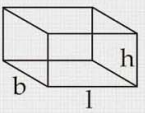
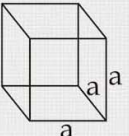
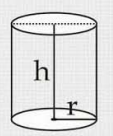
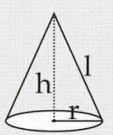
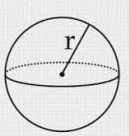
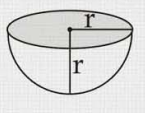


Volume and Surface Area of Solids

Ex 20.A

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$

Q1.

Answer :

Volume of a cylinder = $\pi r^2 h$

Lateral surface = $2\pi rh$

Total surface area = $2\pi r(h+r)$

(i) Base radius = 7 cm; height = 50 cm

Now, we have the following:

Volume = $\frac{22}{7} \times 7 \times 7 \times 50 = 7700 \text{ cm}^3$

Lateral surface area = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 50 = 2200 \text{ cm}^2$

Total surface area = $2\pi r(h+r) = 2 \times \frac{22}{7} \times 7(50+7) = 2508 \text{ cm}^2$

(ii) Base radius = 5.6 m; height = 1.25 m

Now, we have the following:

Volume = $\frac{22}{7} \times 5.6 \times 5.6 \times 1.25 = 123.2 \text{ m}^3$

Lateral surface area = $2\pi rh = 2 \times \frac{22}{7} \times 5.6 \times 1.25 = 44 \text{ m}^2$

Total surface area = $2\pi r(h+r) = 2 \times \frac{22}{7} \times 5.6(1.25+5.6) = 241.12 \text{ m}^2$

(iii) Base radius = 14 dm = 1.4 m, height = 15 m

Now, we have the following:

Volume = $\frac{22}{7} \times 1.4 \times 1.4 \times 15 = 92.4 \text{ m}^3$

Lateral surface area = $2\pi rh = 2 \times \frac{22}{7} \times 1.4 \times 15 = 132 \text{ m}^2$

Total surface area = $2\pi r(h+r) = 2 \times \frac{22}{7} \times 1.4(15+1.4) = 144.32 \text{ cm}^2$

Q2.

Answer :

$$r = 1.5 \text{ m}$$

$$h = 10.5 \text{ m}$$

$$\text{Capacity of the tank} = \text{volume of the tank} = \pi r^2 h = \frac{22}{7} \times 1.5 \times 1.5 \times 10.5 = 74.25 \text{ m}^3$$

$$\text{We know that } 1 \text{ m}^3 = 1000 \text{ L}$$

$$\therefore 74.25 \text{ m}^3 = 74250 \text{ L}$$

Q3.

Answer :

$$\text{Height} = 7 \text{ m}$$

$$\text{Radius} = 10 \text{ cm} = 0.1 \text{ m}$$

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 0.1 \times 0.1 \times 7 = 0.22 \text{ m}^3$$

$$\text{Weight of wood} = 225 \text{ kg/m}^3$$

$$\therefore \text{Weight of the pole} = 0.22 \times 225 = 49.5 \text{ kg}$$

Q4.

Answer :

$$\text{Diameter} = 2r = 140 \text{ cm}$$

$$\text{i.e., radius, } r = 70 \text{ cm} = 0.7 \text{ m}$$

$$\text{Now, volume} = \pi r^2 h = 1.54 \text{ m}^3$$

$$\Rightarrow \frac{22}{7} \times 0.7 \times 0.7 \times h = 1.54$$

$$\therefore h = \frac{1.54 \times 7}{0.7 \times 0.7 \times 22} = \frac{154 \times 7}{154 \times 7} = 1 \text{ m}$$

Q5.

Answer :

$$\text{Volume} = \pi r^2 h = 3850 \text{ cm}^3$$

$$\text{Height} = 1 \text{ m} = 100 \text{ cm}$$

$$\text{Now, radius, } r = \sqrt{\frac{3850}{\pi \times h}} = \sqrt{\frac{3850 \times 7}{22 \times 100}} = 1.75 \times 7 = 3.5 \text{ cm}$$

$$\therefore \text{Diameter} = 2(\text{radius}) = 2 \times 3.5 = 7 \text{ cm}$$

Q6.

Answer :

$$\text{Diameter} = 14 \text{ m}$$

$$\text{Radius} = \frac{14}{2} = 7 \text{ m}$$

$$\text{Height} = 5 \text{ m}$$

$$\therefore \text{Area of the metal sheet required} = \text{total surface area}$$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 7(5 + 7) \text{ m}^2$$

$$= 44 \times 12 \text{ m}^2$$

$$= 528 \text{ m}^2$$

Q7.

Answer :

$$\text{Circumference of the base} = 88 \text{ cm}$$

$$\text{Height} = 60 \text{ cm}$$

$$\text{Area of the curved surface} = \text{circumference} \times \text{height} = 88 \times 60 = 5280 \text{ cm}^2$$

$$\text{Circumference} = 2\pi r = 88 \text{ cm}$$

$$\text{Then radius} = r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 60 = 36960 \text{ cm}^3$$

Q8.

Answer :

Length = height = 14 m

Lateral surface area = $2\pi rh = 220 \text{ m}^2$

Radius = $r = \frac{220}{2\pi h} = \frac{220 \times 7}{2 \times 22 \times 14} = \frac{10}{4} = 2.5 \text{ m}$

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times 2.5 \times 2.5 \times 14 = 275 \text{ m}^3$

Q9.

Answer :

Height = 8 cm

Volume = $\pi r^2 h = 1232 \text{ cm}^3$

Now, radius = $r = \sqrt{\frac{1232}{\pi h}} = \sqrt{\frac{1232 \times 7}{22 \times 8}} = \sqrt{49} = 7 \text{ cm}$

Also, curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 8 = 352 \text{ cm}^2$

$\therefore$ Total surface area

$= 2\pi r(h + r) = \left(2 \times \frac{22}{7} \times 7 \times 8\right) + \left(2 \times \frac{22}{7} \times (7)^2\right) = 352 + 308 = 660 \text{ cm}^2$

Q10.

Answer :

We have: $\frac{\text{radius}}{\text{height}} = \frac{7}{2}$

i.e., $r = \frac{7}{2} h$

Now, volume = $\pi r^2 h = \pi \left(\frac{7}{2} h\right)^2 h = 8316 \text{ cm}^3$

$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h^3 = 8316$

$\Rightarrow h^3 = \frac{8316 \times 2}{11 \times 7} = 108 \times 2 = 216$

$\Rightarrow h = \sqrt[3]{216} = 6 \text{ cm}$

Then $r = \frac{7}{2} h = \frac{7}{2} \times 6 = 21 \text{ cm}$

$\therefore$ Total surface area = $2\pi r(h + r) = 2 \times \frac{22}{7} \times 21 \times (6 + 21) = 3564 \text{ cm}^2$

Q11.

Answer :

Curved surface area = $2\pi rh = 4400 \text{ cm}^2$

Circumference = $2\pi r = 110 \text{ cm}$

Now, height = $h = \frac{\text{curved surface area}}{\text{circumference}} = \frac{4400}{110} = 40 \text{ cm}$

Also, radius, $r = \frac{4400}{2\pi h} = \frac{4400 \times 7}{2 \times 22 \times 40} = \frac{35}{2}$

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 = 22 \times 5 \times 35 \times 10 = 38500 \text{ cm}^3$

Q12.

Answer :

For the cubic pack:

Length of the side, $a = 5 \text{ cm}$

Height = 14 cm

$$\text{Volume} = a^2 h = 5 \times 5 \times 14 = 350 \text{ cm}^3$$

For the cylindrical pack:

Base radius = $r = 3.5 \text{ cm}$

Height = 12 cm

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 12 = 462 \text{ cm}^3$$

We can see that the pack with a circular base has a greater capacity than the pack with a square base.

$$\text{Also, difference in volume} = 462 - 350 = 112 \text{ cm}^3$$

Q13.

Answer :

Diameter = 48 cm

Radius = 24 cm = 0.24 m

Height = 7 m

Now, we have:

$$\text{Lateral surface area of one pillar} = \pi dh = \frac{22}{7} \times 0.48 \times 7 = 10.56 \text{ m}^2$$

$$\text{Surface area to be painted} = \text{total surface area of 15 pillars} = 10.56 \times 15 = 158.4 \text{ m}^2$$

$$\therefore \text{Total cost} = \text{Rs } (158.4 \times 2.5) = \text{Rs } 396$$

Q14.

Answer :

$$\text{Volume of the rectangular vessel} = 22 \times 16 \times 14 = 4928 \text{ cm}^3$$

Radius of the cylindrical vessel = 8 cm

$$\text{Volume} = \pi r^2 h$$

As the water is poured from the rectangular vessel to the cylindrical vessel, we have:

Volume of the rectangular vessel = volume of the cylindrical vessel

$$\therefore \text{Height of the water in the cylindrical vessel} = \frac{\text{volume}}{\pi r^2} = \frac{4928 \times 7}{22 \times 8 \times 8} = \frac{28 \times 7}{8} = \frac{49}{2} = 24.5 \text{ cm}$$

Q15.

Answer :

Diameter of the given wire = 1 cm

Radius = 0.5 cm

Length = 11 cm

$$\text{Now, volume} = \pi r^2 h = \frac{22}{7} \times 0.5 \times 0.5 \times 11 = 8.643 \text{ cm}^3$$

The volumes of the two cylinders would be the same.

Now, diameter of the new wire = 1 mm = 0.1 cm

Radius = 0.05 cm

$$\therefore \text{New length} = \frac{\text{volume}}{\pi r^2} = \frac{8.643 \times 7}{22 \times 0.05 \times 0.05} = 1100.02 \text{ cm} \cong 11 \text{ m}$$

Q16.

Answer :

Length of the edge, $a = 2.2 \text{ cm}$

$$\text{Volume of the cube} = a^3 = (2.2)^3 = 10.648 \text{ cm}^3$$

$$\text{Volume of the wire} = \pi r^2 h$$

Radius = 1 mm = 0.1 cm

As volume of cube = volume of wire, we have:

$$h = \frac{\text{volume}}{\pi r^2} = \frac{10.648 \times 7}{22 \times 0.1 \times 0.1} = 338.8 \text{ cm}$$

Q17.

Answer :

Diameter = 7 m

Radius = 3.5 m

Depth = 20 m

$$\text{Volume of the earth dug out} = \pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 20 = 770 \text{ m}^3$$

$$\text{Volume of the earth piled upon the given plot} = 28 \times 11 \times h = 770 \text{ m}^3$$

$$\therefore h = \frac{770}{28 \times 11} = \frac{70}{28} = 2.5 \text{ m}$$

Q18.

Answer :

Inner diameter = 14 m

i.e., radius = 7 m

Depth = 12 m

$$\text{Volume of the earth dug out} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 12 = 1848 \text{ m}^3$$

Width of embankment = 7 m

Now, total radius = $7 + 7 = 14 \text{ m}$

Volume of the embankment = total volume – inner volume

$$= \pi r_o^2 h - \pi r_i^2 h = \pi h (r_o^2 - r_i^2)$$

$$= \frac{22}{7} h (14^2 - 7^2) = \frac{22}{7} h (196 - 49)$$

$$= \frac{22}{7} h \times 147 = 21 \times 22h$$

$$= 462 \times h \text{ m}^3$$

Since volume of embankment = volume of earth dug out, we have:

$$1848 = 462 h$$

$$\Rightarrow h = \frac{1848}{462} = 4 \text{ m}$$

$\therefore$ Height of the embankment = 4 m

Q19.

Answer :

Diameter = 84 cm

i.e., radius = 42 cm

Length = 1 m = 100 cm

$$\text{Now, lateral surface area} = 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 100 = 26400 \text{ cm}^2$$

$\therefore$ Area of the road

$$= \text{lateral surface area} \times \text{no. of rotations} = 26400 \times 750 = 19800000 \text{ cm}^2 = 1980 \text{ m}^2$$

Q20.

Answer :

Thickness of the cylinder = 1.5 cm

External diameter = 12 cm

i.e., radius = 6 cm

also, internal radius = 4.5 cm

Height = 84 cm

Now, we have the following:

$$\text{Total volume} = \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 84 = 9504 \text{ cm}^3$$

$$\text{Inner volume} = \pi r^2 h = \frac{22}{7} \times 4.5 \times 4.5 \times 84 = 5346 \text{ cm}^3$$

$$\text{Now, volume of the metal} = \text{total volume} - \text{inner volume} = 9504 - 5346 = 4158 \text{ cm}^3$$

$$\therefore \text{Weight of iron} = 4158 \times 7.5 = 31185 \text{ g} = 31.185 \text{ kg} \quad [\text{Given: } 1 \text{ cm}^3 = 7.5 \text{ g}]$$

Q21.

Answer :

Length = 1 m = 100 cm

Inner diameter = 12 cm

Radius = 6 cm

Now, inner volume = $\pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 100 = 11314.286 \text{ cm}^3$

Thickness = 1 cm

Total radius = 7 cm

Now, we have the following:

Total volume = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 100 = 15400 \text{ cm}^3$

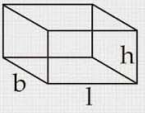
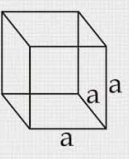
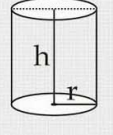
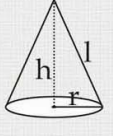
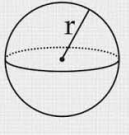
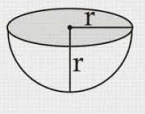
Volume of the tube = **total volume** – **inner volume** = $15400 - 11314.286 = 4085.714 \text{ cm}^3$

Density of the tube = 7.7 g/cm³

∴ Weight of the tube = **volume** × **density** = $4085.714 \times 7.7 = 31459.9978 \text{ g} = 31.459 \text{ kg}$

Volume and Surface Area of Solids

Ex 20.B

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$

Q1.

Answer :

Volume of a cuboid = $(\text{Length} \times \text{Breadth} \times \text{Height})$ cubic units

Total surface area = $2(lb + bh + lh)$ sq units

Lateral surface area = $[2(l + b) \times h]$ sq units

(i) Length = 22 cm, breadth = 12 cm, height = 7.5 cm

Volume = $(\text{Length} \times \text{Breadth} \times \text{Height}) = (22 \times 12 \times 7.5) = 1980 \text{ cm}^3$

Total surface area

= $2(lb + bh + lh) = 2[(22 \times 12) + (22 \times 7.5) + (12 \times 7.5)] = 2[264 + 165 + 90] = 1038 \text{ cm}^2$

Lateral surface area = $[2(l + b) \times h] = 2(22 + 12) \times 7.5 = 510 \text{ cm}^2$

(ii) Length = 15 m, breadth = 6 m, height = 9 dm = 0.9 m

Volume = $(\text{Length} \times \text{Breadth} \times \text{Height}) = (15 \times 6 \times 0.9) = 81 \text{ m}^3$

Total surface area = $2(lb + bh + lh)$

= $2[(15 \times 6) + (15 \times 0.9) + (6 \times 0.9)] = 2[90 + 13.5 + 5.4] = 217.8 \text{ m}^2$

Lateral surface area = $[2(l + b) \times h] = 2(15 + 6) \times 0.9 = 37.8 \text{ m}^2$

(iii) Length = 24 m, breadth = 25 cm = 0.25 m, height = 6 m

$$\text{Volume} = (\text{Length} \times \text{Breadth} \times \text{Height}) = (24 \times 0.25 \times 6) = 36 \text{ m}^3$$

$$\text{Total surface area} = 2(lb + bh + lh)$$

$$= 2[(24 \times 0.25) + (24 \times 6) + (0.25 \times 6)] = 2[6 + 144 + 1.5] = 303 \text{ m}^2$$

$$\text{Lateral surface area} = [2(l + b) \times h] = 2(24 + 0.25) \times 6 = 291 \text{ m}^2$$

(iv) Length = 48 cm = 0.48 m, breadth = 6 dm = 0.6 m, height = 1 m

$$\text{Volume} = (\text{Length} \times \text{Breadth} \times \text{Height}) = (0.48 \times 0.6 \times 1) = 0.288 \text{ m}^3$$

Total surface area

$$= 2(lb + bh + lh) = 2[(0.48 \times 0.6) + (0.48 \times 1) + (0.6 \times 1)] = 2[0.288 + 0.48 + 0.6] = 2.736 \text{ m}^2$$

$$\text{Lateral surface area} = [2(l + b) \times h] = 2(0.48 + 0.6) \times 1 = 2.16 \text{ m}^2$$

Q2.

Answer :

$$1 \text{ m} = 100 \text{ cm}$$

Therefore, dimensions of the tank are:

$$2 \text{ m } 75 \text{ cm} \times 1 \text{ m } 80 \text{ cm} \times 1 \text{ m } 40 \text{ cm} = 275 \text{ cm} \times 180 \text{ cm} \times 140 \text{ cm}$$

$$\therefore \text{Volume} = \text{Length} \times \text{Breadth} \times \text{Height} = 275 \times 180 \times 140 = 6930000 \text{ cm}^3$$

$$\text{Also, } 1000 \text{ cm}^3 = 1 \text{ L}$$

$$\therefore \text{Volume} = \frac{6930000}{1000} = 6930 \text{ L}$$

Q3.

Answer :

$$1 \text{ m} = 100 \text{ cm}$$

$$\therefore \text{Dimensions of the iron piece} = 105 \text{ cm} \times 70 \text{ cm} \times 1.5 \text{ cm}$$

$$\text{Total volume of the piece of iron} = (105 \times 70 \times 1.5) = 11025 \text{ cm}^3$$

1 cm³ measures 8 gms.

$\therefore$ Weight of the piece

$$= 11025 \times 8 = 88200 \text{ g} = \frac{88200}{1000} = 88.2 \text{ kg} \quad (\text{because } 1 \text{ kg} = 1000 \text{ g})$$

Q4.

Answer :

$$1 \text{ cm} = 0.01 \text{ m}$$

$$\text{Volume of the gravel used} = \text{Area} \times \text{Height} = (3750 \times 0.01) = 37.5 \text{ m}^3$$

Cost of the gravel is Rs 6.40 per cubic meter.

$$\therefore \text{Total cost} = (37.5 \times 6.4) = \text{Rs } 240$$

Q5.

Answer :

$$\text{Total volume of the hall} = (16 \times 12.5 \times 4.5) = 900 \text{ m}^3$$

It is given that 3.6 m³ of air is required for each person.

The total number of persons that can be accommodated in that hall

$$= \frac{\text{Total volume}}{\text{Volume required by each person}} = \frac{900}{3.6}$$

$$= 250 \text{ people}$$

Q6.

Answer :

$$\text{Volume of the cardboard box} = (120 \times 72 \times 54) = 466560 \text{ cm}^3$$

$$\text{Volume of each bar of soap} = (6 \times 4.5 \times 4) = 108 \text{ cm}^3$$

$$\begin{aligned} \text{Total number of bars of soap that can be accommodated in that box} \\ = \frac{\text{Volume of the box}}{\text{Volume of each soap}} = \frac{466560}{108} = 4320 \text{ bars} \end{aligned}$$

Q7.

Answer :

$$\text{Volume occupied by a single matchbox} = (4 \times 2.5 \times 1.5) = 15 \text{ cm}^3$$

$$\text{Volume of a packet containing 144 matchboxes} = (15 \times 144) = 2160 \text{ cm}^3$$

$$\text{Volume of the carton} = (150 \times 84 \times 60) = 756000 \text{ cm}^3$$

$$\text{Total number of packets in a carton} = \frac{\text{Volume of the carton}}{\text{Volume of a packet}} = \frac{756000}{2160} = 350 \text{ packets}$$

Q8.

Answer :

$$\text{Total volume of the block} = (500 \times 70 \times 32) = 1120000 \text{ cm}^3$$

$$\text{Total volume of each plank} = 200 \times 25 \times 8 = 40000 \text{ cm}^3 = 200 \times 25 \times 8 = 40000 \text{ cm}^3$$

$$\therefore \text{Total number of planks that can be made} = \frac{\text{Total volume of the block}}{\text{Volume of each plank}} = \frac{1120000}{40000} = 28 \text{ planks}$$

Q9.

Answer :

$$\text{Volume of the brick} = 25 \times 13.5 \times 6 = 2025 \text{ cm}^3$$

$$\text{Volume of the wall} = 800 \times 540 \times 33 = 14256000 \text{ cm}^3$$

$$\text{Total number of bricks} = \frac{\text{Volume of the wall}}{\text{Volume of each brick}} = \frac{14256000}{2025} = 7040 \text{ bricks}$$

Q10.

Answer :

$$\text{Volume of the wall} = 1500 \times 30 \times 400 = 18000000 \text{ cm}^3$$

$$\text{Total quantity of mortar} = \frac{1}{12} \times 18000000 = 1500000 \text{ cm}^3$$

$$\therefore \text{Volume of the bricks} = 18000000 - 1500000 = 16500000 \text{ cm}^3$$

$$\text{Volume of a single brick} = 22 \times 12.5 \times 7.5 = 2062.5 \text{ cm}^3$$

$$\therefore \text{Total number of bricks} = \frac{\text{Total volume of the bricks}}{\text{Volume of a single brick}} = \frac{16500000}{2062.5} = 8000 \text{ bricks}$$

Q11.

Answer :

$$\text{Volume of the cistern} = 11.2 \times 6 \times 5.8 = 389.76 \text{ m}^3 = 389.76 \times 1000 = 389760 \text{ litres}$$

$$\begin{aligned} \text{Area of the iron sheet required to make this cistern} &= \text{Total surface area of the cistern} \\ &= 2(11.2 \times 6 + 11.2 \times 5.8 + 6 \times 5.8) = 2(67.2 + 64.96 + 34.8) = 333.92 \text{ cm}^2 \end{aligned}$$

Q12.

Answer :

$$\text{Volume of the block} = 0.5 \text{ m}^3$$

We know:

$$1 \text{ hectare} = 10000 \text{ m}^2$$

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}} = \frac{0.5}{10000} = 0.00005 \text{ m} = 0.005 \text{ cm} = 0.05 \text{ mm}$$

Q13.

Answer :

$$\text{Rainfall recorded} = 5 \text{ cm} = 0.05 \text{ m}$$

$$\text{Area of the field} = 2 \text{ hectare} = 2 \times 10000 \text{ m}^2 = 20000 \text{ m}^2$$

Total rain over the field =

$$\text{Area of the field} \times \text{Height of the field} = 0.05 \times 20000 = 1000 \text{ m}^3$$

Q14.

Answer :

$$\text{Area of the cross-section of river} = 45 \times 2 = 90 \text{ m}^2$$

$$\text{Rate of flow} = 3 \text{ km/hr} = \frac{3 \times 1000}{60} = 50 \frac{\text{m}}{\text{min}}$$

$$\text{Volume of water flowing through the cross-section in one minute} = 90 \times 50 = 4500 \text{ m}^3 \text{ per minute}$$

Q15.

Answer :

Let the depth of the pit be d m.

$$\text{Volume} = \text{Length} \times \text{width} \times \text{depth} = 5 \text{ m} \times 3.5 \text{ m} \times d \text{ m}$$

But,

$$\text{Given volume} = 14 \text{ m}^3$$

$$\therefore \text{Depth} = d = \frac{\text{volume}}{\text{length} \times \text{width}} = \frac{14}{5 \times 3.5} = 0.8 \text{ m} = 80 \text{ cm}$$

Q16.

Answer :

Capacity of the water tank = 576 litres = 0.576 m^3

Width = 90 cm = 0.9 m

Depth = 40 cm = 0.4 m

$$\text{Length} = = \frac{\text{capacity}}{\text{width} \times \text{depth}} = \frac{0.576}{0.9 \times 0.4} = 1.600 \text{ m}$$

Q17.

Answer :

Volume of the beam = 1.35 m^3

Length = 5 m

Thickness = 36 cm = 0.36 m

$$\text{Width} = = \frac{\text{volume}}{\text{thickness} \times \text{length}} = \frac{1.35}{5 \times 0.36} = 0.75 \text{ m} = 75 \text{ cm}$$

Q18.

Answer :

Volume = height $\times$ area

Given:

Volume = 378 m^3

Area = 84 m^2

$$\therefore \text{Height} = \frac{\text{volume}}{\text{area}} = \frac{378}{84} = 4.5 \text{ m}$$

Q19.

Answer :

Length of the pool = 260 m

Width of the pool = 140 m

Volume of water in the pool = 54600 cubic metres

$$\therefore \text{Height of water} = \frac{\text{volume}}{\text{length} \times \text{width}} = \frac{54600}{260 \times 140} = 1.5 \text{ metres}$$

Q20.

Answer :

External length = 60 cm

External width = 45 cm

External height = 32 cm

External volume of the box = $60 \times 45 \times 32 = 86400 \text{ cm}^3$

Thickness of wood = 2.5 cm

$$\therefore \text{Internal length} = 60 - (2.5 \times 2) = 55 \text{ cm}$$

$$\text{Internal width} = 45 - (2.5 \times 2) = 40 \text{ cm}$$

$$\text{Internal height} = 32 - (2.5 \times 2) = 27 \text{ cm}$$

$$\text{Internal volume of the box} = 55 \times 40 \times 27 = 59400 \text{ cm}^3$$

$$\text{Volume of wood} = \text{External volume} - \text{Internal volume} = 86400 - 59400 = 27000 \text{ cm}^3$$

Q21.

Answer :

External length = 36 cm
External width = 25 cm
External height = 16.5 cm

$$\text{External volume of the box} = 36 \times 25 \times 16.5 = 14850 \text{ cm}^3$$

Thickness of iron = 1.5 cm

$\therefore$ Internal length = $36 - (1.5 \times 2) = 33$ cm
Internal width = $25 - (1.5 \times 2) = 22$ cm
Internal height = $16.5 - 1.5 = 15$ cm (as the box is open)

$$\text{Internal volume of the box} = 33 \times 22 \times 15 = 10890 \text{ cm}^3$$

$$\text{Volume of iron} = \text{External volume} - \text{Internal volume} = 14850 - 10890 = 3960 \text{ cm}^3$$

Given:

$$1 \text{ cm}^3 \text{ of iron} = 8.5 \text{ grams}$$

$$\text{Total weight of the box} = 3960 \times 8.5 = 33660 \text{ grams} = 33.66 \text{ kilograms}$$

Q22.

Answer :

External length = 56 cm
External width = 39 cm
External height = 30 cm

$$\text{External volume of the box} = 56 \times 39 \times 30 = 65520 \text{ cm}^3$$

Thickness of wood = 3 cm

$\therefore$ Internal length = $56 - (3 \times 2) = 50$ cm
Internal width = $39 - (3 \times 2) = 33$ cm
Internal height = $30 - (3 \times 2) = 24$ cm

$$\text{Capacity of the box} = \text{Internal volume of the box} = 50 \times 33 \times 24 = 39600 \text{ cm}^3$$

$$\text{Volume of wood} = \text{External volume} - \text{Internal volume} = 65520 - 39600 = 25920 \text{ cm}^3$$

Q23.

Answer :

External length = 62 cm
External width = 30 cm
External height = 18 cm

$$\therefore \text{External volume of the box} = 62 \times 30 \times 18 = 33480 \text{ cm}^3$$

Thickness of the wood = 2 cm

Now, internal length = $62 - (2 \times 2) = 58$ cm
Internal width = $30 - (2 \times 2) = 26$ cm
Internal height = $18 - (2 \times 2) = 14$ cm

$$\therefore \text{Capacity of the box} = \text{internal volume of the box} = (58 \times 26 \times 14) \text{ cm}^3 = 21112 \text{ cm}^3$$

Q24.

Answer :

External length = 80 cm

External width = 65 cm

External height = 45 cm

$$\therefore \text{External volume of the box} = 80 \times 65 \times 45 = 234000 \text{ cm}^3$$

Thickness of the wood = 2.5 cm

$$\text{Then internal length} = 80 - (2.5 \times 2) = 75 \text{ cm}$$

$$\text{Internal width} = 65 - (2.5 \times 2) = 60 \text{ cm}$$

$$\text{Internal height} = 45 - (2.5 \times 2) = 40 \text{ cm}$$

$$\text{Capacity of the box} = \text{internal volume of the box} = (75 \times 60 \times 40) \text{ cm}^3 = 180000 \text{ cm}^3$$

$$\text{Volume of the wood} = \text{external volume} - \text{internal volume} = (234000 - 180000) \text{ cm}^3 = 54000 \text{ cm}^3$$

It is given that 100 cm^3 of wood weighs 8 g.

$$\therefore \text{Weight of the wood} = \frac{54000}{100} \times 8 \text{ g} = 4320 \text{ g} = 4.32 \text{ kg}$$

Q25.

Answer :

(i) Length of the edge of the cube = $a = 7 \text{ m}$

Now, we have the following:

$$\text{Volume} = a^3 = 7^3 = 343 \text{ m}^3$$

$$\text{Lateral surface area} = 4a^2 = 4 \times 7 \times 7 = 196 \text{ m}^2$$

$$\text{Total Surface area} = 6a^2 = 6 \times 7 \times 7 = 294 \text{ m}^2$$

(ii) Length of the edge of the cube = $a = 5.6 \text{ cm}$

Now, we have the following:

$$\text{Volume} = a^3 = 5.6^3 = 175.616 \text{ cm}^3$$

$$\text{Lateral surface area} = 4a^2 = 4 \times 5.6 \times 5.6 = 125.44 \text{ cm}^2$$

$$\text{Total Surface area} = 6a^2 = 6 \times 5.6 \times 5.6 = 188.16 \text{ cm}^2$$

(iii) Length of the edge of the cube = $a = 8 \text{ dm } 5 \text{ cm} = 85 \text{ cm}$

Now, we have the following:

$$\text{Volume} = a^3 = 85^3 = 614125 \text{ cm}^3$$

$$\text{Lateral surface area} = 4a^2 = 4 \times 85 \times 85 = 28900 \text{ cm}^2$$

$$\text{Total Surface area} = 6a^2 = 6 \times 85 \times 85 = 43350 \text{ cm}^2$$

Q26.

Answer :

Let a be the length of the edge of the cube.

$$\text{Total surface area} = 6a^2 = 1176 \text{ cm}^2$$

$$\Rightarrow a = \sqrt{\frac{1176}{6}} = \sqrt{196} = 14 \text{ cm}$$

$$\therefore \text{Volume} = a^3 = 14^3 = 2744 \text{ cm}^3$$

Q27.

Answer :

Let a be the length of the edge of the cube.

$$\text{Then volume} = a^3 = 729 \text{ cm}^3$$

$$\text{Also, } a = \sqrt[3]{729} = 9 \text{ cm}$$

$$\therefore \text{Surface area} = 6a^2 = 6 \times 9 \times 9 = 486 \text{ cm}^2$$

Q28.

Answer :

$$1 \text{ m} = 100 \text{ cm}$$

$$\text{Volume of the original block} = 225 \times 150 \times 27 = 911250 \text{ cm}^3$$

$$\text{Length of the edge of one cube} = 45 \text{ cm}$$

$$\text{Then volume of one cube} = 45^3 = 91125 \text{ cm}^3$$

$$\therefore \text{Total number of blocks that can be cast} = \frac{\text{volume of the block}}{\text{volume of one cube}} = \frac{911250}{91125} = 10$$

Q29.

Answer :

Let a be the length of the edge of a cube.

$$\text{Volume of the cube} = a^3$$

$$\text{Total surface area} = 6a^2$$

If the length is doubled, then the new length becomes $2a$.

$$\text{Now, new volume} = (2a)^3 = 8a^3$$

$$\text{Also, new surface area} = 6(2a)^2 = 6 \times 4a^2 = 24a^2$$

$\therefore$ The volume is increased by a factor of 8, while the surface area increases by a factor of 4.

Q30.

Answer :

$$\text{Cost of wood} = \text{Rs } 500/\text{m}^3$$

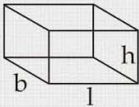
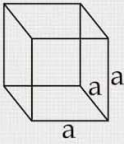
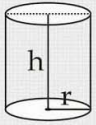
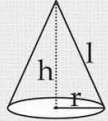
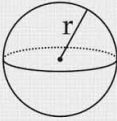
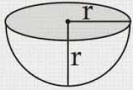
$$\text{Cost of the given block} = \text{Rs } 256$$

$$\therefore \text{Volume of the given block} = a^3 = \frac{256}{500} = 0.512 \text{ m}^3 = 512000 \text{ cm}^3$$

$$\text{Also, length of its edge} = a = \sqrt[3]{0.512} = 0.8 \text{ m} = 80 \text{ cm}$$

Volume and Surface Area of Solids

Ex 20.C

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$

Q1.

Answer :

(b) 17

$$\text{Length of the diagonal of a cuboid} = \sqrt{l^2 + b^2 + h^2}$$

$$\therefore \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ cm}$$

Q2.

Answer :

(b) 125 cm^3

Total surface area = $6a^2 = 150 \text{ cm}^2$, where a is the length of the edge of the cube.

$$\Rightarrow 6a^2 = 150$$

$$\Rightarrow a = \sqrt{\frac{150}{6}} = \sqrt{25} = 5 \text{ cm}$$

$$\therefore \text{Volume} = a^3 = 5^3 = 125 \text{ cm}^3$$

Q3.

Answer :

(c) 294 cm^2

$$\text{Volume} = a^3 = 343 \text{ cm}^3$$

$$\Rightarrow a = \sqrt[3]{343} = 7 \text{ cm}$$

$$\therefore \text{Total surface area} = 6a^2 = 6 \times 7 \times 7 = 294 \text{ cm}^2$$

Q4.

Answer :

(c) 294 cm^2

$$\text{Volume} = a^3 = 343 \text{ cm}^3$$

$$\Rightarrow a = \sqrt[3]{343} = 7 \text{ cm}$$

$$\therefore \text{Total surface area} = 6a^2 = 6 \times 7 \times 7 = 294 \text{ cm}^2$$

Q5.

Answer :

(c) 6400

$$\text{Volume of each brick} = 25 \times 11.25 \times 6 = 1687.5 \text{ cm}^3$$

$$\text{Volume of the wall} = 800 \times 600 \times 22.5 = 10800000 \text{ cm}^3$$

$$\therefore \text{No. of bricks} = \frac{10800000}{1687.5} = 6400$$

Q6.

Answer :

(c) 1000

$$\text{Volume of the smaller cube} = (10 \text{ cm})^3 = 1000 \text{ cm}^3$$

$$\text{Volume of box} = (100 \text{ cm})^3 = 1000000 \text{ cm}^3 \quad [1 \text{ m} = 100 \text{ cm}]$$

$$\therefore \text{Total no. of cubes} = \frac{100 \times 100 \times 100}{10 \times 10 \times 10} = 1000$$

Q7.

Answer :

(a) 48 cm^3

Let a be the length of the smallest edge.

Then the edges are in the proportion $a : 2a : 3a$.

$$\text{Now, surface area} = 2(a \times 2a + a \times 3a + 2a \times 3a) = 2(2a^2 + 3a^2 + 6a^2) = 22a^2 = 88 \text{ cm}^2$$

$$\Rightarrow a = \sqrt{\frac{88}{22}} = \sqrt{4} = 2$$

Also, $2a = 4$ and $3a = 6$

$$\therefore \text{Volume} = a \times 2a \times 3a = 2 \times 4 \times 6 = 48 \text{ cm}^3$$

Q8.

Answer :

(b) 1: 9

$$\frac{\text{Volume } 1}{\text{Volume } 2} = \frac{1}{27} = \frac{a^3}{b^3}$$

$$\Rightarrow a = \frac{b}{\sqrt[3]{27}} = \frac{b}{3} \text{ or } b = 3a \text{ or } \frac{b}{a} = 3$$

$$\text{Now, } \frac{\text{surface area } 1}{\text{surface area } 2} = \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \frac{(b/3)^2}{b^2} = \frac{1}{9}$$

$$\therefore \text{Ratio of the surface areas} = 1 : 9$$

Q9.

Answer :

(c) 164 sq cm

$$\text{Surface area} = 2(10 \times 4 + 10 \times 3 + 4 \times 3) = 2(40 + 30 + 12) = 164 \text{ cm}^2$$

Q10.

Answer :

(c) 36 kg

$$\text{Volume of the iron beam} = 9 \times 0.4 \times 0.2 = 0.72 \text{ m}^3$$

$$\therefore \text{Weight} = 0.72 \times 50 = 36 \text{ kg}$$

Q11.

Answer :

(a) 2 m

$$42000 \text{ L} = 42 \text{ m}^3$$

$$\text{Volume} = lbh$$

$$\therefore \text{Height } (h) = \frac{\text{volume}}{lb} = \frac{42}{6 \times 3.5} = \frac{6}{6 \times 0.5} = 2 \text{ m}$$

Q12.

Answer :

(b) 88

$$\text{Volume of the room} = 10 \times 8 \times 3.3 = 264 \text{ m}^3$$

$$\text{One person requires } 3 \text{ m}^3.$$

$$\therefore \text{Total no. of people that can be accommodated} = \frac{264}{3} = 88$$

Q13.

Answer :

(a) 30000

$$\text{Volume} = 3 \times 2 \times 5 = 30 \text{ m}^3 = 30000 \text{ L}$$

Q14.

Answer :

(b) 1390 cm²

$$\text{Surface area} = 2(25 \times 15 + 15 \times 8 + 25 \times 8) = 2(375 + 120 + 200) = 1390 \text{ cm}^2$$

Q15.

Answer :

(d) 64 cm²

$$\text{Diagonal of the cube} = a\sqrt{3} = 4\sqrt{3} \text{ cm}$$

$$\text{i.e., } a = 4 \text{ cm}$$

$$\therefore \text{Volume} = a^3 = 4^3 = 64 \text{ cm}^3$$

Q16.

Answer :

(b) 486 sq cm

$$\text{Diagonal} = \sqrt{3}a \text{ cm} = 9\sqrt{3} \text{ cm}$$

i.e., $a = 9$

$$\therefore \text{Total surface area} = 6a^2 = 6 \times 81 = 486 \text{ cm}^2$$

Q17.

Answer :

(d) If each side of the cube is doubled, its volume becomes 8 times the original volume.

Let the original side be a units.

Then original volume = a^3 cubic units

Now, new side = $2a$ units

Then new volume = $(2a)^3$ sq units = $8a^3$ cubic units

Thus, the volume becomes 8 times the original volume.

Q18.

Answer :

(b) becomes 4 times.

Let the side of the cube be a units.

Surface area = $6a^2$ sq units

Now, new side = $2a$ units

New surface area = $6(2a)^2$ sq units = $24a^2$ sq units.

Thus, the surface area becomes 4 times the original area.

Q19.

Answer :

(a) 12 cm

$$\text{Total volume} = 6^3 + 8^3 + 10^3 = 216 + 512 + 1000 = 1728 \text{ cm}^3$$

$$\therefore \text{Edge of the new cube} = \sqrt[3]{1728} = 12 \text{ cm}$$

Q20.

Answer :

(d) 625 cm^3

Length of the cuboid so formed = 25 cm

Breadth of the cuboid = 5 cm

Height of the cuboid = 5 cm

$$\therefore \text{Volume of cuboid} = 25 \times 5 \times 5 = 625 \text{ cm}^3$$

Q21.

Answer :

(d) 44 m^3

Diameter = 2 m

Radius = 1 m

Height = 14 m

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 1 \times 1 \times 14 = 44 \text{ m}^3$$

Q22.

Answer :

(b) 12 m

Diameter = 14 m

Radius = 7 m

Volume = 1848 m³

$$\text{Now, volume} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times h = 1848 \text{ m}^3$$

$$\therefore h = \frac{1848}{22 \times 7} = 12 \text{ m}$$

Q23.

Answer :

(c) 4 : 3

Here,

$$\begin{aligned} \frac{\text{Total surface area}}{\text{Lateral surface area}} &= \frac{2\pi r(h+r)}{2\pi rh} \\ &= \frac{h+r}{h} \\ &= \frac{20+60}{60} \\ &= \frac{4}{3} \\ &= 4 : 3 \end{aligned}$$

Q24.

Answer :

(d) 640

$$\text{Total no. of coins} = \frac{\text{volume of cylinder}}{\text{volume of each coin}} = \frac{\pi \times 3 \times 3 \times 8}{\pi \times 0.75 \times 0.75 \times 0.2} = 640$$

Q25.

Answer :

(b) 84 m

$$\text{Length} = \frac{\text{volume}}{\pi r^2} = \frac{66 \times 7}{22 \times 0.05 \times 0.05} = 8400 \text{ cm} = 84 \text{ m}$$

Q26.

Answer :

(a) 1100 cm³

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 14 = 1100 \text{ cm}^3$$

Q27.

Answer :

(a) 1837 cm²

Diameter = 7 cm

Radius = 3.5 cm

Height = 80 cm

$$\therefore \text{Total surface area} = 2\pi r(r+h) = 2 \times \frac{22}{7} \times 3.5(3.5+80) = 22(83.5) = 1837 \text{ cm}^2$$

Q28.

Answer :

(b) 396 cm³

Here, curved surface area = $2\pi rh = 264 \text{ cm}^3$

$$\Rightarrow r = \frac{264 \times 7}{2 \times 22 \times 14} = 3 \text{ cm}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 3 \times 3 \times 14 = 396 \text{ cm}^3$$

Q29.

Answer :

(a) 770 cm^3

Diameter = 14 cm

Radius = 7 cm

Now, curved surface area = $2\pi rh = 220 \text{ cm}^2$

$$\Rightarrow h = \frac{220 \times 7}{2 \times 22 \times 7} = 5 \text{ cm}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 5 = 770 \text{ cm}^3$$

Q30.

Answer :

(c) 20:27

We have the following :

$$\frac{r_1}{r_2} = \frac{2}{3}$$

$$\frac{h_1}{h_2} = \frac{5}{3}$$

$$\therefore \frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{20}{27}$$