Sample Question Paper - 8
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate the integral: $\int \frac{1}{(x^2+2)(x^2+5)} dx$ **[2]**

OR

Prove that: $\int\limits_0^{\pi/2} \frac{\sqrt{\cot x}}{(\sqrt{\tan x}+\sqrt{\cot x})} dx = \frac{\pi}{4}$

2. Find the differential equation of the family of all straight lines. **[2]**
3. If $\vec{a}$ and $\vec{b}$ represent two adjacent sides of a parallelogram, then write vectors representing its diagonals **[2]**
4. Find the vector equation f the plane passing through the point (1,1,1) and parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 5.$ **[2]**
5. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to agree in stating the same fact? **[2]**
6. Determine P(E|F): A dice is thrown three times.E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses. **[2]**

Section B

7. Evaluate: $\int_0^{\pi/2} \frac{dx}{(1+\cos^2 x)}$ **[3]**
8. Solve the initial value problem: $(y^4 - 2x^3 y) dx + (x^4 - 2xy^3) dy = 0$, $y(1) = 1$ **[3]**

OR

Find the particular solution of the differential equation $x(1 + y^2)dx - y(1 + x^2)dy = 0$, given that y = 1, when x = 0.

9. If $\overrightarrow{a}$ makes equal angles with the coordinate axes and has magnitude 3, find the angle between $\overrightarrow{a}$ and each of the three coordinate axes. **[3]**
10. Find the shortest distance between the lines whose vector equations are **[3]**
$\vec{r} = (1-t)\,\hat{i} + (t-2)\,\hat{j} + (3-2t)\,\hat{k}$

$\vec{r} = (s+1)\,\hat{i} + (2s-1)\,\hat{j} - (2s+1)\,\hat{k}$

OR

using vectors, find the value of x such that the four points A(x, 5, -1), B(3, 2, 1), C(4, 5, 5) and D(4, 2, – 2) are coplanar.

Section C

11. Prove that: $\int_{0}^{\pi/2} \log(\tan x + \cot x)dx = \pi(\log 2)$. **[4]**

12. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts. **[4]**

OR

Using integration, find the area of the region given below:
$\{(x, y) : 0 \leqslant y \leqslant x^2 + 1, 0 \leqslant y \leqslant x + 1, 0 \leqslant x \leqslant 2\}$.

13. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, -4, -5) and B(2, -3, 1) intersects the plane 2x + y + z = 7. **[4]**

CASE-BASED/DATA-BASED

14. In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03. **[4]**

Based on the above information, answer the following questions.

i. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Govind is

ii. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$?

Solution

MATHEMATICS 041

Class 12 - Mathematics

Section A

1. Here we have,

$I=\int \frac{dx}{(x^2+2)(x^2+5)}$

Put x^2 = t

$\therefore \frac{1}{(x^2+2)(x^2+5)}=\frac{1}{(t+2)(t+5)}$

Let $\frac{1}{(t+2)(t+5)}=\frac{A}{t+2}+\frac{B}{t+5}$

$\Rightarrow \frac{1}{(t+2)(t+5)}=\frac{A(t+5)+B(t+2)}{(t+2)(t+5)}$

$\Rightarrow$ 1 = A(t + 5) + B(t + 2)

Putting t = -5

$\therefore$ 1 = B(-5 + 2)

$\Rightarrow B=-\frac{1}{3}$

Putting t = -2

$\therefore$ 1 = A(-2 + 5) + B $\times$ 0

$\Rightarrow A=\frac{1}{3}$

$\therefore I=\frac{1}{3}\int \frac{dx}{x^2+2}-\frac{1}{3}\int \frac{dx}{x^2+5}$

$=\frac{1}{3}\int \frac{dx}{x^2+(\sqrt{2})^2}-\frac{1}{3}\int \frac{dx}{x^2+(\sqrt{5})^2}$

$=\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$

OR

Let $y=\int\limits_0^{\pi/2} \frac{\sqrt{\cot x}}{(\sqrt{\tan x}+\sqrt{\cot x})}dx$

$y=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\sin x}{\cos x}}+\sqrt{\frac{\cos x}{\sin x}}}dx$

$y=\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}dx$ .... (i)

Using theorem of definite integral

$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we have

$y=\int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$

$y=\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx$ .... (ii)

Adding eq.(i) and (ii)

$2y=\int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}dx+\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx$

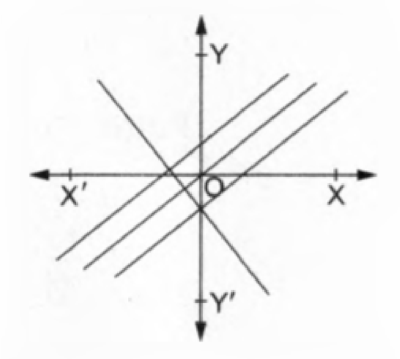
$2y=\int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}dx$

$2y=\int_0^{\pi/2} 1dx$

$2y=(x)_0^{\frac{\pi}{2}}$

$y=\frac{\pi}{4}$

2. The general equation of the family of all straight lines is given by y = mx + c, where m and c are parameters.



Now,we have to solve y = mx + c $\Rightarrow \frac{dy}{dx}$ = m

$\Rightarrow \frac{d^2y}{dx^2}$ = 0

Therefore, the required differential equation is $\frac{d^2y}{dx^2}$ = 0

3. Given that $\vec{a}, \vec{b}$ represent the two adjacent sides of a parallelogram

In $\triangle ABC$, using triangle law, we get

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\vec{a} + \vec{b} = \overrightarrow{AC}$

In $\triangle ABD$, using triangle law, we get

$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$

$\vec{b} + \overrightarrow{DB} = \vec{a}$

$\vec{a} - \vec{b} = \overrightarrow{DB}$

$\therefore$ Diagonals $\overrightarrow{AC} = \vec{a} + \vec{b}$

$\overrightarrow{DB} = \vec{a} - \vec{b}$

4. Position vector of the point (1,1,1) is

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Any plane parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) - 5 = 0$ is given by $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) + d = 0$

since it passes through the point having position vector ,we have

$\hat{i} + \hat{j} + \hat{k} \therefore (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k}) + d = 0$

$\Rightarrow$ 2 - 1 + 2 + d = 0 $\Rightarrow$ d = 3

Therefore,the required equation of plane $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) - 3 = 0$

$\Rightarrow \vec{r} \cdot (2i - \hat{\jmath} + 2\hat{k}) = 3.$

5. Let E be the event that A speaks truth and F be the event that B speaks truth. Therefore, E and F are independent events such that,

P(E) = $\frac{60}{100} = \frac{3}{5}$ and P(F) = $\frac{90}{100} = \frac{9}{10}$

A and B will agree in stating the same fact in the following mutually exclusive ways:

i. A and B both speak truth
ii. A and B both tell a lie.

Therefore, required probability is given by,

P (A and B agree) = $P((E \cap F) \cup (\bar{E} \cap \bar{F}))$

= $P(E \cap F) + P(\bar{E} \cap \bar{F})$

= $P(E)P(F) + P(\bar{E})P(\bar{F})$ = $\frac{3}{5} \times \frac{9}{10} + \frac{2}{5} \times \frac{1}{10} = \frac{29}{50} = \frac{58}{100}$

Therefore, A and B will agree in 58% cases.

6. Since a dice has six faces. Therefore $n\left(S\right) = 6 \times 6 \times 6 = 216$

E = (1, 2, 3, 4, 5, 6) x (1, 2, 3, 4, 5, 6) x (4)

F = (6) x (5) x (1, 2, 3, 4, 5, 6)

$\Rightarrow n\left(F\right)$ = 1 x 1 x 6 = 6

P (F) = $\frac{n(F)}{n(S)} = \frac{6}{216}$

$\therefore E \cap F$ = (6, 5, 4)

$n(E \cap F)$ = 1

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{216}$

And $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}$

Section B

7. Let I = $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos^2 x} dx$

Dividing by cos^{2}x in numerator and denominator, we get

I = $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\sec^2 x+\tan^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+2\tan^2 x} dx$

Consider I = $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2+b^2\tan^2 x} dx$

Put, tan x = t

$\Rightarrow$ sec^{2}xdx = dt

I = $\int_0^{\frac{\pi}{2}} \frac{1}{a^2+b^2t^2} dt$

= $\frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2}+t^2} dt$

Let t = $\frac{a}{b}$ tanθ

= tan x

I = $\frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b}\sec^2\theta}{\frac{a^2}{b^2}+\frac{a^2}{b^2}\tan^2\theta} d\theta$

= $\frac{1}{ab}\theta = \frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\tan x\right)\Big|_0^{\frac{\pi}{2}}$

= $\frac{\pi}{2ab}$

Here, a = 1 and b = $\sqrt{2}$

Hence,

I = $\frac{\pi}{2\sqrt{2}}$

8. The given differential equation is,

(y^4 - 2x^3 y) dx + (x^4 - 2xy^3) dy = 0

$\frac{dy}{dx} = \frac{2x^3y-y^4}{x^4-2xy^3}$

It is a homogeneous equation

Put y = vx and $x\frac{dv}{dx} + v = \frac{dy}{dx}$

So, $x\frac{\mathrm{dv}}{\mathrm{dx}} + \mathrm{v} = \frac{2\mathrm{x}^3\mathrm{vx}-\mathrm{v}^4\mathrm{x}^3}{\mathrm{x}^4-2\mathrm{xv}^3\mathrm{x}^3}$

$x\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{2\mathrm{v}-\mathrm{v}^4}{1-2\mathrm{v}^3} - \mathrm{v}$

$x\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{\mathrm{v}^3+\mathrm{v}}{1-2\mathrm{v}^3}$

$\frac{1-2v^3}{v^3+v} dv = \frac{dx}{x}$

$\int \frac{1-2v^3}{v^3+v} dv = \int \frac{1}{x} dx$ ...(a)

$\frac{1-2v^3}{v(v+1)(v^2-v+1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{Cv+D}{v^2-v+1}$

1 - 2v^2 = A(v^3 + 1) + Bv(v^2 - v + 1) + (Cv + D) (v^2 + v)

1 - 2v^2 = v^3 (A + B + C) + v^2 (-B + C + D) + v(B + D) + A

Comparing coefficients of like power of v,

A = 1 ...(i)

B + D = 0 ...(ii)

-B + C + D = 0 ...(iii)

A + B + C = -2 ...(iv)

Solving eq. (i), (ii), (iii) and (iv) we get

A = 1, B = -1, C = -2, D = 1

Using eq. (a)

$\int \frac{1}{v}dv - \int \frac{1}{v+1}dv = \int \frac{2v-1}{v^2-v+1}dv = \int \frac{1}{x}dx$

log v - log (v + 1) - log (v^2 - v + 1) = log xc

log $\left(\frac{v}{v^3+1}\right)$ = log cx

$\left(\frac{v}{v^3+1}\right)$ = cx

Using the value of v, we get

$\frac{y}{x^3+y^3} = cx$

At, x = 1, and y = 1, we have

$c = \frac{1}{2}$

$\therefore \ \frac{y}{x^3+y^3} = \frac{x}{2}$

OR

Given differential equation -

x(1 + y^2) dx - y(1 + x^2) dy = 0

⇒ x(1 + y^2) dx = y(1 + x^2)dy

Therefore,on separating the variables, we get,

$\frac{y}{(1+y^2)}dy = \frac{x}{(1+x^2)}dx$

On integrating both sides, we get

$\int \frac{y}{1+y^2}dy = \int \frac{x}{(1+x^2)}dx$

$\Rightarrow \quad \frac{1}{2}\log\left|1+y^2\right| = \frac{1}{2}\log\left|1+x^2\right| + C$ ...(i)

$$\left[\begin{array}{l} \text{put } 1+y^2 = u \Rightarrow 2ydy = du \\ \text{then } \int \frac{y}{1+y^2}dy = \int \frac{1}{2u}du = \frac{1}{2}\log|u| \\ \text{and put } 1+x^2 = v \Rightarrow 2xdx = dv \\ \text{then } \int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{1}{v}dv = \frac{1}{2}\log|v| \end{array}\right]$$

Also, given that y = 1, when x = 0.

On substituting the values of x and y in Eq. (i), we get,

$\frac{1}{2}\log|1+(1)^2| = \frac{1}{2}\log|1+(0)^2| + C$

$\Rightarrow \quad \frac{1}{2}\log 2 = C \ [\because \log 1 = 0]$

On putting $C = \frac{1}{2}\log 2$ in Eq. (i), we get

$\frac{1}{2}\log\left|1+y^2\right| = \frac{1}{2}\log\left|1+x^2\right| + \frac{1}{2}\log 2$

$\Rightarrow \quad \log\left|1+y^2\right| = \log\left|1+x^2\right| + \log 2$

$\Rightarrow \quad \log\left|1+y^2\right| - \log\left|1+x^2\right| = \log 2$

$\Rightarrow \quad \log\left|\frac{1+y^2}{1+x^2}\right| = \log 2 \ \left[\because \log m - \log n = \log\frac{m}{n}\right]$

$\Rightarrow \frac{1+y^2}{1+x^2} = 2$

$\Rightarrow \quad 1+y^2 = 2+2x^2$

$\Rightarrow \quad y^2 - 2x^2 - 1 = 0$

which is the required particular solution of given differential equation.

9. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and let A be the angle between $\vec{a}$ and each of the coordinate axes.

Then, A is the angle between $\vec{a}$ and each one of $\vec{i}$, $\vec{j}$ and

$\therefore$ cos A = $\frac{\vec{a}\cdot\hat{i}}{|\vec{a}||\hat{i}|} = \frac{a_1}{3} \Rightarrow$ a_1 = 3 cos A [$\because \vec{a}\cdot\hat{i}$ = a_1, $|\vec{a}| = 3, |\hat{i}|$ = 1]

Similarly, a_2 = 3 cos A and a_3 = 3 cos A.

Now, $|\vec{a}| = 3, |\overrightarrow{a}|^2 = 9$

$\Rightarrow a_1^2 + a_2^2 + a_3^2\ 9$

$\Rightarrow 9\cos^2 A + 9\cos^2 A + 9\cos^2 A = 9$

$\Rightarrow 27\cos^2 A = 9$

$\Rightarrow \cos^2 A = \frac{1}{3}$

$\Rightarrow \cos A = \frac{1}{\sqrt{3}}$

$\Rightarrow A = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

10. $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$

$\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$= 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$

$= \sqrt{29}$

$(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (2\hat{i} - 4\hat{j} - 3\hat{k}).(\hat{j} - 4\hat{k}) = -4 + 12 = 8$

$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{8}{\sqrt{29}}$ units.

OR

A(x, 5, –1), B(3, 2, 1), C(4, 5, 5), D(4, 2, –2)

$$\left.\begin{aligned} \overrightarrow{BA} &= (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overrightarrow{BC} &= \hat{l}i + 3\hat{j} + 4\hat{k} \\ \overrightarrow{BD} &= 1\hat{i} + 0\hat{j} - 3\hat{k} \end{aligned}\right\}$$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$

i.e., (x - 3) (-9) -3(-7) -2(-3) = 0

x = 6

Section C

11. $y = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx$

$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx$

Let the given integral be, y = $-\left(\int_0^{\frac{\pi}{2}} \log\sin x dx + \int_0^{\frac{\pi}{2}} \log\cos x dx\right)$

Let, I = $\int_0^{\frac{\pi}{2}} \log\sin x dx$ .... (i)

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - x\right)dx$

$I = \int_0^{\frac{\pi}{2}} \log\cos x dx$

Adding eq.(1) and eq.(2)

$2I = \int_0^{\frac{\pi}{2}} \log \sin x dx + \int_0^{\frac{\pi}{2}} \log \cos x dx$

$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx$

$2I = \int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 dx$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = π

$2I = \frac{1}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi}{2} \log 2$

$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin x dx - \frac{\pi}{2} \log 2$

$2I = I - \frac{\pi}{2} \log 2$

$I = \int_0^{\frac{\pi}{2}} \log \sin x dx = -\frac{\pi}{2} \log 2$

Similarly, $\int_0^{\frac{\pi}{2}} \log \cos x dx = -\frac{\pi}{2} \log 2$

y = $-\left(\int_0^{\frac{\pi}{2}} \log \sin x dx + \int_0^{\frac{\pi}{2}} \log \cos x dx \right)$

$y = \frac{\pi}{2} \log 2 + \frac{\pi}{2} \log 2$

$y = \pi \log 2$

Hence proved.

12. The given curves are $y^2 = 4x$ and $x^2 = 4y$

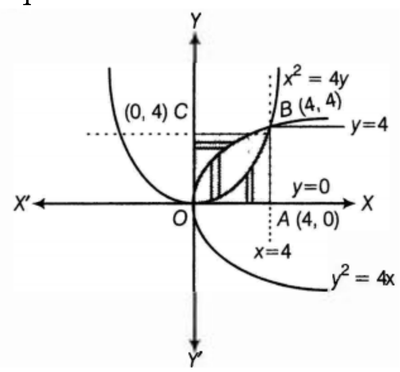
Let OABC be the square whose sides are represented by following equations

Equation of OA is y = 0

Equation of AB is x = 4

Equation of BC is y = 4

Equation of CO is x = 0



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection.

The Area bounded by these curves

$= \int_0^4 \left[y_{(\text{ parabola } y^2=4x)} - y_{(\text{ parabola } x^2=4y)} \right] dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$

$= \left[2 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{12} \right]_0^4$

$= \left[\frac{4}{3} x^{3/2} - \frac{x^3}{12} \right]_0^4$

$= \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot \left(2^2\right)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$

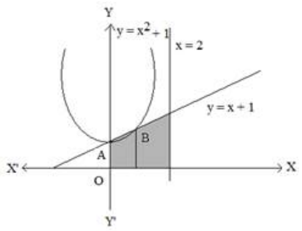
$= \frac{32}{3} - \frac{16}{3}$

$= \frac{16}{3}$ sq units

Hence, area bounded by curves $y^2 = 4x$ and x = 4y is $\frac{16}{3}$ sq units ......(i)

Area bounded by curve x^2 = 4y and the lines x = 0, x = 4 and X-axis
$= \int_0^4 y_{(\text{ parabola } x^2=4y)} dx$
$= \int_0^4 \frac{x^2}{4} dx$
$= \left[\frac{x^3}{12}\right]_0^4$
$= \frac{64}{12}$
$=\frac{16}{3}$sq units ....(ii)
The area bounded by curve y^2 = 4x, the lies y = 0, y = 4 and Y-axis
$= \int_0^4 x_{(\text{ parabola } y^2=4x)} dy$
$= \int_0^4 \frac{y^2}{4} dy$
$= \left[\frac{y^3}{12}\right]_0^4$
$= \frac{64}{12}$
$= \frac{16}{3}$sq units ....(iii)
From Equations. (i), (ii) and (iii), area bounded by the parabolas y^2 = 4x and x^2 = 4y divides the area of square into three equal parts.

OR



$y = x^2 + 1$.....(1)
y = x + 1.......(2)
Solving (1) and(2),we get, x = 1 and y = 2.
Area $= \int_0^1 \left(x^2+1\right) dx + \int_1^2 (x+1) dx$
$= [\frac{x^3}{3} + x]_0^1 + [\frac{x^2}{2} + x]_1^2$
$= [(\frac{1}{3}+1) - 0] + [(2+2) - (\frac{1}{2}+1)]$
$= \frac{23}{6}$

13. The direction ratios of line joining $A(3,-4,-5)$ *and* $B(2,-3,1)$ are
$[(2-3),(-3+4),(1+5)] = (-1,1,6)$
The equation of line passing through (3, -4, -5) and having Direction ratios (-1, 1, 6) is given by
$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \left[\because \frac{x-x_1}{a} - \frac{y-y}{b} - \frac{z-z_1}{c}\right]$
Suppose $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda(\text{ say })$
$\Rightarrow x = -\lambda + 3, y = \lambda - 4$ and $z = 6\lambda - 5$
The general point on the line is given by
$(3-\lambda, \lambda-4, 6\lambda-5)$
Line intersect the plane $2x + y + z = 7$. So,
General point on the line $(3-\lambda, \lambda-4, 6\lambda-5)$satisfy the equation of plane.
$\therefore 2(3-\lambda) + \lambda - 4 + 6\lambda - 5 = 7$
$\Rightarrow 6 - 2\lambda + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow 5\lambda = 10$
$\lambda = 2$
The point of intersection of line and plane is
$(3-2, 2-4, 6\times 2-5) = (1,-2,7)$.
Distance between (3, 4, 4) and (1, -2, 7)
$= \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2}$
$= \sqrt{4+36+9} = \sqrt{49} = 7units$

CASE-BASED/DATA-BASED

14. Let A be the event of commiting an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

 i. Using Bayes' theorem, we have

 $P(E_1 \mid A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$

 $= \frac{0.5\times 0.06}{0.5\times 0.06+0.2\times 0.04+0.3\times 0.03} = \frac{30}{47}$

 $\therefore$ Required probability $= P\left(\bar{E}_1 \mid A\right)$

 $= 1 - P(E_1 \mid A) = 1 - \frac{30}{47} = \frac{17}{47}$

 ii. $\sum_{i=1}^{3} P(E_i \mid A) = P(E_1 \mid A) + P(E_2 \mid A) + P(E_3 \mid A)$

 = 1 [$\because$ Sum of posterior probabilities is 1]