SAMPLE QUESTION PAPER (STANDARD) - 06

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If A(5, 3), B(11, -5) and P(12, y) are the vertices of a right triangle right angled at P, then y = **[1]**

a) -1, 4 b) 2, 4

c) -2, 4 d) 2, -4

2. The length of the tangent from an external point P to a circle of radius 5 cm is 10 cm. The distance of the point from the centre of the circle is **[1]**

a) 12 cm b) $\sqrt{125}$

c) $\sqrt{104}$cm d) 8 cm

3. The probability that it will rain on a particular day is 0.76. The probability that it will not rain on that day is **[1]**

a) 0.24 b) 0.76

c) 0 d) 1

4. The point which lies on the perpendicular bisector of the line segment joining the points A (-2, -5) and B (2, 5) is **[1]**

a) (2, 0) b) (–2, 0)

c) (0, 2) d) (0, 0)

5. The area of the triangle formed by x + 3y = 6, 2x – 3y = 12 and the y-axis is **[1]**

a) 15 sq. units b) 18 sq. units

c) 16 sq. units d) 12 sq. units

6. The points A (-4, 0), B(4, 0) and C(0, 3) are the vertices of a **[1]**

a) isosceles triangle
b) scalene triangle
c) equilateral triangle
d) right triangle

7. If a two digit number is chosen at random, then the probability that the number chosen is a multiple of 3, is **[1]**

a) $\frac{3}{10}$
b) $\frac{29}{100}$
c) $\frac{7}{25}$
d) $\frac{1}{3}$

8. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged then the water level rises by **[1]**

a) 4 cm
b) 5 cm
c) 3 cm
d) 6 cm

9. Ram and Shyam are friends. The probability that both will have the birthday on the same day is **[1]**

a) $\frac{2}{365}$
b) $\frac{364}{365}$
c) $\frac{1}{365}$
d) $\frac{4}{365}$

10. Let b = a + c. Then the equation $ax^2 + bx + c = 0$ has equal roots if **[1]**

a) a = -c
b) a = c
c) a = -2c
d) a = 2c

11. A quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, if **[1]**

a) $b^2 - 4ac = 0$
b) $b^2 - 4ac < 0$
c) $b^2 - 4ac > 0$
d) None of these

12. If a sinθ + b cosθ = c, then the value of a cosθ - b sinθ is **[1]**

a) $\sqrt{a^2 + b^2 - c^2}$
b) $\sqrt{a^2 + b^2 + c^2}$
c) $\sqrt{a^2 - b^2 + c^2}$
d) None of these

13. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is **[1]**

a) an even number
b) an odd prime number
c) an odd number
d) a prime number

14. If x is a positive integer such that the distance between points P (x, 2) and Q (3, - 6) is 10 units, then x = **[1]**

a) 3
b) 9
c) -9
d) -3

15. The angle of elevation of the sun when the shadow of a pole of height ‘h’ metres is $\sqrt{3}$h metres long is **[1]**

a) 60°
b) 45°
c) None of these
d) 30°

16. If $\sum f_i u_i$ = 29, $\sum f_i$ = 30, a = 47.5 and h = 15, then the value of $\overline{x}$ is **[1]**

a) 63
b) 26
c) 64
d) 62

17. The sum of the exponents of the prime factors in the prime factorisation of 196, is **[1]**

a) 2
b) 1
c) 4
d) 6

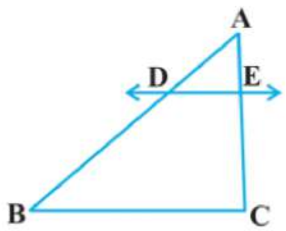
18. The father's age is six times his son's age. Four years later, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively **[1]**

a) 6 and 36
b) 4 and 24
c) 3 and 24
d) 5 and 30

19. **Assertion (A):** For any two positive integers a and b, HCF(a, b) $\times$ LCM(a, b) = a $\times$ b **[1]**
Reason (R): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle$ABC such that DE||BC then the value of x is 11, when AD = 4cm, DB = (x - 4)cm, AE = 8cm and EC = (3x - 19)cm. **[1]**
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.



a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Cards bearing numbers 1, 3, 5, ..., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing **[2]**

i. a prime number less than 15
ii. a number divisible by 3 and 15.

22. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincide: $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$ **[2]**

23. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 +bx + c, then evaluate: $\alpha^2\beta + \alpha\beta^2$ **[2]**

24. Name the type of triangle PQR formed by the points P $(\sqrt{2}, \sqrt{2})$, Q $(-\sqrt{2}, -\sqrt{2})$ and R $(-\sqrt{6}, \sqrt{6})$. **[2]**

OR

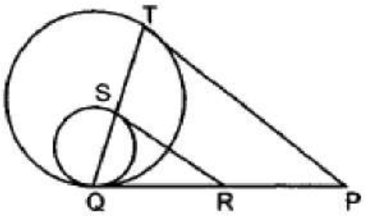
Let A $\rightarrow$ (4, 2), B $\rightarrow$ (6, 5) and C $\rightarrow$ (1, 4) be the vertices of triangle ABC. The median from A meets BC at D. Find the coordinates of the point D.

25. From an external point P, two tangents PA and PB are drawn to the circle with centre O. Prove that OP is the perpendicular bisector of AB. **[2]**

OR

In the following figure, PQ is the common tangent to both the circles. SR and PT are tangent to both the circles. If SR

= 4 cm, PT = 7 cm, then find RP.



Section C

26. If tan A = $\frac{3}{4}$, then show that sin A cos A = $\frac{12}{25}$. **[3]**

27. Two candles of equal height but different thickness are lighted. First candle burns off in 6 hours and the second candle in 8 hours. How long, after lighting both, will the first candle be half the height of the second ? **[3]**

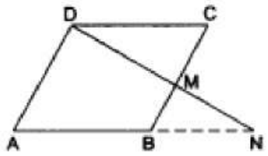
28. Show that $2-\sqrt{3}$ is an irrational number. **[3]**

OR

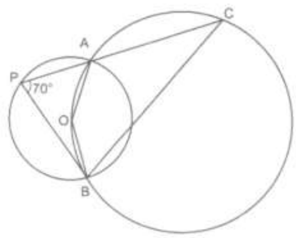
Prove that $\sqrt{5}+\sqrt{3}$ is irrational.

29. M is a point on the side BC of a parallelogram ABCD. DM when produced meets AB at N. Prove that. **[3]**

i. $\frac{DM}{MN}=\frac{DC}{BN}$.
ii. $\frac{DN}{DM}=\frac{AN}{DC}$

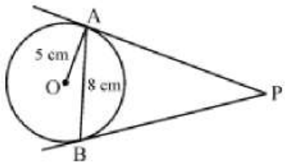


30. In a given figure, two circles intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If ∠APB = 70°, find ∠ACB. **[3]**



OR

In a given figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



31. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. **[3]**

Section D

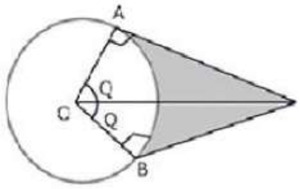
32. Represent the situation in the form of the quadratic equation: A train travels a distance of 480 km's at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. **[5]**

OR

The hypotenuse of a right triangle is $3\sqrt{10}$cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}$cm. How long are the legs of the triangle?

33. In trapezium ABCD, AB ∥ DC and DC = 2AB. EF ∥ AB, where E and F lie on BC and AD respectively, such that $\frac{BE}{EC}=\frac{4}{3}$. Diagonal DB intersects EF at G. Prove that 7EF = 11 AB. **[5]**

34. An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

35. An incomplete distribution is given below: **[5]**

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	-	65	-	25	18

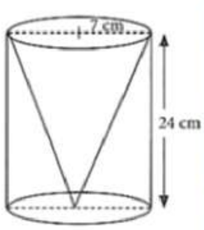
You are given that the median value is 46 and the total number of items is 230.

i. Using the median formula fill up missing frequencies.
ii. Calculate the AM of the completed distribution.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

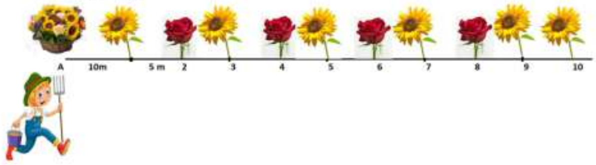
(iii) Find the external curved surface area of the cylinder?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) Write the above information in the progression and find first term and common difference.

(ii) Find the distance covered by Dinesh to plant the first 5 plants and return to basket.

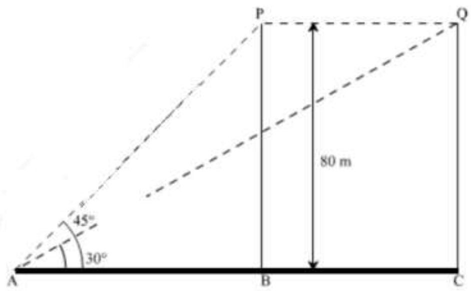
OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants.

(iii) Find the distance covered by Dinesh to plant all 10 plants and return to basket.

38. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

OR

Find the distance between initial position of bird and observer?

(iii) Find the distance between second position of bird and observer?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 06

Class 10 - Mathematics

Section A

1. **(d)** 2, -4

Explanation: A(5, 3), B(11, -5) and P(12, y) are the vertices of a right triangle, right-angled at P

$\therefore AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ [BY P.G.T]

$= (11 - 5)^2 + (-5 - 3)^2 = (6)^2 + (-8)^2$

$= 36 + 64 = 100$

Similarly $BP^2 = (12 - 11)^2 + (y + 5)^2 = (1)^2 + y^2 + 10y + 25$

$= y^2 + 10y + 26$

and $AP^2 = (12 - 5)^2 + (y - 3)^2 = (7)^2 + (y - 3)^2$

$= 49 + y^2 - 6y + 8 = y^2 - 6y + 58$

$\because \triangle ABP$ is a right triangle

$\therefore AB^2 = BP^2 + AP^2$

$100 = y^2 + 10y + 26 + y^2 - 6y + 58$

$100 = 2y^2 + 4y + 84$

$\Rightarrow 2y^2 + 4y + 84 - 100 = 0 \Rightarrow 2y^2 + 4y - 16 = 0$

$\Rightarrow y^2 + 2y - 8 = 0$ (Dividing by 2)

$\Rightarrow y^2 + 4y - 2y - 8 = 0 \left\{\begin{array}{l} \because -8 = 4 \times (-2) \\ 2 = 4 - 2 \end{array}\right\}$

$\Rightarrow y(y + 4) -2(y + 4) = 0$

$\Rightarrow (y + 4)(y - 2) = 0$

Either y + 4 = 0, then y = -4

or y - 2 = 0, then y = 2

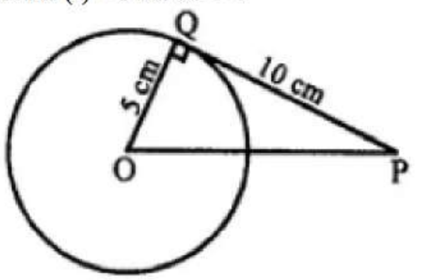
y = 2, -4

2. **(b)** $\sqrt{125}$

Explanation:

Length of a tangent to the circle from an external point = 10 cm

Radius (r) = 5 cm OP = ?



OQ is radius and QP is tangent

$OQ \perp QP$

In right $\triangle OPQ$,

$OP^2 = OQ^2 + QP^2$ (Pythagoras Theorem) $= (5)^2 + (10)^2 = 25 + 100 = 125$

$OP = \sqrt{125}$ cm

3. **(a)** 0.24

Explanation: Given: P (It will rain on a particular day) = 0.76

$\therefore$ P (It will not rain on a particular day) = 1 - P (It will rain particular day)

= 1 - 0.76 = 0.24

4. **(d)** (0, 0)

Explanation: As we know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid - point of the line segment.

As mid - point of any line segment which passes through the points

(x_1, y_1) and (x_2, y_2) is;

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So mid - point of the line segment joining the points A (- 2, - 5) and B (2, 5) will be;

$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

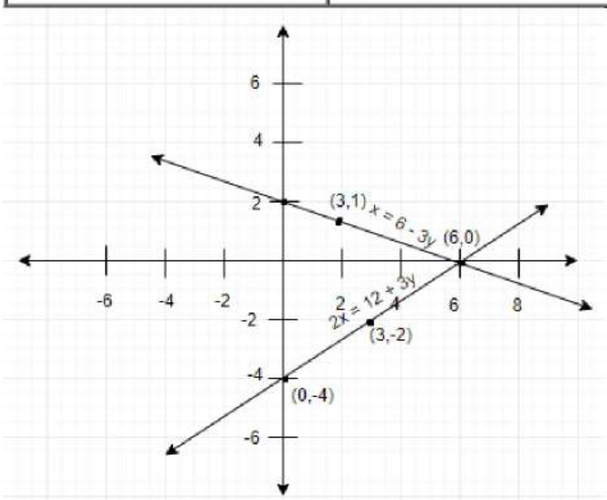
5. **(b)** 18 sq. units

Explanation: Here are the two solutions of each of the given equations. $x + 3y = 6$

x	0	3	6
y	2	1	0

$2x - 3y = 12$

x	0	3	6
y	-4	-2	0



∴ Area of triangle = $\frac{1}{2}$ × Base × Height = $\frac{1}{2} \times 6 \times 6$ = 18 sq. units

6. **(a)** isosceles triangle

Explanation: $AB^2 = (4 + 4)^2 + (0 - 0)^2 = 8^2 + 0^2 = 64 + 0 = 64$

⇒ AB=$\sqrt{64}$= 8 units

$BC^2 = (0-4)^2 + (3-0)^2 = (-4)^2 + 3^2 = 16 + 9 = 25$

⇒ BC = $\sqrt{25}$ = 5 units.

$AC^2 = (0 + 4)^2 + (3 - 0)^2 = 4^2 + 3^2 = 16 + 9 = 25$

⇒ A C = $\sqrt{25}$ = 5 units.

∴ △ABC is isosceles.

7. **(d)** $\frac{1}{3}$

Explanation: Total number of two digit numbers are 10 to 99

= 99 - 9 = 90

Multiples of 3 will be 12, 15, 18, 21, ..., 99 = 33 - 3 = 30

∴ Probability = $\frac{30}{90} = \frac{1}{3}$

8. **(c)** 3 cm

Explanation: Increase in volume of water = volume of the sphere

$\Rightarrow \pi \times 18 \times 18 \times h = \frac{4}{3}\pi \times 9 \times 9 \times 9$

⇒ h = $\left(\frac{4}{3} \times \frac{9\times9\times9}{18\times18}\right)$cm = 3 cm

9. **(c)** $\frac{1}{365}$

Explanation: Assuming a non-leap year

Ram can have the birthday on any day of the 365 days of the year

Shyam has a different birthday if his birthday is on any of the remaining 364 days of the year

Therefore P (Ram and Shyam have different birthdays) = $\frac{364}{365}$

and so, P (Ram and Shyam have birthdays on the same day) = 1 - P(Ram and Shyam have different birthdays)

= 1 - $\frac{364}{365}$

= $\frac{1}{365}$

10. **(b)** a = c

Explanation: Since, If $ax^2 + bx + c = 0$ has equal roots, then

$b^2 - 4ac = 0$

$\Rightarrow (a + c)^2 - 4ac = 0$ ... [Given: b = a + c]

$\Rightarrow a^2 + c^2 + 2ac - 4ac = 0$

$\Rightarrow a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - c)^2 = 0$

$\Rightarrow a - c = 0$

$\Rightarrow a = c$

11. **(a)** $b^2 - 4ac = 0$

Explanation: A quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, if $b^2 - 4ac = 0$.

12. **(a)** $\sqrt{a^2 + b^2 - c^2}$

Explanation: Given: a $\sin\theta$ + b $\cos\theta$ = c

Squaring both sides, we get

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2\cos^2\theta - b^2\sin\theta - 2ab \sin\theta \cos\theta = a^2 + b^2 - c^2$

$\Rightarrow (a \cos\theta - b \sin\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a \cos\theta - b \sin\theta = \sqrt{a^2 + b^2 - c^2}$

13. **(a)** an even number

Explanation: Let p_1 and p_2 be 5 two odd primes.

Then,

$p_1^2 - p_2^2 = (p_1 - p_2)(p_1 + p_2)$

We know that sum and difference of two odd numbers is even

$\therefore$ $(p_1 - p_2)$ *and* $(p_1 + p_2)$ are even numbers.

Also, we know that product of even numbers is an even number, therefore

$p_1^2 - p_2^2 = (p_1 - p_2)(p_1 + p_2)$, is an even number.

14. **(b)** 9

Explanation: Distance between P(x, 2) and Q(3, -6) = 10 units

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 10$

$\Rightarrow \sqrt{(3 - x)^2 + (-6 - 2)^2} = 10$

$\Rightarrow \sqrt{(3 - x)^2 + (-8)^2} = 10$

$\Rightarrow \sqrt{(3 - x)^2 + 64} = 10$

Squaring both sides,

$(3 - x)^2 + 64 = 100$

$\Rightarrow 9 + x^2 - 6x + 64 - 100 = 0$

$\Rightarrow x^2 - 6x - 27 = 0$

$\Rightarrow x^2 - 9x + 3x - 27 = 0 \left\{ \begin{matrix} \because -27 = -9 \times 3 \\ -6 = -9 + 3 \end{matrix} \right\}$

$\Rightarrow$ x(x - 9) + 3(x - 9) = 0

(x - 9) (x + 3) = 0

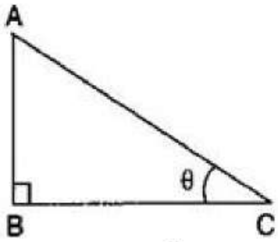
Either x - 9 = 0, then x = 9 or x + 3 = 0, then x = -3

x is positive integer

Hence x = 9

15. **(d)** 30°

Explanation:



Let Height of the pole = AB = h meters and length of the shadow of the pole = BC = $\sqrt{3}h$ meters.

$\therefore \tan\theta = \frac{\mathrm{AC}}{\mathrm{BC}} \Rightarrow \tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^{\circ} \Rightarrow \theta = 30^{\circ}$

16. **(d)** 62

Explanation: $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

= $47.5 + \frac{29}{30} \times 15$

= 47.5+$\frac{29}{2}$

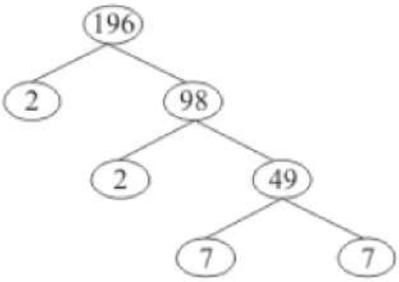
= 47.5 + 14.5

= 62

17. **(c)** 4

Explanation:

Using the factor tree for prime factorisation, we have:



Therefore,

$196 = 2 \times 2 \times 7 \times 7$

$196 = 2^2 \times 7^2$

The exponents of 2 and 7 are 2 and 2 respectively.

Thus the sum of the exponents is 4.

18. **(a)** 6 and 36

Explanation: Let 'x' year be the present age of father and 'y' year be the present age of son.

Four years later, given condition becomes,

(x + 4) = 4(y + 4)

x + 4 = 4y + 16

x - 4y - 12 = 0 ...(i)

and initially, x = 6y ...(ii)

On putting the value of from Eq. (ii) in Eq. (i), we get

6y - 4y - 12 = 0

2y = 12

Hence, y = 6

Putting y = 6, we get x = 36.

Hence, present age of father is 36 years and age of son is 6 years.

19. **(c)** A is true but R is false.

Explanation: We have,

LCM(a, b) × HCF(a, b) = a × b
LCM × 5 = 150
LCM = $\frac{150}{5}$ = 30
LCM = 30

20. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation: If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side. This is the Converse of the Basic Proportionality theorem.
So, the Reason is correct.
By Basic Proportionality theorem, we have $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$
$\Rightarrow$ 4(3x - 19) = 8(x - 4)
$\Rightarrow$ 12x - 76 = 8x - 32
$\Rightarrow$ 4x = 44 $\Rightarrow$ x = 11 cm
So, Assertion is correct.
But reason (R) is not the correct explanation of assertion (A).

Section B

21. Total number of cards $= 18$
$Probability = \frac{favourable\ outcome}{total\ outcomes}$
i. Prime numbers less than 15 $= 3, 5, 7, 11, 13$
P(a prime number less than 15) = $\frac{5}{35}$
ii. Number divisible by 3 and 5 = 15
P(a number divisible by 3 and 5) = $\frac{1}{35}$

22. Given equations are
$9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$
Comparing equation $9x + 3y + 12 = 0$ with $a_1x + b_1y + c_1 = 0$
and $18x + 6y + 24 = 0$ with
$a_2x + b_2y + c_2 = 0$,
We get, $a_1 = 9$, $b_1 = 3$, $c_1 = 12$, $a_2 = 18$, $b_2 = 6$, $c_2 = 24$
We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$
Hence, lines are coincident.

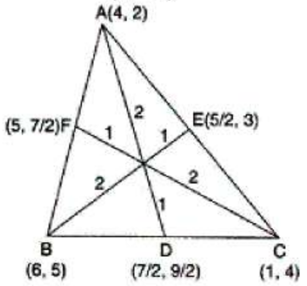
23. α,β are zeros of ax^2+bx+c
Then α+β= $-\frac{b}{a}$ and αβ=$\frac{c}{a}$
$\alpha^2\beta + \alpha\beta^2$
$= \alpha\beta(\alpha + \beta)$
$= \frac{c}{a}\left(\frac{-b}{a}\right)$
$= \frac{-bc}{a^2}$.

24. $P(\sqrt{2}, \sqrt{2}), Q(-\sqrt{2}, -\sqrt{2})$ and $R(-\sqrt{6}, \sqrt{6})$ are the vertices of ΔPQR.
Now,
$PQ = \sqrt{(-\sqrt{2} - \sqrt{2})^2 + (-\sqrt{2} - \sqrt{2})^2} = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} = = \sqrt{8 + 8} = \sqrt{16} = 4\ units$
$QR = \sqrt{(-\sqrt{6} + \sqrt{2})^2 + (\sqrt{6} + \sqrt{2})^2} = \sqrt{6 + 2 - 2\sqrt{12} + 6 + 2 + 2\sqrt{12}} = \sqrt{16} = 4\ units$
$PR = \sqrt{(-\sqrt{6} - \sqrt{2})^2 + (\sqrt{6} - \sqrt{2})^2} = \sqrt{6 + 2 + 2\sqrt{12} + 6 + 2 - 2\sqrt{12}} = \sqrt{16} = 4\ units$
Since PQ = QR = PR, ΔPQR is an equilateral triangle.

OR

The median from A meets BC at D
$\therefore$ D is the mid-point of BC
$\therefore D \rightarrow \left(\frac{6+1}{2}, \frac{5+4}{2}\right)$ [Using mid-point formula]

$\Rightarrow D \rightarrow \left(\frac{7}{2}, \frac{9}{2}\right)$



25. Suppose OP intersects AB at C.

In triangles PAC and PBC, we have

PA = PB [∵ Tangents from an external point are equal]

$\angle APC = \angle BPC$ [∵ PA and PB are equally inclined to O P]

and, PC = PC [Common]

So, by SAS-criterion of similarity, we obtain

$\Delta PAC \cong \Delta PBC$

⇒ AC = BC and $\angle ACP = \angle BCP$

But, $\angle ACP + \angle BCP$= 180°

∴ $\angle ACP = \angle BCP$ = 90°

Hence, $OP \perp AB$

OR

∵ PT = PQ

∴ PQ = 7 cm

Also SR = QR

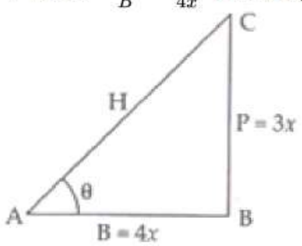
∴ QR =4

Now, RP = PQ - QR = 7 - 4 = 3 cm

Section C

26. Given, tan A = $\frac{3}{4}$

⇒ tan A = $\frac{P}{B} = \frac{3x}{4x}$ [from figure]



$H^2 = P^2 + B^2$ [By Pythagoras theorem]

$= (3x)^2 + (4x)^2$

$= 9x^2 + 16x^2$

⇒ $H^2 = 25x^2$

or, $H^2=(5x)^2$

⇒ H = 5x

Therefore,

sin A = $\frac{P}{H} = \frac{3x}{5x} = \frac{3}{5}$

and cos A = $\frac{B}{H} = \frac{4x}{5x} = \frac{4}{5}$

Now, LHS = sin A cos A

$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$ = RHS

Hence, proved.

27. Let height of each candle = $x\ unit.$

First candle burns off in 6 hours.

Second candle burns off in 8 hours.

Height of 1st candle after burning for 1 hr = $\frac{x}{6} unit$

and height of 2nd candle after burning for 1 hr = $\frac{x}{8} unit$

Let the required time $= y\ hrs.$

Length of 1st candle burnt after y hrs = $\frac{y\times x}{6} unit$

Height of 1st candle left = $\left(x - \frac{xy}{6}\right)$

Length of 2nd candle burnt after y hrs = $\left(\frac{y\times x}{8}\right) unit$

Height of 2nd candle left = $\left(x - \frac{xy}{8}\right)$

According to the question,

Height of 1st candle = $\frac{1}{2}\times$ Height of 2nd candle

$\Rightarrow \quad x - \frac{xy}{6} = \frac{1}{2}\left(x - \frac{xy}{8}\right)$

$\Rightarrow \quad x\left(1 - \frac{y}{6}\right) = \frac{1}{2}x\left(1 - \frac{y}{8}\right)$

$1 - \frac{y}{6} = \frac{1}{2}\left(1 - \frac{y}{8}\right)$

$\Rightarrow \quad 2 - \frac{y}{3} = 1 - \frac{y}{8}$

$2 - 1 = \frac{y}{3} - \frac{y}{8}$

$1 = \frac{8y-3y}{24}$

$\Rightarrow 24 = 5y$

$\Rightarrow y = \frac{24}{5}$

$y = 4.8 hours = 4\ hours\ 48\ minutes.$

28. Let us assume that $2 - \sqrt{3}$ is rational.

Then, there exist positive co-primes a and b such that

$2 - \sqrt{3} = \frac{a}{b}$

$\sqrt{3} = 2 - \frac{a}{b}$

As 2 and $\frac{a}{b}$ are rational number .

So, $\sqrt{3}$ is also rational number .

But $\sqrt{3}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $2 - \sqrt{3}$ is rational wrong .

Hence $2 - \sqrt{3}$ is irrational

OR

Let $\sqrt{5} + \sqrt{3}$ be rational number equal to $\frac{a}{b}$. there exist co-prime integers a and b such that

$\sqrt{5} + \sqrt{3} = \frac{a}{b}$

$\Rightarrow \quad \sqrt{5} = \frac{a}{b} - \sqrt{3}$

$\Rightarrow \quad (\sqrt{5})^2 = \left(\frac{a}{b} - \sqrt{3}\right)^2$ [Squaring both sides] we get,

$\Rightarrow \quad 5 = \frac{a^2}{b^2} - \frac{2a\sqrt{3}}{b} + 3$

$\Rightarrow \quad 2 = \frac{a^2}{b^2} - \frac{2\sqrt{3}a}{b}$

$\sqrt{3} = (a^2 - 2b^2)\frac{b}{2ab}$

Since a,b are integers, therefore $(a^2 - 2b^2)\frac{b}{2ab}$ is a rational number

which is a contradiction as $\sqrt{3}$ is an irrational number.

Hence, $\sqrt{5} + \sqrt{3}$ is irrational.

29. Given: ABCD is a parallelogram

To Prove:

i. $\frac{DM}{MN} = \frac{DC}{BN}$.
ii. $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: In $\triangle DMC$ and $\triangle NMB$, we have

$\angle \text{DMC} = \angle \text{NMB}$ (Vertically opposite angle)

$\angle DCM = \angle NBM$ (Alternate angles)

By AA - Similarity criteria, we have

$\triangle \text{DMC} \sim \triangle \text{NMB}$

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

which completes the proof of part (i).

Now, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we obtain

$\frac{MN}{DM}+1=\frac{BN}{DC}+1$

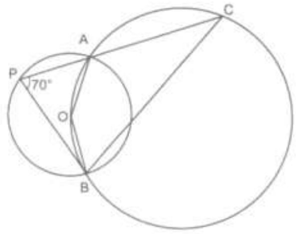
$\Longrightarrow \frac{MN+DM}{DM}=\frac{BN+DC}{DC}$

$\Longrightarrow \frac{MN+DM}{DM}=\frac{BN+AB}{DC}$ [$\because$ ABCD is a parallelogram]

$\Rightarrow \frac{DN}{DM}=\frac{AN}{DC}$

30. Consider the smaller circle whose centre is given as O.

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.



Therefore,we have,

$\angle AOB = 2\angle APB$

$= 2(70^o)$

$\angle AOB = 140^o$

Now consider the larger circle and the points A,C, B and O along its circumference. AOBC from a cycle quadrilateral.

In a cyclic quadrilateral , the opposite angles are supplementary, meaning that the opposite angles add up to 180^o.

$\angle AOB + \angle ACB = 180^o$

$\angle ACB = 180^o - \angle AOB$

$= 180^o - 140^o$

$\angle ACB = 40^o$

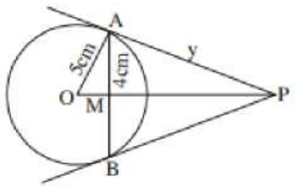
Therefore, the measure of angle ACB is 40^o.

OR

According to the question, radius of circle is = 5 cm.

Also, AB = 8 cm

Now, AM = $\frac{AB}{2}$ =4 cm



In $\triangle$OMA,

By using pythagoras theorem, we get

$OA^2 = OM^2 + AM^2$

$\therefore$ OM = $\sqrt{5^2-4^2}$ = 3 cm

Let AP = y cm, PM = x cm

In $\triangle$OAP,

By using pythagoras theorem, we get

$OP^2 = OA^2 = AP^2$

$(x + 3)^2 = y^2 + 25$

$\Rightarrow x^2 + 9 + 6x = y^2 + 25$ ...(i)

In $\triangle$AMP,

By using pythagoras theorem, we get

$x^2 + 4^2 = y^2$ ....(ii)

Substituting eq.(ii) in eq.(i), we get

$\Rightarrow x^2 + 6x + 9 = x^2 + 16 + 25$

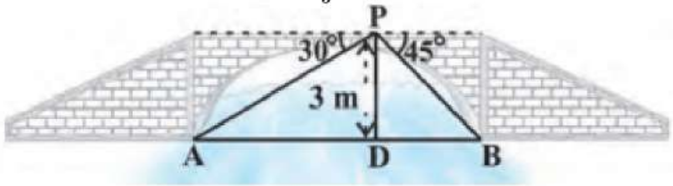
$\Rightarrow$ 6x = 32

$\Rightarrow$ x = $\frac{32}{6}=\frac{16}{3}$ cm

Now, $y^2 = x^2 + 16 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow$ y = $\frac{20}{3}$ cm.

Therefore, length of AP = y = $\frac{20}{3}$ cm.

31. 

In the above-given fig, A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e.,

DP = 3 m. Here, we are interested to determine the width of the river, which is the length of the side AB of the

△ APB.

Now, AB = AD + DB

In right △ APD, ∠ A = 30°

So, tan 30° = $\frac{PD}{AD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{3}{AD}$ or AD = $3\sqrt{3}$ m

Also, in right △ PBD, ∠ B = 45°. So, BD = PD = 3 m

Now, AB = BD + AD = 3 + $3\sqrt{3}$ = $3(1+\sqrt{3})$m

Therefore, the width of the river is $3(\sqrt{3}+1)$m

Section D

32. Distance travelled by the train = 480 km

Let the speed of the train be x kmph

Time taken for the journey = $\frac{480}{x}$

Given speed is decreased by 8 kmph

Hence the new speed of train = (x – 8) kmph

Time taken for the journey = $\frac{480}{x-8}$

$\frac{480}{x-8} = \frac{480}{x} + 3$

$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow \frac{480(x-x+8)}{x(x-8)} = 3$

$\Rightarrow \frac{480\times 8}{x(x-8)} = 3$

$\Rightarrow 3x(x-8) = 480 \times 8$

$\Rightarrow x(x-8) = 160 \times 8$

$\Rightarrow x^2 - 8x - 1280 = 0$

OR

Suppose, the smaller side of the right triangle be x cm and the larger side be y cm. Then,

$\therefore x^2 + y^2 = \left(3\sqrt{10}\right)^2$ [Using pythagoras theorem]

$\Rightarrow x^2 + y^2 = 90$ ....(i)

If the smaller side is tripled and the larger side be doubled, the new hypotenuse is $9\sqrt{5}$ cm.

$\therefore (3x)^2 + (2y)^2 = \left(9\sqrt{5}\right)^2$ [Using pythagoras theorem]

$\Rightarrow 9x^2 + 4y^2 = 405$ ....(ii)

Putting $y^2 = 90 - x^2$ in equation (ii), we get

$9x^2 + 4(90 - x^2) = 405$

$\Rightarrow 9x^2 + 360 - 4x^2 = 405$

$\Rightarrow 5x^2 = 405 - 360$

$\Rightarrow 5x^2 = 45$

$\Rightarrow x^2 = 9$

$\Rightarrow$ x = ± 3

But, length of a side can not be negative. Therefore, $x = 3$

Putting $x = 3$ in (i), we get

$(3)^2 + y^2 = 90$

$\Rightarrow y^2 = 90 - 9$

$\Rightarrow y^2 = 81$

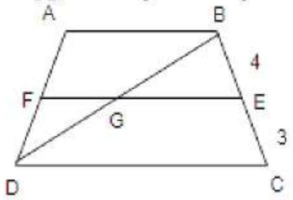
$\Rightarrow$ y = $\pm$9

But, length of a side can not be negative. Therefore, $y = 9$

Hence, the length of the smaller side is 3 cm and the length of the larger side is 9 cm.

33. $\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{AD}{FD} = \frac{AF+FD}{3} = \frac{4+3}{3} = \frac{7}{3}$



In $\triangle DFG$ and $\triangle DAB$

$\angle$FDG $\cong$ $\angle$ADB ($\because$ common angle)

$\angle$ DFG $\cong$ $\angle$DAB ($\because$ FG $\parallel$ AB)

So,$\triangle DFG \sim \triangle DAB$ (AA similarity)

Also, $\triangle BGE \sim \triangle BDC$

$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$ and $\frac{GE}{DC} = \frac{BE}{BC}$

$\Rightarrow \frac{3}{7} = \frac{FG}{AB}$ and $\frac{GE}{2AB} = \frac{4}{7}$

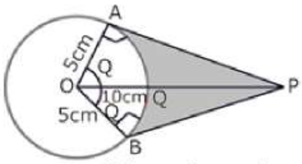
Now, $EF = FG + GE = \frac{3}{7}AB + \frac{8}{7}AB$

$\Rightarrow EF = (\frac{3}{7} + \frac{8}{7})AB$

$\Rightarrow EF = (\frac{3+8}{7})AB$

$\Rightarrow EF = (\frac{11}{7})AB$

$\Rightarrow 7EF = 11AB$

34. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta$ = 60^o

$\Rightarrow \angle$AOB = 2 $\times$ θ = 120^o

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times5}{360}cm = \frac{10\pi}{3}cm$ $\left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= 2π $\times$ 5 cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360}cm^2 = \frac{25\pi}{3}cm^2$ $\left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = 25$\sqrt{3}$ cm^2

$[\because AP = \sqrt{100\ - 25} = \sqrt{75}\ = 5\sqrt{3}\ cm]$

Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3}\ - \pi]\ cm^2$

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77$cm

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$

$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} = \sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s=\frac{a+b+c}{2}=\frac{33+56+65}{2}=77 \mathrm{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2=\sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2=\sqrt{77 \times 44 \times 21 \times 12}=\sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4}=\sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$

$\Rightarrow \quad \Delta_2=7 \times 11 \times 4 \times 3=924 \mathrm{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2=\Delta_1+\Delta_2$

$\Rightarrow \quad \pi r^2$ = 924 + 924

$\Rightarrow \quad \frac{22}{7} \times r^2=1848$

$\Rightarrow \quad r^2=1848 \times \frac{7}{22}=3 \times 4 \times 7 \times 7 \Rightarrow r=\sqrt{3 \times 2^2 \times 7^2}=2 \times 7 \times \sqrt{3}=14 \sqrt{3} \mathrm{cm}$

35. i.

Class intereval	Frequency	Cumulative frequency
10-20	12	12
20-30	30	42
30-40	x	42 + x(F)
40-50	65(f)	107 + x
50-60	y	107 + x + y
60-70	25	132 + x + y
70-80	18	150 + x + y
	N = 230	

let the unknown frequencies are 'x' and 'y'.

Median = 46

Then, median class = 40 - 50

$\therefore$ Median = l + $\frac{\frac{N}{2}-F}{f} \times$ n

Here,

L = Lower limit of median class

cf = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

$\therefore$ l = 40, h = 50 - 40 = 10, f = 65, F = 42 + x

$\Rightarrow$ 46 = 40 + $\frac{115-(42+x)}{65} \times$ 10

$\Rightarrow$ 46 - 40 = $\frac{115-(42+x)}{65} \times$ 10

$\Rightarrow$ $\frac{6 \times 65}{10}$ = 73 - x

$\Rightarrow$ x = 73 - 39 = 34

Given

N = 230

$\Rightarrow$ 12 + 30 + 34 + 65 + y + 25 + 18 = 230

$\Rightarrow$ y = 230 - 184 = 46

ii.

Class interval	Mid value	Frequency	f_x
10-20	15	12	180
20-30	25	30	750
30-40	35	34	1190
40-50	45	65	2925

50-60	55	46	2530
60-70	65	25	1625
70-80	75	18	1350
		N = 230	Σf_x **= 10550**

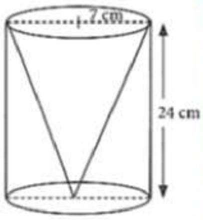
$\therefore$ Mean = $\frac{\sum fx}{N}$

$= \frac{10550}{230}$ = 45.87

Section E

36. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Given height of cone = 24cm and radius of base = r = 7cm

Slant height of conical cavity,

l = $\sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$ = 25 cm

(ii) we know that r = 7cm, l = 25 cm

Curved surface area of conical cavity = πrl

= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

(iii)For cylinder height = h = 24cm, radius of base = r = 7cm

External curved surface area of cylinder

= 2πrh = 2 $\times$ $\frac{22}{7}$ $\times$ 7 $\times$ 24 = 1056 cm^2

OR

Curved surface area of conical cavity = πrl

= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

External curved surface area of cylinder

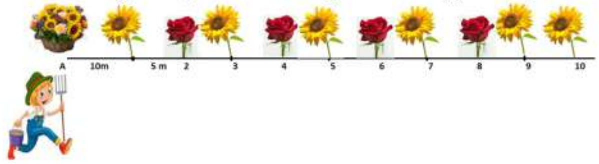
= 2πrh = 2 $\times$ $\frac{22}{7}$ $\times$ 7 $\times$ 24 = 1056 cm^2

$\frac{curved\ surface\ area\ of\ cone}{curved\ surface\ area\ of\ cylinder} = \frac{550}{1056} = \frac{275}{528}$

hence required ratio = 275:528

37. **Read the text carefully and answer the questions:**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) The distance covered by Dinesh to pick up the first flower plant and the second flower plant,

= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30

therefore, the distance covered for planting the first 5 plants

= 20 + 30 + 40 + ... 5 terms

This is in AP where the first term a = 20

and common difference d = 30 - 20 = 10

(ii) We know that a = 20, d = 10 and number of terms = n = 5 so,

$S_n = \frac{n}{2}[2a + (n-1)d]$

So, the sum of 5 terms

$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times 80 = 200$ m

Hence, Dinesh will cover 200 m to plant the first 5 plants.

OR

Total distance covered by Ramesh 650 m

Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes

Time taken to plant all 10 plants = 15 $\times$ 10 = 150 minutes

Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

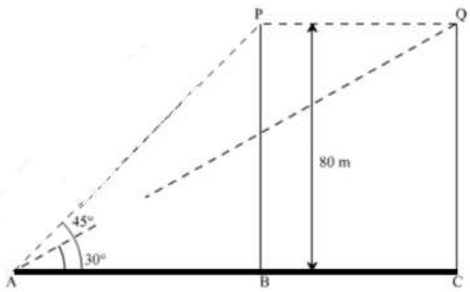
(iii) As a = 20, d = 10 and here n = 10

So, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130 = 650$ m

So, hence Ramesh will cover 650 m to plant all 10 plants.

38. **Read the text carefully and answer the questions:**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle$ABP

$\tan 45^o = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird

In $\triangle$AQC

$\tan 30^o = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow AC = 80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow BC = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

$\sin 45^o = \frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow AP = 80\sqrt{2}m$

(iii)The distance between second position of bird and observer.

In $\triangle AQC$

$\sin 30^{o} = \frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow AQ = 160$ m