Exercise 1.4

Chapter 1 Functions and Limits Exercise 1.4 1E

Consider the values in the table show the volume V of water remaining in the tank (in gallons) after t minutes

5	10	15	20	25	30	t (min)
694	444	250	111	28	0	V (gal.)

(a)

Consider the point $P(15,250)$ on the graph of V

To find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25, \text{and } 30$:

When t = 5, then $P = (15,250), Q = (5,694)$

Slope of the secant line $PQ = \dfrac{694-250}{5-15}$

$= \dfrac{-444}{10}$

$= \boxed{-44.4}$

When t = 10, then $P = (15,250), Q = (10,444)$

Slope of the secant line $PQ = \dfrac{444-250}{10-15}$

$= \dfrac{-194}{5}$

$= \boxed{-38.8}$

When t = 20, then $P = (15,250), Q = (20,111)$

Slope of the secant line $PQ = \dfrac{111-250}{20-15}$

$= \dfrac{-139}{5}$

$= \boxed{-27.8}$

When t = 25, $P = (15,250), Q = (25,28)$

Slope of the secant line $PQ = \dfrac{28-250}{25-15}$

$= \dfrac{-222}{10}$

$= \boxed{-22.2}$

When t = 30, $P = (15,250), Q = (30,0)$

Slope of the secant line $PQ = \dfrac{0-250}{30-15}$

$= \dfrac{-250}{15}$

$= \boxed{-16.\overline{6}}$

(b)

To estimate the slope of the tangent line at P by averaging the slopes of two secant lines:

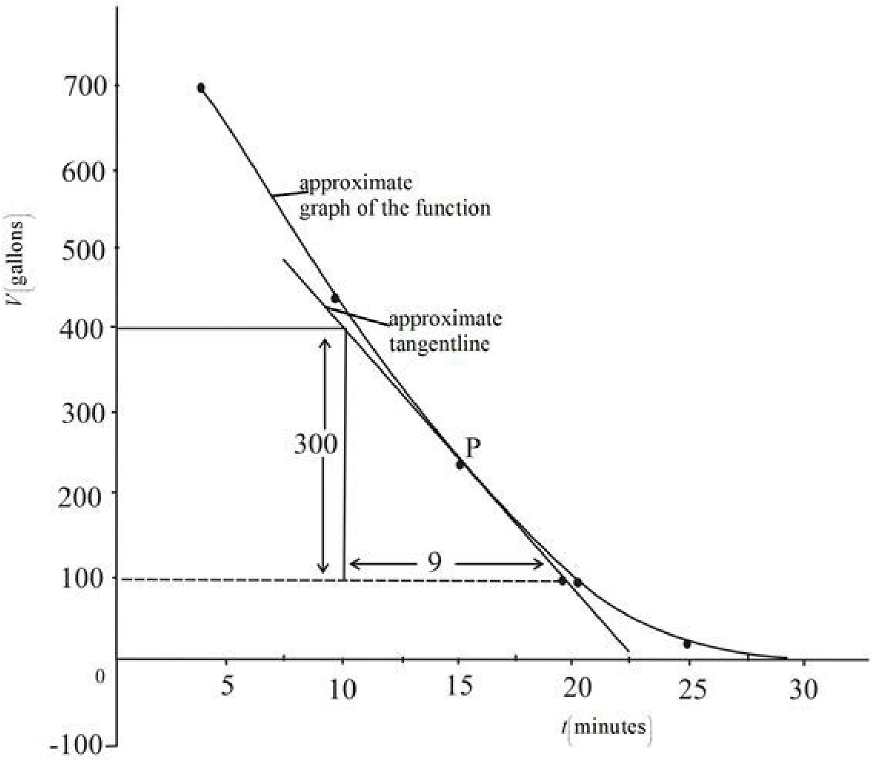
Using the values of that correspond to the points closest to P (t = 10 and t = 20)

Slope of the tangent at $P \approx \dfrac{-38.8+(-27.8)}{2}$

$= \boxed{-33.3}$

(c)

Sketch a graph of the function to estimate the slope of the tangent line at P.



From the graph of slope of the tangent line at $P = \dfrac{-300}{9}$

$= \boxed{-33.\bar{3}}$

Chapter 1 Functions and Limits Exercise 1.4 2E

t (min)	36	38	40	42	44
Hard beats	2530	2661	2806	2948	3080

If P is the point (42, 2948) on the graph of heart beats. We have to find out the slopes of the secant line PQ when Q is the point on the graph with $t = 36, 38, 40, 44$

(A)

We have $P = (42, 2948)$ & $Q = (36, 2530)$

Then slope $= \dfrac{2530-2948}{36-42} = 69.66667$

(B)

We have $P = (42, 2948)$ & $Q = (38, 2661)$

Then slope $= \dfrac{2661-2948}{38-42} = \boxed{71.75}$

(C)

We have $P = (42, 2948)$ & $Q = (40, 2806)$

The slope $= \dfrac{2806-2948}{40-42} = \boxed{71.00}$

(D)

We have $P=(42,2948)$ & $Q=(44,3080)$

Slope $=\dfrac{3080-2948}{44-42}=66.00$

Points	Slope of secant lines
(36, 2530)	69.67
(38, 2661)	71.75
(40, 2806)	71.00
(44, 3080)	66.00

Table-1

From the table-1, we see that heart rate is decreasing from 71 to 66 heartbeats/minute after being stable for a while. The heart rate is dropping.

Chapter 1 Functions and Limits Exercise 1.4 3E

The point $P(2,-1)$ lies on the curve $y=\dfrac{1}{1-x}$.

(a) Point Q: $\left(x,\dfrac{1}{1-x}\right)$

We have to find the slope of secant PQ when the value of *x* is given.

(i) For $x=1.5$

Point Q: $\left(1.5,\dfrac{1}{1-1.5}\right)=\left(1.5,\dfrac{1}{-0.5}\right)=(1.5,-2)$

$$\begin{aligned}\text{Slope of } PQ &= \frac{-2-(-1)}{1.5-2}\\ &= \frac{-2+1}{-0.5}\\ &= \frac{-1}{-0.5}\\ &= 2\end{aligned}$$

(ii) For $x=1.9$

Point Q: $\left(1.9,\dfrac{1}{1-1.9}\right)=\left(1.9,\dfrac{1}{-0.9}\right)=\left(1.9,-\dfrac{10}{9}\right)$

$$\begin{aligned}\text{Slope of } PQ &= \frac{\left(-\frac{10}{9}\right)-(-1)}{1.9-2}\\ &= \frac{\left(-\frac{1}{9}\right)}{-0.1}\\ &= \frac{1}{9}\times 10\\ &= 1.111111\end{aligned}$$

(iii) For $x=1.99$

Point Q: $\left(1.99,\dfrac{1}{1-1.99}\right)=\left(1.99,\dfrac{1}{-0.99}\right)=\left(1.99,-\dfrac{100}{99}\right)$

$$\begin{aligned}\text{Slope of } PQ &= \frac{\left(-\frac{100}{99}\right)-(-1)}{1.99-2}\\ &= \frac{\left(-\frac{1}{99}\right)}{-0.01}\\ &= \frac{1}{99}\times 100\\ &= 1.010101\end{aligned}$$

(iv) For $x = 1.999$

Point Q: $\left(1.999, \frac{1}{1-1.999}\right) = \left(1.999, \frac{1}{-0.999}\right) = \left(1.999, -\frac{1000}{999}\right)$

$$\text{Slope of } PQ = \frac{\left(-\frac{1000}{999}\right)-(-1)}{1.999-2}$$
$$= \frac{\left(-\frac{1}{999}\right)}{-0.001}$$
$$= \frac{1}{999} \times 1000$$
$$= 1.001001$$

(v) For $x = 2.5$

Point Q: $\left(2.5, \frac{1}{1-2.5}\right) = \left(2.5, \frac{1}{-1.5}\right) = \left(2.5, -\frac{2}{3}\right)$

$$\text{Slope of } PQ = \frac{\left(-\frac{2}{3}\right)-(-1)}{2.5-2}$$
$$= \frac{\left(\frac{1}{3}\right)}{0.5}$$
$$= \frac{1}{3} \times \frac{10}{5}$$
$$= \frac{2}{3}$$
$$= 0.666667$$

(vi) For $x = 2.1$

Point Q: $\left(2.1, \frac{1}{1-2.1}\right) = \left(2.1, \frac{1}{-1.1}\right) = \left(2.1, -\frac{10}{11}\right)$

$$\text{Slope of } PQ = \frac{\left(-\frac{10}{11}\right)-(-1)}{2.1-2}$$
$$= \frac{\left(\frac{1}{11}\right)}{0.1}$$
$$= \frac{1}{11} \times 10$$
$$= \frac{10}{11}$$
$$= 0.909091$$

(vii) For $x = 2.01$

Point Q: $\left(2.01, \frac{1}{1-2.01}\right) = \left(2.01, \frac{1}{-1.01}\right) = \left(2.01, -\frac{100}{101}\right)$

$$\text{Slope of } PQ = \frac{\left(-\frac{100}{101}\right)-(-1)}{2.01-2}$$
$$= \frac{\left(\frac{1}{101}\right)}{0.01}$$
$$= \frac{1}{101} \times 100$$
$$= \frac{100}{101}$$
$$= 0.990099$$

(viii) For $x = 2.001$

Point Q: $\left(2.001, \frac{1}{1-2.001}\right) = \left(2.001, \frac{1}{-1.001}\right) = \left(2.001, -\frac{1000}{1001}\right)$

$$\text{Slope of } PQ = \frac{\left(-\frac{1000}{1001}\right) - (-1)}{2.001 - 2}$$
$$= \frac{\left(\frac{1}{1001}\right)}{0.001}$$
$$= \frac{1}{1001} \times 1000$$
$$= \frac{1000}{1001}$$
$$= 0.999001$$

(b) We observe that when the value of *x* approaches then the value of the slope of secant line *PQ* approaches 1.

Therefore, slope of the tangent line to the curve at $P(2,-1)$ = 1

(c) Slope of the line, $m = 1$

Point P: $(2,-1)$

Using point-slope form, equation of tangent line is

$$y - (-1) = 1(x - 2)$$
$$\Rightarrow y + 1 = x - 2$$
$$\Rightarrow \boxed{y = x - 3}$$

Chapter 1 Functions and Limits Exercise 1.4 4E

Given curve
$y = \cos \pi x$

Point P: $(0.5, 0)$

(a) Point Q: $(x, \cos \pi x)$

We have to find the slope of secant PQ when the value of x is given

(i) For $x = 0$

Point Q: $(0, \cos(\pi \times 0)) = (0, \cos 0)$
$= (0, 1)$

Slope of PQ
$$= \frac{1-0}{0-0.5}$$
$$= \frac{1}{-0.5}$$
$$= -2$$

(ii) For $x = 0.4$

Point Q: $(0.4, \cos(\pi \times 0.4)) = (0.4, \cos 0.4\pi)$
$= (0.4, 0.309017)$

Slope of PQ
$$= \frac{0.309017 - 0}{0.4 - 0.5}$$
$$= \frac{0.309017}{-0.1}$$
$$= -3.09017$$

(iii) For $x = 0.49$

Point Q: $\left(0.49, \cos(\pi \times 0.49)\right) = \left(0.49, \cos 0.49\pi\right)$

$= \left(0.49, 0.03141076\right)$

Slope of PQ

$= \dfrac{0.03141076 - 0}{0.49 - 0.5}$

$= \dfrac{0.03141076}{-0.01}$

$= -3.141076$

(iv) For $x = 0.499$

Point Q: $\left(0.49, \cos(\pi \times 0.499)\right) = \left(0.499, \cos 0.499\pi\right)$

$= \left(0.499, 0.00314159\right)$

Slope of PQ

$= \dfrac{0.00314159 - 0}{0.499 - 0.5}$

$= \dfrac{0.00314159}{-0.001}$

$= -3.14159$

(v) For $x = 1$

Point Q: $\left(1, \cos(\pi \times 1)\right) = \left(1, \cos \pi\right)$

$= \left(1, -1\right)$

Slope of PQ

$= \dfrac{-1 - 0}{1 - 0.5}$

$= \dfrac{-1}{0.5}$

$= -2$

(vi) For $x = 0.6$

Point Q: $\left(0.6, \cos(\pi \times 0.6)\right) = \left(0.6, \cos 0.6\pi\right)$

$= \left(0.6, -0.30901699\right)$

Slope of PQ

$= \dfrac{-0.30901699 - 0}{0.6 - 0.5}$

$= \dfrac{-0.30901699}{0.1}$

$= -3.0901699$

≈ -3.090170

(vii) For $x = 0.51$

Point Q: $\left(0.51, \cos(\pi \times 0.51)\right) = \left(0.51, \cos 0.51\pi\right)$

$= \left(0.51, -0.03141076\right)$

Slope of PQ

$= \dfrac{-0.03141076 - 0}{0.51 - 0.5}$

$= \dfrac{-0.03141076}{0.01}$

$= -3.141076$

(viii) For $x = 0.501$

Point Q: $\left(0.501, \cos(\pi \times 0.501)\right) = \left(0.501, \cos 0.501\pi\right) = \left(0.501, -0.00314159\right)$

Slope of PQ

$= \dfrac{-0.00314159 - 0}{0.501 - 0.5}$

$= \dfrac{-0.00314159}{0.001}$

$= -3.14159$

(b) We observe that when the value of x approaches 0.5 then the value of the slope of secant line PQ approaches -3.14159.

Therefore, slope of the tangent line to the curve at $P(0.5, 0)$ = -3.14159

(c) Slope of the line, $m = -3.14159$

Point P: $(0.5, 0)$

Using point-slope form, equation of tangent line is

$y-(0) = -3.14159(x-0.5)$

$\Rightarrow \boxed{y = -3.14159x + 1.570795}$

(d) Given equation of the curve: $y = \cos \pi x$

Equation of the tangent at $(0.5, 0)$: $y = -3.14159x + 1.570795$

Equation of the first secant line passing through $(0,1)$ and with slope -2 is

$y-(1) = -2(x-0)$

$\Rightarrow y = -2x + 1$

Equation of the second secant line passing through $(0.5, 0)$ and with slope -3.09017 is

$y-(0) = -3.09017(x-0.5)$

$\Rightarrow y = -3.09017x + 1.545085$

Required graph is

Chapter 1 Functions and Limits Exercise 1.4 5E

$y(t) = 40t - 16t^2$

At $t = 2 \Rightarrow y(2) = 40 \times 2 - 16(2^2) = 16$

(A)

Average velocity in the time interval [2, 2+h] is

$$V_{ave} = \frac{y(2+h) - y(2)}{h}$$

$$= \frac{\left[40(2+h) - 16(2+h)^2\right] - 16}{h}$$

$$= \frac{-24h - 16h^2}{h}$$

$$= -24 - 16h \quad \text{Where } h \neq 0$$

(1) $h = 0.5$ then time interval is [2, 2.5]

Then $V_{ave} = -24 - 16(0.5) = \boxed{-32 \text{ ft/s}}$

(2) $h = 0.1$ then $V_{ave} = -24 - 16(0.1) = \boxed{-25.6 \text{ ft/s}}$

(3) $h = 0.05$ then $V_{ave} = -24 - 16(0.05) = \boxed{-24.8 \text{ ft/s}}$

(4) $h = 0.01$ then $V_{ave} = -24 - 16(0.01) = \boxed{-24.16 \text{ ft/s}}$

(B)

As h approaches 0 the average velocity approaches -24

So the instantaneous velocity when $t = 2$ is $\boxed{-24. \text{ ft/s}}$

Chapter 1 Functions and Limits Exercise 1.4 6E

Consider that the height of the rock is modeled by $y = 10t - 1.86t^2$

a)

(i)

The average velocity in the interval $[1,2]$ is given as

$$\text{Average velocity} = \frac{\text{Change in position}}{\text{Time elapsed}}$$

$$= \frac{y(2) - y(1)}{2-1}$$

$$= \frac{\left[10(2) - 1.86(2)^2\right] - \left[10(1) - 1.86(1)^2\right]}{1}$$

Simplify the above step, then

$$\begin{aligned}\text{Average velocity} &= \frac{12.56-8.14}{1} && \text{Simplify}\\ &= \frac{4.42}{1} && \text{Simplify.}\\ &= 4.42 && \text{Dividing.}\end{aligned}$$

Thus, the average velocity is $\boxed{4.42 \text{ m/s}}$.

(ii)

The average velocity in the interval $[1,1.5]$ is given as

$$\begin{aligned}\text{Average velocity} &= \frac{\text{Change in position}}{\text{Time elapsed}}\\ &= \frac{y(1.5)-y(1)}{1.5-1}\\ &= \frac{\left[10(1.5)-1.86(1.5)^2\right]-\left[10(1)-1.86(1)^2\right]}{0.5}\end{aligned}$$

Simplify the above step, then

$$\begin{aligned}\text{Average velocity} &= \frac{10.815-8.14}{0.5} && \text{Simplify.}\\ &= \frac{2.675}{0.5} && \text{Simplify.}\\ &= 5.35 && \text{Dividing}\end{aligned}$$

Thus, the average velocity is $\boxed{5.35 \text{ m/s}}$.

(iii)

The average velocity in the interval $[1,1.1]$ is given as

$$\begin{aligned}\text{Average velocity} &= \frac{\text{Change in position}}{\text{Time elapsed}}\\ &= \frac{y(1.1)-y(1)}{1.5-1}\\ &= \frac{\left[10(1.1)-1.86(1.1)^2\right]-\left[10(1)-1.86(1)^2\right]}{0.1}\end{aligned}$$

Simplify the above step, then

$$\begin{aligned}\text{Average velocity} &= \frac{8.7494-8.14}{0.1} && \text{Simplify.}\\ &= \frac{0.6094}{0.1} && \text{Simplify.}\\ &= 6.094 && \text{Dividing}\end{aligned}$$

Thus, the average velocity is $\boxed{6.094 \text{ m/s}}$.

(iv)

The average velocity in the interval $[1,1.01]$ is given as

$$\begin{aligned}\text{Average velocity} &= \frac{\text{Change in position}}{\text{Time elapsed}}\\ &= \frac{y(1.01)-y(1)}{1.01-1}\\ &= \frac{\left[10(1.01)-1.86(1.01)^2\right]-\left[10(1)-1.86(1)^2\right]}{0.01}\end{aligned}$$

Simplify the above step, then

$$\begin{aligned}\text{Average velocity} &= \frac{8.202614-8.14}{0.01} && \text{Simplify.}\\ &= \frac{0.062614}{0.01} && \text{Simplify.}\\ &= 6.2614 && \text{Dividing}\end{aligned}$$

Thus, the average velocity is $\boxed{6.2614 \text{ m/s}}$.

(v)

The average velocity in the interval $[1,1.001]$ is given as

$$\begin{aligned}\text{Average velocity} &= \frac{\text{Change in position}}{\text{Time elapsed}}\\ &= \frac{y(1.001)-y(1)}{1.001-1}\\ &= \frac{\left[10(1.001)-1.86(1.001)^2\right]-\left[10(1)-1.86(1)^2\right]}{0.001}\end{aligned}$$

Simplify the above step, then

$$\begin{aligned}\text{Average velocity} &= \frac{8.14627814-8.14}{0.001} && \text{Simplify.}\\ &= \frac{0.00627814}{0.001} && \text{Simplify.}\\ &= 6.27814 && \text{Dividing}\end{aligned}$$

Thus, the average velocity is $\boxed{6.27814 \text{ m/s}}$.

b)

The average velocities shown in the table form as

Time interval	Average velocity
$1 \le y \le 2$	$4.42 m/s$
$1 \le y \le 0.5$	$5.35\ m/s$
$1 \le y \le 0.1$	$6.094\ m/s$
$1 \le y \le 0.01$	$6.2614\ m/s$
$1 \le y \le 0.001$	$6.27814\ m/s$

The instantaneous velocity is defined as the limiting value of the average velocity as the time period goes shorten and shorten.

Thus from the above table it is observe that the limiting value of the average velocity is

6 m/s.

Thus, the average velocity is $\boxed{6 \text{ m/s}}$.

Chapter 1 Functions and Limits Exercise 1.4 7E

(a)

Consider the table,

t(seconds)	0	1	2	3	4	5
s(meters)	0	1.4	5.1	10.7	17.7	25.8

Need to find the average velocities for each time period.

(i)

The average velocity in the interval $[1,3]$ is,

$$\frac{v(3)-v(1)}{3-1}=\frac{10.7-1.4}{2}$$
$$=\boxed{4.65\,\text{meter/second}}$$

(ii)

The average velocity in the interval $[2,3]$ is,

$$\frac{v(3)-v(2)}{3-2}=\frac{10.7-5.1}{2}$$
$$=\boxed{2.80\,\text{meter/second}}$$

(iii)

The average velocity in the interval $[3,5]$ is,

$$\frac{v(5)-v(3)}{5-3}=\frac{25.8-10.7}{2}$$
$$=\boxed{7.55\,\text{meter/second}}$$

(iv)

The average velocity in the interval $[3,4]$ is,

$$\frac{v(4)-v(3)}{4-3}=\frac{17.7-10.7}{2}$$
$$=\boxed{7\,\text{meter/second}}$$

(b)

The graph of s as a function of t is shown below:

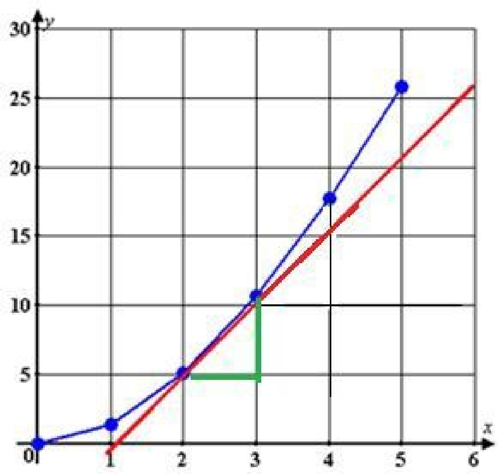


Figure 1

Need to estimate the instantaneous velocity at $t=3$.

From the figure1, it should be clear that the best points to choose are

$(2,5.1)$ and $(4,17.7)$.

So the instantaneous velocity is

$$\frac{\Delta s}{\Delta t}=\frac{17.7-5.1}{4-2}$$
$$=\frac{12.6}{2}$$
$$=6.3\,\text{meter/second}$$

Hence, the instantaneous velocity when $t=3$ is approximately $\frac{\Delta s}{\Delta t}=\boxed{6.3\,\text{meter/second}}$

Let the displacement of a particle moving back and forth along a straight line is given by the equation of motion

$$s = 2\sin \pi t + 3\cos \pi t$$

(a)

(i) To find the average velocity during the time period $[1,2]$

So,

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$
$$= \frac{s(2)-s(1)}{2-1}$$
$$= \frac{[2\sin \pi(2)+3\cos \pi(2)]-[2\sin \pi(1)+3\cos \pi(1)]}{1}$$
$$= [2\sin 2\pi + 3\cos 2\pi]-[2\sin \pi + 3\cos \pi]$$
$$= [2(0)+3(1)]-[2(0)+3(-1)]$$
$$= 3+3$$
$$= 6$$

Hence the average velocity during the time period $[1,2]$ is $\boxed{6 \text{ cm/s}}$.

(ii) To find the average velocity during the time period $[1,1.1]$

So,

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$
$$= \frac{s(1.1)-s(1)}{1.1-1}$$
$$= \frac{[2\sin \pi(1.1)+3\cos \pi(1.1)]-[2\sin \pi(1)+3\cos \pi(1)]}{0.1}$$
$$= \frac{[2(-0.31)+3(-0.95)]-[2(0)+3(-1)]}{0.1}$$
$$= \frac{[-0.62-2.85]+3}{0.1}$$
$$= \frac{-3.47+3}{0.1}$$
$$= \frac{-0.47}{0.1}$$
$$= -4.7$$

Hence the average velocity during the time period $[1,1.1]$ is $\boxed{-4.7 \text{ cm/s}}$.

(iii) To find the average velocity during the time period $[1,1.01]$

So,

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$
$$= \frac{s(1.01)-s(1)}{1.01-1}$$
$$= \frac{[2\sin \pi(1.01)+3\cos \pi(1.01)]-[2\sin \pi(1)+3\cos \pi(1)]}{0.01}$$
$$= \frac{[-3.0613]-[-3]}{0.01}$$
$$= \frac{-3.0613+3}{0.01}$$
$$= -\frac{0.0613}{0.01}$$
$$= -6.13$$

Hence the average velocity during the time period $[1,1.01]$ is $\boxed{-6.13 \text{ cm/s}}$.

(iv) To find the average velocity during the time period $[1,1.001]$

So,

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$
$$= \frac{s(1.001)-s(1)}{1.001-1}$$
$$= \frac{[2\sin\pi(1.001)+3\cos\pi(1.001)]-[2\sin\pi(1)+3\cos\pi(1)]}{0.001}$$

$$= \frac{[-3.0062]-[-3]}{0.001}$$
$$= \frac{-3.0062+3}{0.001}$$
$$= -\frac{0.0062}{0.001}$$
$$= -6.2$$

Hence the average velocity during the time period $[1,1.01]$ is $\boxed{-6.2 \text{ cm/s}}$.

(b)

To estimate the instantaneous velocity of the particle when $t=1$

Take limit of the function $s(t)$ at $t=1$ to get instantaneous velocity

It appears that as shorten the time period, the average velocity is becoming closer to

-6 cm/s. The instantaneous velocity when $t=1$ is defined to be the limiting value of these average velocities over shorter and shorter time periods that start at $t=1$. Thus the instantaneous velocity of the particle when $t=1$ is

$\boxed{t=-6 \text{ cm/s}}$

Chapter 1 Functions and Limits Exercise 1.4 9E

Consider the curve,

$$y=\sin\left(\frac{10\pi}{x}\right).$$

The point on this curve is $P(1,0)$, and let Q be any point on the given curve.

(a)

The objective is to determine the slope of the secant line PQ for $x = 2,\ 1.5,\ 1.4,\ 1.3,\ 1.2,\ 1.1,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ \text{and}\ 0.9$.

The slope of the secant line PQ for $x=2$ is,

$$m=\frac{\sin\left(\frac{10\pi}{x}\right)-0}{x-1}$$
$$=\frac{\sin\left(\frac{10\pi}{2}\right)}{2-1}$$
$$=\sin(5\pi)$$
$$=0$$

The slope of the secant line PQ for $x=1.5$ is,

$$m=\frac{\sin\left(\frac{10\pi}{1.5}\right)}{1.5-1}$$
$$\approx\frac{0.866025}{0.5}$$
$$\approx 1.7321$$

The slope of the secant line PQ for $x = 1.4$ is,

$$m = \frac{\sin\left(\frac{10\pi}{1.4}\right)}{1.4 - 1}$$
$$\approx \frac{-0.4339}{0.4}$$
$$\approx -1.0847$$

The slope of the secant line PQ for $x = 1.3$ is,

$$m = \frac{\sin\left(\frac{10\pi}{1.3}\right)}{1.3 - 1}$$
$$\approx \frac{-0.823}{0.3}$$
$$\approx -2.7433$$

The slope of the secant line PQ for $x = 1.2$ is,

$$m = \frac{\sin\left(\frac{10\pi}{1.2}\right)}{1.2 - 1}$$
$$\approx \frac{0.866025}{0.2}$$
$$\approx 4.3301$$

The slope of the secant line PQ for $x = 1.1$ is,

$$m = \frac{\sin\left(\frac{10\pi}{1.1}\right)}{1.1 - 1}$$
$$\approx \frac{-0.281733}{0.1}$$
$$\approx -2.8173$$

The slope of the secant line PQ for $x = 0.5$ is,

$$m = \frac{\sin\left(\frac{10\pi}{0.5}\right)}{0.5 - 1}$$
$$= \frac{0}{-0.5}$$
$$= 0$$

The slope of the secant line PQ for $x = 0.6$ is,

$$m = \frac{\sin\left(\frac{10\pi}{0.6}\right)}{0.6 - 1}$$
$$\approx \frac{0.866025}{-0.4}$$
$$\approx -2.1651$$

The slope of the secant line PQ for $x = 0.7$ is,

$$m = \frac{\sin\left(\frac{10\pi}{0.7}\right)}{0.7 - 1}$$
$$\approx \frac{0.781831}{-0.3}$$
$$\approx -2.6061$$

The slope of the secant line PQ for $x = 0.8$ is,

$$m = \frac{\sin\left(\frac{10\pi}{0.8}\right)}{0.8 - 1}$$
$$= \frac{1}{-0.2}$$
$$= -5$$

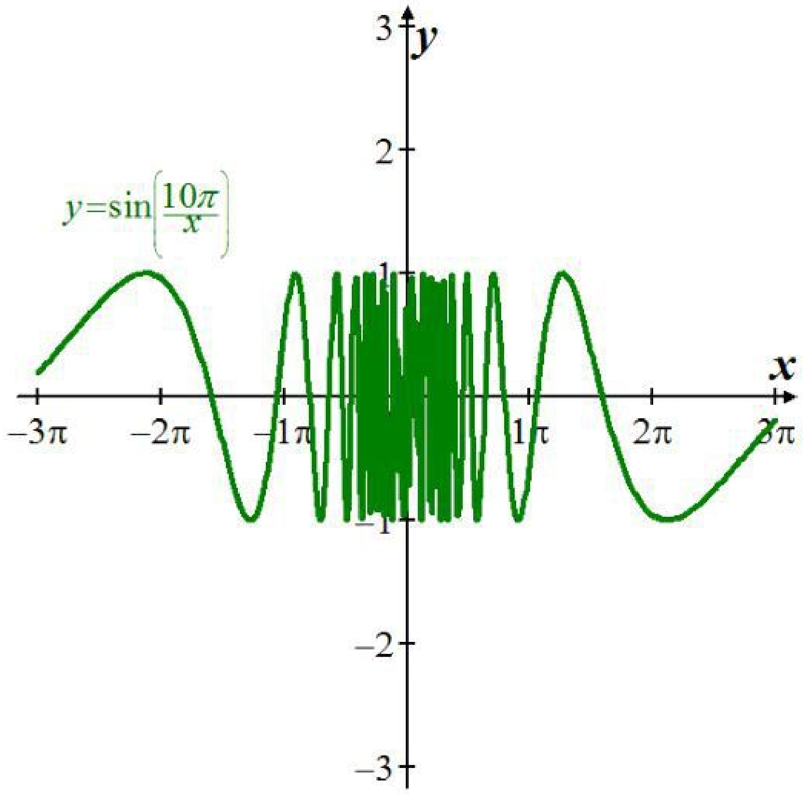
The slope of the secant line PQ for $x = 0.9$ is,

$$m = \frac{\sin\left(\frac{10\pi}{0.9}\right)}{0.9-1}$$
$$\approx \frac{-0.342020}{-0.1}$$
$$\approx 3.4202$$

Observe that the slopes are not approaching a limit.

(b)

Sketch the graph of the curve: $y = \sin\left(\frac{10\pi}{x}\right)$ as shown in the below figure:



From the graph, it can be observed that the slopes of the secant lines cannot approach to any point.

Thus, the slopes of the secant lines are not close to the slope of the tangent line at *P*.

(c)

As the slopes of the secant lines are not close to the slope of the tangent line at *P*, the slope of the tangent line at *P* cannot be estimated from them.