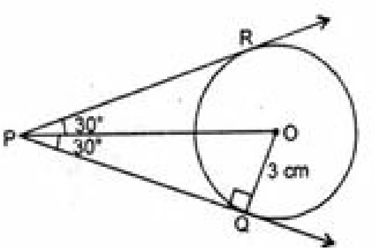
CBSE Test Paper 05

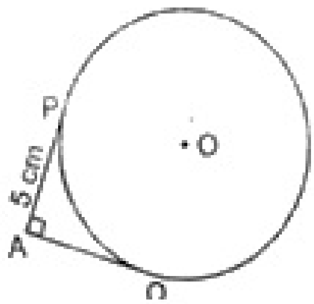
Chapter 10 Circle

1. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to: **(1)**



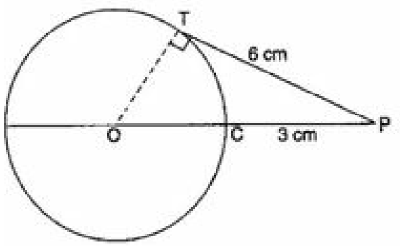
   a. 6 cm
   b. $3\sqrt{3}$
   c. 3 cm
   d. $\frac{3}{2}\sqrt{3}$ cm

2. In the given fig., if O is the center of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle$POQ is equal to : **(1)**



   a. 75°
   b. 100°
   c. 90°
   d. 80°

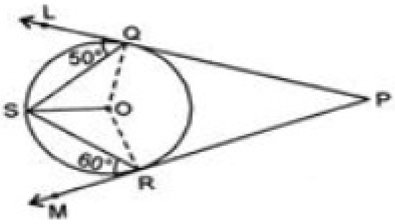
3. In the given figure, O is the centre of the circle and PT is a tangent at T. If PC = 3 cm and PT = 6 cm, then the radius of the circle is equal to: **(1)**



   a. 6 cm

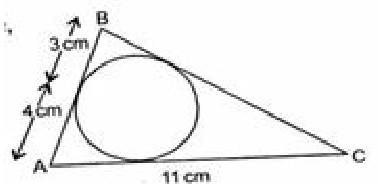
b. 5 cm
c. 7 cm
d. 4.5 cm

4. In the given figure, PQL and PRM are tangents to the circle with centre O at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50^o$ and $\angle SRM = 60^o$. Then $\angle QSR$ is equal to: **(1)**



a. 50°
b. 40°
c. 60°
d. 70°

5. In the given figure, if $\triangle$ABC is circumscribing a circle, then the length of BC is: **(1)**



a. 10 cm
b. 7 cm
c. 11 cm
d. 18 cm

6. Distance between two parallel lines is 14 cm. Find the radius of the circle which will touch both the lines. **(1)**

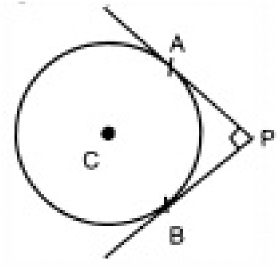
7. If a circle can be inscribed in a parallelogram how will the parallelogram change? **(1)**

8. What do you say about the line which is perpendicular to the radius of the circle through the point of contact? **(1)**

9. How many common tangents can be drawn to two circles intersecting at two distinct points? **(1)**

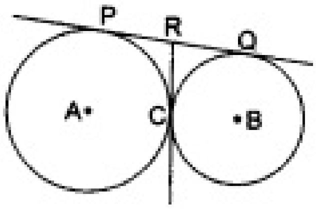
10. In fig., PA and PB are two tangents drawn from an external point P to a circle with

centre C and radius 4 cm. If PA $\perp$ PB, then find the length of each tangent. **(1)**

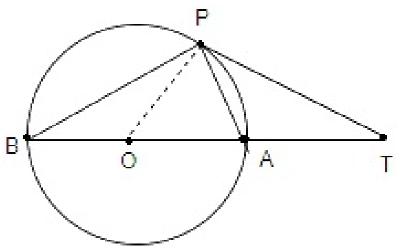


11. A tangent PQ at a point P on a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length PQ. **(2)**

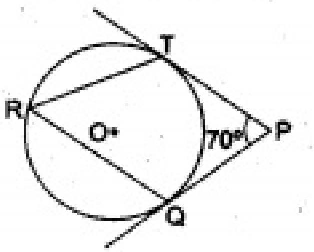
12. In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q. **(2)**



13. In the given figure, O is the centre of a circle, BOA is its diameter and the tangent at the point P meets BA extended at T. If $\angle PBO = 30^o$, then find $\angle PTA.$ **(2)**



14. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, find $\angle TRQ.$ **(3)**



15. Prove that the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc. **(3)**

16. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. **(3)**

17. A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary. **(3)**

18. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC. **(4)**

19. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. **(4)**

20. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ. **(4)**

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Solution

1. b. $3\sqrt{3}$

   Explanation: Refer fig

   PQ and PR are two tangents to a circle

   PQ = PR

   PO bisects the angle between two tangents

   therefore angle $\angle$ OPQ = $\angle$ OPR = 30°

   In right angled triangle OPQ

   $\tan 30^\circ = \frac{\mathrm{OQ}}{\mathrm{PQ}}$

   $\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{\mathrm{PQ}}$

   $\Rightarrow \mathrm{PQ} = 3\sqrt{3}$ cm = PR

2. b. 100°

   Explanation: Since OP is perpendicular to PR,

   then $\angle \mathrm{OPR} = 90^\circ$

   $\Rightarrow \angle \mathrm{RPQ} + \angle \mathrm{QPO} = 90^\circ$

   $\Rightarrow 50^\circ + \angle \mathrm{QPO} = 90^\circ$

   $\Rightarrow \angle \mathrm{QPO} = 40^\circ$

   Now, OP = OQ {Radii of same circle]

   $\therefore \angle \mathrm{OPQ} = \angle \mathrm{OQP} = 40^\circ$ [Angles opposite to equal sides]

   In triangle OPQ,

   $\angle \mathrm{POQ} + \angle \mathrm{OPQ} + \angle \mathrm{OQP} = 180^\circ$

   $\Rightarrow \angle$POQ + 40° + 40° = 180°

   $\Rightarrow$ $\angle$POQ = 100°

3. d. 4.5 cm

   Explanation: In right angled triangle OTP,

   Let the radius of the circle be r cm, then OT = OC = r

   $OP^2 = OT^2 + PT^2$

$\Rightarrow (r + 3)^2 = r^2 + 6^2$

$\Rightarrow r^2 + 6r + 9 = r^{2 + 36}$

$\Rightarrow 6r = 27 \Rightarrow r = 4.5cm$

4. d. 70°

   Explanation: Here $\angle OQS = \angle OQL - \angle SQL = 90° - 50° = 40°$

   And $\angle ORS = \angle ORM - \angle SQM$ $90° - 60° = 30°$ Since OS = OQ [Radii]

   $\Rightarrow \angle OSQ = \angle OQS = 40°$ [Angles opposite to equal sides] Again, since OS = OR [Radii]

   $\Rightarrow \angle OSR = \angle ORS = 30°$ [Angles opposite to equal sides]

   $\therefore \angle QSR = \angle OSQ + \angle OSR = 40° + 30° = 70°$

5. a. 10 cm

   Explanation: Let point of contact of tangent AB be P, point of contact of tangent BC be Q and point of contact of tangent AC be R.

   Since, Tangents from an external points are equal.

   $\therefore$ BP = BQ = 3cm

   PA = AR = 4 cm

   $\Rightarrow$ CR = 11 - 4 = 7 cm

   CR = QC = 7 cm

   $\therefore$ BC = CQ + BQ = 7 + 3 = 10 cm

6. Circle touches both the parallel lines

   Given, Distance between the parallel lines = 14 cm

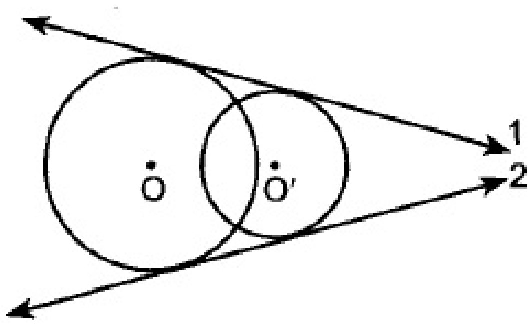
   We know that, Diameter of circle = Distance between the parallel lines

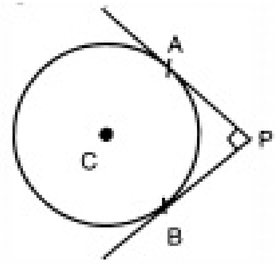
   $\therefore$Radius = $\frac{14}{2}$ = 7 cm

7. It changes into a rectangle or a square.

8. The line which is perpendicular to the radius of the circle through the point of contact will be tangent to the circle. A line which intersects a circle at any one point is called the tangent.

9. 2 common tangents can be drawn to two circles intersecting at two distinct points.



10. 

Construction: Join AC and BC

Now, $AC \perp AP \text{ and } CB \perp BP$

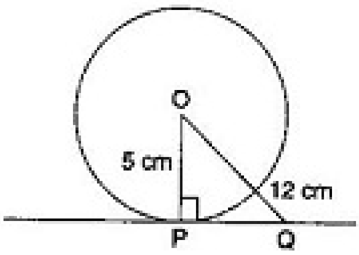
$\angle APB = 90^0$

Therefore, CAPB will be a square

CA = AP = PB = BC = 4 cm

$\therefore$ Length of tangent = 4 cm.

11. $\because$ PQ is the tangent and OP is the radius through the point of contact.



$\therefore \angle OPQ = 90^\circ$

[The tangent at any point of a circle is perpendicular to the radius through the point of contact]

By Pythagoras theorem in right $\triangle OPQ$,

$\mathrm{OQ}^2 = \mathrm{OP}^2 + \mathrm{PQ}^2$

$\Rightarrow \quad (12)^2 = (5)^2 + \mathrm{PQ}^2$

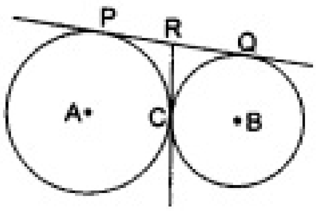
$\Rightarrow \quad 144 = 25 + \mathrm{PQ}^2$

$\Rightarrow \quad PQ^2 = 144 - 25$

$\Rightarrow \quad \mathrm{PQ}^2 = 119$

$\Rightarrow \quad PQ = \sqrt{119}\mathrm{cm}$

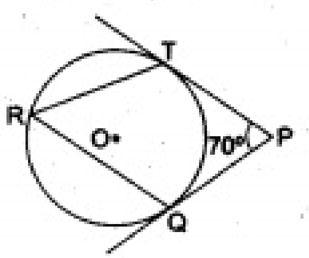
Hence, the length PQ is $\sqrt{119}$ cm.

12. 

In the given figure, PR and CR are both tangents drawn to the same circle from an external point R.
$\therefore$ PR = CR. ...(i)
Also, QR and CR are both tangents drawn to the same circle (second circle) from an external point R
QR = CR ... (ii)
From (i) and (ii), we get
PR = QR [each equal to CR].
R is the midpoint of PQ,
i.e., the common tangent to the circles at C, bisects the common tangent at P and Q.

13. If $\angle PBO = 30^o$
Then $\angle OPB = 30^o$ [angles opposite to equal sides are equal]
and $\angle OPT = 90^o$ [angle between radius and tangent]
$\angle BPT = \angle OPT + \angle OPB$
$\angle BPT = 90^o + 30^o = 120^o$
Now, $\angle PTA = 180^o - (\angle OBP + \angle BPT)$ [angle sum property of a triangle]
$\angle PTA = 180^o - (30^o + 120^o)$
$\angle PTA = 180^o - (150^o)$
$\therefore \angle PTA = 30^o$

14. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, then,we have to find $\angle TRQ$.



We know that the radius and tangent are perpendicular at their point of contact.

$\angle OTP = \angle OQP = 90^\circ$

Now, In quadrilateral OQPT

$\angle QOT + \angle OTP + \angle OQP + \angle TPQ = 360^\circ$ [Angle sum property of a quadrilateral]

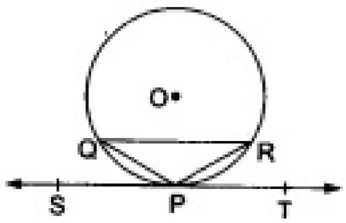
$\angle QOT + 90 + 90 + 70 = 360$

$250 + \angle QOT = 360 =$

$\angle QOT = 110^\circ$

We know that the angle subtended by an arc at the centre is double of the angle subtended by the arc at any point on the circumference of the circle.

$\angle TRQ = \frac{1}{2}\angle QOT \Rightarrow \angle TRQ = \frac{1}{2} \times 110 = 55$

15. 

Point P is the midpoint of arc $\widehat{QR}$ of a circle with centre O.

ST is the tangent to the circle at point P.

TO prove :Chord $\mathrm{QR} \| \mathrm{ST}$

Proof: P is the midpoint of $\widehat{QR}$

$\Rightarrow \widehat{QP} = \widehat{PR}$

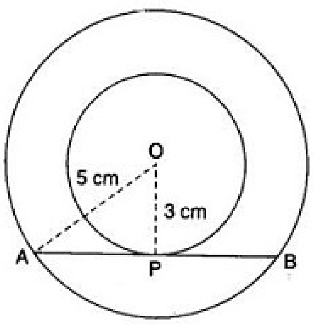
$\Rightarrow$ chord QP = chord PR [$\because$ in a circle, if two arcs are equal, then their corresponding chords are equal]

$\therefore \quad \angle PQR = \angle PRQ$

$\Rightarrow \angle TPR = \angle PRQ$ [as , $\angle PQR = \angle TPR$, angles in alternate segments]

$\Rightarrow \quad QR \| ST$, [$\because \angle TPR$ and $\angle PRQ$ are alternate interior $angles$]

16. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA

Then, $\angle$OPA = 90^0 [$\because$The tangent at any point of a circle is perpendicular to th radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$ .... By Pythagoras theorem

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow P^2 = 25 - 9$

$\Rightarrow AP^2 = 16$

$\Rightarrow AP = \sqrt{16} = 4$ cm

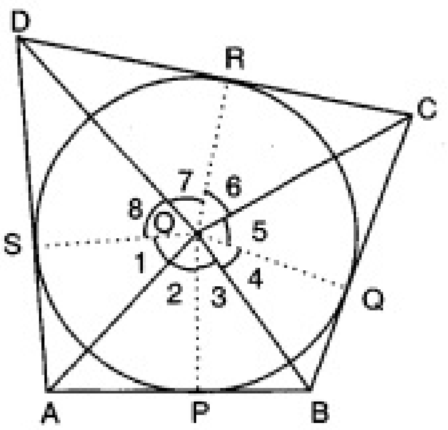
SInce the perpendicular from the centre of a circle to a chord bisects the chord, therfore,

AP = BP = 4 cm

$\therefore$ AB = AP + BP = AP + AP = 2AP = 2(4) = 8 cm

Hence, the required length is 8 cm.

17.



GIVEN: A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE$\angle AOB + \angle COD$= 180° and, $\angle AOD + \angle BOC$ = 180°

CONSTRUCTION Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

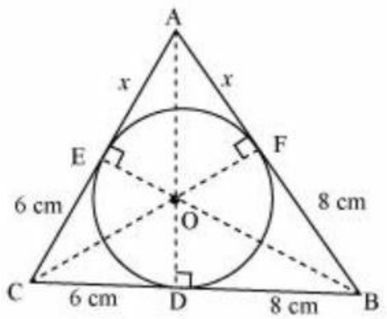
$$\begin{bmatrix} \text{Sum of all the angles} \\ \text{subtended at a point is } 360^\circ \end{bmatrix}$$

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^\circ$

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\Rightarrow \quad \angle AOB + \angle COD = 180^\circ \begin{bmatrix} \because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD \\ \angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC \end{bmatrix}$

and, $\angle AOD + \angle BOC = 180^\circ$

18. 

Let the sides BC, CA, AB of $\triangle$ABC touch the incircle at D, E, F respectively.

Join the centre O of the circle with A, B, C, D, E, F

Since, tangents to a circle from an external point are equal

$\therefore$ CE = CD = 6 cm

BF = BD = 8 cm

AE = AF = x cm (say)

OE = OF = OD = 4 cm [Radii of the circle]

AB = (x + 8) cm and AC = (x + 6) cm and CB = 6 + 8 = 14 cm

Area of $\triangle$OAB = $\frac{1}{2}$(8 + x) × 4 = (16 + 2x) cm^2 ....(i)

area of $\triangle$OBC = $\frac{1}{2}$×14 × 4 = 28 cm^2 ....(ii)

area of $\triangle$OCA = $\frac{1}{2}$(6 + x) × 4 = (12 + 2x) cm^2 ....(iii)

$\therefore$ area of $\triangle$ABC = 16 + 2x + 12 + 2x + 28 = (4x + 56) cm^2 ....(iv)

Again, perimeter of $\triangle$ABC = AC + AB + BC

= 6 + x + (8 + x) + (6 + 8)

= 28 + 2x = 2(14 + x) cm

S = $\frac{2(14+x)}{2}$ = 14 + x

Area of $\triangle$ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(14+\ x)(14+\ x-14)(14+\ x-6-x)(14+\ x-8-x)}$

$= \sqrt{(14\ +x)48x}$

$\sqrt{672x + 48x^2}$ ....(v)

$\therefore$ (4x + 56) = $\sqrt{672x + 48x^2}$ [By 4 and 5]

$\Rightarrow$ $(4x + 56)^2 = 672x + 48x^2$

$\Rightarrow$ $16(x + 14)^2 = 16(42x + 3x^2)$

$\Rightarrow$ $(x + 14)^2 = (42x + 3x^2)$

$\Rightarrow$ $x^2 + 28x + 196 = 3x^2 + 42x$

(x + 14) (x -7) = 0

x = 7 , x = -14

But x = -14 is not possible

$\therefore$ x = 7

AB = x + 8 = 7 + 8 = 15 cm

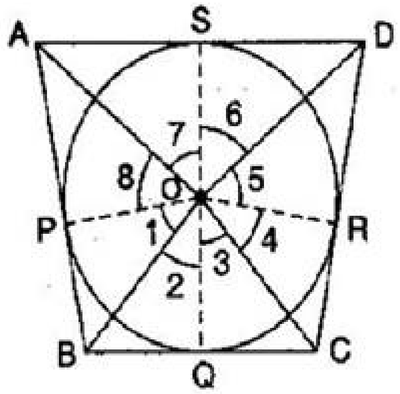
and AC = x + 6 = 7 + 6 = 13 cm

19. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove:

i. $\angle AOB + \angle COD = 180^o$
ii. $\angle BOC + \angle AOD = 180^o$

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ .... (i)

CQ = CR

DR = DS

In $\triangle$OBP and $\triangle$OBQ,

OP = OQ [Radii of the same circle]
OB = OB [Common]
BP = BQ [From eq. (i)]
$\therefore \triangle$OPB $\cong \triangle$OBQ [By SSS congruence criterion]
$\therefore \angle 1 = \angle 2$ [By C.P.C.T.]
Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to 360^{o}.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^{o}$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^{o}$

$\Rightarrow 2 (\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^{o}$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^{o}$

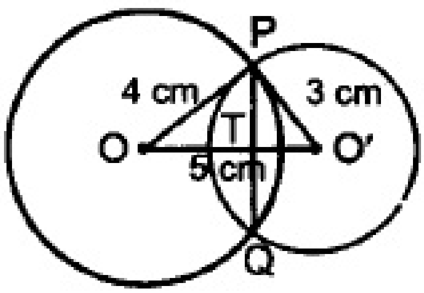
$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180^{o}$

$\Rightarrow \angle AOB + \angle COD = 180^{o}$

Similarly we can prove that

$\angle BOC + \angle AOD = 180^{o}$

20. Given, OP is tangent of the circle having center O'



So, $\angle OPO' = 90^{o}$
In right angled $\triangle$OPO'
OP = 4 cm
O'P = 3 cm
By pythagoras theorem, we get
$$\mathrm{OO'}^{2} = \mathrm{OP}^{2} + \mathrm{O'P}^{2}$$
$$= 4^{2} + 3^{2}$$
$$= 16 + 9 = 25$$
$$OO' = 5cm.$$
Let O'T = x , then OT = 5 - x

In right angled $\triangle$PTO

By pythagoras theorem, we get

$OP^2 = OT^2 + PT^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

$PT^2 = 4^2 - (5-x)^2$...(i)

In right angled $\triangle$PTO'

By pythagoras theorem, we get

$O'P^2 = O'T^2 + PT^2$

$\Rightarrow PT^2 = O'P^2 - O'T^2$

$PT^2 = 3^2 - x^2$ ...(ii)

From (i) and (ii) , we get

$3^2 - x^2 = 4^2 - (5-x)^2$

$9 - x^2 = 16 - 25 - x^2 + 10x$

$18 = 10x$

$\Rightarrow x = \frac{18}{10} = 1.8$

Substitute x in (ii), we get

$PT^2 = 3^2 - 1.8^2 = 9 - 3.24 = 5.76$

$PT = \sqrt{5.76} = 2.4$

$\Rightarrow \mathrm{PQ} = 2\mathrm{PT}$

$\quad = 2 \times 2.4$

$\therefore PQ = 4.8cm.$