## Long Answer Type Questions

## [4 MARKS]

**Que 1. Prove that the tangent to a circle is perpendicular to the radius through the point of contact.**

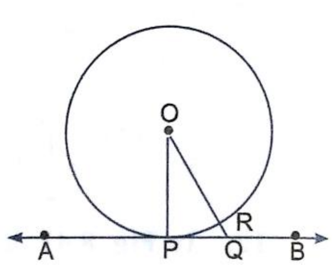

Fig. 8.43

**Sol. Given:** A circle C (O, r) and a tangent AB at a point P.
**To prove:** $OP \perp AB$.
**Construction:** Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.
**Proof:** We know that among all line segment joining the point O to point on AB, the shortest one is perpendicular to AB. So, to prove that OP ⊥ AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.
Clearly, OP = OR [Radii of the same circle]
Now, OQ = OR + RQ
⇒ OQ > OR
⇒ OQ > OP [$\because OP = OR$]
Thus, OP is shorter than any other segment joining O to any point on AB.
Hence, $OP \perp AB$.

**Que 2. Prove that the length of two tangent drawn from an external point to a circle are equal.**

**Sol. Given:** AP and AQ are two tangent from a point A to a circle C (O, r).
**To prove:** AP = AQ
**Construction:** Join OP, OQ and OA.
**Proof:** In order to prove that AP = AQ, we shall first prove that $\Delta OPA \cong \Delta OQA$.
Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.
∴ $OP \perp AP \text{ and } OQ \perp AQ$.
⇒ $\angle OPA = \angle OQA = 90°$ …(i)
Now, in right triangles OPA and OQA, we have
OP = OQ [Radii of a circle]
$\angle OPA = \angle OQA$ [Each 90°]
and OA = OA [Common]

So, by RHS-criterion of congruence, we get

$\Delta OPA \cong \Delta OQA \quad \Rightarrow \quad AP = AQ$ [CPCT]

Hence, lengths of two tangents from an external point are equal.

**Que 3. Prove that the parallelogram circumscribing a circle is a rhombus.**

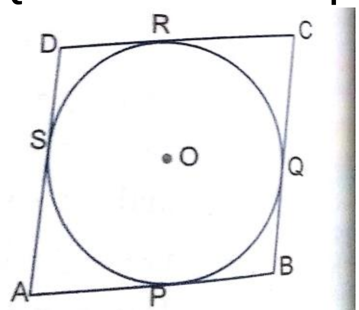

Fig. 8.45

**Sol.** Let ABCD be a parallelogram such that its sides touch a circle with centre O.
We know that the tangent to a circle from an exterior point are equal in length.
Therefore, we have

AP = AS [Tangents from A] …(i)

BP = BQ [Tangents from B] …(ii)

CR = CQ [Tangents from C] …(iii)

And DR = DS [Tangents from D] …(iv)

Adding (i), (ii), (iii) and (iv), we have

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

⇒ AB + CD + = AD + BC

⇒ AB + AB = BC + BC [$\because$ $ABCD\ is\ a\ paralleogram\ \therefore\ AB = CD, BC = DA$]

⇒ 2AB = 2BC ⇒ AB = BC

Thus, AB = BC = CD = AD

Hence, ABCD is a rhombus.

**Que 4. In Fig.8.46, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.**

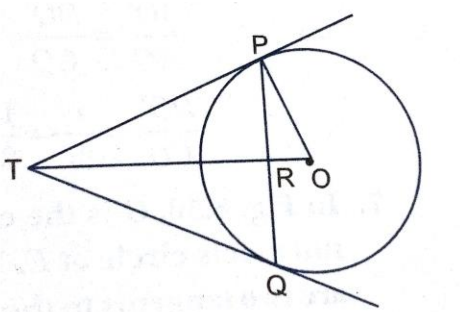

Fig. 8.46

**Sol. Given:** PQ = 16 cm

PO = 10 cm

**To find:** TP

$PR = RQ = \frac{16}{2} = 8cm$ [Perpendicular from the center bisects the chord]

$In\ \Delta OPR$

$$OR = \sqrt{OP^2 - PR^2}$$
$$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$
$$= \sqrt{36} = 6\ cm$$

Let $\angle POR\ be\ \theta$

In $\Delta POR,$ $\tan\theta = \frac{PR}{RO} = \frac{8}{6}$

$$\tan\theta = \frac{4}{3}$$

We know, OP ⊥ TP (Point of contact of a tangent is perpendicular to the line from the centre)

In $\Delta OTP,$ $\tan\theta = \frac{OP}{TP}$ $\Rightarrow \frac{4}{3} = \frac{10}{TP}$

$$TP = \frac{10\times3}{4} = \frac{15}{2} = 7.5\ cm.$$