$

$= 2 + 2tan^2A$

$= 2(1 + tan^2A)$ [As $1 + tan^2A = Sec^2A$]

$= 2Sec^2A$

= R.H.S.

(iv) $sec^2A + cosec^2A = sec^2A.cosec^2A$

L.H.S. = $sec^2A + cosec^2A$

$= \frac{1}{cos^2A} + \frac{1}{sin^2A}$

$= \frac{(sin^2A + cos^2A)}{(sin^2ACos^2A)}$

$= \frac{1}{(sin^2ACos^2A)}$

$= sec^2ACosec^2A$

= R.H.S.

17. (i) $\frac{(1+sinA)}{cosA} + \frac{cosA}{(1+sinA)} = 2secA$

(ii) $\frac{tanA}{(secA-1)} + \frac{tanA}{(secA+1)} = 2cosecA$

Solution:

L.H.S. = $\frac{(1+sinA)}{cosA} + \frac{cosA}{(1+sinA)}$

$= \frac{(1+sinA)(1+sinA)+cos^2A}{cosA\,(1+sinA)}$

$= \frac{1+sinA+sinA+sin^2A+cos^2A}{cosA\,(1+sinA)}$

$= \frac{1+2sinA+1}{cosA\,(1+sinA)}$

$= \frac{2+2sinA}{cosA\,(1+sinA)}$

$= \frac{2(1+sinA)}{cosA(1+sinA)}$

$= \frac{2}{cosA}$

= 2secA

= R.H.S.

(ii) $\frac{tanA}{(secA-1)} + \frac{tanA}{(secA+1)} = 2\ cosecA$

L.H.S. = $\frac{tanA}{(secA-1)} + \frac{tanA}{(secA+1)}$

= tanA$\left(\frac{1}{secA-1} + \frac{1}{secA+1}\right)$

= tanA $\left(\frac{secA+1+secA-1}{(secA-1)(secA+1)}\right)$

= $\frac{tanA\times 2secA}{sec^2A-1}$

= $\frac{2secAtanA}{tan^2A}$

= $\frac{2secA}{tanA}$

= $\frac{2\times 1\times cosA}{cosA\times sinA}$

= $\frac{2}{sinA}$

= $2cosecA$

= R.H.S.

18. (i) $\frac{cosecA}{(cosecA-1)} + \frac{cosecA}{(cosecA+1)} = 2sec^2A$

(ii) $cotA - tanA = \frac{(2cos^2A-1)}{(sinA-cosA)}$

(iii) $\frac{(cotA-1)}{(2-sec^2A)} = \frac{cotA}{(1+tanA)}$

Solution:

(i) $\frac{cosecA}{(cosecA-1)} + \frac{cosecA}{(cosecA+1)} = 2sec^2A$

Solution:

L.H.S. = (i) $\frac{cosecA}{(cosecA-1)} + \frac{CosecA}{(cosecA+1)} = 2sec^2A$

$= cosecA\left[\frac{1}{(cosecA-1)} + \frac{1}{(cosecA+1)}\right]$

$= cosecA\left[\frac{cosecA+1+cosecA-1}{(cosecA-1)(cosecA+1)}\right]$

$= \frac{cosecA \times 2cosecA}{Cosec^2A-1} = \frac{2Cosec^2A}{Cot^2A}$

$= 2sec^2A$

= R.H.S.

(ii) $cotA - tanA = \frac{(2cos^2A-1)}{(sinA-cosA)}$

$= \frac{cosA}{sinA}$ - $\frac{sinA}{cosA}$ $= \frac{cos^2A - sin^2A}{sinAcosA}$

$= \frac{cos^2A-(1-cos^2A)}{sinAcosA}$

$= \frac{cos^2A-1+ cos^2A}{sinAcosA}$

$=\frac{2cos^2A-1}{sinAcosA} =$ R.H.S.

(iii) $\frac{(cotA-1)}{(2-sec^2A)} = \frac{cotA}{(1+tanA)}$

$$= \frac{\frac{cosA-sinA}{sinA}}{\frac{2cos^2A-1}{cos^2A}}$$

$$= \frac{cosA-sinA}{sinA} \times \frac{cos^2A}{2cos^2A-1}$$

$$= \frac{cos^2A\ (cosA-sinA)}{sinA(2cos^2A-1)}$$

$$= \frac{cos^2A(cosA-sinA)}{sinA[2cos^2A-(sin^2A+cos^2A)]}$$

$$= \frac{cos^2A(cosA-sinA)}{sinA[2cos^2A-sin^2A-cos^2A]}$$

$$= \frac{cos^2A(cosA-sinA)}{sinA\ (cos^2A-sin^2A)}$$

$$= \frac{cos^2A\ (cosA-sinA)}{sinA\ (cosA+sinA)(cosA-sinA)}$$

$$= \frac{cos^2A}{sinA\ (cosA+sinA)}$$

R.H.S. $= \frac{cotA}{1+tanA} = \frac{\frac{cosA}{sinA}}{1+\frac{sinA}{cosA}}$

$= \frac{\frac{cosA}{sinA}}{\frac{cosA+sinA}{cosA}}$

$= \frac{cosA}{sinA} \times \frac{cosA}{cosA+sinA}$

$= \frac{Cos^2A}{sinA(cosA+sinA)}$

$\therefore L.H.S. = R.H.S.$

19. (i) $tan^2\theta - sin^2\theta = tan^2\theta sin^2\theta$

(ii) $\frac{cos\theta}{(1-tan\theta)}$ - $\frac{sin^2\theta}{(cos\theta-sin\theta)} = cos\theta + sin\theta$

Solution:

(i) L.H.S. $= tan^2\theta - sin^2\theta$

$= \frac{sin^2\theta}{cos^2\theta}$ - $sin^2\theta$

$= \frac{sin^2\theta - sin^2\theta cos^2\theta}{cos^2\theta}$

$= \frac{sin^2\theta\,(1-cos^2\theta)}{cos^2\theta}$

$= \ sin^2\theta \ \times \frac{sin^2\theta}{cos^2\theta}$

$= sin^2\theta \times tan^2\theta$

$= \ tan^2\theta sin^2\theta$

= R.H.S.

(ii) $\frac{cos\theta}{(1-tan\theta)}$ - $\frac{sin^2\theta}{(cos\theta-sin\theta)} = cos\theta + sin\theta$

L.H.S. $\frac{cos\theta}{(1-tan\theta)}$ - $\frac{sin^2\theta}{(cos\theta-sin\theta)}$

$= \ \frac{cos\theta}{1-\frac{sin\theta}{cos\theta}} - \ \frac{sin^2\theta}{(cos\theta-sin\theta)}$

$= \ \frac{cos\theta}{\frac{cos\theta-sin\theta}{cos\theta}} - \ \frac{sin^2\theta}{(cos\theta-sin\theta)}$

$= \ \ \frac{cos^2\theta}{cos\theta-sin\theta}$ - $\frac{sin^2\theta}{(cos\theta-sin\theta)}$

$= \frac{cos^2\theta- sin^2\theta}{cos\theta-sin\theta}$

$= \ \frac{(cos\theta+sin\theta)(cos\theta-sin\theta)}{cos\theta-sin\theta}$

$= cos\theta + sin\theta$

= R.H.S.

20. (i) $cosec^4\theta - cosec^2\theta = \ cot^4\theta + cot^2\theta$
(ii) $2sec^2\theta - sec^4\theta - 2cosec^2\theta + cosec^4\theta = cot^4\theta - tan^4\theta$

Solution:

(ii) L.H.S. = $cosec^4\theta - cosec^2\theta$
= $cosec^2\theta(cosec^2\theta - 1)$
= $cosec^2\theta cot^2\theta[cosec^2\theta - 1 = cot^2\theta]$
= $(cot^2\theta + 1)cot^2\theta$
= $\ cot^4\theta + \ cot^2\theta$
= R.H.S.

(ii) L.H.S. =

$$2sec^2\theta - sec^4\theta - 2cosec^2\theta + cosec^4\theta = cot^4\theta - tan^4\theta$$

= 2 $(tan^2\theta + 1) - (tan^2\theta + 1)^2 - 2(1 + cot^2\theta) + (1 + cot^2\theta)^2$

$$\begin{Bmatrix} \because \ sec^2\theta \ = tan^2\theta + 1 \\ Cosec^2\theta \ = 1 + cot^2\theta \end{Bmatrix}$$

= 2 $tan^2\theta + 2 - (tan^4\theta + 2tan^2\theta + 1) - 2 - 2cot^2\theta + (1 + 2cot^2\theta + cot^4\theta)$

=2 $tan^2\theta + 2 - tan^4\theta - 2tan^2\theta - 1 - 2 - 2cot^2\theta + 1 + 2cot^2\theta + cot^4\theta$

=$cot^4\theta - \ tan^4\theta = R.H.S.$

21.

(i) $\frac{1+cos\theta-sin^2\theta}{sin\theta\ (1+cos\theta)} = \cot\theta$

(ii) $\frac{(tan^3\theta-1)}{(tan\theta-1)} = sec^2\theta + tan\theta$

Solution:

(i) L.H.S. = $\frac{1+cos\theta-sin^2\theta}{sin\theta\ (1+cos\theta)}$

$= \frac{cos\theta+ cos^2\theta}{sin\theta\ (1+cos\theta)}$

$\{\because 1 - \ sin^2\theta = \ Cos^2\theta\}$

$= \frac{cos\theta(1+cos\theta)}{Sin\theta(1+cos\theta)} = \frac{cos\theta}{Sin\theta} = \cot\theta$

= R.H.S.

(ii) $\frac{(tan^3\theta-1)}{(tan\theta-1)} = sec^2\theta + tan\theta$

L.H.S. = $\frac{(tan\theta-1)}{tan\theta-1}\ (tan^2\theta + tan\theta + 1)$

$\{\because a^3 - b^3 = (a-b)(a^2 + ab + \ b^2)\}$

$= \ tan^2\theta + tan\theta + 1$

$= tan^2\theta + 1 + \ tan\theta$

$= \ sec^2\theta + tan\theta$

= R.H.S. $\{\because sec^2\theta = tan^2\theta + 1\}$

22. (i) $\frac{(1+cosecA)}{cosecA} = \frac{cos^2A}{(1-sinA)}$

(ii) $\sqrt{\frac{1-cosA}{1+cosA}} = \frac{sinA}{1+cosA}$

Solution:

(i) $\frac{(1+cosecA)}{cosecA} = \frac{cos^2A}{(1-sinA)}$

L.H.S. $= \frac{(1+cosecA)}{cosecA}$

$= \frac{1+\frac{1}{sinA}}{\frac{1}{sinA}}$

$= \frac{sinA+1}{sinA} \times \frac{sinA}{1}$

$= sinA + 1$

R.H.S. $= \frac{\boldsymbol{cos^2A}}{(\boldsymbol{1-sinA})} = \frac{\boldsymbol{1-sin^2A}}{\boldsymbol{1-sinA}}$

$= \frac{(\boldsymbol{1+sinA})(\boldsymbol{1-sinA})}{\boldsymbol{1-sinA}} = 1 + sinA = sinA + 1$

$\therefore L.H.S. = R.H.S.$

(ii) $\sqrt{\frac{1-cosA}{1+cosA}} = cosecA - cotA$

L.H.S. $= \sqrt{\frac{1-cosA}{1+cosA}}$

Rationalising the denominators

$$= \sqrt{\frac{(1-cosA)(1-cosA)}{(1+cosA)(1+cosA)}}$$

$$= \sqrt{\frac{(1-cosA)^2}{1-Cos^2A}}$$

$$= \sqrt{\frac{(1-cosA)^2}{Sin^2A}}$$

$$= \frac{1-cosA}{sinA}$$

$$= \frac{1}{sinA} - \frac{cosA}{sinA}$$

$$= cosecA - cotA = R.H.S.$$

23. (i) $\sqrt{\frac{1+sinA}{1-sinA}} = tanA + SecA$

(ii) $\sqrt{\frac{1-cosA}{1+cosA}} = cosecA - cotA$

Solution:

(i) $\sqrt{\frac{1+sinA}{1-sinA}} = tanA + SecA$

L.H.S.= $\sqrt{\frac{1+sinA}{1-sinA}}$

Rationalising the denominators

$$= \sqrt{\frac{(1+sinA)(1-sinA)}{(1-sinA)(1+sinA)}}$$

$$= \sqrt{\frac{(1+sinA)^2}{1-Sin^2A}} = \sqrt{\frac{(1+sinA)^2}{Cos^2A}}$$

$$= \frac{1+sinA}{cosA}$$

$$= \frac{1}{cosA} + \frac{sinA}{CosA}$$

$$= secA + tanA = tanA + secA = R.H.S.$$

(ii) $\sqrt{\frac{1-cosA}{1+cosA}}$ = cosecA - cotA

L.H.S. = $\sqrt{\frac{1-cosA}{1+cosA}}$

Rationalising the denominators

$$= \sqrt{\frac{(1-cosA)(1-cosA)}{(1+cosA)(1-cosA)}}$$

$$= \sqrt{\frac{(1-cosA)^2}{1-cos^2A}}$$

$$= \sqrt{\frac{(1-cosA)^2}{Sin^2A}}$$

$= \frac{1-cosA}{sinA}$

$= \frac{1}{sinA}$ - $\frac{cosA}{sinA}$

$= cosecA - cotA = R.H.S.$

24. (i) $\sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}} = 2cosecA$

(ii) $\frac{cosAcotA}{(1-sinA)} = 1 + cosecA$

Solution:

(i) $\sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}}$ **= 2cosecA**

L.H.S. = $\sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}}$

$= \frac{\sqrt{secA-1}}{\sqrt{secA+1}} + \frac{\sqrt{secA+1}}{\sqrt{secA-1}}$

$= \frac{secA-1+secA+1}{\sqrt{(secA+1)(secA-1)}} = \frac{2secA}{\sqrt{sec^2A-1}}$

$\{\because sec^2A - 1 = tan^2A\}$

$= \frac{2secA}{\sqrt{tan^2A}} = \frac{2secA}{tanA}$

$= \frac{2\times cosA}{cosA\times sinA} = \frac{2}{sinA}$

$= 2cosecA$

= R.H.S.

(ii) $\frac{\boldsymbol{cosAcotA}}{(\mathbf{1}-\boldsymbol{sinA})} = \mathbf{1} + \boldsymbol{cosecA}$

L.H.S. = $\frac{cosAcotA}{(1-sinA)} \quad = \frac{cosAcosA}{sinA(1-sinA)}$

$$\left\{cosA = \frac{cosA}{sinA}\right\}$$

$=\frac{Cos^2A}{SinA(1-sinA)} = \frac{1-sin^2A}{SinA(1-sinA)}$

$$\{\because Cos^2A = 1 - sin^2A \ \}$$

$= \frac{(1+sinA)(1-sinA)}{sinA(1-sinA)} = \frac{1+sinA}{sinA}$

$= \frac{1}{sinA} + \frac{sinA}{sinA}$

$= cosecA + 1$

$= 1 + cosecA$ R.H.S.

25. (i) $\frac{(\mathbf{1}+\boldsymbol{tanA})}{\boldsymbol{sinA}} + \frac{(\mathbf{1}+\boldsymbol{cotA})}{\boldsymbol{cosA}} = \mathbf{2}(\boldsymbol{secA} + \boldsymbol{cosecA})$

(ii) $sec^4A - tan^4A = 1 + 2tan^2A$

Solution:

(i) $\frac{(\mathbf{1}+\boldsymbol{tanA})}{\boldsymbol{sinA}} + \frac{(\mathbf{1}+\boldsymbol{cotA})}{\boldsymbol{cosA}} = \mathbf{2}(\boldsymbol{secA} + \boldsymbol{cosecA})$

L.H.S. = $\frac{(\mathbf{1}+\boldsymbol{tanA})}{\boldsymbol{sinA}} + \frac{(\mathbf{1}+\boldsymbol{cotA})}{\boldsymbol{cosA}}$

$= \frac{1+\left(\frac{sinA}{cosA}\right)}{sinA} + \frac{1+\left(\frac{cosA}{sinA}\right)}{CosA}$

$= \frac{cosA+sinA}{sinA\times cosA} + \frac{sinA+cosA}{cosA \times sinA}$

$= 2\left[\frac{cosA+sinA}{cosAsinA}\right]$

$= 2 \left[\frac{cosA}{cosAsinA} + \frac{sinA}{sinAcosA}\right]$

$= 2\left[\frac{1}{sinA} + \frac{1}{cosA}\right]$

$= 2[cosecA + secA]$

$= 2\ (secA + cosecA)$

= R.H.S.

(ii) $sec^4A - tan^4A = 1 + 2tan^2A$

L.H.S. = $sec^4A - tan^4A$

$= (sec^2A - tan^2A)(sec^2A + tan^2A)$

$= (1 + tan^4A - tan^4A)(1 + tan^4A + tan^4A)$

$[As\ sec^2A = tan^4A + 1\ ]$

$= 1(1 + 2tan^2A)$

$= 1 + 2tan^2A = R.H.S.$

26. (i) $\text{cosec}^6 A - \cot^6 A = 3cot^2 A\text{cosec}^2 A + 1$

(ii) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$

Solution:

(i) $\text{cosec}^6 A - \cot^6 A = 3cot^2 A\text{cosec}^2 A + 1$

L.H.S. = $\text{cosec}^6 A - \cot^6 A$

= $(\text{cosec}^2 A)^3 - (cot^2 A)^3$

= $(\text{cosec}^2\theta - cot^2 A)^3$ +3 $\text{cosec}^2 Acot^2 A(\text{cosec}^2\theta - cot^2 A)$

= $(1)^3 + 3\text{cosec}^2 Acot^2 A \times 1$

= $\mathbf{1 + 3}cot^2 A\text{cosec}^2 A + 1$

= R.H.S.

(ii) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$

Solution:

= $(sec^2 A)^3 - (tan^2 A)^3$

= $(\sec^2\theta - tan^2 A)^3$ +3$\sec^2 Atan^2 A(\sec^2\theta - tan^2 A)$

= $(1)^3 + 3\sec^2 Atan^2 A \times 1$

= 1 + $3\sec^2 Atan^2 A$

= $1 + 3[(1 + tan^2 A)\ tan^2 A]$

= 1 + $3[tan^2 A + tan^4 A]$

= 1 + $3tan^2 A + 3\ tan^4 A$

= R.H.S.

27. (i) $\boldsymbol{\frac{cot\theta - cosec\theta - 1}{cot\theta - cosec\theta + 1} = \frac{1 + cos\theta}{sin\theta}}$

(ii) $\boldsymbol{\frac{sin\theta}{cot\theta + cosec\theta} = 2 + \frac{sin\theta}{cot\theta + cosec\theta}}$

Solution:

(i) $\frac{cot\theta - cosec\theta - 1}{cot\theta - cosec\theta + 1} = \frac{1 + cos\theta}{sin\theta}$

L.H.S. = $\frac{cot\theta - cosec\theta - 1}{cot\theta - cosec\theta + 1}$

$$= \frac{\frac{cos\theta}{sin\theta} + \frac{1}{sin\theta} - 1}{\frac{cos\theta}{sin\theta} - \frac{1}{sin\theta} + 1}$$

$$= \frac{cos\theta + 1 - sin\theta}{sin\theta} \times \frac{sin\theta}{cos\theta - 1 + sin\theta}$$

$$= \frac{cos\theta + 1 - sin\theta}{cos\theta - 1 + sin\theta}$$

$$= \frac{cos\theta + (1 - sin\theta)}{cos\theta - (1 - sin\theta)}$$

$$= \frac{[cos\theta + (1 - sin\theta)][cos\theta + (1 - sin\theta)]}{[cos\theta - (1 - sin\theta)][cos\theta + (1 - sin\theta)]}$$

$$= \frac{(cos\theta + 1 - sin\theta)^2}{cos^2\theta - (1 + sin^2\theta - 2sin\theta)}$$

$$= \frac{cos^2\theta\ + sin^2\theta + 1 + 2cos\theta - 2sin\theta - 2sin\theta cos\theta}{cos^2\theta - 1 - sin^2\theta + 2sin\theta}$$

$$= \frac{1 + 1 + 2cos\theta - 2sin\theta - 2sin\theta cos\theta}{1 - sin^2\theta - 1 - sin^2\theta + 2sin\theta}$$

$$= \frac{2 + 2cos\theta - 2sin\theta - 2sin\theta cos\theta}{1 - sin^2\theta - 1 - sin^2\theta + 2sin\theta}$$

$$= \frac{2 + 2cos\theta - 2sin\theta - 2sin\theta cos\theta}{2sin\theta - 2sin^2\theta}$$

$$= \frac{2\ (\ 1 + cos\theta) - 2sin\theta(1 + cos\theta)}{2sin\theta(1 - sin\theta)}$$

$$= \frac{1 + cos\theta}{sin\theta}$$

= R.H.S.

(ii) $\frac{\boldsymbol{sin\theta}}{\boldsymbol{cot\theta + cosec\theta}} \mathbf{= 2 +} \frac{\boldsymbol{sin\theta}}{\boldsymbol{cot\theta + cosec\theta}}$

L.H.S. = $\frac{sin\theta}{\frac{cos\theta}{sin\theta}} + sin\theta = \frac{\boldsymbol{sin\theta}}{\frac{\boldsymbol{cos\theta + 1}}{\boldsymbol{sin\theta}}}$

$$= \frac{sin^2\theta}{1 + cos\theta}$$

$= \frac{1-cos^2\theta}{1+cos\theta} = \frac{(1+cos\theta)(1-cos\theta)}{1+cos\theta}$

$= 1 - cos\theta$

R.H.S. = $2 + \frac{sin\theta}{cot\theta + cosec\theta}$

$= 2 + \frac{sin\theta}{\frac{cos\theta}{sin\theta} - \frac{1}{sin\theta}}$

$= 2 + \frac{sin^2\theta}{cos\theta - 1}$

$= \frac{2cos\theta - 2 + \left(1 - cos^2\theta\right)}{cos\theta - 1}$

$= \frac{2(cos\theta - 1) + (1 + \cos\theta)(1 - cos\theta)}{cos\theta - 1}$

$= \frac{(\cos\theta - 1)(2 - 1 - \cos\theta)}{(cos\theta - 1)}$

$= 1 - cos\theta$

L.H.S. = R.H.S.

28. (i) $(\boldsymbol{sin\theta + cos\theta})(\boldsymbol{sec\theta + cosec\theta}) = \mathbf{2} + \boldsymbol{sec\theta cosec\theta}$

(ii) $(\boldsymbol{cosecA - sinA})(\boldsymbol{secA - cosA})\boldsymbol{sec^2A} = \boldsymbol{tanA}$

Solution:

(i) $(sin\theta + cos\theta)(sec\theta + cosec\theta) = 2 + sec\theta cosec\theta$

L.H.S. = $(sin\theta + cos\theta)(sec\theta + cosec\theta)$

= $(sin\theta + cos\theta)\left(\frac{1}{cos\theta} + \frac{1}{sin\theta}\right)$

= $\frac{(sin\theta+cos\theta)(sin\theta+cos\theta)}{(sin\theta cos\theta)}$

= $\frac{sin^2\theta+sin\theta cos\theta+cos\theta sin\theta+Cos^2\theta}{sin\theta cos\theta}$

= $\frac{1+2sin\theta cos\theta}{sin\theta cos\theta}$

= $\frac{1}{sin\theta cos\theta} + \frac{2sin\theta cos\theta}{sin\theta cos\theta}$

= $cosec\theta\ sec\theta + 2$

= 2 + $sec\theta cosec\theta$.

= R.H.S.

(ii) $(\boldsymbol{cosecA - sinA})(\boldsymbol{secA - cosA})\boldsymbol{sec^2A} = \boldsymbol{tanA}$

L.H.S. = $(\boldsymbol{cosecA - sinA})(\boldsymbol{secA - cosA})\boldsymbol{sec^2A}$

= $\left(\frac{1}{sinA} - sinA\right)\left(\frac{1}{CosA} - CosA\right)\frac{1}{Cos^2A}$

$= \left(\frac{1-Sin^2A}{sinA}\right)\left(\frac{1-Cos^2A}{CosA}\right)\frac{1}{Cos^2A}$

$= \frac{Cos^2A}{sinA} . \frac{Sin^2A}{cosA} . \frac{1}{Cos^2A} = \frac{SinA}{cosA}$

= tanA

= R.H.S.

29. (i) $\frac{sin^3A + Cos^3A}{sinA+cosA} + \frac{sin^3A - Cos^3A}{sinA-cosA} = 2$

(ii) $\frac{tan^2A}{1+tan^2A} + \frac{Cot^2A}{1+ Cot^2A} = 1$

Solution:

(i) $\frac{sin^3A + Cos^3A}{sinA+cosA} + \frac{sin^3A - Cos^3A}{sinA-cosA} = 2$

L.H.S. $= \frac{sin^3A + Cos^3A}{sinA+cosA} + \frac{sin^3A - Cos^3A}{sinA-cosA}$

$= \frac{(sinA+cosA)(sin^2A - sinAcosA + cos^2A)}{(sinA+cosA)} + \frac{(sinA-cosA)(sin^2A + sinAcosA + cos^2A)}{(sinA-cosA)}$

$= (1 - sinAcosA) + (1 + sinAcosA)$

$[\because sin^2A + cos^2A = 1]$

$= 1 - sinAcosA + 1 + sinAcosA = 2$ = R.H.S.

(ii) $\frac{tan^2A}{1+tan^2A} + \frac{Cot^2A}{1+ Cot^2A} = 1$

Solution:

L.H.S. $= \frac{tan^2A}{1+tan^2A} + \frac{\frac{1}{tan^2A}}{1+\frac{1}{tan^2A}}$

$= \frac{tan^2A}{1+tan^2A} + \frac{\frac{1}{tan^2A}}{\frac{tan^2A+1}{tan^2A}}$

$= \frac{tan^2A}{1+tan^2A} + \frac{tan^2A}{tan^2A\,(tan^2A+1)}$

$= \frac{tan^2A}{1+tan^2A} + \frac{1}{1+tan^2A}$

$= \frac{1+tan^2A}{1+tan^2A}$

$= 1$

= R.H.S.

30. (i) $\frac{1}{(secA+tanA)} - \frac{1}{cosA} = \frac{1}{cosA} - \frac{1}{(secA-tanA)}$

(ii) $(sinA + secA)^2 + (cosA + cosecA)^2 = (1 + secAcosecA)^2$

(iii) $\frac{(tanA+sinA)}{(tanA-sinA)} = \frac{(secA+1)}{(secA-1)}$

Solution:

(i) $\frac{1}{(secA+tanA)} - \frac{1}{cosA} = \frac{1}{cosA} - \frac{1}{(secA-tanA)}$

L.H.S. $= \frac{1}{(secA+tanA)} - \frac{1}{cosA}$

$$= \frac{1}{\left(\frac{1}{cosA}+\frac{sinA}{cosA}\right)} - \frac{1}{cosA}$$

$$= \frac{cosA}{(1+sinA)} - \frac{1}{cosA}$$

$$= \frac{cos^2A-1-sinA}{cosA(1+sinA)} = \frac{-Sin^2-sinA}{cosA(1+sinA)}$$

$$= \frac{-sinA\,(1+sinA)}{cosA(1+sinA)} = -tanA$$

$$R.H.S. = \frac{1}{cosA} - \frac{1}{(secA-tanA)}$$

$$= \frac{1}{cosA} - \frac{1}{\frac{1}{cosA}-\frac{sinA}{cosA}}$$

$$= \frac{1}{cosA} - \frac{cosA}{1-sinA}$$

$$= \frac{1-sinA-Cos^2A}{cosA(1-sinA)}$$

$$= \frac{Sin^2A-sinA}{cosA(1-sinA)}$$

$= \frac{-sinA+ Sin^2 A}{\boldsymbol{cosA(1-sinA)}}$

$= \frac{-sinA(1-sinA)}{\boldsymbol{cosA(1-sinA)}}$

$= \frac{-sinA}{cosA}$ = - tanA

∴ L.H.S. = R.H.S.

(ii) $\boldsymbol{(sinA + secA)^2 + (cosA + cosecA)^2 = (1 + secAcosecA)^2}$

Solution: $\boldsymbol{(sinA + secA)^2 + (cosA + cosecA)^2}$

$= sin^2 A + sec^2 A + 2sinAsecA + cos^2 A + cosec^2 A + 2secAcosecA$

$= (sin^2 A + cos^2 A) + (sec^2 A + cosec^2 A) + 2sinA \times \frac{1}{cosA} + 2 \times cosA \times \frac{1}{sinA}$

$= 1 + \left[\frac{1}{cos^2 A} + \frac{1}{sin^2 A}\right] + \frac{2sin^2 A+2cos^2 A}{sinAcosA}$

$= 1 + \left[\frac{sin^2 A+cos^2 A}{cos^2 A.sin^2 A}\right] + \frac{2[sin^2 A+2cos^2 A]}{sinAcosA}$

$= 1 + \frac{1}{cos^2 A.sin^2 A} + \frac{2}{sinA.cosA}$ $[\because sin^2\theta + cos^2\theta = 1]$

$= \left(1 + \frac{1}{cosA.sinA}\right)^2$ $[\because (a+b)^2 = a^2 + (b)^2 + 2ab]$

$= (1 + cosecAsecA)^2$

= R.H.S.

(iii) $\frac{tanA+sinA}{tanA-sinA} = \frac{secA+1}{secA-1}$

L.H.S. $= \frac{tanA+sinA}{tanA-sinA} = \frac{\frac{sinA}{cosA}+sinA}{\frac{sinA}{cosA}-sinA}$

$= \frac{\frac{sinA+sinA\,cosA}{cosA}}{\frac{sinA-sinAcosA}{cosA}}$

$= \frac{sinA\,(1+cosA)}{sinA(1-cosA)} = \frac{1+cosA}{1-cosA}$

On dividing each term by cosA, we have

$= \frac{\frac{1}{cosA}+1}{\frac{1}{cosA}-1} = \frac{secA+1}{secA-1} =$ R.H.S.

31. If $sin\theta + cos\theta = \sqrt{2}\sin(90^{\circ} - \theta)$, show that $cot\theta = \sqrt{2} + 1$

Solution:

Given, $sin\theta + cos\theta = \sqrt{2}\sin(90^{\circ} - \theta)$

$sin\theta + cos\theta = \sqrt{2}\,cos\theta$

On dividing by sin θ, we have

$1 + cot\theta = \sqrt{2}\,\text{cot}\theta$

$1 = \sqrt{2}\,\text{cot}\theta - cot\theta$

$(\sqrt{2} - 1)\,cot\theta = 1$

$cot\theta = \frac{1}{(\sqrt{\mathbf{2}} - 1)}$

(Rationalising thedenominators)

$= \frac{1\times((\sqrt{\mathbf{2}}+1))}{((\sqrt{\mathbf{2}}-1)(\sqrt{\mathbf{2}}+1))}$

$= \frac{(\sqrt{\mathbf{2}}+1)}{(\sqrt{\mathbf{2}}^2)-(1^2)} = \frac{\sqrt{\mathbf{2}}+1}{2-1} = \frac{\sqrt{\mathbf{2}}+1}{1}$

$= \sqrt{\mathbf{2}} + 1$ = R.H.S.

Hence, cotθ= $\sqrt{\mathbf{2}} + 1$

32. If $7\boldsymbol{sin^2\theta + 3cos^2\theta = 4, \theta^\circ \leq \theta \leq 90^\circ}$, then find thevalue of θ.

Solution:

Given,

$7sin^2\theta + 3cos^2\theta = 4, \theta^\circ \leq \theta \leq 90^\circ$

$3\ sin^2\theta\ + 3Cos^2\theta + 4\ sin^2\theta = 4$

$3(sin^2\theta\ + Cos^2\theta) + 4sin^2\theta = 4$

$3(1) + 4sin^2\theta = 4$

$4sin^2\theta = 4 - 3$

$sin^2\theta = \frac{1}{4}$

Taking square-root on both sides, we get

$sin\theta = \frac{1}{2}$

Thus, $\theta = 30^\circ$

33. If $\boldsymbol{sec\theta + tan\theta = m \ \ and\ sec\theta - tan\theta = n}$, prove that $\boldsymbol{mn = 1}$.

Solution:

Given,

$sec\theta + tan\theta = m$

$sec\theta - tan\theta = n$

Now,

$mn = (sec\theta + tan\theta)(sec\theta - tan\theta)$

$= sec^2\theta - tan^2\theta = 1$

Thus, mn = 1

34. If $\boldsymbol{x - asec\theta + btan\theta}$ and $\boldsymbol{y = atan\theta + bsec\theta}$, prove that $\boldsymbol{x^2 - y^2 = a^2 - b^2}$

Solution:

Given,

$x = asec\theta + btan\theta$

$y = atan\theta + bsec\theta$

Now,

$x^2 - y^2 = (asec\theta + btan\theta)^2 - (atan\theta + bsec\theta)^2$

$=(a^2sec^2\theta + b^2tan^2\theta + 2ab\ sec\theta\ tan\theta) - (a^2tan^2\theta + b^2sec^2\theta + 2ab\ sec\theta tan\theta)$

$= a^2sec^2\theta + b^2tan^2\theta + 2ab\ sec\theta\ tan\theta - a^2tan^2\theta - b^2sec^2\theta - 2ab\ sec\theta tan\theta$

$= \ a^2(sec^2\theta - tan^2\theta) - \ b^2(sec^2\theta - tan^2\theta)$

$= \ a^2 \times 1 - \ b^2 \times 1$ $\{ \ sec^2\theta - tan^2\theta = 1\}$

$= \ a^2 - b^2$

= Hence provded.

35. If $x = h + acos\theta$ and $y = k + asin\theta$, prove that $(x-h)^2 + (y-k)^2 = a^2$

Solution:

Given,

$x = h + acos\theta$

$y = k + asin\theta$

Now,

$x - h = acos\theta$

$y - k = asin\theta$

On squaring and adding we get

$(x-h)^2 + (y-k)^2 = a^2cos^2\theta + a^2sin^2\theta$

$= a^2(sin^2\theta \ + Cos^2\theta \ )$

$= a^2(1)$

[Since, $sin^2\theta \ + Cos^2\theta = \ $]

= Hence proved.

Chapter Test

1. If θ is an acute angle cosec $\theta = \sqrt{5}$, find the value of $cot\theta - cos\theta$.

(ii) If θ is an acute angle and tan $\theta = \frac{8}{15}$, find the value of $sec\theta + cosec\theta$.

Solution:

Given, θ is an acute angle and cosec $\theta = \sqrt{5}$

So,

Sin $\theta = \frac{1}{\sqrt{5}}$

And, cos$\theta = \sqrt{1 - \ sin^2\theta}$

$$\cos\theta = \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\left(1 - \left(\frac{1}{5}\right)\right)}$$

$$= \sqrt{\frac{4}{5}}$$

$$cos\theta = \frac{2}{\sqrt{5}}$$

Now,

$$cot\theta - cos\theta = \left(\frac{cos\theta}{sin\theta}\right) - cos\theta$$

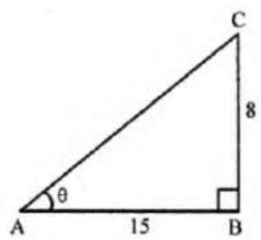
$$= \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \sqrt{1 - \frac{1}{5}}$$

$= \sqrt{\frac{5-1}{5}}$

$= \sqrt{\frac{4}{5}}$

$= \frac{2}{\sqrt{5}}$



(ii) Given, θ is an acute angle and $\tan\theta = \frac{8}{15}$

In Fig. we have

$\tan\theta = \frac{BC}{AB} = \frac{8}{15}$

So, BC = 8 and AB = 15

By Pythagoras theorem, we have

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(5^2 + 8^2)}$

$= \sqrt{25 + 64}$

$= \sqrt{289}$

$\Rightarrow AC = 17$

Now,

Sec $\theta = \frac{AC}{AB} = \frac{17}{15}$

$cosec\theta = \frac{AC}{BC} = \frac{17}{8}$

So,

Sec $\theta + Cosec\theta = \frac{17}{15} + \frac{17}{8}$

$= \frac{(136+255)}{120}$

$= \frac{391}{120}$

$= 3\frac{31}{120}$

2. Evaluate the following :

(i) $2 \times \left(\frac{cos^2 20^\circ + cos^2 70^\circ}{sin^2 25^\circ + sin^2 65^\circ}\right) - tan45^\circ + tan13^\circ tan23^\circ tan30^\circ tan67^\circ tan77^\circ$

(ii) $\frac{sin^2 22^\circ + sin^2 68^\circ}{Cos^2 22^\circ + Cos^2 68^\circ} + sin^2 63^\circ + cos^2 63^\circ sin27^\circ$

Solution;

(i) $2 \times \left(\frac{cos^2 20^\circ + cos^2 70^\circ}{sin^2 25^\circ + sin^2 65^\circ}\right) - tan45^\circ + tan13^\circ tan23^\circ tan30^\circ tan67^\circ tan77^\circ$

$= 2\left[\frac{cos^2 (90^\circ - 70^\circ) + cos^2 70^\circ}{sin^2 25^\circ + sin^2 (90^\circ - 25^\circ)}\right]$ - $tan45^\circ + tan13^\circ tan77^\circ tan23^\circ tan67^\circ tan30^\circ$

$= 2\left[\frac{Sin^2 70^\circ + Cos^2 70}{sin^2 25^\circ + Cos^2 25^\circ}\right]$ - 1 + $tan13^\circ\, tan(90^\circ - 13^\circ) tan23^\circ \tan(90^\circ - 23^\circ) \times \frac{1}{\sqrt{3}}$

$= 2\left(\frac{1}{1}\right) - 1 + tan13\ Cot13^\circ\ tan23^\circ\ cot23^\circ \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + 1 \times 1 \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + \frac{1}{\sqrt{3}}$

$= 1 + \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}}$

$= \frac{(\sqrt{3}+1)\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{3+\sqrt{3}}{3}$

(ii) $\frac{sec29^\circ}{cosec61^\circ} + 2\cot 8^\circ\ cot17^\circ cot45^\circ cot73^\circ\ cot82^\circ - 3(sin^2 38^\circ + sin^2 52^\circ)$

$= \frac{sec29^\circ}{cosec(90^\circ - 29^\circ)} + 2\cot 8^\circ \times cot(90^\circ - 8^\circ) \times\ cot17^\circ \times cot(90^\circ - 17^\circ) cot45^\circ - 3[sin^2 38^\circ +\ sin^2(90^\circ - 38^\circ)]$

$= \frac{sec29^\circ}{sec29^\circ} + 2\cot 8^\circ \tan 8^\circ \times cot17^\circ tan17^\circ \times 1 - 3(sin^2 38^\circ + Cos^2 38^\circ)$

$= 1 + 2 \times 1 \times 1 \times 1 - 3 \times 1 = 1 + 2 - 3 = 0$

(iii) $\frac{sin^2 22^\circ + sin^2 68^\circ}{Cos^2 22^\circ + Cos^2 68^\circ} + sin^2 63^\circ + cos^2 63^\circ sin 27^\circ$

$= \frac{sin^2 22^\circ + sin^2 (90^\circ - 22^\circ)}{Cos^2 22^\circ + Cos^2 (90^\circ - 22^\circ)} + sin^2 63^\circ + cos^2 63^\circ sin(90^\circ - 63^\circ)$

$= \frac{sin^2 22^\circ + Cos^2 22^\circ}{Cos^2 22^\circ + Sin^2 22^\circ} + sin^2 63^\circ + cos^2 63^\circ \times cos^2 63^\circ$

$= \frac{1}{1} + (sin^2 63^\circ + cos^2 63^\circ)$

= 1 + 1

= 2

3. If $\frac{4}{3}(sec^2 59^\circ + cot^2 31^\circ) - \frac{2}{3} sin 90^\circ + 3tan^2 56^\circ tan^2 34^\circ = \frac{x}{2}$, then find the value of x.

Solution :

Given

$$\frac{4}{3}(sec^2 59^\circ + cot^2 31^\circ) - \frac{2}{3} sin 90^\circ + 3tan^2 56^\circ tan^2 34^\circ = \frac{x}{2}$$

$\Rightarrow \frac{4}{3}[sec^2 59^\circ - tan^2 59^\circ] - \frac{2}{3} \times 1 + 3\, tan^2 56^\circ\, Cot^2 56^\circ = \frac{x}{3}$

$= \frac{4}{3} \times 1 - \frac{2}{3} + 3 \times 1 = \frac{x}{3}$

$= \frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3}$

$= \frac{4-2+9}{3} = \frac{x}{3}$

$\frac{11}{3} = \frac{x}{3}$

$\Rightarrow x = \frac{11 \times 3}{3} = 11$

$\therefore \quad x = 11$

4. (i) $\frac{\boldsymbol{cosA}}{\boldsymbol{(1-sinA)}} + \frac{\boldsymbol{CosA}}{\boldsymbol{(1+sinA)}}$ **= 2 secA**

(ii) $\frac{\boldsymbol{cosA}}{\boldsymbol{(CosecA+1)}} + \frac{\boldsymbol{CosA}}{\boldsymbol{(CosecA-1)}}$ **= 2tanA**

Solution:

(i) $\frac{\boldsymbol{cosA}}{\boldsymbol{(1-sinA)}} + \frac{\boldsymbol{CosA}}{\boldsymbol{(1+sinA)}}$ **= 2 secA**

L.H.S. = $\frac{\boldsymbol{cosA}}{\boldsymbol{(1-sinA)}} + \frac{\boldsymbol{CosA}}{\boldsymbol{(1+sinA)}}$

= cosA $\left[\frac{\boldsymbol{1}}{\boldsymbol{(1-sinA)}} + \frac{\boldsymbol{1}}{\boldsymbol{(1+sinA)}}\right]$

= cosA $\left[\frac{\boldsymbol{1-sinA+ 1-sinA}}{\boldsymbol{(1-sinA)\ (1+sinA)}}\right]$

= cosA $\left[\frac{2}{2-Sin^2A}\right]$ = $\frac{2cosA}{Cos^2A}$

= $\frac{2}{cosA}$ = 2secA

= R.H.S.

(ii) $\frac{cosA}{cosecA+1} + \frac{cosA}{cosecA-1} = 2tanA$

L.H.S. $= \frac{cosA}{cosecA+1} + \frac{cosA}{cosecA-1}$

$= cosA \left[\frac{1}{cosecA+1} + \frac{1}{cosecA-1}\right]$

$= cosA \left[\frac{cosecA-1+cosecA+1}{(cosecA+1)(cosecA-1)}\right]$

$= \frac{cosA[2cosecA]}{cosec^2A-1} = \frac{2cosA}{sinA(Cot^2SA)}$

$= \frac{2cotA}{cot^2A} = \frac{2}{cotA}$

$= 2tanA$

= R.H.S.

5. **(i)** $\frac{(\boldsymbol{cos\theta - sin\theta})(\boldsymbol{1+tan\theta})}{\boldsymbol{2cos^2\theta - 1}} = \boldsymbol{sec\theta}$

(ii) $(\boldsymbol{cosec\theta - sin\theta})(\boldsymbol{sec\theta - cos\theta})(\boldsymbol{tan\theta + cot\theta}) = \boldsymbol{1}$

Solution:

(i) $\frac{(\boldsymbol{cos\theta - sin\theta})(\boldsymbol{1+tan\theta})}{\boldsymbol{2cos^2\theta - 1}} = sec\theta$

L.H.S. $= \frac{(cos\theta - sin\theta)(1+tan\theta)}{2cos^2\theta - 1}$

$= \frac{(cos\theta - sin\theta)(cos\theta + sin\theta)}{cos\theta(2cos^2\theta - 1)}$

$= \frac{Cos^2\theta - Sin^2\theta}{cos\theta(2cos^2\theta - 1)} = \frac{Cos^2\theta - (1 - Cos^2\theta)}{cos\theta\ (2Cos^2\theta - 1)}$

$= \frac{Cos^2\theta - 1 + Cos^2\theta}{cos\theta(2Cos^2\theta - 1)} = \frac{2Cos^2\theta - 1}{cos\theta\ (2Cos^2\theta - 1)}$

$= \frac{1}{cos\theta}$

$=$ secθ

= R.H.S.

(ii) $(cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta)$ = 1.

L.H.S. = $(cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta)$

$= \left(\frac{1}{sin\theta} - sin\theta\right)\left(\frac{1}{cos\theta} - cos\theta\right)\left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)$

$= \left(\frac{1 - sin^2\theta}{sin\theta}\right)\left(\frac{1 - cos^2\theta}{cos\theta}\right)\left(\frac{sin^2\theta + cos^2\theta}{sin\theta cos\theta}\right)$

$= \frac{(cos^2\theta \times sin^2\theta)}{sin\theta cos\theta} \times \frac{1}{sin\theta cos\theta}$

$= \frac{sin^2\theta cos^2\theta}{sin\theta cos\theta}$

= 1

= R.H.S.

6. (i) $sin^2\theta + cos^4\theta = cos^2\theta + sin^4\theta$

(ii) $\frac{cot\theta}{cosec\theta+1} + \frac{cosec\theta+1}{cot\theta}$ = 2sec θ

Solution:

Given,

(i) $sin^2\theta + cos^4\theta = cos^2\theta + sin^4\theta$

L.H.S. = $sin^2\theta + cos^4\theta$

= (1-$cos^2\theta$) + $cos^4\theta$

= $cos^4\theta - cos^2\theta + 1$

= $cos^2\theta(cos^2\theta - 1) + 1$

= $cos^2\theta(-sin^2\theta)$ + 1

= 1 - $sin^2\theta cos^2\theta$

Now,

R.H.S. = $cos^2\theta + sin^4\theta$

= $(1 - sin^2\theta) + sin^4\theta$

= $sin^4\theta - sin^2\theta + 1$

= $sin^2\theta(sin^2\theta - 1) + 1$

= $sin^2\theta(-cos^2\theta) + 1$

= 1 - $sin^2\theta cos^2\theta$

Hence, L.H.S. = R.H.S.

$= \frac{cot\theta}{cosec\theta+1} + \frac{cosec\theta+1}{cot\theta} = 2\sec\theta$

$=$ L.H.S. $= \frac{cot\theta}{cosec\theta+1} + \frac{cosec\theta+1}{cot\theta}$

$= \frac{\frac{cos\theta}{sin\theta}}{\frac{1}{sin\theta}+1} + \frac{\frac{1}{sin\theta}+1}{\frac{cos\theta}{sin\theta}}$

7. (i) $\boldsymbol{sec^4A(1 - sin^4A) - 2tan^2A = 1}$

(ii) $\boldsymbol{\frac{1}{sinA+cosA+1} + \frac{1}{sinA+cosA-1} = secA + cosecA}$

Solution:

(i) $sec^4A(1 - sin^4A) - 2tan^2A = 1$

L.H.S. $= sec^4A(1 - sin^4A) - 2tan^2A$

$= \frac{1}{cos^4A}(1 + sin^2A)(1 - sin^2A) - 2tan^2A$

$[\because\ a^2 - b^2 = (a + b)(a - b)]$

$= \frac{(1+sin^2A)cos^2A}{cos^4A} - 2\frac{sin^2A}{cos^2A}$

$= \frac{(1+sin^2A)}{cos^2A} - \frac{2sin^2A}{cos^2A}$

$= \frac{1+sin^2A-2sin^2A}{cos^2A}$

$= \frac{1-sin^2A}{cos^2A}$

$= \frac{cos^2A}{cos^2A}$

$= 1$

$(\because 1 - sin^2A = cos^2A)$

= R.H.S.

(ii) $\frac{1}{sinA+cosA+1} + \frac{1}{sinA+cosA-1} = secA + cosecA$

L.H.S. = $\frac{1}{sinA+cosA+1} + \frac{1}{sinA+cosA-1}$

$= \frac{sinA+cosA-1+sinA+cosA+1}{(sinA+cosA+1)(sinA+cosA-1)}$

$= \frac{2(sinA+cosA)}{(sinA+cosA)^2-(1)^2}$

$= \frac{2(sinA+cosA)}{sin^2A+cos^2A+2sinAcosA-1}$

$= \frac{2(sinA+cosA)}{2sinAcosA}$

$= \frac{sinA+cosA}{sinAcosA} = \frac{sinA}{sinAcosA} + \frac{CosA}{sinAcosA}$

$= \frac{1}{cosA} + \frac{1}{sinA}$

$= secA + cosecA = R.H.S.$

8. (i) $\frac{sin^3\theta+cos^3\theta}{sin\theta cos\theta}+sin\theta cos\theta=1$

(ii) $(secA-tanA)^2(1+sinA)=1-sinA$

Solution:

L.H.S. = $\frac{sin^3\theta+cos^3\theta}{sin\theta cos\theta}+sin\theta cos\theta$

= $\frac{(sin\theta+cos\theta)(sin^2\theta-sin\theta cos\theta+cos^2\theta)}{(sin\theta+cos\theta)}+sin\theta cos\theta$

= $sin^2\theta+\ cos^2\theta-sin\theta cos\theta+sin\theta cos\theta$

= 1

= R.H.S.

(ii) $(secA-tanA)^2(1+sinA)=1-sinA$

L.H.S. = $\left(\frac{1}{cosA}-\frac{sinA}{cosA}\right)^2(1+sinA)$

= $\left(\frac{1-sinA}{cosA}\right)^2(1+sinA)$

= $\frac{(1-sinA)^2(1+sinA)}{1-sin^2A}$

= $\frac{(1-sinA)^2(1+sinA)}{(1-sinA)(1+sinA)}$

= 1 - sin A

= R.H.S.

9. (i) $\frac{cosA}{1-tanA} - \frac{sin^2A}{cosA-sinA} = sinA + cosA$

(ii) $(secA - cosecA)(1 + tanA + cotA) = tanAsecA - cotAcosecA$

(iii) $\frac{tan^2\theta}{tan^2\theta-1} - \frac{cosec^2\theta}{sec^2\theta-cosec^2\theta} = \frac{1}{sin^2\theta-cos^2\theta}$

Solution:

(i) $\frac{cosA}{1-tanA} - \frac{sin^2A}{cosA-sinA} = sinA + cosA$

L.H.S. = $\frac{cosA}{1-tanA} - \frac{sin^2A}{cosA-sinA}$

$= \frac{cosA}{1-\frac{sinA}{cosA}} - \frac{sin^2A}{cosA-sinA}$

$= \frac{cosA \times cosA}{cosA-sinA} - \frac{sin^2A}{cosA-sinA}$

$= \frac{cos^2A}{cosA-sinA} - \frac{sin^2A}{cosA-sinA}$

$= \frac{cos^2A - sin^2A}{cosA-sinA}$

$= \frac{(cosA+sinA)(cosA-sinA)}{(cosA-sinA)} = cosA + sinA$

= R.H.S.

(ii) $(secA - cosecA)\ (1 + tanA + cotA) = tanAsecA - cotAcosecA$

L.H.S. = $(secA - cosecA)\ (1 + tanA + cotA)$

$= \left(\frac{1}{cosA} - \frac{1}{sinA}\right)\left(1 + \frac{sinA}{cosA} + \frac{cosA}{sinA}\right)$

$= \frac{sinA - cosA}{sinAcosA} \times \frac{sinAcosA + sin^2A + cos^2A}{sinAcosA}$

$= \frac{(sinA - cosA)(sinAcosA + 1)}{sin^2Acos^2A}$

R.H.S. = $tanAsecA - cotAcosecA$

$= \frac{sinA}{cosA} \cdot \frac{1}{cosA} - \frac{cosA}{sinA} \cdot \frac{1}{sinA}$

$= \frac{sinA}{cos^2A} - \frac{cosA}{sin^2A}$

$= \frac{sin^3A - cos^3A}{sin^2Acos^2A}$

$= \frac{(sinA - cosA)(sin^2A + cos^2A + sinAcosA)}{sin^2Acos^2A}$

$= \frac{(sinA - cosA)(sinAcosA + 1)}{sin^2Acos^2A}$

∴ L.H.S. = R.H.S.

(iii) $\frac{tan^2\theta}{tan^2\theta-1}+\frac{cosec^2\theta}{sec^2\theta-cosec^2\theta}=\frac{1}{sin^2\theta-cos^2\theta}$

L.H.S. = $\frac{tan^2\theta}{tan^2\theta-1}+\frac{cosec^2\theta}{sec^2\theta-cosec^2\theta}$

$$=\frac{\frac{Sin^2\theta}{Cos^2\theta}}{\frac{Sin^2\theta}{Cos^2\theta}-1}+\frac{\frac{1}{Sin^2\theta}}{\frac{1}{Cos^2\theta}-\frac{1}{Sin^2\theta}}$$

$$=\frac{Sin^2\theta}{Cos^2\theta}\times\frac{Cos^2\theta}{Sin^2\theta-Cos^2\theta}+\frac{\frac{1}{Sin^2\theta}}{\frac{Sin^2\theta-Cos^2\theta}{Sin^2\theta Cos^2\theta}}$$

$$=\frac{Sin^2\theta}{Sin^2\theta-Cos^2\theta}+\frac{Sin^2\theta Cos^2\theta}{Sin^2\theta(Sin^2\theta-Cos^2\theta)}$$

$$=\frac{Sin^2\theta}{Sin^2\theta-Cos^2\theta}+\frac{Cos^2\theta}{Sin^2\theta-Cos^2\theta}$$

$$=\frac{Sin^2\theta+Cos^2\theta}{Sin^2\theta-Cos^2\theta}$$

$$=\frac{1}{Sin^2\theta-Cos^2\theta}$$

= R.H.S.

10. $\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}=\frac{2}{sin^2A-cos^2A}=\frac{2}{1-2cos^2A}=\frac{2sec^2A}{tan^2A-1}$

Solution:

L.H.S. = $\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}$

$=\frac{(sinA+cosA)^2+(sinA-cosA)^2}{sin^2A-cos^2A}$

$=\frac{2(sin^2A+cos^2A)}{sin^2A-cos^2A}$

$=\frac{2\times 1}{sin^2A-cos^2A}$

$=\frac{2}{sin^2A-cos^2A}$

= R.H.S.

Now, we have

$=\frac{2}{sin^2A-cos^2A}$

$=\frac{2}{1-cos^2A-cos^2A}$

$=\frac{2}{1-2cos^2A}$

= R. H.S.

now, we have

$$= \frac{2}{sin^2A - cos^2A}$$

$$= \frac{2}{sin^2A - (1 - sin^2A)}$$

$$= \frac{2}{sin^2A - cos^2A}$$

$$= \frac{\frac{2}{cos^2A}}{\frac{sin^2A}{cos^2A} - \frac{cos^2A}{cos^2A}}$$

$$= \frac{2sec^2A}{tan^2A - 1}$$

= R.H.S.

Hence,

$$\frac{sinA + cosA}{sinA - cosA} + \frac{sinA - cosA}{sinA + cosA} = \frac{2}{sin^2A - cos^2A} = \frac{2}{1 - 2cos^2A} = \frac{2sec^2A}{tan^2A - 1}$$

11. $\mathbf{2[sin^6\theta + Cos^6\theta] - 3(sin^4\theta + Cos^4\theta) + 1 = 0}$

Solution:

Given,

$2[sin^6\theta + Cos^6\theta] - 3(sin^4\theta + Cos^4\theta) + 1 = 0$

L.H.S. =

$2[sin^6\theta + Cos^6\theta] - 3(sin^4\theta + Cos^4\theta) + 1$

L.H.S. = $2[sin^6\theta + Cos^6\theta] - 3(sin^4\theta + Cos^4\theta) + 1$

= $2[(sin^2\theta)^3 + (Cos^2\theta)^3] - 3(sin^4\theta + Cos^4\theta) + 1$

= $2[(sin^2\theta + Cos^2\theta)(sin^4\theta + Cos^4\theta - sin^2\theta Cos^2\theta)] - 3(sin^4\theta + Cos^4\theta) + 1$

= $2\ (sin^4\theta + Cos^4\theta - sin^2\theta Cos^2\theta) - 3(sin^4\theta + Cos^4\theta) + 1$

= $2sin^4\theta + 2Cos^4\theta - 2sin^2\theta Cos^2\theta - 3sin^4\theta - 3Cos^4\theta + 1$

= $1 - sin^4\theta - Cos^4\theta - 2sin^2\theta Cos^2\theta$

= $1 - [sin^4\theta + Cos^4\theta + 2sin^2\theta Cos^2\theta]$

= 1 – 1

= 0

= R.H.S.

12. If $cot\theta + cos\theta = m, cot\theta - \cos\theta = n,$ then prove that $(m^2 - n^2)^2 = 16.$

Solution:

Given,

$cot\theta + cos\theta = m$ ……….(i)

$cot\theta + cos\theta = n$ ………(ii)

Adding (i) and (ii), we get

$2cot\theta = m + n$

$\Rightarrow\ cot\theta = \frac{m+n}{2}$

$\therefore\ tan\theta = \frac{2}{m+n}$ ……(iii)

Subtracting (ii) from (i),

$2cos\theta = m - n \Rightarrow cos\theta = \frac{m-n}{2}$

$\therefore sec\theta = \frac{2}{m-n}$ ……..(iv)

Now, squaring and subtracting (iii) from (iv), we have

$sec^2\theta - tan^2\theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$

$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$

$\Rightarrow\ 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$

$= 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$

$= \frac{4(4mn)}{(m^2-n^2)^2} = 1$

$\therefore (m^2 - n^2)^2 = 16\,mn.$

$2cot\theta = m + n$

$\Rightarrow cot\theta = \dfrac{m+n}{2}$

$\therefore \; tan\theta = \frac{2}{m+n}$ ....(iii)

Subtracting (ii) fron (i),

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Now, squaring and subtracting (iii) from (iv), we have

$sec^2\theta - tan^2\theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$

$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$

$\Longrightarrow \; 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$

$= 4\left[\frac{(m+n)^2-(m-n)^2}{(m+n)^2(m-n)^2}\right] = 1$

$= \frac{4\,(4mn)}{(m^2-n^2)^2} = 1$

$= \frac{16mn}{(m^2-n^2)^2} = 1$

$\therefore (m^2 - n^2)^2 = 16mn$

13. If $\boldsymbol{sec\theta + tan\theta = p, prove\ that}$ $\mathbf{sin}\boldsymbol{\theta} = \frac{(p^2-1)}{(p^2+1)}$

Solution:

Given,

$sec\theta + tan\theta = p$

$\frac{1}{cos\theta} + \frac{sin\theta}{cos\theta} = p$

$\frac{1+sin\theta}{cos\theta} = p$

Squaring on both sides,

$\frac{(1+sin\theta)^2}{cos^2\theta} = p^2$

$\Rightarrow \frac{(1+sin\theta)^2}{1-sin^2\theta} = p^2$

$\frac{(1+sin\theta)^2}{(1-sin\theta)(1+sin\theta)} = p^2$

$\Rightarrow \frac{(1+sin\theta)}{(1-sin\theta)} = \frac{p^2}{1}$

Applying componendo and dividendo

$\frac{1+sin\theta+1-sin\theta}{1+sin\theta-1+sin\theta} = \frac{p^2+1}{p^2-1}$

$= \frac{2}{2sin\theta} = \frac{p^2+1}{p^2-1}$

$\Rightarrow \frac{1}{sin\theta}$

$= \frac{p^2+1}{p^2-1}$

14. If $tanA = ntanB\ and\ sinA = msinB$, prove that $cos^2A = \frac{(m^2-1)}{(n^2-1)}$

Solution:

Given,

$tanA = ntanB\ and\ sinA = m\sin B$

$n = \frac{tanA}{tanB}$

$m = \frac{sinA}{sinB}$

$\frac{1}{sinB} = \frac{m}{sinA} \Rightarrow cosecB = \frac{m}{sinA}$

$\frac{1}{tanB} = \frac{m}{tanA} \Rightarrow cotB = \frac{n}{tanA}$

Now, $cosec^2B - cot^2B = 1$

$\Rightarrow \frac{m^2}{sin^2A} - \frac{n^2}{tan^2A} = 1$

$= \frac{m^2}{sin^2A} - \frac{n^2cos^2A}{sin^2A} = 1$

$= m^2 - n^2cos^2A = sin^2A$

$= m^2 - n^2cos^2A = 1 - cos^2A$

$= m^2 - 1 = cos^2A + n^2cos^2A$

$= m^2 - 1 = (n^2 - 1)cos^2A$

$$\Rightarrow cos^2A = \frac{m^2 - 1}{n^2 - 1}$$

15. If secA = $\boldsymbol{x+\frac{1}{4}x}$, then prove that secA + tanA =$\boldsymbol{2x\ or\ \frac{1}{2}x}$

Solution:

Given, secA = $x+\frac{1}{4}x$

we know that

$tanA = \pm\sqrt{sec^2A-1}$

$= \pm\sqrt{\left(x+\frac{1}{4x}\right)^2-1}$

$= \pm\sqrt{x^2+\frac{1}{16x^2}+\frac{1}{2}-1}$

$= \pm\sqrt{x^2+\frac{1}{16x^2}-\frac{1}{2}}$

$= \pm\left(x-\frac{1}{4x}\right)$

$\therefore secA + tanA = x+\frac{1}{4x}+x-\ \frac{1}{4x} = 2x$

Or $x+\frac{1}{4x}-x+\frac{1}{4x}=\frac{1}{2x}$

Or $x+\frac{1}{4x}-x+\frac{1}{4x}=\frac{1}{2x}$

$tanA = \ \pm\sqrt{sec^2A-1}$

$= \pm\sqrt{\left(x+\frac{1}{4x}\right)^2-1}$

$= \pm\sqrt{x^2+\frac{1}{16x^2}+\ \frac{1}{2}-1}$

$= \pm\sqrt{x^2 + \frac{1}{16x^2} - \frac{1}{2}}$

$= \pm\left(x - \frac{1}{4x}\right)$

$\therefore secA + tanA = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

Or $x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$

16. When $0^\circ < \theta < 90^\circ$, Solve the following equations:

(i) $2cos^2\theta + sin\theta - 2 = 0$

(ii) $3cos\theta = 2sin^2\theta$

(iii) $sec^2\theta - 2tan\theta = 0$

(iv) $tan^2\theta = 3(sec\theta - 1)$

Solution:

Given, $0^\circ < \theta < 90^\circ$

(i) $2cos^2\theta + sin\theta - 2 = 0$

$2(1 - sin^2\theta) + sin\theta - 2 = 0$

$2 - 2sin^2\theta + sin\theta - 2 = 0$

$-2sin^2\theta + sin\theta = 0$

$sin\theta(1 - 2sin\theta) = 0$

So, either $sin\theta = 0$ or 1 - $2sin\theta = 0$

If $sin\theta = 0$

$\Rightarrow \theta = 0^\circ$

And, if $1 - 2sin\theta = 0$

$sin\theta = \frac{1}{2}$

$\Rightarrow \ \theta = 30^{\circ}$

Thus, $\theta = 0^{\circ}$ or 30°

(ii) $3 \ cos\theta = 2sin^2\theta$

$3 \ cos\theta = 2(1 - cos^2\theta)$

$3 \ cos\theta = 2 - 2cos^2\theta$

$2cos^2\theta \ + \ 3 \ cos\theta - 2 = 0$

$2cos^2\theta + 4 \ cos\theta - cos\theta - 2 = 0$

$2cos\theta(cos\theta + 2) - 1 \ (cos\theta + 2)$

$(2cos\theta - 1)(cos\theta + 2) = 0$

So, either $2cos\theta - 1 = 0$ or $cos\theta + 2 = 0$

If $2cos\theta - 1 = 0$

$cos\theta = \frac{1}{2}$

$\Rightarrow \ \theta = 60^{\circ}$

And, for $cos\theta + 2 = 0$

$\Rightarrow \ cos\theta = \ -2$ which is not possible being out of range.

Thus, $\theta = 60^{\circ}$

(iii) $sec^2\theta - 2tan\theta = 0$

$(1 + tan^2\theta) - 2tan\theta = 0$

$tan^2\theta - 2tan\theta + 1 = 0$

$(tan\theta - 1)^2 = 0$

$tan\theta - 1 = 0$

$\Rightarrow tan\theta = 1$

Thus, $\theta = 45^\circ$

(iv) $tan^2\theta = 3(sec\theta - 1)$

$= (sec^2\theta - 1) = 3sec\theta - 3$

$sec^2\theta - 1 - 3sec\theta + 3 = 0$

$sec^2\theta - 3sec\theta + 2 = 0$

$sec^2\theta - 2sec\theta - sec\theta + 2 = 0$

$sec\theta(sec\theta - 2) - 1(sec\theta - 2) = 0$

$(sec\theta - 1)(sec\theta - 2) = 0$

So, either $sec\theta - 1 = 0$ or $sec\theta - 2 = 0$

$If \quad sec\theta - 1 = 0$

$sec\theta = 1$

$\Rightarrow \theta = 0^\circ$

And, if $sec\theta - 2 = 0$

$sec\theta = 2$

$\Rightarrow \theta = 60^\circ$

Thus, $\theta = 0^\circ$ or 60°