DAY TWENTY FIVE

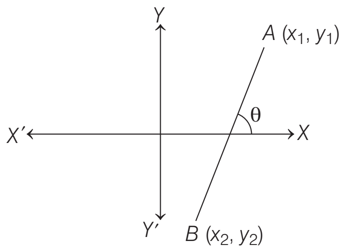
Straight Line

Learning & Revision for the Day

- Concept of Straight Line
- Angle between Two Lines
- Conditions for Concurrence of Three Lines
- Distance of a Point from a Line

Concept of Straight Line

Any curve is said to be a **straight line**, if for any two points taken on the curve, each and every point on the line segment joining these two points lies on the curve.



The slope of a line AB is $m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

Various Forms of Equations of a Line

The equation of a line in the general form can be written as $ax + by + c = 0$

1. **Slope Intercept Form** The equation of a line with slope m and making an intercept c on Y-axis is $y = mx + c$
2. **Point Slope Form** The equation of a line which passes through the point (x_1, y_1) and has the slope m is $y - y_1 = m(x - x_1)$.
3. **Two Points Form** The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is
$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$
4. **Intercept Form of a Line** The equation of a line which cuts off intercepts a and b respectively from the X and Y-axes is $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

5. **Normal or Perpendicular Form** The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with positive direction of X-axis in anti-clockwise sense is

$$x\cos\alpha + y\sin\alpha = p, \text{ where } 0 \le \alpha \le 2\pi.$$

6. **General Equation of a Line to the Normal Form** The general equation of a line is

$$Ax + By + C = 0$$

Now, to reduce the general equation of a line to normal form, we first shift the constant term on the RHS and make it positive, if it is not so and then divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\Rightarrow \left(\frac{A}{\sqrt{A^2+B^2}}\right)x + \left(\frac{B}{\sqrt{A^2+B^2}}\right)y = \left(\frac{-C}{\sqrt{A^2+B^2}}\right)$$

Now, take $\cos\alpha = \dfrac{A}{\sqrt{A^2+B^2}}$, $\sin\alpha = \dfrac{B}{\sqrt{A^2+B^2}}$ and

$p = \dfrac{-c}{\sqrt{A^2+B^2}}$, which gives the required normal form.

7. **Intersection of lines** Let the equation of lines be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then their point of intersection is $\left(\dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

8. **Distance Form or Parametric form** The equation of the straight line passing through (x_1, y_1) and making an angle θ with the positive direction of X-axis is $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$,

where r is the distance of any point (x, y) on the line from the point (x_1, y_1).

Angle between Two Lines

The acute angle θ between the lines having slopes m_1 and m_2 is given by $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1m_2}\right|$.

Condition of Parallel Lines

Let m_1, m_2 be slope of two lines, then lines are parallel, if $m_1 = m_2$.

Equation of any line parallel to $ax + by + c = 0$ can be taken as $ax + by + \lambda = 0$

Condition of Perpendicular Lines

Let m_1, m_2 be slope of two lines, then the lines are perpendicular, if $m_1m_2 = -1$

If one line is parallel to X-axis, then its perpendicular line is parallel to Y-axis

Equation of the line perpendicular to $ax + by + c = 0$ is taken as $bx - ay + \lambda = 0$

Straight line $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are right angle if $aa' + bb' = 0$

Conditions for Concurrence of Three Lines

1. Three lines are said to be concurrent, if they pass through a common point i.e. they meet at a point.
2. If three lines are concurrent, the point of intersection of two lines lies on the third line.
3. The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$, are concurrent iff $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

   This is the required condition of concurrence of three lines.

Distance of a Point from a Line

(i) The length of the perpendicular from a point (x_1, y_1) to a line $ax + by + c = 0$ is $\left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

(ii) Distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\dfrac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$.

Important Results

- The foot of the perpendicular (h, k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by $\dfrac{h - x_1}{a} = \dfrac{k - y_1}{b} = -\dfrac{(ax_1 + by_1 + c)}{a^2 + b^2}$
- Foot of perpendicular from (a, b) on $x - y = 0$ is $\left(\dfrac{a + b}{2}, \dfrac{a + b}{2}\right)$.
- Foot of perpendicular from (a, b) on $x + y = 0$ is $\left(\dfrac{a - b}{2}, \dfrac{b - a}{2}\right)$.
- Image (h, k) from (x_1, y_1) w.r.t. the line mirror $ax + by + c = 0$ is given by $\dfrac{h - x_1}{a} = \dfrac{k - y_1}{b} = \dfrac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$
- Area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ $a_1x + b_1y + d_1 = 0$; $a_2x + b_2y + d_2 = 0$ is $\left|\dfrac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$.

Equation of Internal and External Bisectors of Angles between Two Lines

The bisectors of the angles between two straight lines are the locus of a point which is equidistant from the two lines. The equation of the bisector of the angles between the lines

$$a_1x + b_1y + c_1 = 0 \quad \ldots\text{(i)}$$

and

$$a_2x + b_2y + c_2 = 0 \quad \ldots\text{(ii)}$$

are given by, $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

where,

(i) if $a_1a_2 + b_1b_2 > 0$, the positive sign for obtuse and negative sign for acute.

(ii) $a_1a_2 + b_1b_2 < 0$, negative sign for obtuse and positive sign for acute.

Equation of Family of Lines Through the Intersection of Two given Lines

The equation of the family of lines passing through the intersection of the lines

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

$$(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0 .$$

where λ is a parameter.

Important Properties

(i) The **position** of a **point** (x_1, y_1) and (x_2, y_2) relative to the line $ax + by + c = 0$

(a) If $\frac{(ax_1 + by_1 + c)}{ax_2 + by_2 + c} > 0$, then points lie on the same side.

(b) If $\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$, then the points lie on opposite side.

(ii) The equations of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with the given straight line $y = mx + c$ are

$$(y - y_1) = \frac{m \pm \tan\alpha}{1 \mp \tan\alpha}(x - x_1).$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The equation of the line, the reciprocals of whose intercepts on the axes are a and b, is given by:

(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $ax + by = 1$
(c) $ax + by = ab$ (d) $ax - by = 1$

2 The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is -1, is

(a) $\frac{x}{2} + \frac{y}{3} = -1, \frac{x}{-2} + \frac{y}{1} = -1$

(b) $\frac{x}{2} - \frac{y}{3} = -1, \frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} + \frac{y}{3} = 1, \frac{x}{2} + \frac{y}{1} = 1$

(d) $\frac{x}{2} - \frac{y}{3} = 1, \frac{x}{-2} + \frac{y}{1} = 1$

3 The equation of a line passing through $(-4,3)$ and this point divided the portion of line between axes in the ratio $5:3$ internally, is

(a) $9x + 20y + 96 = 0$ (b) $20x + 9y + 96 = 0$
(c) $9x - 20y + 96 = 0$ (d) $20x - 9y - 96 = 0$

4 A straight line through a fixed point $(2,3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(a) $3x + 2y = 6$ (b) $2x + 3y = xy$
(c) $3x + 2y = xy$ (d) $3x + 2y = 6xy$

5 If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is ➔ **JEE Mains 2013**

(a) -3 (b) $-\frac{3}{8}$ (c) $-\frac{3}{2}$ (d) $-\frac{3}{16}$

6 For which values of a and b, intercepts on axes by line $ax + by + 8 = 0$ are equal and opposite in sign of intercepts on axis by line $2x - 3y + 6 = 0$

(a) $a = \frac{8}{3}, b = -4$ (b) $a = \frac{-8}{3}, b = -4$

(c) $a = \frac{8}{3}, b = 4$ (d) $a = \frac{-8}{3}, b = 4$

7 Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is ➔ **JEE Mains 2013**

(a) $y - 3 = 0$ (b) $y + 3 = 0$ (c) $3y + 1 = 0$ (d) $3y - 1 = 0$

8 A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching X-axis, the equation of the reflected ray is

(a) $y = x + \sqrt{3}$ (b) $\sqrt{3}y = x - \sqrt{3}$
(c) $y = \sqrt{3}x - \sqrt{3}$ (d) $\sqrt{3}y = x - 1$

9 The range of values of α such that $(0, \alpha)$ lie on or inside the triangle formed by the lines $3x + y + 2 = 0$, $2x - 3y + 5 = 0$ and $x + 4y - 14 = 0$ is

(a) $1/2 \le \alpha \le 1$ (b) $5/3 \le \alpha \le 7/2$
(c) $5 \le \alpha \le 7$ (d) None of these

10 The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then, the set of all possible values of a is the interval

(a) $(-1, 1]$ (b) $(0, \infty)$
(c) $[1, \infty)$ (d) $(-1, \infty)$

11 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$, $y = nx + 1$ is equal to

(a) $\frac{|m+n|}{(m-n)^2}$ (b) $\frac{2}{|m+n|}$
(c) $\frac{1}{|m+n|}$ (d) $\frac{1}{|m-n|}$

12 If PS is the median of the triangle with vertices $P(2, 2)$, $Q(6,-1)$ and $R(7, 3)$, then equation of the line passing through $(1, -1)$ and parallel to PS is ➔ **JEE Mains 2014**
(a) $4x-7y-11=0$ (b) $2x+9y+7=0$
(c) $4x+7y+3=0$ (d) $2x-9y-11=0$

13 If $A(2,-1)$ and $B(6, 5)$ are two points, then the ratio in which the foot of the perpendicular from $(4,1)$ to AB divides it, is
(a) 8 : 15 (b) 5 : 8 (c) –5 : 8 (d) –8 : 5

14 The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then, the distance between L and K is
(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$ (c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$

15 The nearest point on the line $3x-4y=25$ from the origin is
(a) $(-4,5)$ (b) $(3,-4)$ (c) $(3,4)$ (d) $(3,5)$

16 Two sides of rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$ if its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus ➔ **JEE Mains 2016**
(a) $(-3,-9)$ (b) $(-3,-8)$
(c) $\left(\frac{1}{3},\frac{-8}{3}\right)$ (d) $\left(\frac{-10}{3},\frac{-7}{3}\right)$

17 If the lines $ax+2y+1=0$, $bx+3y+1=0$, $cx+4y+1=0$ are concurrent, then a, b, c are in
(a) AP (b) GP
(c) HP (d) None of these

18 For all real values of a and b lines $(2a+b)x+(a+3b)y+(b-3a)=0$ and $mx+2y+6=0$ are concurrent, then m is equal to
(a) -2 (b) -3 (c) -4 (d) -5

19 If p is the length of perpendicular from origin to the line which intercepts a and b on axes, then
(a) $a^2+b^2=p^2$ (b) $a^2+b^2=\frac{1}{p^2}$
(c) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{2}{p^2}$ (d) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$

20 A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y=-6$ at points P and Q, respectively. Then, the point O divides the segment PQ in the ratio
(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

21 If p is the length the perpendicular from the origin on the line $\frac{x}{a}+\frac{y}{b}=1$ and a^2, p^2, b^2 are in AP then $a^4+b^4=$
(a) 0 (b) 1
(c) data is inconsistent (d) None of these

22 If p and p' be perpendiculars from the origin upon the straight lines $x\sec\theta+y\operatorname{cosec}\theta=a$ and $x\cos\theta-y\sin\theta=a\cos 2\theta$ respectively, then the value of the expression $4p^2+p'^2$ is
(a) a^2 (b) $3a^2$ (c) $2a^2$ (d) $4a^2$

23 If p_1, p_2, p_3, are lengths of perpendiculars from points $(m^2, 2m)$, $(mm', m+m')$ and $(m'^2, 2m')$ to the line $x\cos\alpha+y\sin\alpha+\frac{\sin^2\alpha}{\cos\alpha}=0$, then p_1, p_2, p_3 are in
(a) AP (b) GP
(c) HP (d) None

24 The equation of bisector of acute angle between lines $3x-4y+7=0$ and $12x+5y-2=0$ is
(a) $21x+77y-101=0$ (b) $11x-3y+9=0$
(c) $31x+77y+101=0$ (d) $11x-3x-9=0$

25 A ray of light coming along the line $3x+4y-5=0$ gets reflected from the line $ax+by-1=0$ and goes along the line $5x-12y-10=0$, then
(a) $a=\frac{64}{115}, b=\frac{112}{15}$ (b) $a=-\frac{64}{115}, b=\frac{8}{115}$
(c) $a=\frac{64}{115}, b=-\frac{8}{115}$ (d) $a=-\frac{64}{115}, b=-\frac{8}{115}$

26 The sides BC, CA, AB of ΔABC are respectively $x+2y=1$, $3x+y+5=0$, $x-y+2=0$. The altitude through B is
(a) $x-3y+1=0$ (b) $x-3y+4=0$
(c) $3x-y+4=0$ (d) $x-y+2=0$

27 A variable straight line drawn through the point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the coordinates axes at A and B, the locus of the mid-point of AB is
(a) $2xy(a+b)=ab(x+y)$
(b) $2xy(a-b)=ab(x-y)$
(c) $2xy(a+b)=ab(x-y)$
(d) None of the above

28 The base BC of ΔABC is bisected at the point (p,q) and equations of AB and AC are $px+qy=1$ and $qx+py=1$ respectively, then equation of the median passing through A is
(a) $(2pq-1)(px+qy-1)=(p^2+q^2-1)(qx+py-1)$
(b) $(2pq+1)(px+qy-1)=(p^2+q^2-1)(qx+py-1)$
(c) $(2pq+1)(px+qy-1)=(p^2+q^2+1)(qx+py-1)$
(d) None of the above

29 If P is a point (x, y) on the line $y = -3x$ such that P and the point $(3, 4)$ are on the opposite sides of the line $3x - 4y = 8$, then

(a) $x > \frac{8}{15}, y < -\frac{8}{5}$ (b) $x > \frac{8}{5}, y < \frac{8}{15}$

(c) $x = \frac{8}{15}, y = -\frac{8}{5}$ (d) None of these

30 If $P\left(1 + \frac{\alpha}{\sqrt{2}}, 2 + \frac{\alpha}{\sqrt{2}}\right)$ be any point on a line, then the range of values of α for which the point P lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$ is

(a) $-\frac{4\sqrt{2}}{3} < \alpha < \frac{5\sqrt{2}}{6}$ (b) $0 < \alpha < \frac{5\sqrt{2}}{6}$

(c) $\frac{-4\sqrt{2}}{3} < \alpha < 0$ (d) None of these

31 If (a, a^2) falls inside the angle made by the lines $x - 2y = 0, x > 0$ and $y = 3x (x > 0)$, then a belongs to :

(a) $(0, 1/2)$ (b) $(3, \infty)$

(c) $(1/2, 3)$ (d) $(-3, -1/2)$

32 The lines passing through $(3, -2)$ and inclined at angle $60°$ with $\sqrt{3}x + y = 1$ is

(a) $y + 2 = 0$ (b) $x + 2 = 0$

(c) $x + y = 2$ (d) $x - y = \sqrt{3}$

Directions (Q. Nos. 33-36) *Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

33 **Statement I** Consider the points $A(0, 1)$ and $B(2, 0)$ and P be a point on the line $4x + 3y + 9 = 0$, then coordinates of P such that $|PA - PB|$ is maximum, is $\left(\frac{-12}{5}, \frac{17}{5}\right)$.

Statement II $|PA - PB| \le |AB|$

34 **Statement I** If point of intersection of the lines $4x + 3y = \lambda$ and $3x - 4y = \mu, \forall \lambda, \mu \in R$ is (x_1, y_1), then the locus of (λ, μ) is $x + 7y = 0, \forall x_1 = y_1$.

Statement II If $4\lambda + 3\mu > 0$ and $3\lambda - 4\mu > 0$, then (x_1, y_1) is in first quadrant.

35 Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers.

Statement I If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.

Statement II $\theta_1 = \theta_2$ for all c_2 and c_3. ➜ **JEE Mains 2013**

36 **Statement I** Each point on the line $y - x + 12 = 0$ is equidistant from the lines $4y + 3x - 12 = 0$, $3y + 4x - 24 = 0$.

Statement II The locus of a point which is equidistant from two given lines is the angular bisector of the two lines.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A line $4x + y = 1$ through the point $A(2, -7)$ meets the line BC whose equation is $3x - 4y + 1 = 0$. The equation to the line AC, so that $AB = AC$ is

(a) $52x - 89y - 519 = 0$ (b) $52x + 89y - 519 = 0$

(c) $52x - 89y + 519 = 0$ (d) $52x + 89y + 519 = 0$

2 If the lines $y = m_r x, r = 1, 2, 3$ cut off equal intercepts on the transversal $x + y = 1$, then $1 + m_1, 1 + m_2, 1 + m_3$ are in:

(a) AP (b) GP

(c) HP (d) None of these

3 In triangle ABC, equation of the right bisectors of the sides AB and AC are $x + y = 0$ and $x - y = 0$ respectively. If $A \equiv (5, 7)$ then equation of side BC is

(a) $7y = 5x$ (b) $5x = y$ (c) $5y = 7x$ (d) $5y = x$

4 Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq units. Then, the orthocentre of this triangle is at the point

(a) $\left(2, -\frac{1}{2}\right)$ (b) $\left(1, \frac{3}{4}\right)$ (c) $\left(1, -\frac{3}{4}\right)$ (d) $\left(2, \frac{1}{2}\right)$

5 A variable line through the point (p, q) cuts the x and y axes at A and B respectively. The lines through A and B parallel to Y – axis and the X – axis respectively meet at P. If the locus of P is $3x + 2y - xy = 0$, then

(a) $p = 2, q = 3$ (b) $p = 3, q = 2$

(c) $p = -2, q = -3$ (d) $p = -3, q = -2$

6 If the three lines $x - 3y = p, ax + 2y = q$ and $ax + y = r$ from a right angled triangle, then ➜ **JEE Mains 2013**

(a) $a^2 - 9a + 18 = 0$

(b) $a^2 - 6a - 12 = 0$

(c) $a^2 - 6a - 18 = 0$

(d) $a^2 - 9a + 12 = 0$

7 A light ray emerging from the point source placed at $P(1, 3)$ is reflected at a point Q in the axis of x. If the reflected ray passes through the point $R(6, 7)$, then the abscissa of Q is **→ JEE Mains 2013**

(a) 1 (b) 3

(c) $\frac{7}{2}$ (d) $\frac{5}{2}$

8 Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a

(a) straight line parallel to X-axis **→ JEE Mains 2015**

(b) straight line parallel to Y-axis

(c) circle of radius $\sqrt{2}$

(d) circle of radius $\sqrt{3}$

9 A square of side a lies above the X-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(\text{where, } 0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction X-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(b) $y(\cos\alpha + \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

10 A variable line 'L' is drawn through $O(0,0)$ to meet the line $L_1: y - x - 10 = 0$ and $L_2: y - x - 20 = 0$ at the point A and B respectively. A point P is taken on 'L' such that $\frac{2}{OP} = \frac{1}{OA} + \frac{1}{OB}$. Locus of '$P$' is

(a) $3x + 3y = 40$ (b) $3x + 3y + 40 = 0$

(c) $3x - 3y = 40$ (d) $3y - 3x = 40$

11 Consider the family of lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belongs to both families is

(a) $25x - 62y + 86 = 0$ (b) $62x - 25y + 86 = 0$

(c) $25x - 62y = 86$ (d) $5x - 2y - 7 = 0$

12 Equation of the straight line which belongs to the system of straight lines $a(2x + y - 3) + b(3x + 2y - 5) = 0$ and is farthest from the point $(4, -3)$ is

(a) $4x + 11y - 15 = 0$ (b) $3x - 4y + 1 = 0$

(c) $7x + y - 8 = 0$ (d) None of these

13 One diagonal of a square is along the line $8x - 15y = 0$ and its one vertex $(1,2)$, then equations of a side passing through this vertex are

(a) $7x + 23y - 53 = 0$, $23x - 7y - 9 = 0$

(b) $7x - 23y - 53 = 0$, $23x + 7y - 9 = 0$

(c) $7x + 23y + 53 = 0$, $23x - 7y + 9 = 0$

(d) $7x + 23y + 53 = 0$, $23x + 7y + 9 = 0$

14 The equations of the straight lines through $(-2, -7)$ and having intercept of length 3 between the lines $4x + 3y = 12$ and $4x + 3y = 3$ is

(a) $7x - 24y - 182 = 0$ (b) $7x + 24y + 182 = 0$

(c) $7x + 24y - 182 = 0$ (d) None of these

15 If angle between lines $ax + by + p = 0$ and $x\cos\alpha + y\sin\alpha = p$ is $\frac{\pi}{4}$ and these lines with other line $x\sin\alpha - y\cos\alpha = 0$ are concurrent, then $a^2 + b^2 =$

(a) 1 (b) 2 (c) 3 (d) 4

16 Straight lines $y = mx + c_1$ and $y = mx + c_2$, where $m \in R^+$ meet the X-axis at A_1 and A_2 respectively and Y-axis at B_1 and B_2 respectively. It is given that point A_1, A_2, B_1 and B_2 are concyclic. Locus of intersection of lines A_1B_2 and A_2B_1 is

(a) $y = x$ (b) $y + x = 0$

(c) $xy = 1$ (d) $xy + 1 = 0$

ANSWERS

SESSION 1	**1** (b)	**2** (d)	**3** (c)	**4** (c)	**5** (d)	**6** (d)	**7** (d)	**8** (b)	**9** (b)	**10** (c)
	11 (d)	**12** (b)	**13** (b)	**14** (d)	**15** (b)	**16** (c)	**17** (a)	**18** (a)	**19** (d)	**20** (b)
	21 (c)	**22** (a)	**23** (b)	**24** (b)	**25** (c)	**26** (b)	**27** (a)	**28** (a)	**29** (a)	**30** (a)
	31 (c)	**32** (a)	**33** (d)	**34** (b)	**35** (a)	**36** (a)				
SESSION 2	**1** (d)	**2** (c)	**3** (a)	**4** (d)	**5** (a)	**6** (a)	**7** (d)	**8** (c)	**9** (b)	**10** (d)
	11 (d)	**12** (b)	**13** (a)	**14** (b)	**15** (b)	**16** (b)				

Hints and Explanations

SESSION 1

1 If a_1, b_1 are intercepts of the line on the axes, then

$$1/a_1 = a, 1/b_1 = b$$

$$\Rightarrow \quad a_1 = 1/a, b_1 = 1/b$$

∴ Equation of the line is

$$x/a_1 + y/b_1 = 1 \text{ or } ax + by = 1$$

2 Let x –intercept $= a$

and y –intercept $= b$

Since, $a + b = -1 \Rightarrow b = -(a+1)$

∴ Equation of line is $\frac{x}{a} - \frac{y}{a+1} = 1$

Clearly,

$$\frac{4}{a} - \frac{3}{a+1} = 1 \Rightarrow \frac{4a + 4 - 3a}{a(a+1)} = 1$$

$$\Rightarrow a + 4 = a^2 + a \Rightarrow a = \pm 2$$

Hence, equation of line is

$$\frac{x}{2} - \frac{y}{3} = 1 \text{ or } \frac{x}{-2} + \frac{y}{1} = 1.$$

3 Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

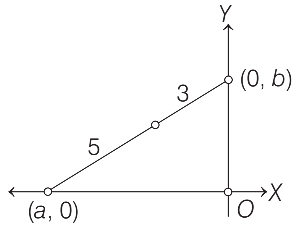
∴ According to given condition, we have

$$C(-4, 3) \equiv C\left(\frac{3a}{8}, \frac{5b}{8}\right)$$

$$\Rightarrow \quad a = -\frac{32}{3} \text{ and } b = \frac{24}{5}$$

∴ Equation of line is

$$-\frac{3x}{32} + \frac{5y}{24} = 1$$

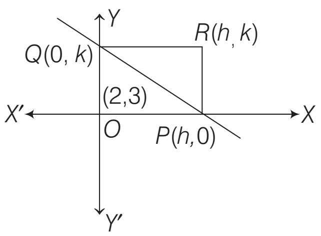


$$\Rightarrow -9x + 20y - 96 = 0$$

$$\Rightarrow \quad 9x - 20y + 96 = 0$$

4 Equation of PQ is

$$\frac{x}{h} + \frac{y}{k} = 1$$



Since, it is passes through the points (2,3)

∴ $\frac{2}{h} + \frac{3}{k} = 1 \Rightarrow 2k + 3h = hk$

So, locus is $3x + 2y = xy$

5 We have, $\frac{x}{4} + \frac{y}{3} = 1$

For line L, x-intercept $= 2 \times 4 = 8$

y-intercept $= \frac{1}{2} \times 3 = \frac{3}{2}$

∴ Line L is $\frac{x}{8} + \frac{y}{3/2} = 1$, Slope, $m = -\frac{3}{16}$

6 $ax + by + 8 = 0$

$$\Rightarrow \quad ax + by = -8$$

$$\Rightarrow \quad \frac{x}{-\frac{8}{a}} + \frac{y}{-\frac{8}{b}} = 1 \text{ (intercept form)}$$

Also, $\quad 2x - 3y = -6 \Rightarrow -\frac{x}{3} + \frac{y}{2} = 1$

According to given condition, we have

$$-\frac{8}{a} = -(-3) \text{ and } -\frac{8}{b} = -2$$

$$\Rightarrow \quad a = -\frac{8}{3} \text{ and } b = 4$$

7 On solving both the equations, we get

$$\frac{8y}{3} + y^2 = 1$$

$$\Rightarrow \quad 3y^2 + 8y - 3 = 0$$

$$\Rightarrow \quad (3y - 1)(y + 3) = 0$$

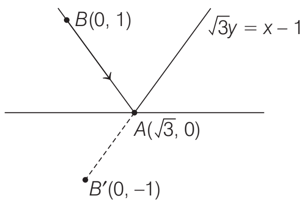
$$\Rightarrow \quad y = -3, \frac{1}{3} \text{ here } y \neq -3$$

At $y = \frac{1}{3}$, $x = \pm 2\sqrt{\frac{2}{3}}$

So, the points of intersection are $\left(2\sqrt{\frac{2}{3}}, \frac{1}{3}\right)$ and $\left(-2\sqrt{\frac{2}{3}}, \frac{1}{3}\right)$.

From option (d); $3y - 1 = 0$ is the required equation which satisfied the intersection points.

8 Take any point $B(0,1)$ on given line.



Equation of AB' is

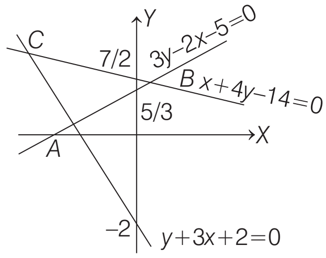
$$y - 0 = \frac{-1-0}{0-\sqrt{3}}(x - \sqrt{3})$$

$$\Rightarrow \quad -\sqrt{3}y = -x + \sqrt{3}$$

$$\Rightarrow \quad x - \sqrt{3}y = \sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$$

9 From figure, for $(0, \alpha)$ to be inside or on the triangle,



$$\frac{5}{3} \le \alpha \le \frac{7}{2}$$

10 As $x + y = |a|$ and $ax - y = 1$.

Intersect in first quadrant.

So, x and y-coordinates are positive.

∴ $x = \frac{1+|a|}{1+a} \ge 0$ and $y = \frac{a|a|-1}{a+1} \ge 0$

$$\Rightarrow \quad 1 + a \ge 0 \text{ and } a|a| - 1 \ge 0$$

$$\Rightarrow \quad a \ge -1 \text{ and } a|a| \ge 1 \quad \ldots\text{(i)}$$

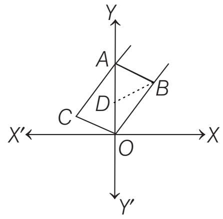
If $-1 \le a < 0 \Rightarrow -a^2 \ge 1$ [not possible]

If $a \ge 0 \Rightarrow a^2 \ge 1 \Rightarrow a \ge 1 \Rightarrow a \in [1, \infty)$

11 Let lines $OB: y = mx$, $CA: y = mx + 1$

$BA: y = nx + 1$ and $OC: y = nx$

So, the point of intersection B of OB and AB has x-coordinate $\frac{1}{m-n}$.



Now, area of a parallelogram

$OBAC = 2 \times$ Area of ΔOBA

$$= 2 \times \frac{1}{2} \times OA \times DB = 2 \times \frac{1}{2} \times \frac{1}{m-n}$$

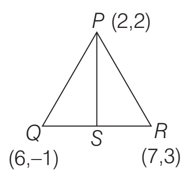
$$= \frac{1}{m-n} = \frac{1}{|m-n|}$$

depending upon whether $m > n$ or $m < n$.

12 Coordinate of

$$S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$$

[since, S is mid-point of line QR]



Slope of the line PS is $\frac{-2}{9}$.

Required equation of line passes through $(1, -1)$ and parallel to PS is

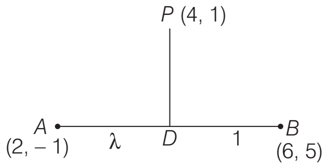
$$y + 1 = \frac{-2}{9}(x - 1)$$

$\Rightarrow \quad 2x + 9y + 7 = 0$

13 Let $P(4,1)$ and $PD \perp AB$.

Equation of AB is $3x - 2y - 8 = 0$

$\therefore$ Equation of PD is $2x + 3y - 11 = 0$



Let line AB is divided by PD in the ratio $\lambda : 1$, then intersecting point $D\left(\frac{6\lambda + 2}{\lambda + 1}, \frac{5\lambda - 1}{\lambda + 1}\right)$ lies on $2x + 3y - 11 = 0$.

$\Rightarrow \quad 2\left(\frac{6\lambda + 2}{\lambda + 1}\right) + 3\left(\frac{5\lambda - 1}{\lambda + 1}\right) - 11 = 0$

$\Rightarrow \quad 16\lambda - 10 = 0 \Rightarrow \lambda : 1 = 5 : 8$

14 Since, the line L is passing through the point $(13, 32)$.

Therefore, $\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20$

The line K is parallel to the line L, its equation must be

$$\frac{x}{5} - \frac{y}{20} = a \quad \text{or} \quad \frac{x}{5a} - \frac{y}{20a} = 1$$

On comparing with $\frac{x}{c} + \frac{y}{3} = 1$, we get

$20a = -3, c = 5a$

$a = \frac{-3}{20}$ and $c = 5 \times \frac{-3}{20} = \frac{-3}{4}$

Hence, the distance between lines

$$= \frac{|a - 1|}{\sqrt{\frac{1}{25} + \frac{1}{400}}} = \frac{\left|\frac{-3}{20} - 1\right|}{\sqrt{\frac{17}{400}}} = \frac{23}{\sqrt{17}}$$

15 The desired point is the foot of the perpendicular from the origin on the line $3x - 4y = 25$.

The equation of a line passing through the origin and perpendicular to $3x - 4y = 25$ is $4x + 3y = 0$.

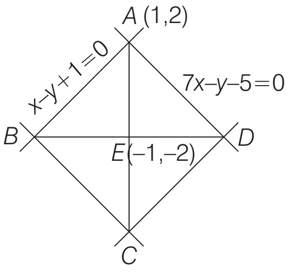
Solving these two equations we get $x = 3, y = -4$.

Hence, the nearest point on the line from the origin is $(3, -4)$.

16 Coordinates of $A \equiv (1,2)$

$\therefore \quad$ Slope of $AE = 2$

$\Rightarrow \quad$ Slope of $BD = -\frac{1}{2}$



$\Rightarrow \quad$ Equation of BD is $\frac{y + 2}{x + 1} = \frac{-1}{2}$

$\Rightarrow \quad x + 2y + 5 = 0$

$\therefore \quad$ Coordinates of $D = \left(\frac{1}{3}, \frac{-8}{3}\right)$

17 It is given that the lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$, $cx + 4y + 1 = 0$ are concurrent

$$\therefore \quad \begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad -a + 2b - c = 0 \Rightarrow 2b = a + c$

$\therefore \; a, b, c$ are in AP.

18 Given equations,

$(2a + b)x + (a + 3b)y + (b - 3a) = 0$

and $mx + 2y + 6 = 0$ are concurrent for all real values of a and b, so they must represent the same line for some values of a and b. Therefore, we get

$$\frac{2a + b}{m} = \frac{(a + 3b)}{2} = \frac{b - 3a}{6}$$

On taking last two ratios,

$$\frac{a + 3b}{2} = \frac{-3a + b}{6} \Rightarrow b = -\frac{3}{4}a$$

On taking first two ratios,

$$m = \frac{2(2a + b)}{a + 3b} = \frac{2\{2a - (3/4)a\}}{a + 3(-3/4)a}$$

$$= -\frac{10}{5} = -2$$

19 The length of perpendicular from $(0,0)$ to line $\frac{x}{a} + \frac{y}{b} = 1$, is $p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$

$$\therefore \quad \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$$

20 Now, distance of origin from $4x + 2y - 9 = 0$ is

$$\frac{|-9|}{\sqrt{4^2 + 2^2}} = \frac{9}{\sqrt{20}}$$

and distance of origin from $2x + y + 6 = 0$ is

$$\frac{|6|}{\sqrt{2^2 + 1^2}} = \frac{6}{\sqrt{5}}$$

$\therefore$ Required ratio $= \frac{9/\sqrt{20}}{6/\sqrt{5}} = \frac{3}{4}$

21 $p = \frac{1}{\sqrt{(1/a^2) + (1/b^2)}} = \frac{ab}{\sqrt{a^2 + b^2}}$

a^2, p^2, b^2 are in AP.

$\Rightarrow \quad \frac{2a^2b^2}{a^2 + b^2} = a^2 + b^2$

$\Rightarrow \quad a^4 + b^4 = 0$ i.e. $a = b = 0$

This is impossible, therefore given information is inconsistent.

22 Since, $p =$ length of the perpendicular from (0,0) on $x\sec\theta + y\operatorname{cosec}\theta = a$

$\therefore \quad p = \frac{a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} = \frac{a\sin 2\theta}{2}$

$\Rightarrow \quad 2p = a\sin 2\theta \quad$ ...(i)

Also, p' = length of perpendicular from (0,0) on $x\cos\theta - y\sin\theta = a\cos 2\theta$

$\therefore \quad p' = \frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}$

$= a\cos 2\theta \quad$ ...(ii)

On squaring and adding Eqs. (i), (ii), we get

$$4p^2 + p'^2 = a^2$$

23 $p_1 = \left|m^2\cos\alpha + 2m\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right|$

$p_2 = \left|mm'\cos\alpha + (m + m')\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right|$

$p_3 = \left|m'^2\cos\alpha + 2m'\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right|$

$$p_2 = \frac{mm'\cos^2\alpha + (m + m')\sin\alpha\cos\alpha + \sin^2\alpha}{\cos\alpha}$$

$$p_1 = \frac{(m\cos\alpha + \sin\alpha)^2}{\cos\alpha}$$

$$p_3 = \frac{(m'\cos\alpha + \sin\alpha)^2}{\cos\alpha}$$

$\therefore p_2 = \sqrt{p_1}\sqrt{p_3} \Rightarrow p_2^2 = p_1 p_3$

Hence, p_1, p_2 and p_3 are in GP.

24 Make constant terms of both equation positive.

$$3x - 4y + 7 = 0$$

and $\quad -12x - 5y + 2 = 0$

Since, $a_1a_2 + b_1b_2 = -36 + 20 < 0$

$\therefore$ Bisector of acute angle is given by with positive sign

$$\frac{3x - 4y + 7}{\sqrt{9 + 16}} = +\left(\frac{-12x - 5y + 2}{\sqrt{144 + 25}}\right)$$

$\Rightarrow \quad 39x - 52y + 91 = -60x - 25y + 10$

$\Rightarrow \quad 99x - 27y + 81 = 0$

$\therefore \quad 11x - 3y + 9 = 0$

25 Equation of bisectors of the given lines are

$$\left(\frac{3x + 4y - 5}{\sqrt{3^2 + 4^2}}\right) = \pm\left(\frac{5x - 12y - 10}{\sqrt{5^2 + (-12)^2}}\right)$$

$\therefore \quad (39x + 52y - 65) = \pm(25x - 60y - 50)$

$$\Rightarrow \quad 14x + 112y - 15 = 0$$
$$\text{or} \quad 64x - 8y - 115 = 0$$
$$\Rightarrow \quad \frac{14}{15}x + \frac{112}{15}y - 1 = 0$$
$$\text{or} \quad \frac{64}{115}x - \frac{8}{115}y - 1 = 0$$
$$\therefore \quad a = \frac{14}{15}, b = \frac{112}{15}$$
$$\text{or} \quad a = \frac{64}{115}, b = -\frac{8}{115}$$

26 The required line is given by

$$x + 2y - 1 + \lambda(x - y + 2) = 0 \quad \ldots(\text{i})$$

It is perpendicular to $3x + y + 5 = 0$

$$\therefore \; 3(1+\lambda) + 2 - \lambda = 0 \Rightarrow \lambda = -\frac{5}{2}$$

From Eq. (i), we get

$$x - 3y + 4 = 0$$

27 The intersection of given lines is

$$\frac{x}{a} + \frac{y}{b} - 1 + \lambda\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0$$

meets the coordinate axes at

$$A\left[\frac{1+\lambda}{\frac{1}{a} + \frac{\lambda}{b}}, 0\right] \text{ and } B\left[0, \frac{1+\lambda}{\frac{1}{b} + \frac{\lambda}{a}}\right]$$

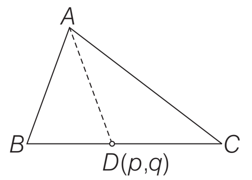
The mid-point of AB is given by

$$2x = \frac{1+\lambda}{\frac{1}{a} + \frac{\lambda}{b}}, \; 2y = \frac{1+\lambda}{\frac{1}{b} + \frac{\lambda}{a}}$$

$$\Rightarrow \; (1+\lambda)\left[\frac{1}{x} + \frac{1}{y}\right] = 2\left[\frac{1}{a} + \frac{\lambda}{b}\right] + 2\left[\frac{1}{b} + \frac{\lambda}{a}\right] = 2(1+\lambda)\left[\frac{1}{a} + \frac{1}{b}\right]$$

$$\therefore \; (x + y)\,ab = 2xy\,(a + b)$$

28



Equation of line AB

$$px + qy = 1$$

Equation of line AC

$$qx + py = 1$$

The equation of line passing through the intersection point of above lines is

$$px + qy - 1 + \lambda(qx + py - 1) = 0$$

which passes through (p, q)

$$\therefore \; p^2 + q^2 - 1 + \lambda(pq + pq - 1) = 0 \quad \ldots(\text{i})$$

$$\Rightarrow \quad \lambda = -\frac{p^2 + q^2 - 1}{2pq - 1}$$

$\therefore$ Substituting the value of λ in Eq. (i), of the line through A is $(px + qy - 1)$

$$-\frac{p^2 + q^2 - 1}{2pq - 1}(qx + py - 1) = 0$$

$$\Rightarrow \; (2pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$$

29 Let $L_1 = 3x - 4y - 8$

At $(3, 4)$, $L_1 = 9 - 16 - 8 = -15 < 0$

For the point $P(x, y)$, we should have $L_1 > 0$.

$$\Rightarrow \quad 3x - 4y - 8 > 0 \qquad [\because y = -3x]$$

$$\Rightarrow \quad 3x - 4(-3x) - 8 > 0$$

$$[\because P(x, y) \text{ lies on } y = -3x]$$

$$\Rightarrow \quad x > 8/15 \text{ and } -y - 4y - 8 > 0$$

$$\Rightarrow \quad y < -8/5$$

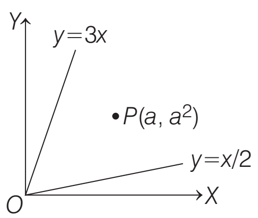
30 Since, $P\left(1 + \frac{\alpha}{\sqrt{2}}, 2 + \frac{\alpha}{\sqrt{2}}\right)$ lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$, therefore

$$\frac{\left(1 + \frac{\alpha}{\sqrt{2}}\right) + 2\left(2 + \frac{\alpha}{\sqrt{2}}\right) - 1}{2\left(1 + \frac{\alpha}{\sqrt{2}}\right) + 4\left(2 + \frac{\alpha}{\sqrt{2}}\right) - 15} < 0$$

$$\Rightarrow \quad \frac{4 + \frac{3\alpha}{\sqrt{2}}}{-5 + \frac{6\alpha}{\sqrt{2}}} < 0 \Rightarrow \frac{\left(\alpha + \frac{4\sqrt{2}}{3}\right)}{\left(\alpha - \frac{5\sqrt{2}}{6}\right)} < 0$$

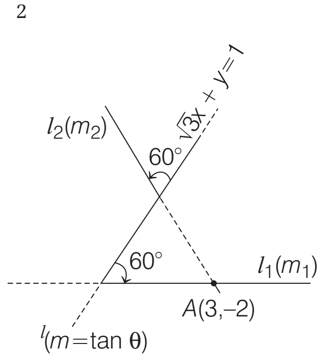
$$\therefore \quad \frac{-4\sqrt{2}}{3} < \alpha < \frac{5\sqrt{2}}{6}$$

31 Clearly, $a^2 - \frac{a}{2} > 0$, $a^2 - 3a < 0$



$$\Rightarrow \quad \frac{1}{2} < a < 3.$$

32



Let l_1 and l_2 are the equations of the lines inclined at an angle of 60° with the line l.

$\therefore$ Slopes of lines are $\tan(\theta \pm 60^\circ)$

Now, equation of lines l_1 and l_2 are

$$y + 2 = \tan(\theta \pm 60^\circ)(x - 3)$$

$$\Rightarrow \quad y + 2 = \frac{\tan\theta \pm \tan 60^\circ}{1 \mp \tan\theta \tan 60^\circ}(x - 3)$$

$$\Rightarrow \quad y + 2 = \frac{-\sqrt{3} \pm \sqrt{3}}{1 \mp (-\sqrt{3})\sqrt{3}}(x - 3)$$

$$\Rightarrow \quad y + 2 = 0$$

$$\text{or} \quad y + 2 = \sqrt{3}(x - 3)$$

33 Equation of line AB is

$$y - 1 = \frac{0 - 1}{2 - 0}(x - 0)$$

$$\Rightarrow \quad x + 2y - 2 = 0$$

Here, $\quad |PA - PB| \le |AB|$

Thus, for $|PA - PB|$ to be maximum, A, B and P must be collinear.

34 The point of intersection of lines $4x + 3y = \lambda$ and $3x - 4y = \mu$ is

$$x_1 = \frac{4\lambda + 3\mu}{25}$$

$$\text{and} \quad y_1 = \frac{3\lambda - 4\mu}{25}$$

$$\because \quad x_1 = y_1 \Rightarrow \frac{4\lambda + 3\mu}{25} = \frac{3\lambda - 4\mu}{25}$$

$$\Rightarrow \quad \lambda + 7\mu = 0$$

Hence, locus of a point (λ, μ) is $x + 7y = 0$.

35 Here, angle between the lines $2x + 3y + c_1 = 0$ and $-x + 5 + c_2 = 0$ is θ_1.

$$\therefore \quad \tan\theta_1 = \left|\frac{1/5 + 2/3}{1 - 2/15}\right| = \left|\frac{13/15}{13/15}\right| = 1 = \tan 45^\circ$$

$$\Rightarrow \quad \theta_1 = 45^\circ$$

Also, the angle between the lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$ is θ_2.

$$\therefore \quad \tan\theta_2 = \left|\frac{1/5 + 2/3}{1 - 2/15}\right| = \left|\frac{13/15}{13/15}\right| = 1 = \tan 45^\circ$$

$$\Rightarrow \quad \theta_2 = 45^\circ$$

Here, we observe that the value of c_1, c_2 and c_3 is not depend on measuring the angle between the lines.

So, c_2 and c_3 are proportional or for all c_2 and c_3 $\theta_1 = \theta_2$

36 Equation of bisector of $4y + 3x - 12 = 0$ and $3y + 4x - 24 = 0$ is

$$\frac{4y + 3x - 12}{\sqrt{16 + 9}} = \pm \frac{3y + 4x - 24}{\sqrt{9 + 16}}$$

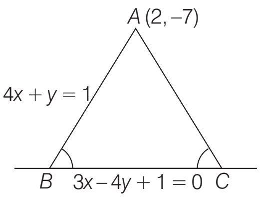
$$\Rightarrow \quad y - x + 12 = 0$$

$$\text{and } 7y + 7x - 36 = 0$$

So, the line $y - x + 12 = 0$ is the angular bisector.

SESSION 2

1



Let m be the slope of AC, then

$$\tan B = \tan C \Rightarrow \frac{\frac{3}{4}+4}{1-3} = \frac{m-\frac{3}{4}}{1+\frac{3m}{4}}$$

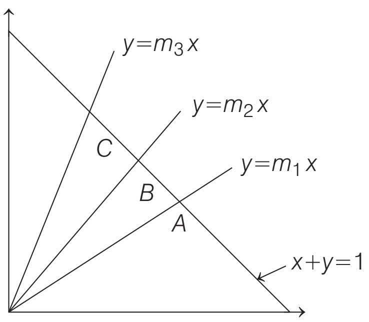
$$\Rightarrow \quad -\frac{19}{8} = \frac{4m-3}{4+3m} \Rightarrow m = -\frac{52}{89}$$

$\therefore$ Equation of AC is

$$y+7 = -\frac{52}{89}(x-2)$$

$$\Rightarrow \quad 52x + 89y + 519 = 0$$

2 $AB = BC \Rightarrow B$ is mid-point of AC.



Clearly, $A = \left(\frac{1}{m_1+1}, \frac{m_1}{m_1+1}\right)$

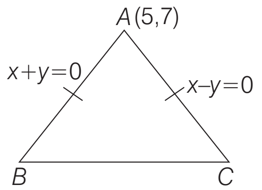
$$B = \left(\frac{1}{m_2+1}, \frac{m_2}{m_2+1}\right)$$

$$C = \left(\frac{1}{m_3+1}, \frac{m_3}{m_3+1}\right)$$

Now, $\frac{2}{m_2+1} = \frac{1}{m_1+1} + \frac{1}{m_3+1}$

$\therefore$ m_1+1, m_2+1, m_3+1 are in HP.

3 Clearly, $B =$ reflection of $A(5,7)$ in the line $x + y = 0$



$$\Rightarrow \quad B \equiv (-7, -5)$$

$C =$ reflection of $A(5,7)$ in the line $x - y = 0$

$$\Rightarrow \quad C = (7,5)$$

Equation of BC is $7y = 5x$.

4 Given, vertices of triangle are $(k, -3k)$, $(5, k)$ and $(-k, 2)$.

$$\therefore \quad \frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 28$$

$$\Rightarrow \quad \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

$$\Rightarrow \quad k(k-2) + 3k(5+k) + 1(10+k^2) = \pm 56$$

$$\Rightarrow \quad k^2 - 2k + 15k + 3k^2 + 10 + k^2 = \pm 56$$

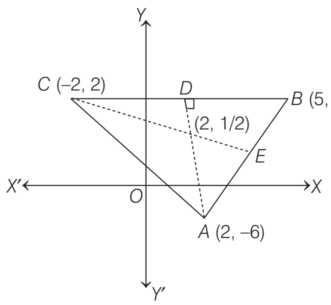
$$\Rightarrow \quad 5k^2 + 13k + 10 = \pm 56$$

$$\Rightarrow \quad 5k^2 + 13k - 66 = 0$$

$$\text{or} \quad 5k^2 + 13k - 46 = 0$$

$$\Rightarrow \quad k = 2 \qquad [\because k \in I]$$

Thus, the coordinates of vertices of triangle are $A(2, -6)$, $B(5, 2)$ and $C(-2, 2)$.



Now, equation of altitude from vertex A is $y-(-6) = \frac{-1}{\left(\frac{2-2}{-2-5}\right)}(x-2)$

$$\Rightarrow \quad x = 2 \qquad \ldots(\text{i})$$

Equation of altitude from vertex C is

$$y - 2 = \frac{-1}{\left[\frac{2-(-6)}{5-2}\right]}[x-(-2)]$$

$$\Rightarrow \quad 3x + 8y - 10 = 0 \qquad \ldots(\text{ii})$$

On solving Eqs. (i) and (ii), we get $x = 2$ and $y = \frac{1}{2}$.

$\therefore$ Orthocentre $= \left(2, \frac{1}{2}\right)$

5 Let the equation of the variable line be $y - q = m(x-p)$, where m is a variable

Then, $A \equiv \left(\frac{mp-q}{m}, 0\right)$

and $B \equiv (0, q - mp)$

Let $P \equiv (x, y)$, then

$$x = \frac{mp-q}{m} \text{ or } m(p-x) = q \qquad \ldots(\text{i})$$

and $y = q - mp$ or $mp = q - y$ $\qquad \ldots(\text{ii})$

On eliminating m from Eqs. (i) and (ii), we get

$$\frac{p-x}{q} = \frac{p}{q-y}$$

$$\Rightarrow \quad pq - qx - py + xy = pq$$

$$\Rightarrow \quad py + qx = xy$$

$$\text{or} \quad \frac{p}{x} + \frac{q}{y} = 1$$

This is the locus of P.

But locus of P is $3x + 2y = xy$ (given)

$$\text{or} \quad \frac{2}{x} + \frac{3}{y} = 1$$

$$\therefore \quad p = 2$$

$$\text{and} \quad q = 3$$

6 ***Case* I** Let line $l_1 \equiv x - 3y = p$ and $l_2 \equiv ax + 2y = p$ are perpendicular, then

$$\frac{1}{3} \times -\frac{a}{2} = -1$$

$$\Rightarrow \quad a = 6$$

***Case* II** Let line $l_2 \equiv ax + 2y = p$ and $l_3 \equiv ax + y = r$ are perpendicular, then

$$\frac{-a}{2} \times -a = -1$$

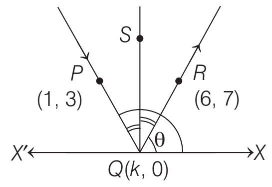
$$\Rightarrow \quad a^2 = -2 \quad [\text{not possible}]$$

***Case* III** Let line $l_3 \equiv ax + y = r$ and $l_1 \equiv x - 3y = p$ are perpendicular, then $-a \times \frac{1}{3} = -1 \Rightarrow a = 3$. So, formation of quadratic equation in a, whose roots are 3 and 6, is

$$a^2 - (6+3)a + (6 \cdot 3) = 0$$

$$\Rightarrow \quad a^2 - 9a + 18 = 0$$

7 Here, $QS \perp OX$



It means QS bisect the $\angle PQR$.

Then, $\angle PQS = \angle RQS$

$$\Rightarrow \quad \angle RQX = \angle PQO = \theta \qquad [\text{let}]$$

$$\Rightarrow \quad \angle XQP = 180° - \theta$$

Slope of $QR = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{7-0}{6-k} \qquad \ldots(\text{i})$$

Slope of $QP = \tan(180° - \theta) = -\tan\theta$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{1-k} \qquad \ldots(\text{ii})$$

$\therefore$ From Eqs. (i) and (ii),

$$\frac{7}{6-k} = -\frac{3}{1-k}$$

$$\Rightarrow \quad 7 - 7k = -18 + 3k$$

$$10k = 25$$

$$\Rightarrow \quad k = \frac{5}{2}$$

Hence, the coordinate of Q is $\left(\frac{5}{2}, 0\right)$.

8 **Key Idea** First of all find the point of inter- section of the lines $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$ (say A). Now, the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$ is the perpendicular bisector of the line joining points $P(2, 3)$ and image $P'(h, k)$. Now, $AP = AP'$ and simplify.

Given line is

$$(2x - 3y + 4) + k(x - 2y + 3) = 0, \quad k \in R \quad \ldots(\text{i})$$

This line will pass through the point of intersection of the lines

$$2x - 3y + 4 = 0 \quad \ldots(\text{ii})$$

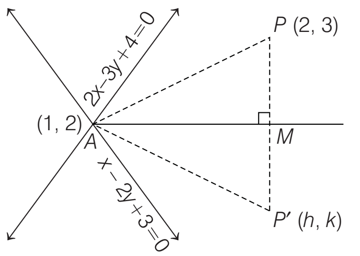
and $\quad x - 2y + 3 = 0 \quad \ldots(\text{iii})$

On solving Eqs. (ii) and (iii), we get

$$x = 1,\ y = 2$$

Thus, point of intersection of lines (ii) and (iii) is $(1, 2)$.

Let M be the mid-point of PP', then AM is perpendicular bisector of PP' (where, A is the point of intersection of given lines).



Clearly, $\quad AP = AP'$

$$\Rightarrow \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{(h-1)^2 + (k-2)^2}$$

$$\Rightarrow \sqrt{2} = \sqrt{h^2 + k^2 - 2h - 4k + 1 + 4}$$

$$\Rightarrow \sqrt{2} = \sqrt{h^2 + k^2 - 2h - 4k + 5}$$

$$\Rightarrow h^2 + k^2 - 2h - 4k + 5 = 2$$

$$\Rightarrow h^2 + k^2 - 2h - 4k + 3 = 0$$

Thus, the required locus is

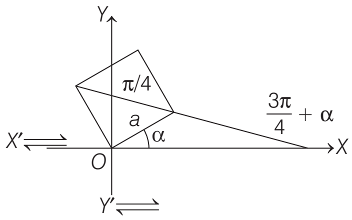
$$x^2 + y^2 - 2x - 4y + 3 = 0$$

which is an equation of circle with radius $= \sqrt{1 + 4 - 3} = \sqrt{2}$

9 Slope of the diagonal $= \tan\left(\frac{3\pi}{4} + \alpha\right)$

$$= \frac{-1 + \tan\alpha}{1 + \tan\alpha}$$

$$= \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$



The equation is

$$\frac{y - a\sin\alpha}{x - a\cos\alpha} = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

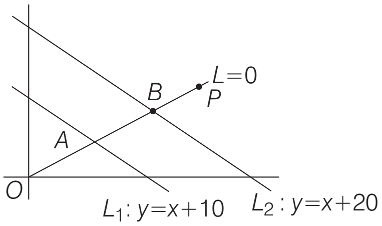
$$\Rightarrow y\,(\cos\alpha + \sin\alpha) - x\,(\sin\alpha - \cos\alpha) = a$$

10 Let equation of the line OAB be

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$$

$$\Rightarrow \quad x = r\cos\theta,\ y = r\sin\theta.$$

For A, let $OA = r_1$ then $A(r_1\cos\theta, r_1\sin\theta)$ lies on L_1.



$$\Rightarrow \quad \frac{1}{OA} = \frac{1}{r_1} = \frac{\sin\theta - \cos\theta}{10}$$

Similarly, $\frac{1}{OB} = \frac{1}{r_2} = \frac{\sin\theta - \cos\theta}{20}$

Let $\quad P = (h, k) = (r\cos\theta, r\sin\theta)$

Then, $\frac{2}{OP} = \frac{2}{r} = \frac{1}{r_1} + \frac{1}{r_2}$

$$\Rightarrow \quad \frac{2}{r} = \frac{\sin\theta - \cos\theta}{10} + \frac{\sin\theta - \cos\theta}{20}$$

$$\Rightarrow \quad 40 = 3r\sin\theta - 3r\cos\theta = 3k - 3h$$

$\Rightarrow$ Locus of P is $3x - 3y + 40 = 0$

$$\Rightarrow \quad 3y - 3x = 40$$

11 Lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ are concurrent at the point of intersection of the lines $5x + 3y - 2 = 0$ and $3x - y - 4 = 0$ i.e. at $A(1, -1)$.

Similarly, lines $x - y + 1 + \lambda_2(2x - y - 2) = 0$ are concurrent at $B(3, 4)$.

The line, that belongs to both families is AB, whose equation is

$$y - 4 = \frac{-1-4}{1-3}(x - 3)$$

i.e. $\quad 5x - 2y - 7 = 0.$

12 $a(2x + y - 3) + b(3x + 2y - 5) = 0$ passes through the point of intersection of the lines $2x + y - 3 = 0$ and $3x + 2y - 5 = 0$.

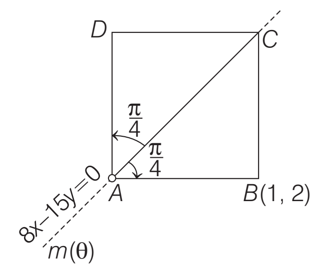
i.e. Through $(1, 1)$.

The line of this family which is farthest from $(4, -3)$ is the line through $(1, 1)$ and perpendicular to the line joining $(1, 1)$ and $(4, -3)$.

$\therefore$ Required line is $y - 1 = 3/4(x - 1)$

i.e. $3x - 4y + 1 = 0$

13



As can be seen from the figure, AB and AD are the line segments inclined at an angle of 45° with the diagonal line

$$AC(8x - 15y = 0)$$

Now, slope of line $AB = m_{AB}$

$$\Rightarrow \quad m_{AB} = \tan\left(\theta - \frac{\pi}{4}\right)$$

where, θ is the angle of inclination of diagonal with the positive X–axis.

$$\therefore \quad m_{AB} = \frac{\tan\theta - 1}{1 + \tan\theta} = \frac{8/15 - 1}{1 + 8/15}$$

$$\Rightarrow \quad m_{AB} = -\frac{7}{23}$$

$\therefore$ Equation of line AB is

$$y - 2 = -\frac{7}{23}(x - 1)$$

$$\Rightarrow \quad 23y - 46 = -7x + 7$$

$$\therefore \quad 7x + 23y = 53$$

Also, slope of line

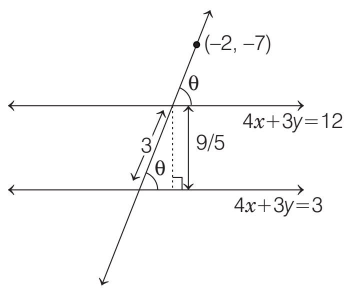
$$AD(\perp AB) = m_{BC} = \frac{23}{7}$$

$$\therefore \quad y - 2 = \frac{23}{7}(x - 1)$$

$$\Rightarrow \quad 23x - 7y = 23 - 14$$

$$\therefore \quad 23x - 7y = 9$$

14 Let m be the slope of the line and angle θ it makes with the parallel line.



$$\therefore \quad \sin\theta = \frac{3}{5} \text{ or } \tan\theta = \frac{3}{4}$$

Hence, slope of the parallel lines is $-\frac{4}{3}$.

$$\therefore \quad \left|\frac{m+\frac{4}{3}}{1-\frac{4m}{3}}\right| = \tan\theta = \frac{3}{4}$$

$$\Rightarrow \quad \frac{3m+4}{3-4m} = \pm\frac{3}{4} \Rightarrow m = -\frac{7}{24}$$

The line is $y + 7 = -\frac{7}{24}(x+2)$ or

$7x + 24y + 182 = 0.$

15 According to the question,

$$\tan\frac{\pi}{4} = \frac{-\frac{a}{b} + \frac{\cos\alpha}{\sin\alpha}}{1 + \frac{a\cos\alpha}{b\sin\alpha}}$$

$$\Rightarrow \quad 1 = \frac{b\cos\alpha - a\sin\alpha}{a\cos\alpha + b\sin\alpha}$$

$$\Rightarrow a\cos\alpha + b\sin\alpha = b\cos\alpha - a\sin\alpha$$

$$\Rightarrow \quad (a-b)\cos\alpha = -(b+a)\sin\alpha$$

$$\Rightarrow \quad \tan\alpha = \frac{b-a}{b+a} \qquad \ldots\text{(i)}$$

Intersection point of $ax + by + p = 0$ and $y = x\tan\alpha$ given by is $ax + bx\tan\alpha = p$

$$\Rightarrow \quad x = \frac{p}{a + b\tan\alpha}$$

and $$y = \frac{p\tan\alpha}{a + b\tan\alpha}$$

Intersection point of $x\cos\alpha + y\sin\alpha = p$ and $y = x\tan\alpha$ is given by

$x\cos\alpha + x\tan\alpha\sin\alpha = p$

$$\Rightarrow \quad x = p\cos\alpha,\ y = p\cos\alpha\tan\alpha$$
$$y = p\sin\alpha$$

According to the question,

$$x = \frac{p}{a + b\tan\alpha} = p\cos\alpha \qquad \ldots\text{(ii)}$$

and $$y = \frac{p\tan\alpha}{a + b\tan\alpha} = p\sin\alpha \qquad \ldots\text{(iii)}$$

$$\therefore \quad \frac{p}{a + b\left\{\frac{b-a}{b+a}\right\}} = \frac{p}{\sec\alpha}$$

$$\Rightarrow \quad \frac{p}{a + b\left(\frac{b-a}{b+a}\right)} = \frac{p}{\sqrt{\sec^2\alpha}}$$

$$= \frac{p}{\sqrt{1+\tan^2\alpha}}$$

$$\Rightarrow \quad \frac{b+a}{ab + a^2 + b^2 - ab} = \frac{1}{\sqrt{1+\left(\frac{b-a}{b+a}\right)^2}}$$

[using Eq. (i)]

$$= \frac{b+a}{\sqrt{(b+a)^2 + (b-a)^2}}$$

$$\Rightarrow \quad a^2 + b^2 = \sqrt{2(a^2+b^2)}$$

$$\Rightarrow \quad (a^2+b^2)^2 = 2(a^2+b^2)$$

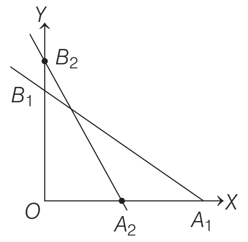
$$\Rightarrow \quad (a^2+b^2)(a^2+b^2-2) = 0$$

$$\Rightarrow \quad a^2 + b^2 \neq 0$$

$$\therefore \quad a^2 + b^2 = 2$$

16 $A_1B_1 \equiv y = mx + c_1$

$A_2B_2 \equiv y = mx + c_2$



$$\therefore \quad A_1 = \left(-\frac{c_1}{m}, 0\right), B_1 = (0, c_1),$$

$$A_2 = \left(-\frac{c_2}{m}, 0\right), B_2 = (0, c_2)$$

Since A_1, A_2, B_1, B_2 are concyclic,

$OA_1 OA_2 = OB_1 OB_2 \Rightarrow \frac{c_1c_2}{m_2} = c_1c_2$

$\therefore \quad m^2 = 1 \Rightarrow m = 1 (m > 0)$

$\therefore \quad A_1 = (-c_1, 0), A_2 = (-c_2, 0),$
$B_1 = (0, c_1), B_2 = (0, c_2)$

Now, $A_1B_2 \equiv -\frac{x}{c_1} + \frac{y}{c_2} = 1$

and $A_2B_1 \equiv -\frac{x}{c_2} + \frac{y}{c_1} = 1$

For point of intersection, consider

$$-\frac{x}{c_1} + \frac{y}{c_2} = -\frac{x}{c_2} + \frac{y}{c_1}$$

$$(c_1 - c_2)x + (c_1 - c_2)y = 0 \Rightarrow x + y = 0$$