* Mechanical Vibration *

To & Fro periodic motions about their eq^m position } Harmonic motion (oscillation) } → Vibration

Mean position
Eq^m position
zero position } Same.

Sin wave Periodic   Cos wave

* e^x, $\log x$ are not Vibration.

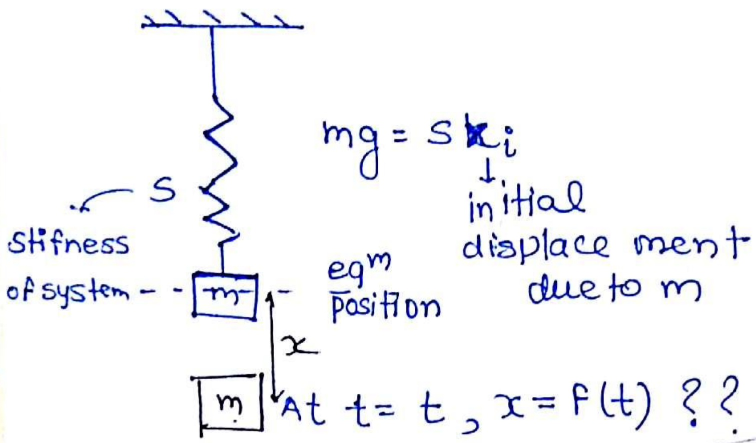
Any Vibration system is a Contribution of (1, 2, 3) → That Vib. system which is not having Rot/Reci mass (Here Vib. are introduced due to some external disturbance initially t=0)

1. Mass : m
   (K.E. storing device)
2. Stiffness : S → Resistance towards deformation)
   (spring const) → depends upon
   (P.E. storing device)
   (i) material
   (ii) Geometrical dim^n
3. Damping (energy loss due to kinetic friction)
4. $F \neq 0$
   (unbalance)

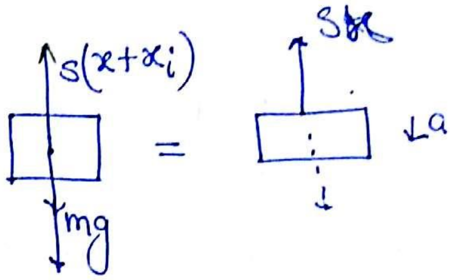
(1, 2, 3, 4) → That Vib. system in which Rot./Reci parts are there (Running M/C)

Natural Vibration (Galileo)

"The Vibration in which there is no kinetic friction at all, as well as there is no external force after the initial release of the system, are know as natural vibration"



At t = t system F.B.D.



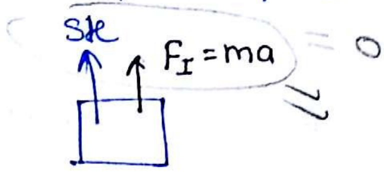
Apply Newton's 2nd law

$$(0 - sx) = ma$$

$$ma + sx = 0$$

D'Alembert's principle

At t = t System F.B.D



$$ma + sx = 0$$

$$m\ddot{x} + sx = 0$$

$$\ddot{x} + \frac{s}{m}x = 0$$

equation of Natural System.

The sol^n will be

$$x = \underline{R} \ \underline{\sin}\left(\sqrt{\frac{s}{m}}\,t + \phi\right)$$ R, ϕ = Constant

Vibration with freq. $\boxed{\omega_n = \sqrt{\frac{s}{m}}}$ rad/s

Amplitude
⇓
Const.

Natural freq. of system.

$\ast$ $\boxed{T_n = \frac{2\pi}{\omega_n}}$ Sec.

$\ast$ $\boxed{f_n = \frac{\omega_n}{2\pi} \ (Hz)}$ Linear.

R, ϕ constant. are found by initial condition.

1. At $t = 0$ $\begin{cases} x_0 = x_0 \\ \dot{x} = 0 \end{cases}$ $R, \phi = ?$

2. At $t = 0$ $\begin{cases} x = 0 \\ \dot{x} = V_0 \end{cases}$ $R, \phi = ?$

3. At $t = 0$ $\begin{cases} x = x_0 \\ \dot{x} = V_0 \end{cases}$ $R, \phi = ?$

Finally eq^n of Natural Vibration

$$* \boxed{\ddot{x} + (\omega_n)^2 x = 0}$$

Prob

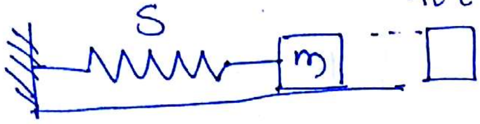
$5\ddot{x} + 3x = 0$ find ω_n

$$\ddot{x} + \tfrac{3}{5} x = 0$$

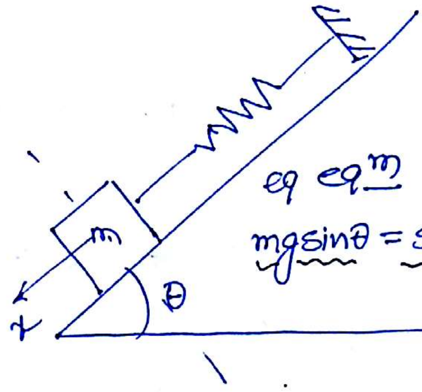
$$\omega_n^2 = 3/5 \Rightarrow \omega_n = \sqrt{3/5} \text{ rad/sec}$$

Note:

$$\omega_n = \sqrt{\frac{S}{m}} \text{ rad/s}$$



$$\omega_n = \sqrt{\frac{S}{m}} \text{ rad/s}$$



eq eq^m

$mg\sin\theta = S x_i$

$$\omega_n = \sqrt{\frac{S}{m}} \text{ rad/s}$$

After x displacement $mg\sin\theta = S x_i$

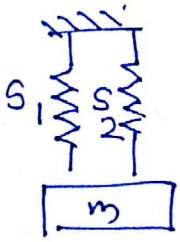
Combination of springs

series:-

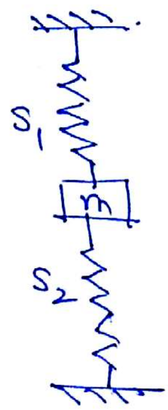
$$* \boxed{\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}}$$

$$\omega_n = \sqrt{\frac{S}{m}} \text{ rad/s}$$

Parallel:-

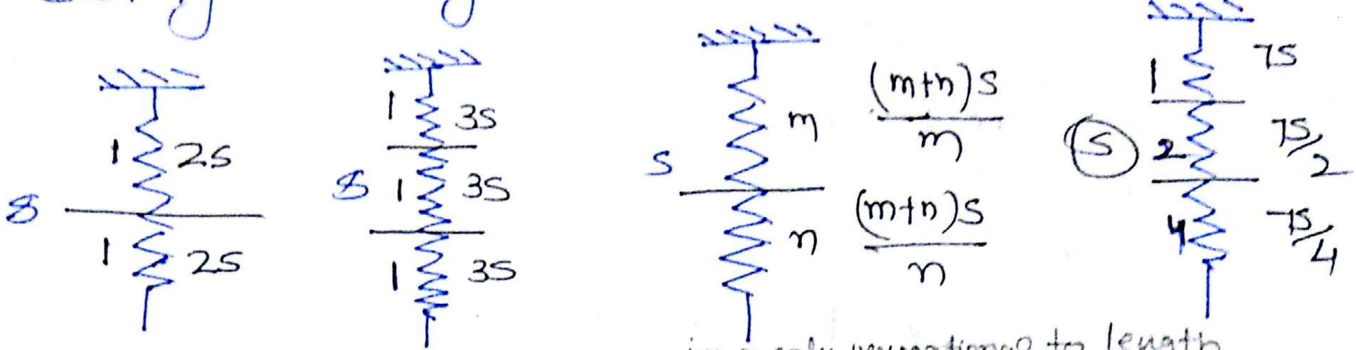


=



$$* \boxed{S = S_1 + S_2}$$

Cutting of springs:-



inversaly perportional to length

* $\boxed{s \propto A}$, $\boxed{s \propto \frac{1}{l}}$ stiffness $\propto \frac{1}{\text{length}}$

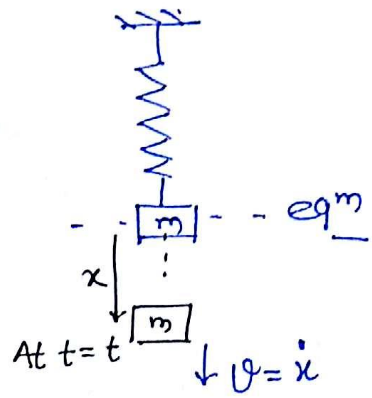
Energy Method :-

In natural vibration

Kinetic energy = 0

Total energy (E) = Const

$$\boxed{\frac{dE}{dt} = 0}$$



At $t = t$

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2$$

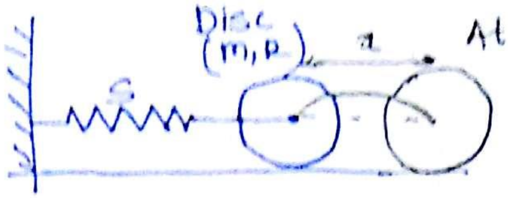
$$\frac{dE}{dt} = \frac{1}{\cancel{2}} . s . \cancel{2}x . \frac{dx}{dt} + \frac{1}{\cancel{2}} . m . \cancel{2}v . \frac{dv}{dt} = 0$$

$$m\ddot{x} + sx = 0$$

$$\ddot{x} + \frac{s}{m} x = 0$$

$$\omega_n = \sqrt{\frac{s}{m}} \ \text{rad/s}$$

Prob:



(Rotation + oscillation)

At t = t

System energy $E = \frac{1}{2}Sx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$E = \frac{1}{2}Sx^2 + \frac{1}{2}\cdot\frac{mR^2}{2}\cdot\frac{v^2}{R^2} + \frac{1}{2}mv^2$$

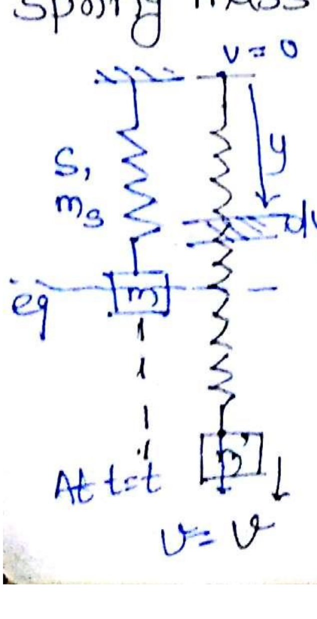
$$E = \frac{1}{2}Sx^2 + \frac{1}{2}\left(\frac{3m}{2}\right)v^2$$

$$\omega_n = \sqrt{\frac{S}{3m/2}} \Rightarrow \omega_n = \sqrt{\frac{2S}{3m}} \text{ rad/s.}$$

* Rings, Hollow Cy. $\boxed{I = mR^2}$ Disc, Solid Cy $\boxed{I = \frac{mR^2}{2}}$

Hollow Sphere $\boxed{I = \frac{2}{3}mR^2}$ Solid Sphere $\boxed{I = \frac{2}{5}mR^2}$

Spring mass system



$\rightarrow u = \frac{v}{l}\cdot y$

$$dm = \left(\frac{m_s}{l}\cdot dy\right)$$

At. $K.E._{spring} = \int_0^l \frac{1}{2}\left(\frac{m_s}{l}\cdot dy\right)\left(\frac{v}{l}\cdot y\right)^2$

$$K.E._{spring} = \frac{1}{6}m_s v^2$$

At t = t

$$E = \frac{1}{2}Sx^2 + \frac{1}{2}mv^2 + \frac{1}{6}m_s v^2$$

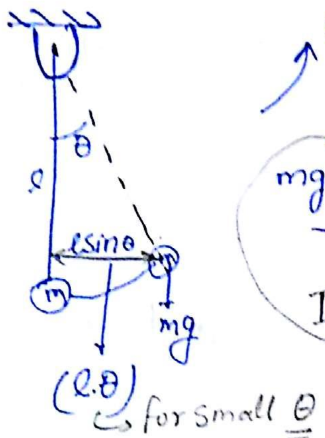
$$E = \frac{1}{2} S x^2 + \frac{1}{2}\left(m + \frac{m_s}{3}\right) v^2$$

$$\boxed{\omega_n = \sqrt{\frac{S}{\left(m + \frac{m_s}{3}\right)}}} \text{ rad/s.}$$

↳ Natural frequency in case of spring

mass = m_s

<u>Torque Method</u> :- ~~Pure rolling~~ (For very small oscillation)
Apply in Pure Rotation



$I = m l^2$

θ mg torque

D'Alemberts

$$mg(l.\theta) + I\alpha = I\ddot{\theta} = 0$$

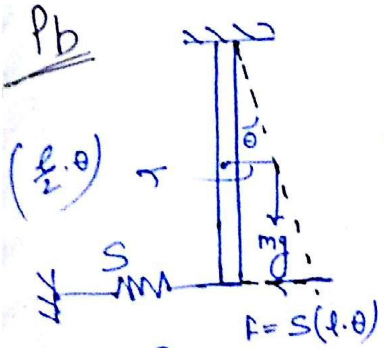
$$I\ddot{\theta} + mg\,l.\theta = 0$$

$$m l^2 \ddot{\theta} + mg\,l\,\theta = 0$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\boxed{\omega_n = \sqrt{g/l}}$$

<u>Pb</u>



$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

$$I = \frac{ml^2}{3}$$

θ

$$mg\left(\frac{l}{2}.\theta\right)$$

$$(S l \theta) l$$

$$I\ddot{\theta}$$

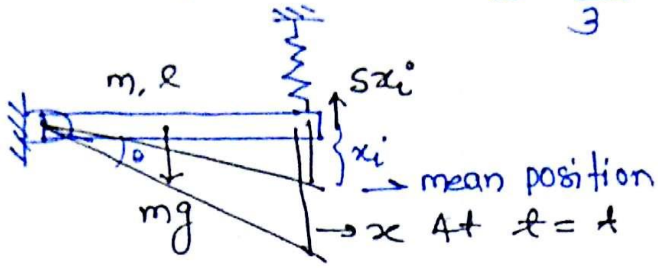
$$mg\frac{l}{2}.\theta + S l^2 \theta + I\ddot{\theta} = 0$$

$$\frac{ml^2}{3}\ddot{\theta} + \left(\frac{mgl}{2} + S l^2\right)\theta = 0$$

$$\ddot{\theta} + \left(\frac{mg\frac{l}{2} + S l^2}{ml^2/3}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{mg\frac{l}{2} + S l^2}{ml^2/3}}$$

Ques Horizontal system. $I = \frac{ml^2}{3}$



$(s l \theta) = F_s$ $T_s = (s l \theta) \cdot l$

at mean position $mg \frac{l}{2} = s x_i l$ eqm eqn

mg torque is cancelled by $s x_i$ torque

Therefor will no be consider.

$$\omega_n = \sqrt{\frac{sl^2}{ml^2/3}} = \sqrt{\frac{3s}{m}}$$

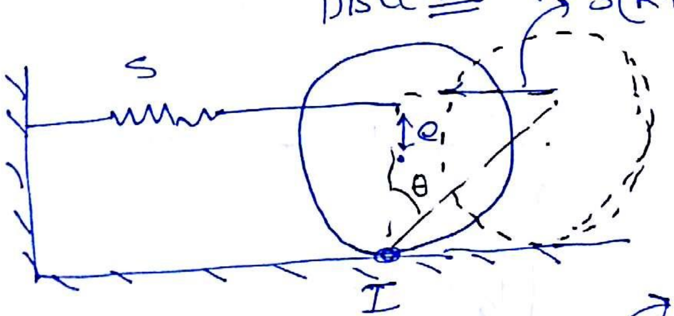
$$I\ddot{\theta} + sl^2\theta = 0$$

$$\frac{ml^2}{3}\ddot{\theta} + sl^2\theta = 0$$

$$\ddot{\theta} + \frac{3s}{m}\theta = 0$$

$$\omega_n = \sqrt{\frac{3s}{m}} \text{ rad/s.}$$

Ques



about I-C

$$I = \frac{mR^2}{2} + mR^2$$

$$I = \frac{3}{2}mR^2$$

Soln Pure Rolling only possible about I center

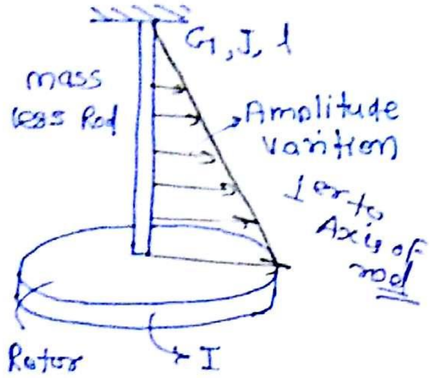
θ
$s(R+e)^2 \cdot \theta$
$I\ddot{\theta}$

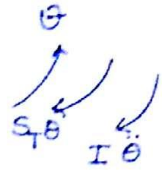
$$I\ddot{\theta} + s(R+e)^2 \cdot \theta = 0$$

$$\ddot{\theta} + \frac{2s(R+e)^2}{3m}\theta = 0$$

$$\omega_n = \sqrt{\frac{2s(R+e)^2}{3m}} \text{ rad/s}$$

Torsional Vibration :-





torsional stifness of Rod

$$S_T = \frac{GJ}{l}$$

$$I\ddot{\theta} + S_T\theta = 0$$

$$\ddot{\theta} + \left(\frac{S_T}{I}\right)\theta = 0$$

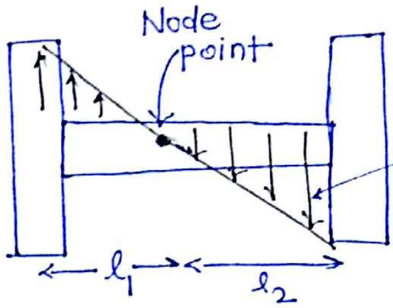
$$\omega_n = \sqrt{\frac{S_T}{I}}$$

Note: If Rod having mass

$$\omega_n = \sqrt{\frac{S_T}{I + \frac{I_{rod}}{3}}}$$

Imp * If Number of Rotor 'n' then Number of Node point $= (n-1)$

Two Rotor System :-



Variation of Amplitude

* At Node point amplitude Variation zero.

$$l_1 + l_2 = l \quad -(1)$$

$$\sqrt{\frac{S_{T1}}{I_1}} = \sqrt{\frac{S_{T2}}{I_2}} \Rightarrow \frac{GJ}{l_1 I_1} = \frac{GJ}{l_2 I_2}$$

$$\frac{l_1}{l_2} = \frac{I_2}{I_1} \quad -(2)$$

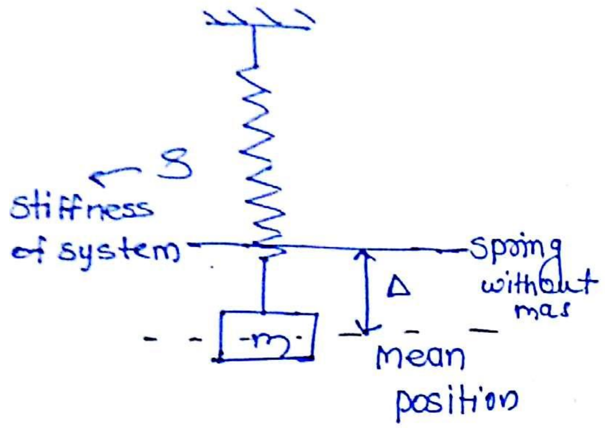
$l_1 = ?$
$l_2 = ?$ location of node point.

Method of Static Deflection of Mass (Δ):-

Imp

Rayleigh Method :-

Basic spring mass system



$$\Delta = \frac{mg}{S}$$

$$\frac{m}{S} = \sqrt{\frac{\Delta}{g}}$$

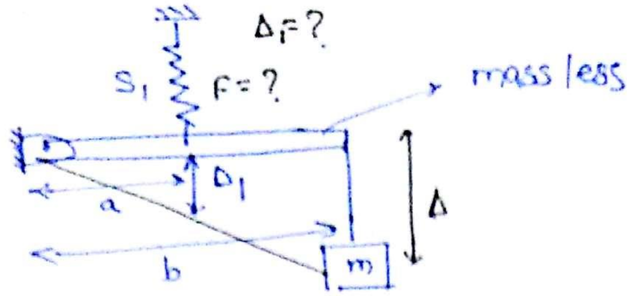
$$\Rightarrow \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{mg/S}} = \sqrt{\frac{S}{m}} = \omega_n$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{g}{\Delta}}$$

Condition:-

1. spring mass system (equivalent)
2. only point load (No contineuos mass)
3. static deflection

Pb1



$$F = mg\,(b/a)$$

$$(mg)\,b = F \cdot a \rightarrow$$

$$\cancel{F \cdot \Delta_1 = mg\,(b/a)} \Rightarrow \cancel{\Delta_1 =}$$

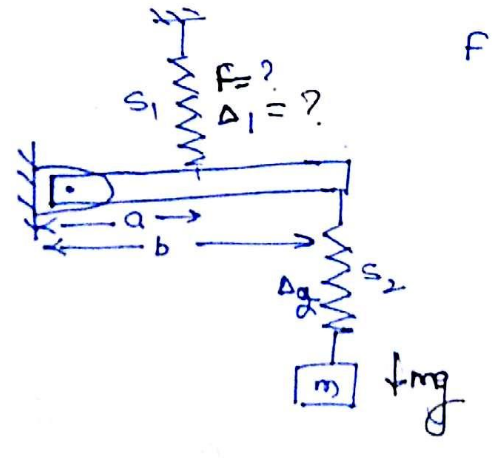
$$F = S_1 \Delta_1 \qquad \Delta_1 = \frac{mg\,(b/a)}{S_1}$$

$$\frac{\Delta_1}{a} = \frac{\Delta}{b} \Rightarrow \Delta = \Delta_1\,(b/a)$$

$$\Delta = \frac{mg}{S_1}(b/a)^2$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} \Rightarrow \omega_n = \sqrt{\frac{g\,S}{mg\,(b/a)^2}} = \sqrt{\frac{S}{m\,(b/a)^2}}$$

Pb2



$$F = mg\,(b/a)$$

$$\Delta_1 = \frac{mg}{S_1}(b/a) \qquad \Big| \qquad \frac{\Delta_1}{a} = \frac{\Delta}{b}$$

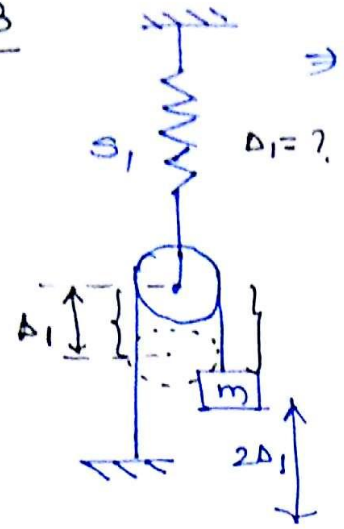
$$\Delta_2 = \frac{mg}{S_2}$$

$$\Delta = \Delta_1\,(b/a) + \Delta_2$$

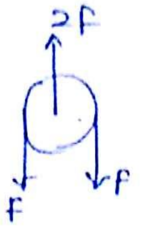
$$\Delta = \frac{mg}{S_1}(b/a)^2 + mg/S_2$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

Pb3



$\Rightarrow$



$$\Delta_1 = \frac{2mg}{S_1}$$

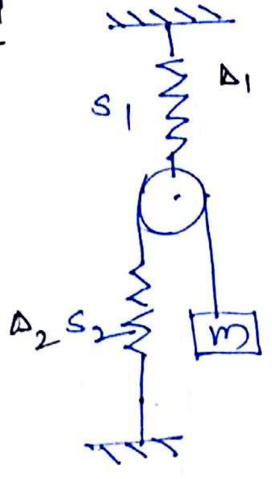
So $\Delta = \Delta_1 + \Delta_1$

$$\Delta = 2\Delta_1 = \frac{4mg}{S_1}$$

$$\Delta = \frac{4mg}{S_1}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} \qquad \omega_n = \sqrt{\frac{g S_1}{4mg}} \Rightarrow \omega_n = \sqrt{\frac{S_1}{4m}}$$

Pb4



$$\Delta_1 = \frac{2mg}{S_1}$$

$$\Delta_2 = \frac{mg}{S_2}$$

$$\Delta = 2\Delta_1 + \Delta_2$$

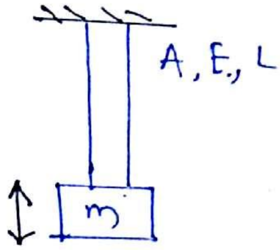
$$\Delta = \frac{4mg}{S_1} + \frac{mg}{S_2}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{1}{4m/S_1 + m/S_2}}$$

✱ Frequency of this system doen't depends on the g i.e. (same on earth & moon)

Longitudinal vibration of beam:-

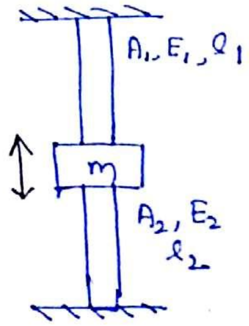
Vib^n Along the length



$$s = \frac{AE}{L} \qquad \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{AE}{mL}}$$

Longitudinal stiffness.
||el to rod axis

*



$$s = s_1 + s_2$$

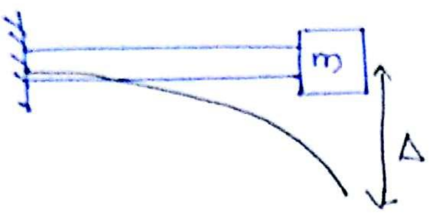
$$s_1 = \frac{A_1E_1}{l_1}, \quad s_2 = \frac{A_2E_2}{L_2}$$

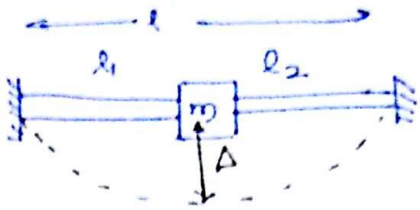
$$S = \frac{A_1E_1}{l_1} + \frac{A_2E_2}{l_2}$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{1}{m}\left(\frac{A_1E_1}{l_1} + \frac{A_2E_2}{l_2}\right)}$$

Transverse Vibration of Beams:-

vibration in beams in the dir^n $\perp^{ar}$ to length.





$$\Delta = \frac{W l^3}{3EI}$$

$W = mg$

$I =$ Area m.I

$$\Delta = \frac{W l_1^3 l_2^3}{3EI\, l^3}$$

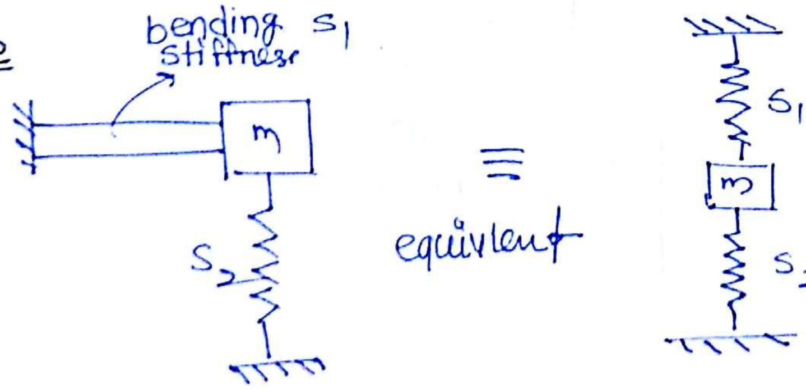
$W = mg$, $l = l_1 + l_2$

$I =$ Area m.I.

$$\omega = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{S}{m}} \text{ —?.}$$

S – bending stiffness.

Pb

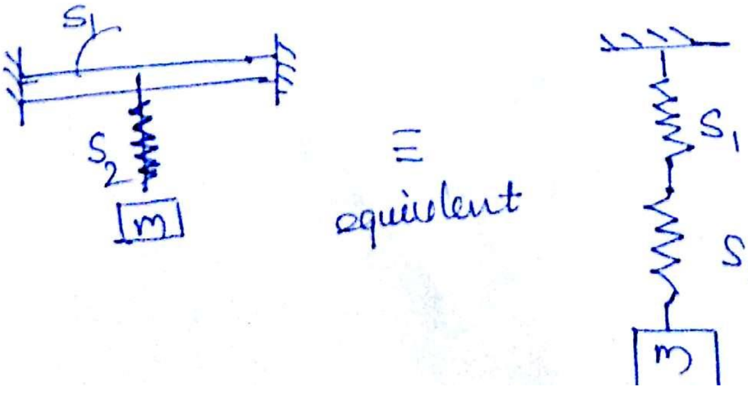


≡ equivlent

$S = S_1 + S_2$

$\omega_n = \sqrt{S/m}$

Pb

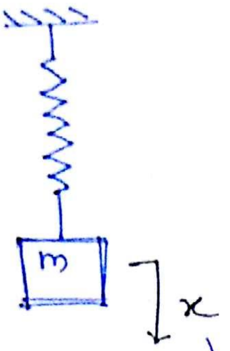


≡ equivalent

$$\omega_n = \sqrt{\frac{S}{m}}$$

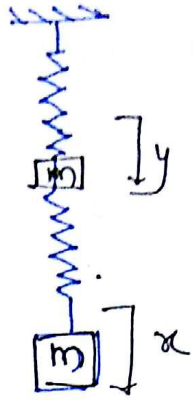
$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}$$

D.O.F. of Vibrating System :— No. of Independent Variables



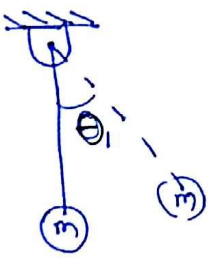
DOF = 1

One Independent Variable



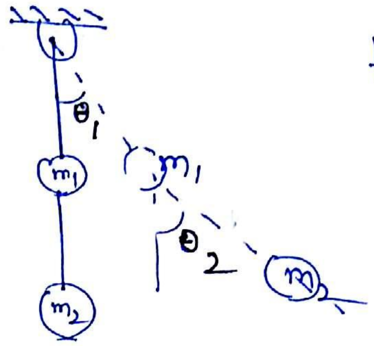
DOF = 2

x, y two independent variable.



θ_1 - one Independend Variable

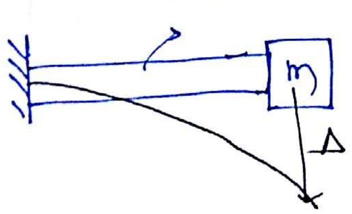
DOF = 1



two independent variable

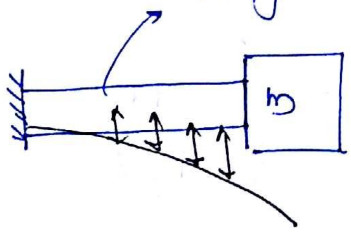
DOF = 2

massless



DOF = 1

Having mass

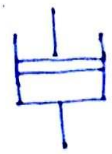


DOF = ∞

Damped system : (kinetic friction $\neq 0$)

Technical name of kinetic friction in any vibration system is "Damping"

And it is represented by the symbol "Damper"

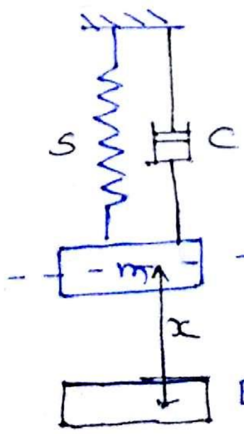
Damper 

Damping in any system
- friction b/w Dry surface (coloumb damping) very high
- Viscous Damping (very less)

$$\text{Damping force} \propto \dot{x}$$

$$= c\,\dot{x}$$

c = coefficient of damping constant for a system.

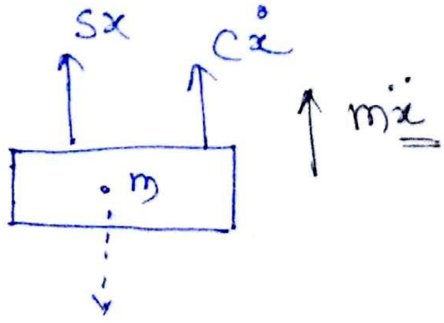


-mean position.

At t=t

$x = f(t) = ??$

At t = t



$$m\ddot{x} + c\dot{x} + sx = 0$$

$$m\ddot{x} + c\dot{x} + sx = 0$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = 0$$

equation of Damped system.

The sol^n will be

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \qquad (\alpha_1 \neq \alpha_2)$$

$$x = (A + Bt)e^{\alpha t} \qquad (\alpha_1 = \alpha_2)$$

Where α_1 & α_2 are roots of Auxillary eq^n.

$$\alpha^2 + \left(\frac{c}{m}\right)\alpha + (\omega_n^2) = 0$$

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\frac{c^2}{m^2} - 4\omega_n^2}}{2}$$

$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}$$

$$\Rightarrow \frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2} = \text{Degree of Dampingness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{(\omega_n)^2}} = \xi = \text{Damping factor } \underline{\text{or}} \text{ Damping Ratio}$$

$$\xi = \sqrt{\frac{c^2/4m^2}{s/m}} \quad + \quad \boxed{\xi = \frac{c}{2\sqrt{sm}}}$$

Damping factor (or) Damping Ratio

$$\Rightarrow 2\xi\omega_n = 2 \cdot \frac{c}{2\sqrt{sm}} \times \sqrt{\frac{s}{m}}$$

$$\boxed{2\xi\omega_n = \left(\frac{c}{m}\right)} \qquad \boxed{\omega_n = \sqrt{\frac{s}{m}}}$$

equation of Damped system

$$\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n^2)x = 0$$

soln $\quad x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$

(or)

$$x = (A+Bt)e^{\alpha t} \qquad (\alpha_1 = \alpha_2 = \alpha)$$

where $\alpha_{1,2} = \left(-\frac{c}{2m}\right) \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}$ $\qquad \boxed{\frac{c}{2m} = \xi\omega_n}$

$$\alpha_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$$

If $\xi > 1$ → over damped system / over damping / Coulamb damping

If $\xi = 1$ → critical damped system / critical damping

If $\xi < 1$ → Under damped System / under damping

Viscous damping.

1 Over damped System:- $(\xi > 1)$

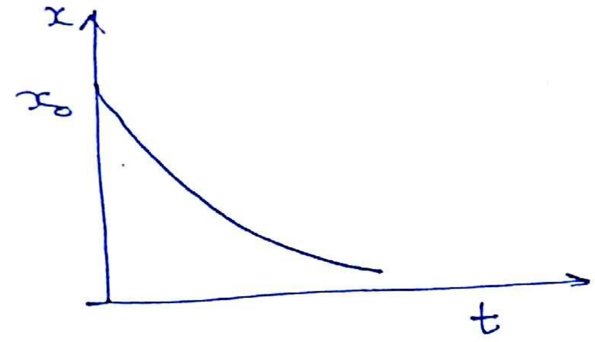
The sol^n will be

$$x = A e^{\alpha_1 t} + B e^{\alpha_2 t}$$

$$x = A e^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_n t} + B e^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}$$

A, B const

* No Vibration (not a Harmonic f^n)



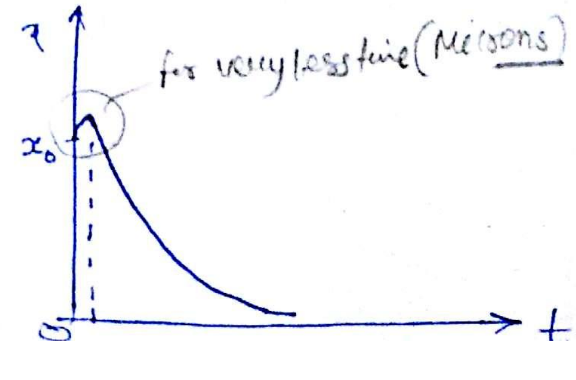
2. Critically damped systems: $(\xi = 1)$

$$\alpha_1 = \alpha_2 = \alpha = -\omega_n$$

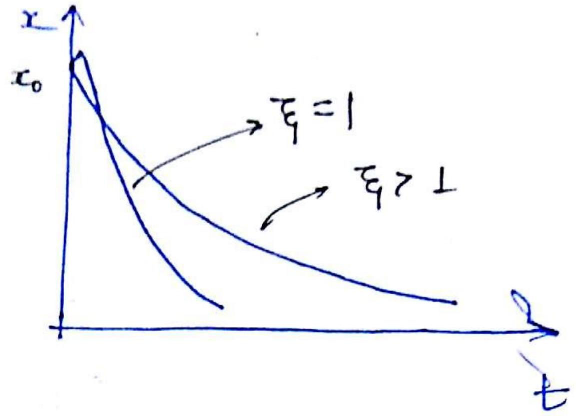
the sol^n will be $x = (A + Bt)\, e^{\alpha t}$

$$x = (A + Bt)\, e^{-\omega_n t}$$ A, B Constant

* No Vibration



Note:- critical damping Response much fast than over damping.



→ Door closers - over damped / Coulomb damping

AK-47 - critical damped. 660 bullet / min

3. Under damped System:- ($\xi < 1$)

$$\alpha_{1,2} = -\xi\omega_n \pm i\sqrt{1-\xi^2}\,\omega_n$$

$$\boxed{\omega_d = \sqrt{1-\xi^2}\cdot\omega_n}$$ frequency of critical damped system.

$$\alpha_{1,2} = -\xi\omega_n \pm i\omega_d$$

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = A\left(e^{(-\xi\omega_n + i\omega_d)t}\right) + Be^{(-\xi\omega_n - i\omega_d)t}$$

$$x = e^{-\xi \omega_n t}\left[\underbrace{(A+B)}_{X\sin\phi}\cos\omega_d t + \underbrace{i(A-B)}_{X\cos\phi}\sin\omega_d t\right]$$

$$x = X e^{-\xi \omega_n t}\sin(\omega_d t + \phi) \Rightarrow \boxed{\text{Vibration}}$$

$X, \phi \rightarrow$ Constant

Amplitude decreasing function of tein

Amplitude $= X e^{-\xi \omega_n t}$

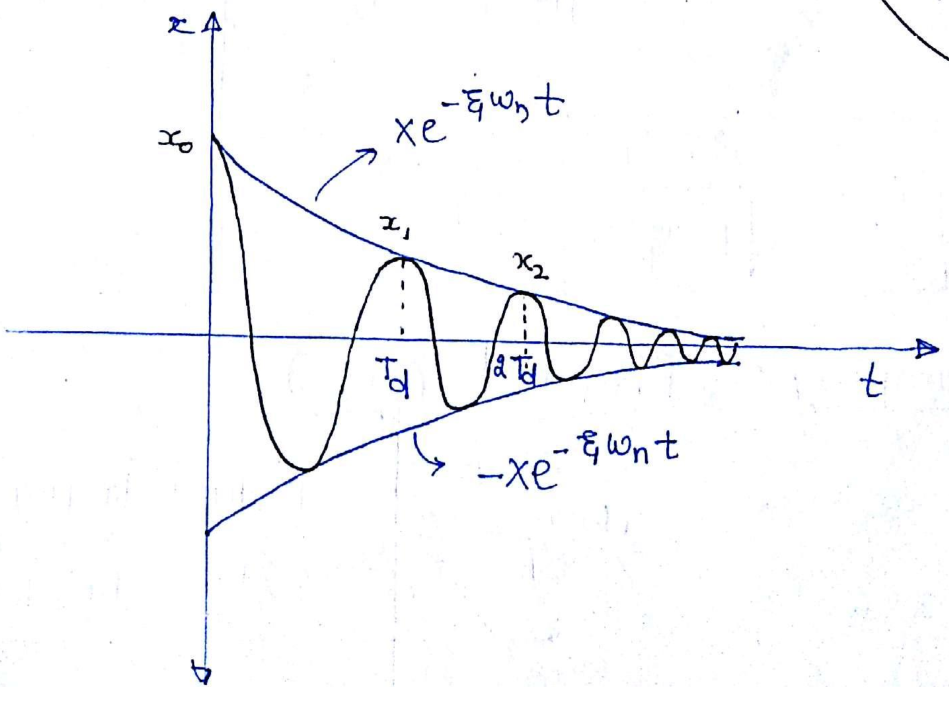
$$\boxed{\omega_d = \sqrt{1-\xi^2}\,.\,\omega_n}$$ Constant

$\hookrightarrow$ frequency of ~~dam~~ under damped system.

Time period $$\boxed{T_d = \frac{2\pi}{\omega_d}}$$ $\Rightarrow$ Constant

$$\boxed{f_d = \frac{\omega_d}{2\pi}}$$ (Hz) $\Rightarrow$ Constant.

$T = \frac{1}{f}$



At $t = 0$ $\quad x_0 = X \sin\phi$

At $t = T_d$ $\quad x_1 = X e^{-\xi \omega_n T_d} . \sin\phi$

At $t = 2T_d$ $\quad x_2 = X e^{-\xi \omega_n (2T_d)} . \sin\phi$

Decrement Ratio

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} - - - - = e^{\xi \omega_n T_d} = \text{Const.}$$

$$= e^{\delta}$$

logarithmic decrement

$$\frac{x_i}{x_{i+1}} = e^{\xi . \omega_n T_d}$$

$$\delta = \ln\left(\frac{x_i}{x_{i+1}}\right) = \xi \omega_n T_d$$

$$\delta = \xi . \omega_n \frac{2\pi}{\sqrt{1-\xi^2} . \omega_n}$$

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

$$2 \times 1 \times \sqrt{\frac{k}{m}} = \frac{C_c}{m}$$

$$\boxed{C_c = 2\sqrt{km}}$$

Critical Damping Coefficient (C_c)

$$2\xi\omega_n = c/m$$

$$2 \times 1 \times \omega_n = \frac{C_c}{m}$$

$$\boxed{\xi = \frac{c}{C_c} = \frac{\text{Actual damping Coeff.}}{\text{Critical damping Coeff.}}}$$

Note:- * The Ratio of displacement of end of $\underline{3^{rd}}$ cycle to the start of $\underline{8^{th}}$ cycle is 2.5

ie $\frac{x_3}{x_7} = 2.5$

* The Ratio of displacement of $\underline{3^{rd}}$ cy. to $\underline{8^{th}}$ cy = 2.5

$\frac{x_3}{x_8} = 2.5$ $\quad \frac{\cancel{x}_3}{\cancel{x}_8} = 2.5$ (take start to start or end to end)

Pb 55 $\quad m = 7.5$ kg

O.B

$T_d = \frac{35}{60}$ sec.

$\omega_d = \frac{2\pi}{T_d}$

$\omega_d = \sqrt{1-\xi^2} \cdot \omega_n$

$\omega_n = \quad ?$

(i) $\omega_n = \sqrt{S/m} \leftarrow ?$

(ii) $2\xi\omega_n = C/m \leftarrow ?$

(iii) $\xi = \frac{C}{C_c}$

$\frac{x_1}{x_7} = 2.5$

$\left(\frac{x_1}{x_2}\right) \cdot \left(\frac{x_2}{x_3}\right) \cdot \left(\frac{x_3}{x_4}\right)\left(\frac{x_4}{x_5}\right)\left(\frac{x_5}{x_6}\right)\left(\frac{x_6}{x_7}\right) = 2.7$

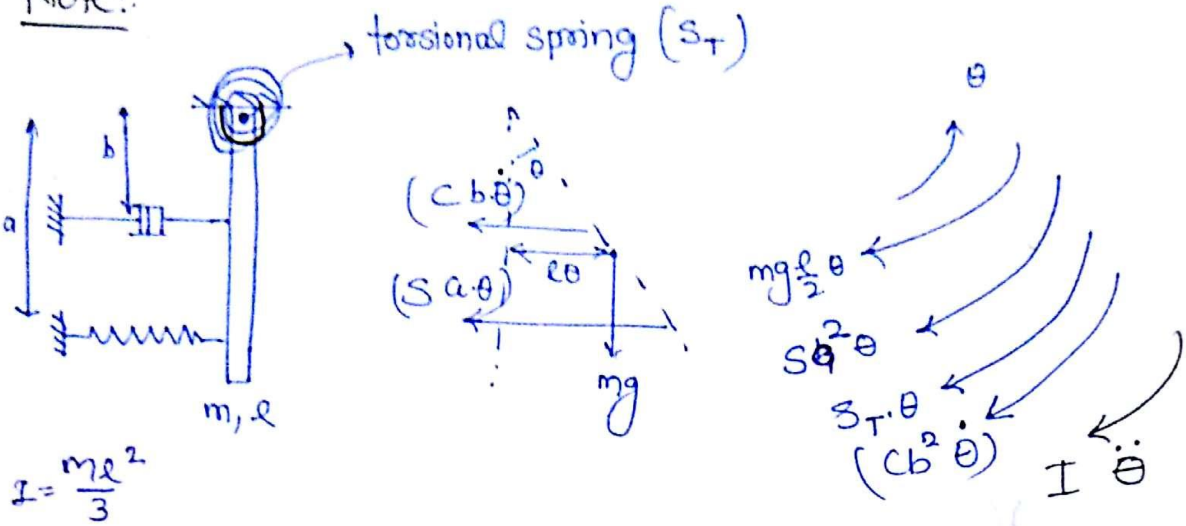
$e^{6\delta} = 2.7$

$6\delta = \ln(2.5)$

$\frac{6 \times 2\pi\xi}{\sqrt{1-\xi^2}} = \ln 2.5$

$\xi =$

Note:



$I = \frac{ml^2}{3}$

$$I\ddot{\theta} + (cb^2)\dot{\theta} + \left(mg\frac{l}{2} + sa^2 + S_T\right)\theta = 0$$

eqn of damped system

$$\cancel{m\ddot{\theta} + c\dot{\theta} + s\theta = 0}$$

$$m\ddot{x} + c\dot{x} + sx = 0$$

Pb1 The damping coefficient in kcib eqn will be $= Cb^2$

Pb2 $\omega_n = ?$ $\boxed{\omega_n = \sqrt{\frac{S}{m}}}$ $\omega_n = \sqrt{\dfrac{mg\frac{l}{2} + Sa^2 + S_T}{I}}$

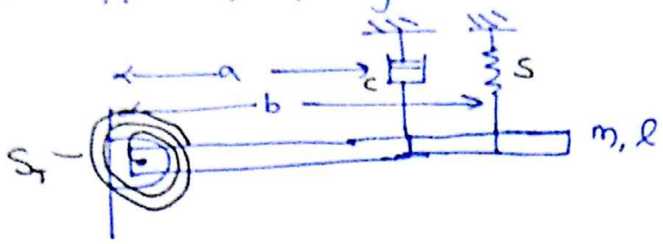
Pb3 $\xi = ?$ $2\xi\omega_n = \dfrac{Cb^2}{I}$

Pb4 $C_c = ?$

$2 \times 1 \times \omega_n = \dfrac{C_c b^2}{I}$

Pb5 $\omega_d = ? \Rightarrow \omega_d = \sqrt{1-\xi^2}\cdot\omega_n$

Note:- Horizontal system



mg torque is cancelled by Sx_i torque

therefore mg torque will not be considered

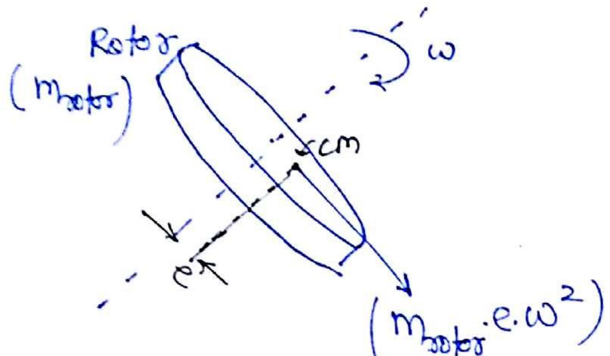
$$I\ddot{\theta} + (Cb^2)\dot{\theta} + (Sa^2 + S_T)\theta = 0$$

Vibration Causing Unbalance Forces in Running Mechanical System:-

m - machine mass which is under Vibration.

⇒ Only two types of unbalance forces:-

Rotating Unbalance:-



At $t = t$

F_{un} in a particular dir^n

$$F_{un} = (m_{rotor} \cdot e \cdot \omega^2) \sin\theta$$

$$F_{un} = (m_{rotor} \cdot e \cdot \omega^2) \sin\omega t$$

$$F_{un} = F_o \sin \omega t$$

F_o = max value

ω = force frequency (excitation freq.)
↳ speed.

Reciprocating Unbalance:-

At $t = t$

$$F_{un} = (m_{rec.} \cdot r \cdot \omega^2) \sin\theta$$

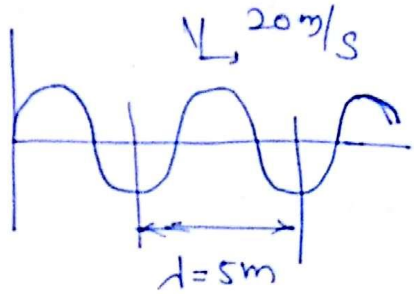
$$F_{un} = (m_{reci} \cdot r \cdot \omega^2) \sin\omega t$$

$$F_{un} = F_o \sin\omega t$$

F_o - max. speed
ω - force freq.
$\left(r \text{ - crank Radius} = \frac{\text{stroke}}{2} \right)$

Ways of ω & F_o Given

wave form:- $F_o \Rightarrow 200N$



$V = f\lambda$

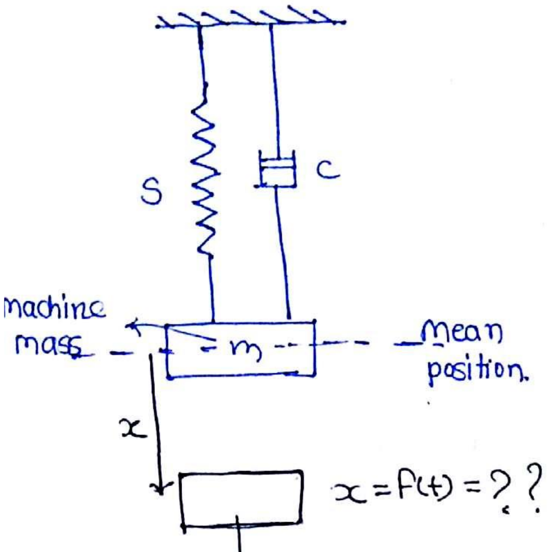
$f = \frac{20}{5} = 4 \text{ Hz}$ $\quad \omega = 2\pi f$

$\omega = 8\pi \text{ rad/s.}$

Direct form:- $F_{un} = 200 \cos 3t$

$F_o = 200 N, \quad \omega = 3 \text{ rad/s.}$

Forced - Damped System (Running machine Analysis)

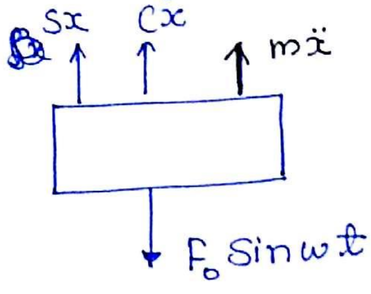


$F_{un} = F_0 \sin\omega t$

F_0 = max. Value

ω = force freq. (Speed)

At t = t system FBD



$$m\ddot{x} + c\dot{x} + Sx = F_0 \sin\omega t$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{S}{m}x = \frac{F_0}{m}\sin\omega t$$

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \left(\frac{F_0}{m}\right)\sin\omega t$$

eqⁿ of forced damped System.

The solⁿ will be

$$x = CF + PI$$

CF : $\rightarrow \xi > 1$] No Vib.

$\rightarrow \xi = 1$]

$\rightarrow \xi < 1$] Vib.

After some time CF $\rightarrow$ 0 (very less time

CF $\rightarrow$ 0

$$\boxed{x = PI}$$

PI

$$PI = \frac{F_0/m \; \sin \omega t}{D^2 + (2\xi\omega_n) D + \omega_n^2}$$

$\left\{ \begin{array}{l} D \rightarrow \text{operator} \\ D^2 = -\omega^2 \end{array} \right.$

$$PI = \frac{F_0/m \; \sin\omega t}{(\omega_n^2 - \omega^2) + (2\xi\omega_n) D} \times \frac{((\omega_n^2 - \omega^2) - (2\xi\omega_n D))}{((\omega_n^2 - \omega^2) - (2\xi\omega_n D))}$$

$$PI = \frac{\frac{F_0}{m}\left[\underbrace{(\omega_n^2 - \omega^2)}_{R\cos\phi} . \sin\omega t - \underbrace{(2\xi\omega\omega_n)}_{R\sin\phi} \cos\omega t\right]}{(\omega_n^2 - \omega^2)^2 + (2\xi\omega\omega_n)^2}$$

$$PI = \frac{F_0/m \left[R \sin(\omega t - \phi) \right]}{R^2}$$

$$PI = \frac{F_0/m \left[\sin(\omega t - \phi) \right]}{\sqrt{\omega_n^2 - \omega^2 + (2\xi\omega\omega_n)^2}}$$

$$\omega_n = \sqrt{\frac{S}{m}}$$

$$PI = \frac{F_0/m \quad \sin(\omega t - \phi)}{S/m \sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

$$PI = \frac{F_0/S \quad \sin(\omega t - \phi)}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

→ Vib with frequency ω

Amplitude (A) ⇒ independent of time

After some time

$CF \rightarrow 0$

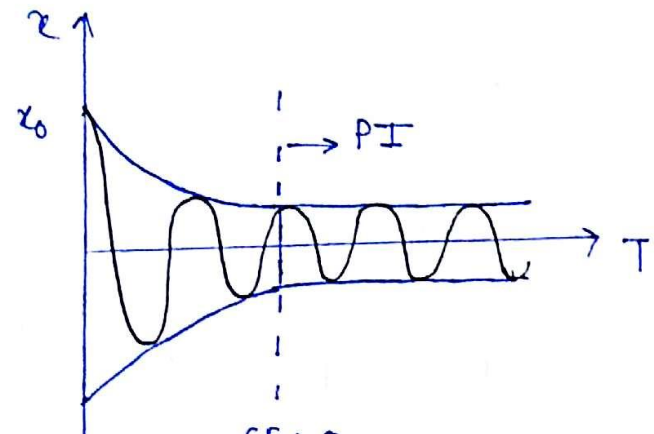
$x = PI \qquad x = A \sin(\omega t - \phi)$

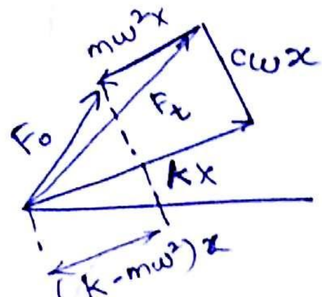
where :-

$$A = \frac{F_0/S}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

→ Amplitude of steady-state vib.
→ Amplitude of forced Vib
→ Amplitude of motion of m/c

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$





* Vibration in Running system will never stop

* To stop the fatigue every Running mechanical Component must have one Running life
↳ fatigue life.

$$A = \frac{F_o/s}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

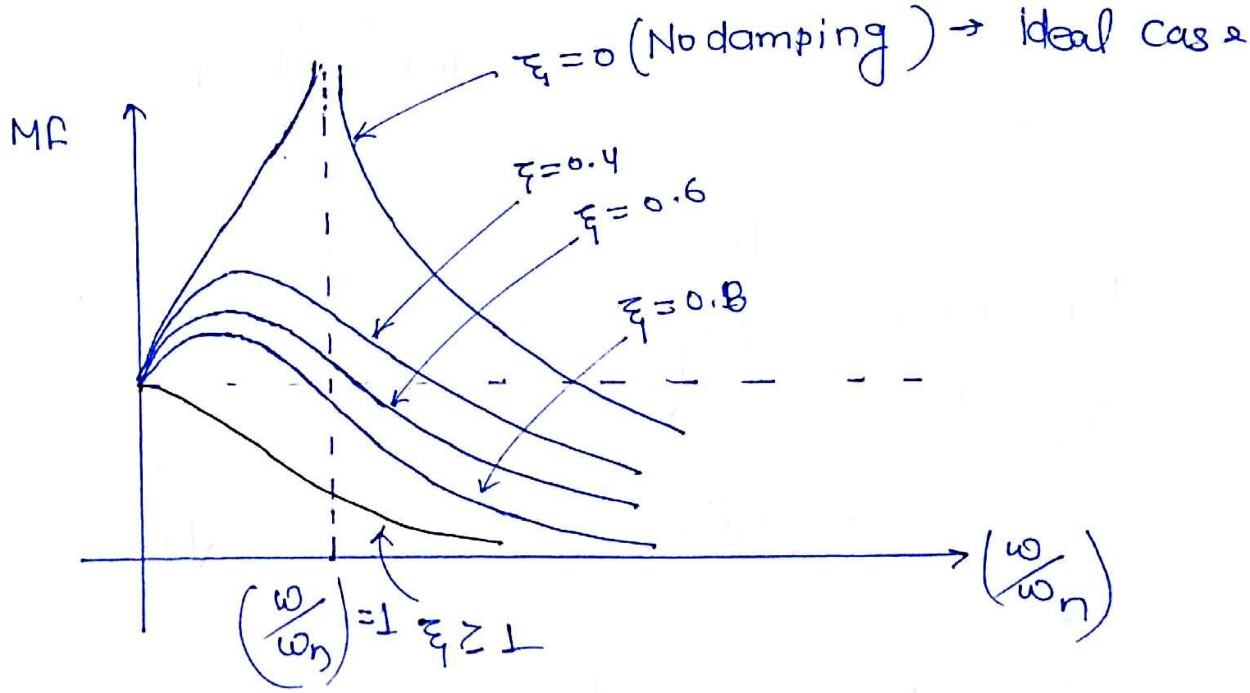
Magnification Factor $= MF = \frac{A}{(F_o/s)}$

strength of A

$$MF = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

it depands upon (i) $\omega/\omega_n \Rightarrow$ withing a system

(ii) $\xi \Rightarrow$ system to system



when $\frac{\omega}{\omega_n} = 0$

$MF = 1$

for all values of ξ

$\left(\frac{\omega}{\omega_n}\right) = 1 \rightarrow$ Resonance

Natural frequency is equal to forced frequency

Ans
(1) Underdaming increase ($\uparrow$) → it means how far ξ from 1

$\Rightarrow \xi \downarrow$

$\Rightarrow$ MF $\uparrow$

$\Rightarrow$ Amplitude increase → Running life will decrease.

(2) A_{max} at (or MF_{max} at)

at $\frac{\omega}{\omega_n} < 1$ Under damping / Viscous damping

at $\frac{\omega}{\omega_n} = 1$ No damping

at $\frac{\omega}{\omega_n} = 0$ over damping / critical damping. Columb damping

(3) $$A_{Resonance} = \frac{F_0/S}{2\xi}$$

$\frac{\omega}{\omega_n} = 1$

$$2\xi\omega_n = \frac{c}{m}$$

$$\xi = \frac{c\sqrt{m}}{2m\sqrt{K}}$$

$$\xi = \frac{c}{2\sqrt{Km}}$$

$$A_{Res} = \frac{F_0\, 2\sqrt{Sm}}{2S \cdot C}$$

$$A_{Res} = \frac{F_0}{C}\sqrt{\frac{m}{S}} = \frac{F_0}{C\omega_n}$$

Note:-

elastic or spring $\quad x = A\sin(\omega t - \phi)$

$\dot{x} = A\omega\cos(\omega t - \phi)$

damping (v) $\quad \dot{x} = A\omega\sin\left(\frac{\pi}{2} + \omega t - \phi\right)$

inertia (a) $\quad \ddot{x} = -A\omega^2\sin(\omega t - \phi)$

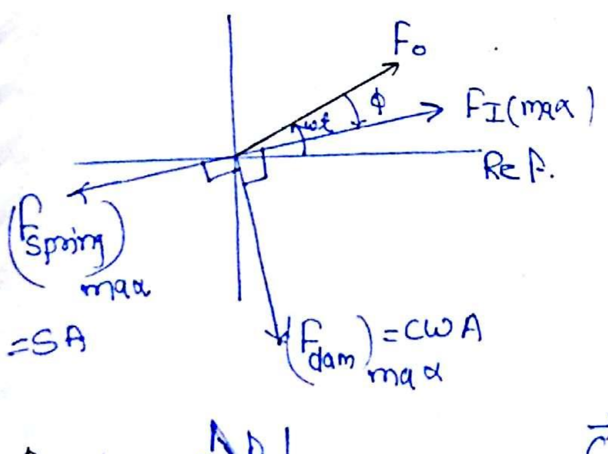
Inertia force $\propto$ acc^n
damping force $\propto$ velocity
spring or elastic force $\propto$ displacement
$m\ddot{x} + c\dot{x} + kx = f_{um}$

The basic equation was:-

$$m\ddot{x} + c\dot{x} + sx = F_0 \sin\omega t$$

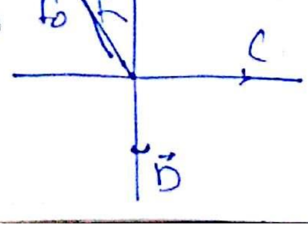
$$\Rightarrow \underbrace{F_0\sin\omega t}_{(F_{on})_{max}} + \underbrace{mA\omega^2\sin(\omega t - \phi)}_{(F_{inertia})_{max}} - \underbrace{c\omega A\sin\left(\frac{\pi}{2} + \omega t - \phi\right)}_{(F_{damping})_{max}} - \underbrace{SA\sin(\omega t - \phi)}_{(F_{elastic})_{max} \text{ or } (F_{spring})_{max}} = 0$$

Phasor:- At $t = t$



At Any moment the max. Response of spring and damping force are mutually $\perp^{er}$ to each other

Problem

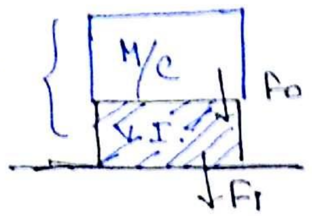


$\vec{C} \to ? \Rightarrow (\text{damping force})_{max}$

$\vec{D} \to ? \Rightarrow (\text{Spring force})_{max}$

Vibration Isolation:- foundation

How to isolate the ground from the vibration of a running machine.

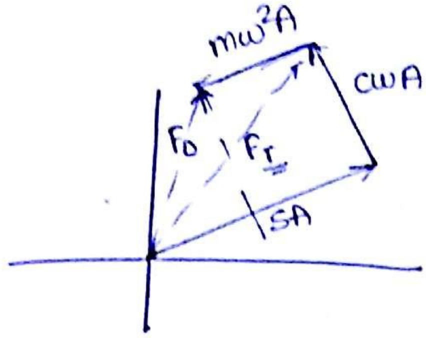


F_T - max transmitted force due to Ground.

$F_T <<<< F_0$ $\quad 0 < \epsilon < 1$

$\epsilon \to 0$ best

Transmissibility:- $\epsilon = \frac{F_T}{F_0}$



$$\Rightarrow \quad F_T = \sqrt{SA + c\omega A}$$

$$F_T = SA\sqrt{1 + \frac{c\omega/m}{S/m}}$$

$$F_T = SA\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$$F_0 = SA\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}$$

$$\epsilon = \frac{F_T}{F_0} = \frac{\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

transmissibility depand on

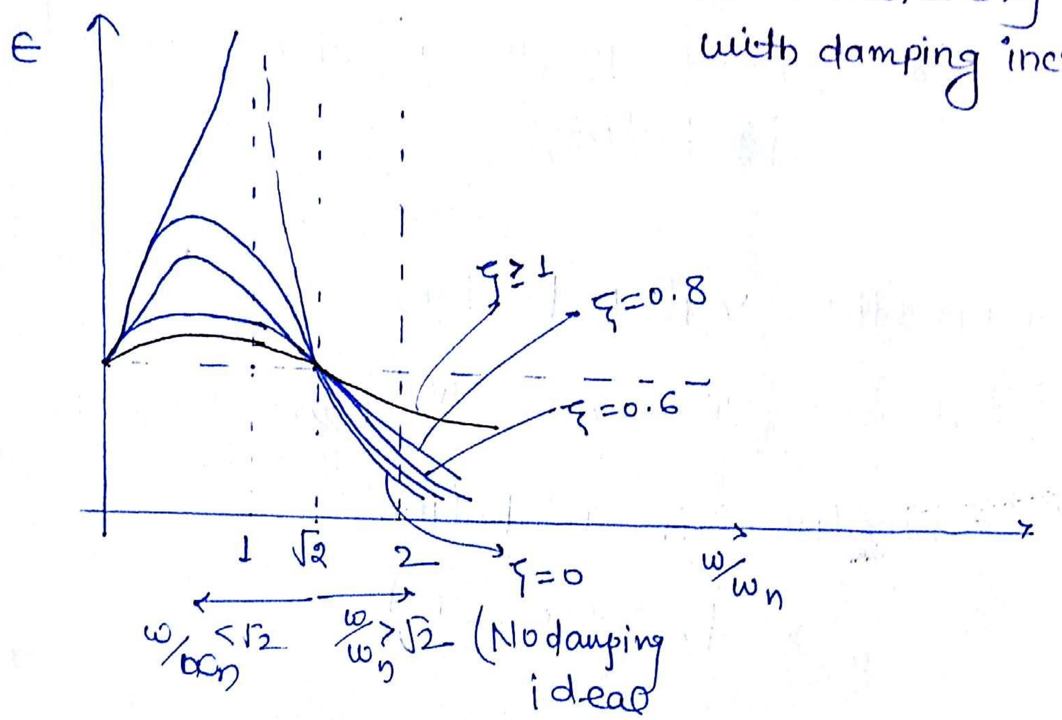
(i) ω/ω_n → withing a system

(ii) ξ → system to system

⇒ if $\omega/\omega_n = 0$, $\epsilon = 1$ } For All value of ξ

⇒ if $\omega/\omega_n = \sqrt{2}$, $\epsilon = 1$

* transmissibility increase with damping increase.



1. Underdamping increase (↑) → ξ ↓

ϵ will ↑ if $\frac{\omega}{\omega_n} < \sqrt{2}$

ϵ will ↓ if $\frac{\omega}{\omega_n} > \sqrt{2}$

ϵ will remain same if $\frac{\omega}{\omega_n} = \sqrt{2}$

2.

2. Vib. isolation will be effective when $0 < \epsilon < 1$

$$\Rightarrow \frac{\omega}{\omega_n} > \sqrt{2}$$

3. If effective Vib. isolation zone

$$\frac{\omega}{\omega_n} > \sqrt{2}, \quad 0 < \epsilon < 1$$

{ No damping is best $(\epsilon \to 0)$
Damping is detriotal (harmful)

* Rubber → best material for vibration isolation.

Pb for effective vib. isolation $\omega_n = ?$

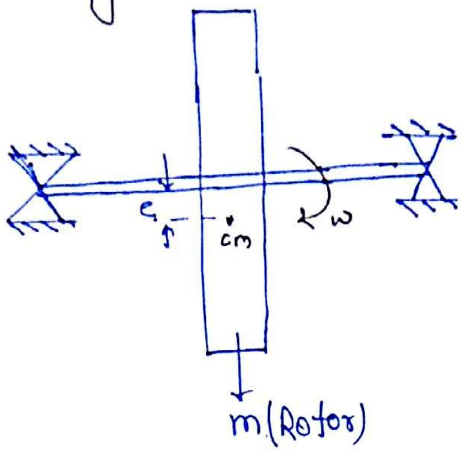
(i) 2ω (ii) 3ω (iii) $\omega/4$ (iv) 10ω

Sol^n for effective Vib. isolation

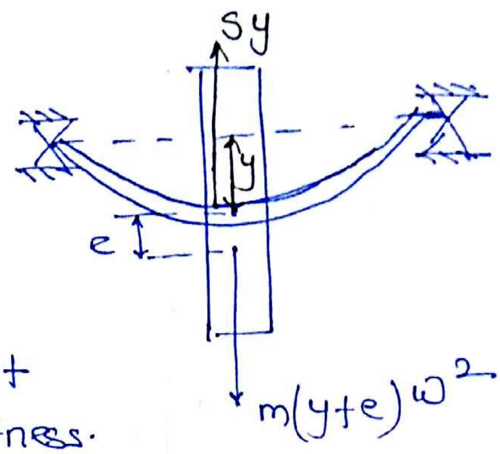
$$\omega/\omega_n > \sqrt{2} \quad (\epsilon < 1)$$

$$\omega_n < \omega/\sqrt{2}$$

Whirling of shaft: (critical)



After some time



S - Shaft stiffness.

$$m(y+e)\omega^2 = S \cdot y$$

$$my\omega^2 + me\omega^2 = Sy$$

$$me\omega^2 = my\omega^2\left(\frac{S}{m\omega^2} - 1\right)$$

$$\boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}}$$

$A(\xi = 0)$

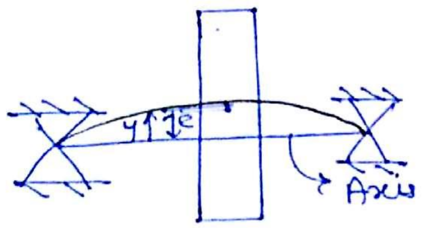
* In some system ~~frequency~~ running life is very less
  - Vibration are too heavy
  - Items are too delicate.

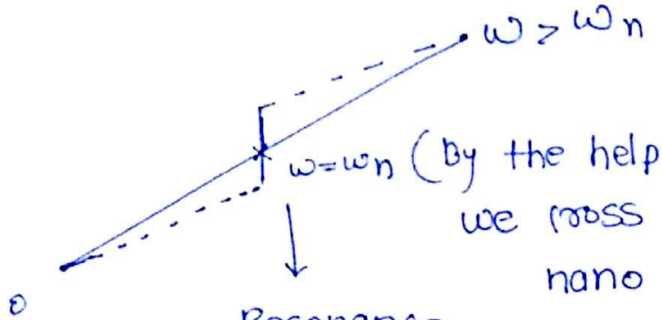
To Increase fatigue life

if $\omega > \omega_n$ $\qquad y = \dfrac{1}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$

$\Rightarrow$ y will become −ve

$\Rightarrow$ shaft bending will be in opposite dir^n





(By the help of Governors we cross this speed in nano time

Resonance (death)

$$\boxed{\text{Whirling speed} = \sqrt{\frac{S}{m}} \ \text{rad/s.}}$$

Fundamental whirling speed + critical speed

Second whirling speed :- Due to self weight only in Horizontal shaft

$$\text{second whirling speed} = \frac{1}{2}\omega_n = \frac{1}{2}\sqrt{S/m}$$

यहाँ भी Amplitude ज्यादा हो जाता है so cross it fast

Problem No. 56

$m = 17$ kg

$s = 1000$ N/m

$\omega_n = \sqrt{\frac{1000}{17}}$

$m_{reci} = 2$ kg

Stroke = 75 mm $\quad N = 500$ rpm

$r = \frac{75}{2000}$ m

$\omega = \frac{2\pi \times 500}{60}$ rad/s =

$\frac{\omega}{\omega_n} = ?$

$F_0 = 2 \times \frac{75}{2000} \times \left(\frac{2\pi \times 500}{60}\right)^2$ N

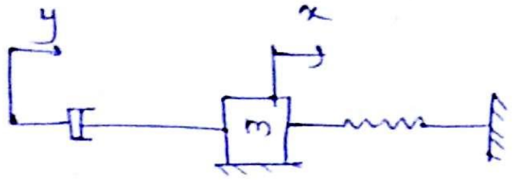
$\xi = 0.2$

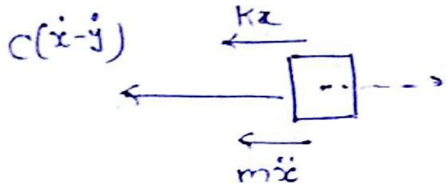
(i) $A = \frac{F_0/s}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$??

(ii) $\epsilon = \frac{\sqrt{1+\left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}} = \frac{F_T}{F_0}$ $F_T \rightarrow ?$

W.B.

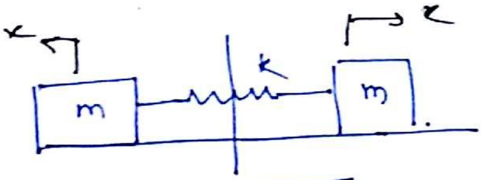
Q.3





$$m\ddot{x} + c(\dot{x}-\dot{y}) + kx = 0$$

Ques 2 DOF problem



$$0, \sqrt{\frac{2K}{m}}$$