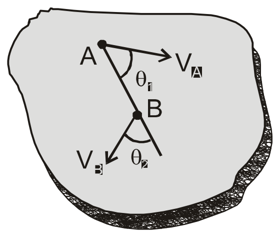
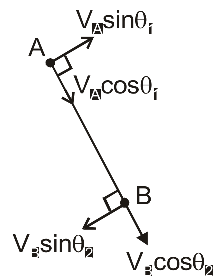
10.RIGID BODY DYNAMICS

1. RIGID BODY :

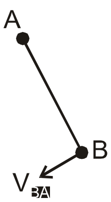


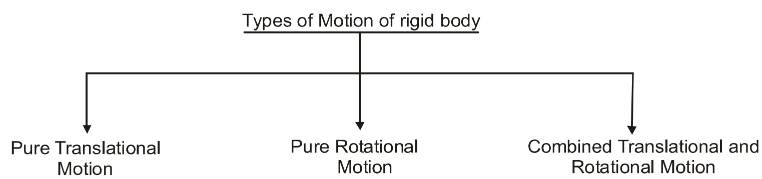


If the above body is rigid

$$V_A \cos\theta_1 = V_B \cos\theta_2$$

V_{BA} = relative velocity of point B with respect to point A.





2. MOMENT OF INERTIA (I) :

Definition : Moment of Inertia is defined as the capability of system to oppose the change produced in the rotational motion of a body.

Moment of Inertia is a scalar positive quantity.

$$I = mr_1^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots$$

$$= I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots$$

SI units of Moment of Inertia is Kgm^2.

Moment of Inertia of :

2.1 A single particle : $I = mr^2$

where m = mass of the particle

r = perpendicular distance of the particle from the axis about which moment of Inertia is to be calculated

2.2 For many particles (system of particles) :

$$I = \sum_{i=1}^{n} m_i r_i^2$$

2.3 For a continuous object :

$$I = \int dm\,r^2$$

where dm = mass of a small element

r = perpendicular distance of the particle from the axis

2.4 For a larger object :

$$I = \int dI_{element}$$

where dI = moment of inertia of a small element

3. TWO IMPORTANT THEOREMS ON MOMENT OF INERTIA :

3.1 Perpendicular Axis Theorem

[Only applicable to plane lamina (that means for 2-D objects only)].

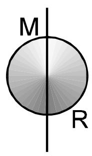
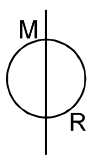
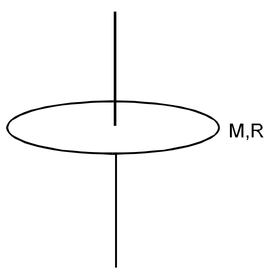
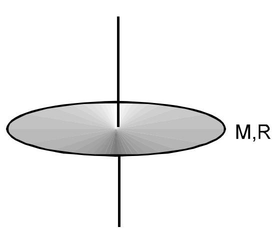
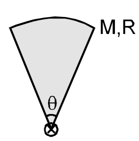
$I_z = I_x + I_y$ (when object is in x-y plane).

3.2 Parallel Axis Theorem

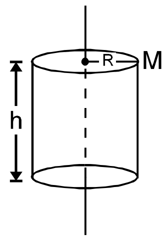
(Applicable to any type of object):

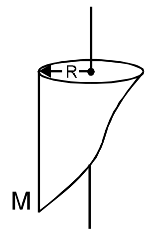
$I_{AB} = I_{cm} + Md^2$

List of some useful formula :

Object	Moment of Inertia
 **Solid Sphere**	$\frac{2}{5}MR^2$ (Uniform)
 **Hollow Sphere**	$\frac{2}{3}MR^2$ (Uniform)
 **Ring.**	MR^2 (Uniform or Non Uniform)
 	$\frac{MR^2}{2}$ (Uniform)

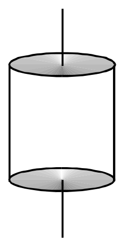
Disc





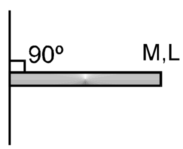
MR^2 (Uniform or Non Uniform)

Hollow cylinder

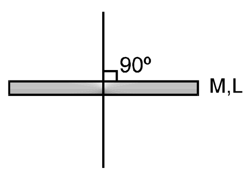


$\frac{MR^2}{2}$ (Uniform)

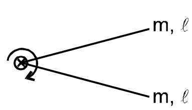
Solid cylinder



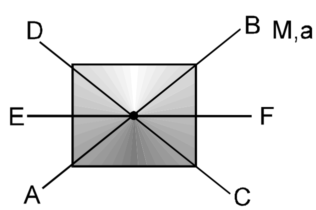
$\frac{ML^2}{3}$ (Uniform)



$\frac{ML^2}{12}$ (Uniform)

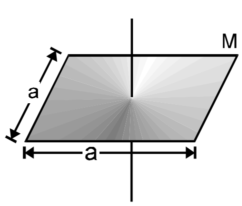


$\frac{2m\ell^2}{3}$ (Uniform)



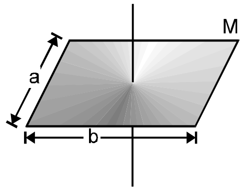
$I_{AB} = I_{CD} = I_{EF} = \frac{Ma^2}{12}$ (Uniform)

Square Plate



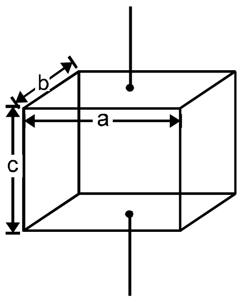
$\frac{Ma^2}{6}$ (Uniform)

Square Plate



$$I = \frac{M(a^2 + b^2)}{12} \text{ (Uniform)}$$

Rectangular Plate



$$\frac{M(a^2 + b^2)}{12} \text{ (Uniform)}$$

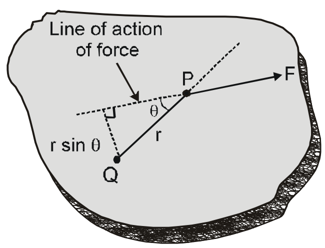
Cuboid

4. **RADIUS OF GYRATION :**

$$I = MK^2$$

5. **TORQUE :**

$\tau = r \quad F$

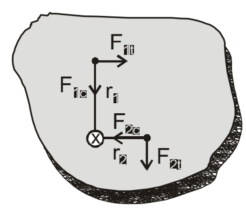


5.5 Relation between 'τ' & 'α ' (for hinged object or pure rotation)

$\tau_{ext})_{Hinge} = I_{Hinge} \quad \alpha$

Where $\tau_{ext})_{Hinge}$ = net external torque acting on the body about Hinge point

I_{Hinge} = moment of Inertia of body about Hinge point



$F_{1t} = M_1 a_{1t} = M_1 r_1 \alpha$

$F_{2t} = M_2 a_{2t} = M_2 r_2 \alpha$

$\tau_{resultant} = F_{1t}\ r_1 + F_{2t}\ r_2 + ...$

$= M_1\ \alpha\ r_1^2 + M_2\ \alpha\ r_2^2 + ...$

$\tau_{resultant}\ )_{external} = I\ \alpha$

Rotational Kinetic Energy = $\frac{1}{2}.I.\omega^2$

$P = Mv_{CM}$ $\qquad F_{external} = Ma_{CM}$

Net external force acting on the body has two parts tangential and centripetal.

$F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2\ r_{CM}$ $\qquad F_t = ma_t = m\alpha\ r_{CM}$

6. ROTATIONAL EQUILIBRIUM :

For translational equilibrium.

$\Sigma F_x = 0$ .... (i)

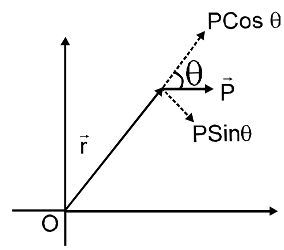
and $\Sigma F_y = 0$ .... (ii)

The condition of rotational equilibrium is

$\Sigma \Gamma_z = 0$

7. ANGULAR MOMENTUM ($\vec{L}$)

7.1 Angular momentum of a particle about a point.



$L = r \quad P$ $L = rp\sin\theta$

$|L| = r_{\perp} \quad P$

$|L| = P_{\perp} \quad r$

7.3 Angular momentum of a rigid body rotating about fixed axis :

$L_H = I_H \omega$

L_H = angular momentum of object about axis H.

I_H = Moment of Inertia of rigid object about axis H.

ω = angular velocity of the object.

7.4 Conservation of Angular Momentum

Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about that point or axis of rotation.

7.5 Relation between Torque and Angular Momentum

$$\tau = \frac{dL}{dt}$$

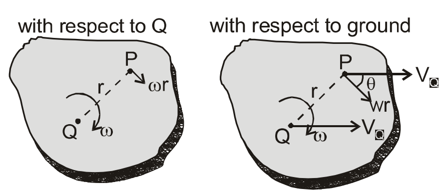
Torque is change in angular momentum

7.6 Impulse of Torque :

$\tau dt = \Delta J$ ΔJ d Change in angular momentum.

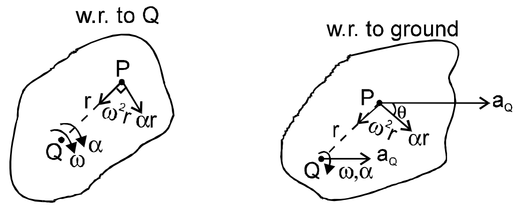
For a rigid body, the distance between the particles remain unchanged during its motion i.e. $r_{P/Q}$ = constant

For velocities



$$V_P = \sqrt{V_Q^2 + (\omega r)^2 + 2\, V_Q\, \omega r \cos\theta}$$

For acceleration :



θ, ω, α are same about every point of the body (or any other point outside which is rigidly attached to the body).

Dynamics :

$$\tau_{cm} = I_{cm}\,\alpha\ ,\ \ F_{ext} = Ma_{cm} \qquad\qquad P_{system} = Mv_{cm}\ ,$$

Total K.E. $= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\,\omega^2$

Angular momentum axis AB = $\vec{L}$ about C.M. + $\vec{L}$ of C.M. about AB

$$L_{AB} = I_{cm}\,\omega + r_{cm} \quad Mv_{cm}$$