Chapter 6

Linear Programming

Contents

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The linear programming originated during World War II, when the British and American military management called upon a group of Scientists to study and plan the war activities, so that maximum damages could be inflicted on the enemy camps at minimum cost and loss.

Because of the success in military operations, it quickly spread in all phases of industry and government organizations. The Russian mathematician L.Kantorovich *has, for a number of years, been interested in the applications of mathematics to programming problems. In 1941,* F.L. Hitchcock *formulated and solved the transportation problem. In 1947* T.C. Koopmans *solved the transportation problem. The minimum cost diet problem was studied by economist* Stigler *in 1945. In 1947,* G.B. Dantzig *suggested an efficient method known as the simplex method, which is an iterative procedure to solve any linear programming problem in a finite number of steps.*

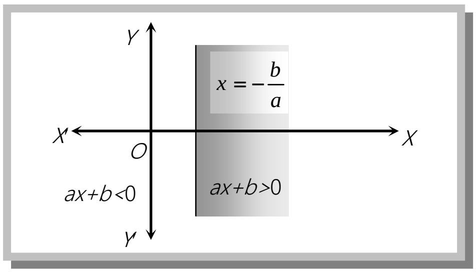
Linear Programming

'Linear Programming' is a scientific tool to handle optimization problems. Here, we shall learn about some basic concepts of linear programming problems in two variables, their applications, advantages, limitations, formulation and graphical method of solution.

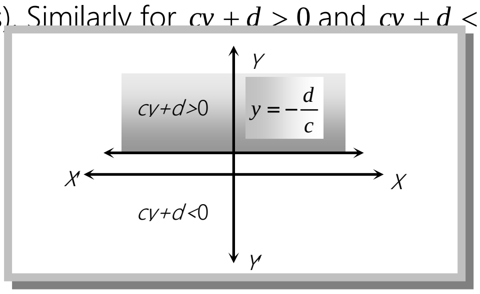
6.1 Linear Inequations

(1) **Graph of linear inequations**

(i) Linear inequation in one variable : $ax+b>0, ax+b<0, cy+d>0$ etc. are called linear inequations in one variable. Graph of these inequations can be drawn as follows :

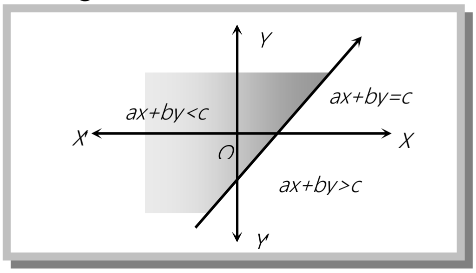


The graph of $ax+b>0$ and $ax+b<0$ are obtained by dividing *xy*-plane in two semi-planes by the line $x=-\frac{b}{a}$ (which is parallel to *y*-axis). Similarly for $cy+d>0$ and $cy+d<0$.



(ii) Linear Inequation in two variables : General form of these inequations are $ax + by > c, ax + by < c$. If any ordered pair (x_1, y_1) satisfies some inequations, then it is said to be a solution of the inequations.

The graph of these inequations is given below (for $c > 0$) :



Working rule

To draw the graph of an inequation, following procedure is followed :

(i) Write the equation $ax + by = c$ in place of $ax + by < c$ and $ax + by > c$.

(ii) Make a table for the solutions of $ax + by = c$.

(iii) Now draw a line with the help of these points. This is the graph of the line $ax + by = c$.

(iv) If the inequation is > or <, then the points lying on this line is not considered and line is drawn dotted or discontinuous.

(v) If the ineuqation is $\geq$ or $\leq$, then the points lying on the line is considered and line is drawn bold or continuous.

(vi) This line divides the plane XOY in two region.

To Find the region that satisfies the inequation, we apply the following rules:

(a) Take an arbitrary point which will be in either region.

(b) If it satisfies the given inequation, then the required region will be the region in which the arbitrary point is located.

(c) If it does not satisfy the inequation, then the other region is the required region.

(d) Draw the lines in the required region or make it shaded.

(2) Simultaneous linear inequations in two variables : Since the solution set of a system of simultaneous linear inequations is the set of all points in two dimensional space which satisfy all the inequations simultaneously. Therefore to find the solution set we find the region of the plane common to all the portions comprising the solution set of given inequations. In case there is no region common to all the solutions of the given inequations, we say that the solution set is void or empty.

(3) **Feasible region :** The limited (bounded) region of the graph made by two inequations is called feasible region. All the points in feasible region constitute the solution of a system of inequations. The feasible solution of a L.P.P. belongs to only first quadrant. If feasible region is empty then there is no solution for the problem.

Example: 1 Inequations $3x - y \geq 3$ and $4x - y > 4$

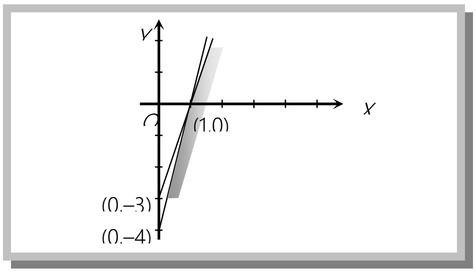
(a) Have solution for positive *x* and *y* (b) Have no solution for positive *x* and *y*

(c) Have solution for all *x* (d) Have solution for all *y*

Solution : (a) Following figure will be obtained on drawing the graphs of given inequations :

From $3x - y \geq 3, \frac{x}{1} + \frac{y}{-3} = 1$

From $4x - y \geq 4, \frac{x}{1} + \frac{y}{-4} = 1$



Clearly the common region of both the inequations is true for positive value of (*x, y*). It is also true for positive values of *x* and negative values of *y*.

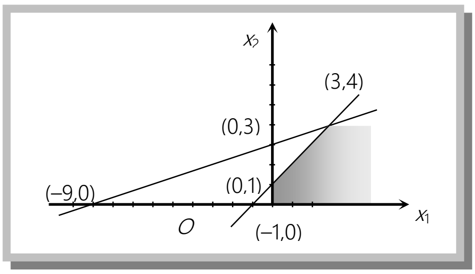
Example: 2 The constraints **[MP PET 1999]**

$- x_1 + x_2 + \leq 1 ; - x_1 + 3x_2 \leq 9 ;\ x, x_2 \geq 0$ define

(a) Bounded feasible space (b) Unbounded feasible space

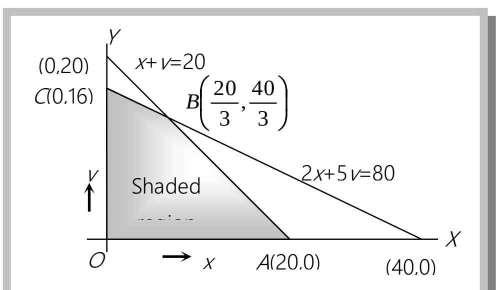
(c) Both bounded and unbounded feasible space (d) None of these

Solution : (b)



It is clear from the graph, the constraints define an unbounded feasible space.

Example: 3 Shaded region is represented by **[MP PET 1997]**

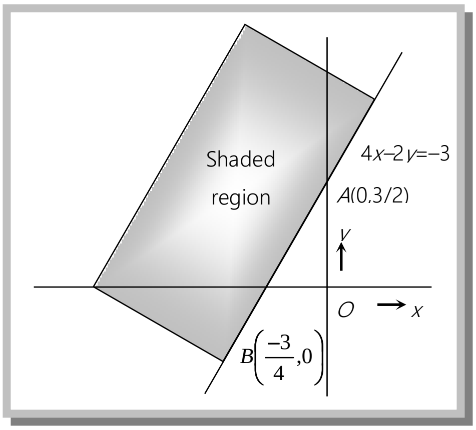


(a) $2x+5y\geq 80, x+y\leq 20, x\geq 0, y\leq 0$ (b) $2x+5y\geq 80, x+y\geq 20, x\geq 0, y\geq 0$

(c) $2x+5y\leq 80, x+y\leq 20, x\geq 0, y\geq 0$ (d) $2x+5y\leq 80, x+y\leq 20, x\leq 0, y\leq 0$

Solution: (c) In all the given equations, the origin is present in shaded area. Answer (c) satisfy this condition.

Example: 4 Shaded region is represented by



(a) $4x-2y\leq 3$ (b) $4x-2y\leq -3$ (c) $4x-2y\geq 3$ (d) $4x-2y\geq -3$

Solution: (b) Origin is not present in given shaded area. So $4x-2y\leq -3$ satisfy this condition.

6.2 Terms of Linear Programming

The term programming means planning and refers to a process of determining a particular program.

(1) **Objective function :** The linear function which is to be optimized (maximized or minimized) is called objective function of the *L.P.P.*

(2) **Constraints or Restrictions :** The conditions of the problem expressed as simultaneous equations or inequalities are called constraints or restrictions.

(3) **Non-negative Constraints :** Variables applied in the objective function of a linear programming problem are always non-negative. The ineqaulities which represent such constraints are called non-negative constraints.

(4) **Basic variables :** The m variables associated with columns of the $m \times n$ non-singular matrix which may be different from zero, are called basic variables.

(5) **Basic solution :** A solution in which the vectors associated to m variables are linear and the remaining $(n-m)$ variables are zero, is called a basic solution. A basic solution is called a degenerate basic solution, if at least one of the basic variables is zero and basic solution is called non-degenerate, if none of the basic variables is zero.

(6) **Feasible solution :** The set of values of the variables which satisfies the set of constraints of linear programming problem (*L.P.P*) is called a feasible solution of the *L.P.P*.

(7) **Optimal solution :** A feasible solution for which the objective function is minimum or maximum is called optimal solution.

(8) **Iso-profit line :** The line drawn in geometrical area of feasible region of *L.P.P*. for which the objective function remains constant at all the points lying on the line, is called iso-profit line.

If the objective function is to be minimized then these lines are called iso-cost lines.

(9) **Convex set :** In linear programming problems feasible solution is generally a polygon in first quadrant . This polygon is convex. It means if two points of polygon are connecting by a line, then the line must be inside to polygon. For example

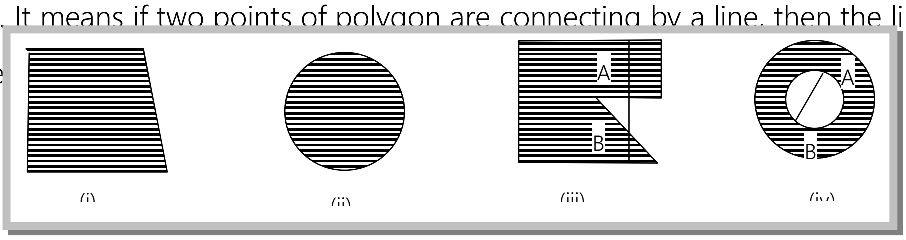


Figure (i) and (ii) are convex set while (iii) and (iv) are not convex set

6.3 Mathemiatical formulation of a Linear programming problem

There are mainly four steps in the mathematical formulation of a linear programming problem, as mathematical model. We will discuss formulation of those problems which involve only two variables.

(1) Identify the decision variables and assign symbols x and y to them. These decision variables are those quantities whose values we wish to determine.

(2) Identify the set of constraints and express them as linear equations/inequations in terms of the decision variables. These constraints are the given conditions.

(3) Identify the objective function and express it as a linear function of decision variables. It might take the form of maximizing profit or production or minimizing cost.

(4) Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

6.4 Graphical solution of two variable Linear programming problem

There are two techniques of solving an *L.P.P.* by graphical method. These are :

(1) Corner point method, and (2) Iso-profit or Iso-cost method.

(1) **Corner point method**

Working rule

(i) Formulate mathematically the *L.P.P.*

(ii) Draw graph for every constraint.

(iii) Find the feasible solution region.

(iv) Find the coordinates of the vertices of feasible solution region.

(v) Calculate the value of objective function at these vertices.

(vi) Optimal value (minimum or maximum) is the required solution.

Note :
- ❑ If there is no possibility to determine the point at which the suitable solution found, then the solution of problem is unbounded.
- ❑ If feasible region is empty, then there is no solution for the problem.
- ❑ Nearer to the origin, the objective function is minimum and that of further from the origin, the objective function is maximum.

(2) **Iso-profit or Iso-cost method :** Various steps of the method are as follows :

(i) Find the feasible region of the *L.P.P.*

(ii) Assign a constant value Z_1 to Z and draw the corresponding line of the objective function.

(iii) Assign another value Z_2 to Z and draw the corresponding line of the objective function.

(iv) If $Z_1 < Z_2, (Z_1 > Z_2)$, then in case of maximization (minimization) move the line P_1Q_1 corresponding to Z_1 to the line P_2Q_2 corresponding to Z_2 parallel to itself as far as possible, until the farthest point within the feasible region is touched by this line. The coordinates of the point give maximum (minimum) value of the objective function.

Note : ❑ The problem with more equations/inequations can be handled easily by this method.

- In case of unbounded region, it either finds an optimal solution or declares an unbounded solution. Unbounded solutions are not considered optimal solution. In real world problems, unlimited profit or loss is not possible.

6.5 To find the Vertices of Simple feasible region without Drawing a Graph

(1) **Bounded region:** The region surrounded by the inequalities $ax + by \leq m$ and $cx + dy \leq n$ in first quadrant is called bounded region. It is of the form of triangle or quadrilateral. Change these inequalities into equation, then by putting $x = 0$ and $y = 0,$ we get the solution also by solving the equation in which there may be the vertices of bounded region.

The maximum value of objective function lies at one vertex in limited region.

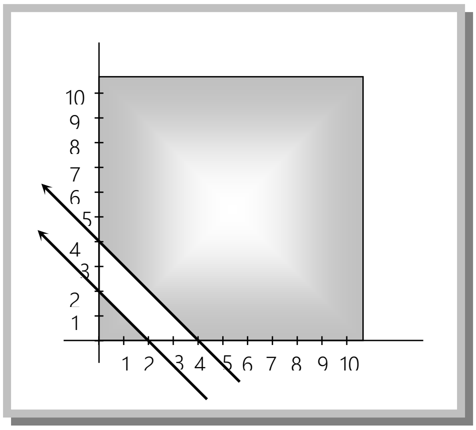
(2) **Unbounded region :** The region surrounded by the inequations $ax + by \geq m$ and $cx + dy \geq n$ in first quadrant, is called unbounded region.

Change the inequation in equations and solve for $x = 0$ and $y = 0$. Thus we get the vertices of feasible region.

The minimum value of objective function lies at one vertex in unbounded region but there is no existence of maximum value.

6.6 Problems having Infeasible Solutions

In some of the linear programming problems, constraints are inconsistent *i.e.* there does not exist any point which satisfies all the constraints. Such type of linear programming problems are said to have *infeasible solution.*



For Example : Maximize $Z = 5x + 2y$

subject to the constraints

$x + y \leq 2\,, 3x + 3y \geq 2\,, x, y \geq 0$

The above problem is illustrated graphically in the fig.

There is no point satisfying the set of above constraints. Thus, the problem is having an infeasible solution.

6.7 Some important points about L.P.P.

(1) If the constraints in a linear programming problem are changed, the problem is to be re-evaluated.

(2) The optimal value of the objective function is attained at the point, given by corner points of the feasible region.

(3) If a *L.P.P.* admits two optimal solutions, it has an infinite number of optimal solutions.

(4) If there is no possibility to determine the point at which the suitable solution can be found, then the solution of problem is unbounded.

(5) The maximum value of objective function lies at one vertex in limited region.

Example: 5 A firm makes pants and shirt. A shirt takes 2 hour on machine and 3 hour of man labour while a pant takes 3 hour on machine and 2 hour of man labour. In a week there are 70 hour of machine and 75 hour of man labour available. If the firm determines to make *x* shirts and *y* pants per week, then for this the linear constraints are

(a) $x \geq 0, y \geq 0, 2x + 3y \geq 70, 3x + 2y \geq 75$ (b) $x \geq 0, y \geq 0, 2x + 3y \leq 70, 3x + 2y \geq 75$

(c) $x \geq 0, y \geq 0, 2x + 3y \geq 70, 3x + 2y + \leq 75$ (d) $x \geq 0, y \geq 0, 2x + 3y \leq 70, 3x + 2y \leq 75$

Solution: (d)

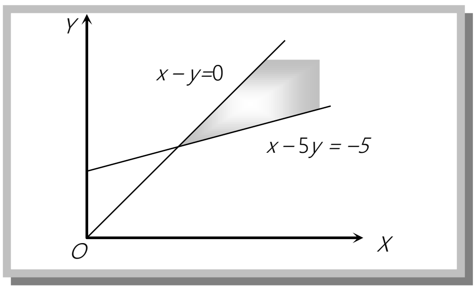
	Working time on machine	*Man labour*
Shirt (*x*)	2 hours	3 hours
Pant (*y*)	3 hours	2 hours
Availability	70 hours	75 hours

Linear constraints are $2x + 3y \leq 70, 3x + 2y \leq 75$ and $x, y \geq 0$

Example: 6 The minimum value of the objective function $Z = 2x + 10y$ for linear constraints $x - y \geq 0, x - 5y \leq -5$ and $x, y \geq 0$ is

(a) 10 (b) 15 (c) 12 (d) 8

Solution: (b) Required region is unbounded whose vertex is $\left(\frac{5}{4}, \frac{5}{4}\right)$



Hence the minimum value of objective function is $= 2 \times \frac{5}{4} + 10 \times \frac{5}{4} = 15$.

Example: 7 Minimize $z = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij}$ **[MP PET 1999]**

Subject to : $\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1........, m$; $\sum_{i=1}^{m} x_{ij} = b_j, j = 1......, n$ is a *LPP* with number of constraints

(a) $m + n$ (b) $m - n$ (c) mn (d) $\frac{m}{n}$

Solution: (a) (I) Condition, $i = 1, x_{11} + x_{12} + x_{13} + ... + x_{1n} \leq a_1$

$i = 2, x_{21} + x_{22} + x_{23} + ... + x_{2n} \leq a_2$

$i = 3, x_{31} + x_{32} + x_{33} + ... + x_{3n} \le a_3$

....

$i = m, x_{m1} + x_{m2} + x_{m3} + ... + x_{mn} \le a_m \rightarrow m$ constraints

(II) Condition $j = 1, x_{11} + x_{21} + x_{31} + ... + x_{m1} = b_1$

$j = 2, x_{12} + x_{22} + x_{32} + ... + x_{m2} = b_2$

....

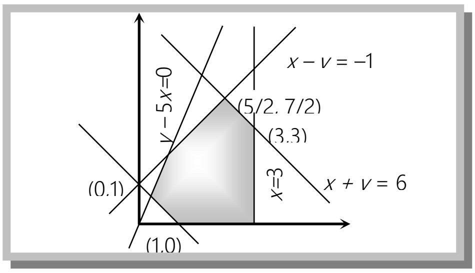
$j = n, x_{1n} + x_{2n} + x_{3n} + ... + x_{mn} = b_n \rightarrow n$ constraints

$\therefore$ Total constraints $= m + n$.

Example: 8 Maximize $z = 3x + 2y,$ subject to $x + y \ge 1, y - 5x \le 0, x - y \ge -1, x + y \le 6, x \le 3$ and $x, y \ge 0$

(a) $x = 3$ (b) $y = 3$ (c) $z = 15$ (d) All the above

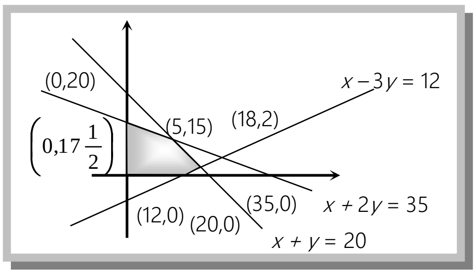
Solution: (d) The shaded region represents the bounded region (3,3) satisfies, so $x = 3, y = 3$ and $z = 15$.



Example: 9 Maximum value of $4x + 5y$ subject to the constraints $x + y \le 20, x + 2y \le 35, x - 3y \le 12$ is [Kurukshetra CEE 1996]

(a) 84 (b) 95 (c) 100 (d) 96

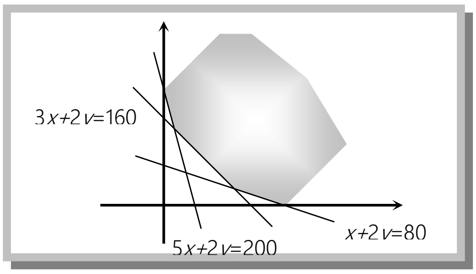
Solution: (b) Obviously, *max.* $4x + 5y = 95$. It is at (5, 15).



Example: 10 The maximum value of $Z = 4x + 3y$ subjected to the constraints $3x + 2y \ge 160$, $5x + 2y \ge 200, x + 2y \ge 80, x, y \ge 0$ is [MP PET 1998]

(a) 320 (b) 300 (c) 230 (d) None of these

Solution: (d) Obviously, it is unbounded. Therefore its maximum value does not exist.



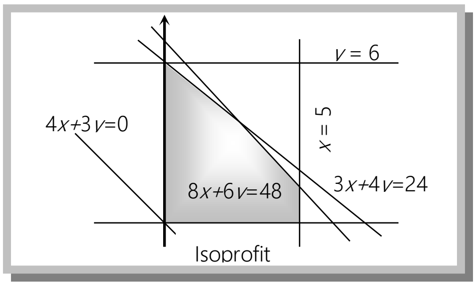
Example: 11 The objective function $Z = 4x + 3y$ can be maximized subjected to the constraints

$3x + 4y \le 24, 8x + 6y \le 48, x \le 5, y \le 6; x, y \ge 0$

(a) At only one point (b) At two points only (c) At an infinite number of points (d) None of these

Solution: (c) Obviously, the optimal solution is found on the line which is parallel to 'isoprofit line'.

Hence it has infinite number of solutions.

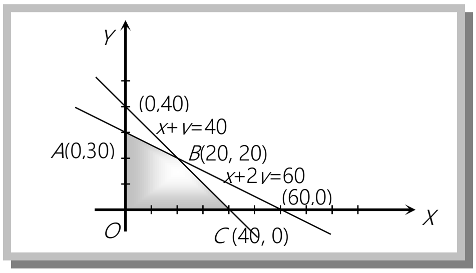


Example: 12 The maximum value of $\mu = 3x + 4y$ subjected to the conditions $x + y \le 40, x + 2y \le 60; x, y \ge 0$ is [MP PET 2002,04]

(a) 130 (b) 120 (c) 40 (d) 140

Solution: (d) Obviously $Max\ \mu = 3x + 4y$ at $(20,20)$

$\mu = 60 + 80 = 140$.



6.8 Advantages and Limitations of L.P.P.

(1) Advantages : Linear programming is used to minimize the cost of production for maximum output. In short, with the help of linear programming models, a decision maker can most efficiently and effectively employ his production factor and limited resources to get maximum profit at minimum cost.

(2) Limitations : (i) The linear programming can be applied only when the objective function and all the constraints can be expressed in terms of linear equations/inequations.

(ii) Linear programming techniques provide solutions only when all the elements related to a problem can be quantified.

(iii) The coefficients in the objective function and in the constraints must be known with certainty and should remain unchanged during the period of study.

(iv) Linear programming technique may give fractional valued answer which is not desirable in some problems.

Assignment

Fundamental concepts, Simultaneous linear inequations

Basic Level

1. The solution set of the inequation $2x + y > 5$, is

(a) Half plane that contains the origin
(b) Open half plane not containing the origin
(c) Whole *xy*–plane except the points lying on the line $2x + y = 5$
(d) None of these

2. Inequation $y - x \leq 0$ represents

(a) The half plane that contains the positive *x*-axis
(b) Closed half plane above the line $y = x$ which contains positive *y*-axis
(c) Half plane that contains the negative *x*-axis
(d) None of these

3. If a point (*h, k*) satisfies an inequation $ax + by \geq 4$, then the half plane represented by the inequation is

(a) The half plane containing the point (*h, k*) but excluding the points on $ax + by = 4$
(b) The half plane containing the point (*h, k*) and the points on $ax + by = 4$
(c) Whole *xy*-plane
(d) None of these

4. If the constraints in a linear programming problem are changed

(a) The problem is to be re-evaluated
(b) Solution is not defined
(c) The objective function has to be modified
(d) The change in constraints is ignored.

5. The optimal value of the objective function is attained at the points

(a) Given by intersection of inequations with the axes only
(b) Given by intersection of inequation with *x*-axis only
(c) Given by corner points of the feasible region
(d) None of these

6. Let X_1 and X_2 are optimal solutions of a *LPP*, then

(a) $X = \lambda X_1 + (1-\lambda)X_2, \lambda \in R$ is also an optimal solution
(b) $X = \lambda X_1 + (1-\lambda)X_2, 0 \leq \lambda \leq 1$ gives an optimal solution
(c) $X = \lambda X_1 + (1+\lambda)X_2, 0 \leq \lambda \leq 1$ gives an optimal solution
(d) $X = \lambda X_1 + (1+\lambda)X_2, \lambda \in R$ gives an optimal solution

7. The position f points *O* (0, 0) and *P* (2, –2) in the region of graph of inequations $2x - 3y < 5$, will be

(a) *O* inside and *P* outside
(b) *O* and *P* both inside
(c) *O* and *P* both outside
(d) *O* outside and *P* inside

8. The solution set of constraints $x + 2y \geq 11, 3x + 4y \leq 30, 2x + 5y \leq 30, x \geq 0, y \geq 0$ includes the point **[MP PET 1993]**

(a) (2, 3)
(b) (1, 1)
(c) (3, 4)
(d) (4, 3)

9. The solution set of linear constraints $x - 2y \geq 0, 2x - y \leq -2$ and $x, y \geq 0$, is

(a) $\left(-\frac{4}{3}, -\frac{2}{3}\right)$
(b) (1, 1)
(c) $\left(0, \frac{2}{3}\right)$
(d) (0, 2)

10. For the constraints of a *L.P.* problem given by $x_1 + 2x_2 \leq 2000, x_1 + x_2 \leq 1500$, $x_2 \leq 600$ and $x_1, x_2 \geq 0$, which one of the following points does not lie in the positive bounded region

(a) (1000, 0)
(b) (0, 500)
(c) (2, 0)
(d) (2000, 0)

11. The graph of $x \leq 2$ and $y \geq 2$ will be situated in the

(a) First and second quadrant
(b) Second and third quadrant
(c) First and third quadrant
(d) Third and fourth quadrant

12. The true statements for the graph of inequations $3x + 2y \leq 6$ and $6x + 4y \geq 20$, is

(a) Both graphs are disjoint
(b) Both do not contain origin
(c) Both contain point (1, 1)
(d) None of these

13. In which quadrant, the bounded region for inequations $x+y\le 1$ and $x-y\le 1$ is situated

(a) I, II (b) I, III (c) II, III (d) All the four quadrants

14. The region represented by the inequation system $x, y\ge 0, y\le 6, x+y\le 3$, is

(a) Unbounded in first quadrant
(b) Unbounded in first and second quadrants
(c) Bounded in first quadrant
(d) None of these

15. If the number of available constraints is 3 and the number of parameters to be optimized is 4, then

(a) The objective function can be optimized (b) The constraints are short in number
(c) The solution is problem oriented (d) None of these

16. The intermediate solutions of constraints must be checked by substituting them back into

(a) Object function (b) Constraint equations (c) Not required (d) None of these

17. A basic solution is called non-degenerate, if

(a) All these basic variables are zero (b) None of the basic variables is zero
(c) At least one of the basic variable is zero (d) None of these

18. Objective function of a *L.P.P.* is

(a) A constraint (b) A function to be optimized
(c) A relation between the variables(d) None of these

19. "*The maximum or the minimum of the objecitve function occurs only at the corner points of the feasible region*". This theorem is known as Fundamental Theorem of

(a) Algebra (b) Arithmetic (c) Calculus (d) Extreme points

20. Which of the terms is not used in a linear programming problem **[MP PET 2000]**

(a) Slack variable (b) Objective function (c) Concave region (d) Feasible region

21. Which of the following is not true for linear programming problems **[Kurukshetra CEE 1998]**

(a) A slack variable is a variable added to the left hand side of a less than or equal to constraint to convert it into an equality
(b) A surplus variable is a variable subtracted from the left hand side of a greater than or equal to constraint to convert it into an equality
(c) A basic solution which is also in the feasible region is called a basic feasible solution
(d) A column in the simplex tableau that contains all of the variables in the solution is called pivot or key column

22. The value of objective function is maximum under linear constraints

(a) At the centre of feasible region (b) At (0, 0)
(c) At any vertex of feasible region (d) The vertex which is at maximum distance from (0, 0)

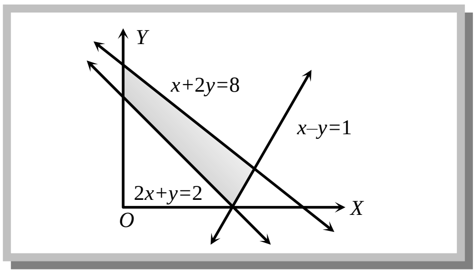
23. Which of the following sets are not convex

(a) $\{(x,y)|\ 3\le x^2+y^2\le 5\}$ (b) $\{(x,y)|\ 3x^2+2y^2\le 6\}$ (c) $\{(x,y)|\ y^2\le x\}$ (d) $\{(x,y|\ x\ge 2, x\le 3\}$

24. Which of the following sets are convex

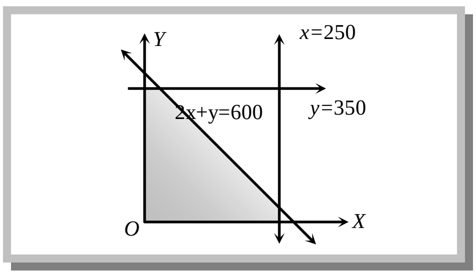
(a) $\{(x,y)|\ x^2+y^2\ge 1\}$ (b) $\{(x,y)|\ y^2\ge x\}$ (c) $\{(x,y)|\ 3x^2+4y^2\ge 5\}$ (d) $\{(x,y)|\ y\ge 2, y\le 4\}$

25. For the following shaded area, the linear constraints except $x\ge 0$ and $y\ge 0$, are



(a) $2x+y\le 2, x-y\le 1, x+2y\le 8$ (b) $2x+y\ge 2, x-y\le 1, x+2y\le 8$
(c) $2x+y\ge 2, x-y\ge 1, x+2y\le 8$ (d) $2x+y\ge 2, x-y\ge 1, x+2y\ge 8$

26. For the following feasible region, the linear constraints except $x\ge 0$ and $y\ge 0$, are



(a) $x \geq 250, y \leq 350, 2x + y = 600$
(b) $x \leq 250, y \leq 350, 2x + y = 600$
(c) $x \leq 250, y \leq 350, 2x + y \geq 600$
(d) $x \leq 250, y \leq 350, 2x + y \leq 600$

27. Which of the following is not a vertex of the positive region bounded by the inequalities $2x + 3y \leq 6, 5x + 3y \leq 15$ and $x, y \geq 0$

(a) (0, 2) (b) (0, 0) (c) (3, 0) (d) None of these

28. The vertex of common graph of inequalities $2x + y \geq 2$ and $x - y \leq 3$, is

(a) (0, 0) (b) $\left(\frac{5}{3}, -\frac{4}{3}\right)$ (c) $\left(\frac{5}{3}, \frac{4}{3}\right)$ (d) $\left(-\frac{4}{3}, \frac{5}{3}\right)$

29. A vertex of bounded region of inequalities $x \geq 0, x + 2y \geq 0$ and $2x + y \leq 4$, is

(a) (1, 1) (b) (0, 1) (c) (3, 0) (d) (0, 0)

30. A vertex of the linear inequalities $2x + 3y \leq 6, x + 4y \leq 4$ and $x, y \geq 0$, is

(a) (1, 0) (b) (1, 1) (c) $\left(\frac{12}{5}, \frac{2}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{12}{5}\right)$

31. Consider the inequalities $x_1 + x_2 \leq 3, 2x_1 + 5x_2 \geq 10, x_1, x_2 \geq 0$, which of the following points lies in the feasible region **[MP PET 2003]**

(a) (2, 2) (b) (1, 2) (c) (2, 1) (d) (4, 2)

32. The region represented by the inequalities $x \geq 6, y \geq 2, 2x + y \leq 10, x \geq 0, y \geq 0$ is

(a) Unbounded (b) A polygon (c) Exterior of a triangle (d) None of these

Formulation of Linear Programming Problem

Basic Level

33. A whole sale merchant wants to start the business of cereal with Rs. 24,000. Wheat is Rs. 400 per quintal and rice is Rs. 600 per quintal. He has capacity to store 200 quintal cereal. He earns the profit Rs. 25 per quintal on wheat and Rs. 40 per quintal on rice. If he store *x* quintal rice and *y* quintal wheat, then for maximum profit the objective function is

(a) $25x + 40y$ (b) $40x + 25y$ (c) $400x + 600y$ (d) $\frac{400}{40}x + \frac{600}{25}y$

34. A firm produces two types of product *A* and *B*. The profit on both is Rs. 2 per item. Every product need processing on machines M_1 and M_2. For A, machines M_1 and M_2 takes 1 minute and 2 minute respectively and that of for B, machines M_1 and M_2 takes the time 1 minute and 1 minute. The machines M_1, M_2 are not available more than 8 hours and 10 hours any of day respectively. If the products made *x* of A and *y* of B, then the linear constraints for the *L.P.P.* except $x \geq 0, y \geq 0$, are

(a) $x + y \leq 480, 2x + y \leq 600$ (b) $x + y \leq 8, 2x + y \leq 10$ (c) $x + y \geq 480, 2x + y \geq 600$ (d) $x + y \leq 8, 2x + y \geq 10$

35. In a test of Mathematics, there are two types of questions to be answered, short answered and long answered. The relevant data are given below

	Time taken to solve	Marks	Number of questions
Short answered questions	5 minutes	3	10
Long answered questions	10 minutes	5	14

The total marks are 100. Student can solve all the questions. To secure maximum marks, student solve *x* short answered and *y* long answered questions in three hours, them the linear constraints except $x \geq 0, y \geq 0$, are

(a) $5x + 10y \leq 180, x \leq 10, y \leq 14$
(b) $x + 10y \geq 180, x \leq 10, y \leq 14$
(c) $5x + 10y \geq 180, x \geq 10, y \geq 14$
(d) $5x + 10y \leq 180, x \geq 10, y \geq 14$

36. A company manufactures two types of telephone sets *A* and *B*. The *A* type telephone set requires 2 hour and *B* type telephone requires 4 hour to make. The company has 800 work hour per day. 300 telephone can pack in a day. The selling prices of *A* and *B* type telephones are Rs. 300 and 400 respectively. For maximum profit company produces *x* telephones of *A* type and *y* telephones of B type. Then except $x \geq 0$ and $y \geq 0$, linear constraints are

(a) $x + 2y \leq 400; x + y \leq 300$
(b) $2x + y \leq 400; x + y \geq 300$

Max $z = 300x + 400y$ Max $z = 400x + 300y$

(c) $2x + y \geq 400; x + y \geq 300$ (d) $x + 2y \leq 400; x + y \geq 300$

Max $z = 300x + 400y$ Max $z = 300x + 400y$

37. In a factory which produces two products *A* and *B*, in manufacturing product *A*, the machine and the carpenter requires 3 hours each and in manufacturing product B, the machine and carpenter requires 5 hour and 3 hour respectively. The machine and carpenter works at most 80 hour and 50 hour per week respectively. The profit on *A* and *B* are Rs. 6 and Rs. 8 respectively. If profit is maximum by manufacturing *x* and *y* units of *A* and *B* type products respectively, then for the function $6x + 8y$, the constraints are

(a) $x \geq 0, y \geq 0, 5x + 3y \leq 80, 3x + 2y \leq 50$

(b) $x \geq 0, y \geq 0, 3x + 5y \leq 80, 3x + 3y \leq 50$

(c) $x \geq 0, y \geq 0, 3x + 5y \geq 80, 2x + 3y \geq 50$

(d) $x \geq 0, y \geq 0, 5x + 3y \geq 80, 3x + 2y \geq 50$

38. The sum of two positive integers is at most 5. The difference between two times of second number and first number is at most 4. If the first number is *x* and second number *y*, then for maximizing the product of these two numbers, the mathematical formulation is

(a) $x + y \geq 5, 2y - x \geq 4, x \geq 0, y \geq 0$

(b) $x + y \geq 5, -2x + y \geq 4, x \geq 0, y \geq 0$

(c) $x + y \leq 5, 2y - x \leq 4, x \geq 0, y \geq 0$

(d) None of these

Advance Level

39. Mohan wants to invest the total amount of Rs. 15,000 in saving certificates and national saving bonds. According to rules, he has to invest at least Rs. 2000 in saving certificates and Rs. 2500 in national saving bonds. The interest rate is 8% on saving certificate and 10% on national saving bonds per annum. He invests Rs. *x* in saving certificates and Rs. *y* in national saving bonds. Then the objective function for this problem is

(a) $0.08x + 0.10y$ (b) $\frac{x}{2000} + \frac{y}{2500}$ (c) $2000x + 2500y$ (d) $\frac{x}{8} + \frac{y}{10}$

40. Two tailors *A* and *B* earn Rs. 15 and Rs. 20 per day respectively *A* can make 6 shirts and 4 pants in a day while *B* can make10 shirts and 3 pants. To spend minimum on 60 shirts and 40 pants, *A* and *B* work *x* and *y* days respectively. Then linear constraints except $x \geq 0, y \geq 0$, are

(a) $15x + 20y \geq 0, 60x + 40y \geq 0$

(b) $15x + 20y \geq 0, 6x + 10y = 10$

(c) $6x + 10y \geq 60, 4x + 3y \geq 40$

(d) $6x + 10y \leq 60, 4x + 3y \leq 40$

41. In the examination of P.E.T. the total marks of mathematics are 300. If the answer is right, marks provided is 3 and if the answer is wrong, marks provided is –1. A student knows the correct answer of 67 questions and remaining questions are doubtful for him. He takes the time $1\frac{1}{2}$ minute to give the correct answer and 3 minute that for doubtful. Total time is 3 hour. In the question paper after every two simple questions, one question is doubtful. He solves the questions one by one, then the number of questions solved by him, is

(a) 67   (b) 90   (c) 79   (d) 80

42. A shopkeeper wants to purchase two articles *A* and *B* of cost price Rs. 4 and Rs. 3 respectively. He thought that he may earn 30 paise by selling article *A* and 10 paise by selling article *B* He has not to purchase total articles of more than Rs. 24. If he purchases the number of articles of *A* and *B*, *x* and *y* respectively, then linear constraints are

(a) $x \geq 0, y \geq 0, 4x + 3y \leq 24$ (b) $x \geq 0, y \geq 0, 30x + 10y \leq 24$ (c) $x \geq 0, y \geq 0, 4x + 3y \geq 24$ (d) $x \geq 0, y \geq 0, 30x + 40y \geq 24$

43. A company manufacturers two types of products *A* and *B*. The storage capacity of its godown is 100 units. Total investment amount is Rs. 30,000. The cost prices of *A* and *B* are Rs. 400 and Rs. 900 respectively. All the products are sold and per unit profit is Rs. 100 and Rs. 120 through *A* and *B* respectively. If *x* units of *A* and *y* units of *B* be produced, then two linear constraints and iso-profit line are respectively

(a) $x + y = 100; 4x + 9y = 300, 100x + 120y = c$

(b) $x + y \leq 100; 4x + 9y \leq 300, x + 2y = c$

(c) $x + y \leq 100; 4x + 9y \leq 300, 100x + 120y = c$

(d) $x + y \geq 100; 9x + 4y \geq 300, 5x + 6y = c$

44. We have to purchase two articles *A* and *B* of cost Rs. 45 and Rs. 25 respectively. I can purchase total article maximum of Rs. 1000. After selling the articles *A* and *B*, the profit per unit is Rs. 5 and 3 respectively. If I purchase *x* and *y* numbers of articles *A* and *B* respectively, then the mathematical formulation of problem is

(a) $x \geq 0, y \geq 0, 45x + 25y \geq 1000, 5x + 3y = c$

(b) $x \geq 0, y \geq 0, 45x + 25y \leq 1000, 5x + 3y = c$

(c) $x \geq 0, y \geq 0, 45x + 25y \leq 1000, 3x + 5y = c$

(d) None of these

Graphical method of solution of Linear programming problems

Basic Level

45. The L.P. problem *Max* $z = x_1 + x_2$, such that $-2x_1 + x_2 \le 1, x_1 \le 2, x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$ has

(a) One solution (b) Three solution

(c) An infinite number of solutions (d) None of these

46. On maximizing $z = 4x + 9y$ subject to $x + 5y \le 200, 2x + 3y \le 134$ and $x, y \ge 0$, $z =$

(a) 380 (b) 382 (c) 384 (d) None of these

47. The point at which the maximum value of $(3x + 2y)$ subject to the constraints $x + y \le 2, x \ge 0, y \ge 0$ is obtained, is **[MP PET 1993]**

(a) (0, 0) (b) (1.5, 1.5) (c) (2, 0) (d) (0, 2)

48. The solution of a problem to maximize the objective function $z = x + 2y$ under the constraints $x - y \le 2, x + y \le 4$ and $x, y \ge 0$, is

(a) $x = 0, y = 4, z = 8$ (b) $x = 1, y = 2, z = 5$ (c) $x = 1, y = 4, z = 9$ (d) $x = 0, y = 3, z = 6$

49. The maximum value of $P = 6x + 8y$ subject to constraints $2x + y \le 30, x + 2y \le 24$ and $x \ge 0, y \ge 0$ is **[MP PET 1994,95]**

(a) 90 (b) 120 (c) 96 (d) 240

50. The maximum value of $P = x + 3y$ such that $2x + y \le 20, x + 2y \le 20, x \ge 0, y \ge 0$, is **[MP PET 1995]**

(a) 10 (b) 60 (c) 30 (d) None of these

51. The point at which the maximum value of $x + y$, subject to the constraints $x + 2y \le 70, 2x + y \le 95, x, y \ge 0$ is obtained, is

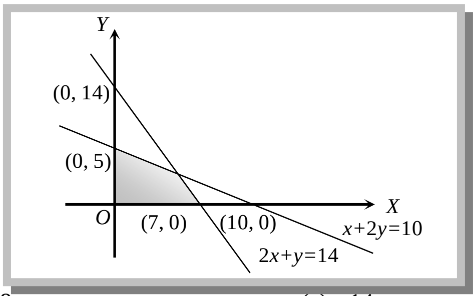
(a) (30, 25) (b) (20, 35) (c) (35, 20) (d) (40, 15)

52. If $3x_1 + 5x_2 \le 15$; $5x_1 + 2x_2 \le 10$; $x_1, x_2 \ge 0$

then the maximum value of $5x_1 + 3x_2$ by graphical method is

(a) $12\frac{7}{19}$ (b) $12\frac{1}{7}$ (c) $12\frac{3}{5}$ (d) 12

53. The maximum value of objective function $c = 2x + 3y$ in the given feasible region, is



(a) 29 (b) 18 (c) 14 (d) 15

54. The maximum value of the objective function $P = 5x + 3y$, subject to the constraints $x \ge 0, y \ge 0$ and $5x + 2y \le 10$ is **[AMU 1990, 92]**

(a) 6 (b) 10 (c) 15 (d) 25

55. The maximum value of $P = 8x + 3y$, subject to the constraints $x + y \le 3, 4x + y \le 6, x \ge 0, y \ge 0$ is **[AMU 1988]**

(a) 9 (b) 12 (c) 14 (d) 16

56. The maximum value of $P = 6x + 11y$ subject to the constraints

$2x + y \le 104$

$x + 2y \le 76$ and $x \ge 0, y \ge 0$ is

(a) 240 (b) 540 (c) 440 (d) None of these

57. For the L.P. problem, Min $z = -x_1 + 2x_2$, such that $-x_1 + 3x_2 \le 0, x_1 + x_2 \le 6, x_1 - x_2 \le 2$ and $x_1, x_2 \ge 0$, then $x_1 =$

(a) 2 (b) 8 (c) 10 (d) 12

58. For the L.P. problem Min $z = 2x_1 + 3x_2$, such that $-x_1 + 2x_2 \le 4, x_1 + x_2 \le 6, x_1 + 3x_2 \ge 9$ and $x_1, x_2 \ge 0$

(a) $x_1 = 1.2$ (b) $x_2 = 2.6$ (c) $z = 10.2$ (d) All of these

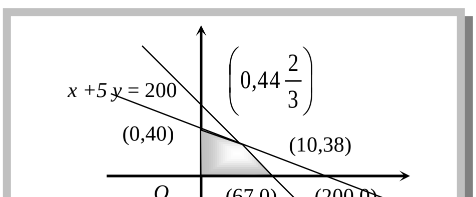
59. For the L.P. problem Min $z = 2x + y$ subject to $5x + 10y \le 50, x + y \ge 1, y \le 4$ and $x, y \ge 0$, $z =$

(a) 0 (b) 1 (c) 2 (d) 1/2

60. For the L.P. problem Min. $z = 2x - 10y$ subject to $x - y \ge 0, x - 5y \ge -5$ and $x, y \ge 0$, $z =$

(a) – 10 (b) – 20 (c) 0 (d) 10

61. The maximum value of objective function $c = 2x + 2y$ in the given feasible region, is



(a) 134 (b) 40 (c) 38 (d) 80

62. The Minimum value of $P = x + 3y$ subject to constraints $2x + y \le 20, x + 2y \le 20, x \ge 0, y \ge 0$ is

(a) 10 (b) 60 (c) 30 (d) None of these

63. Min. $Z = -x_1 + 2x_2$ **[EAMCET 1995]**

Subjected to $x_1 + 3x_2 \le 10$,

$x_1 + x_2 \le 6$, $x_1 - x_2 \le 2$ and $x_1, x_2 \ge 0$ is :

(a) – 4 (b) – 2 (c) 2 (d) None of these

Advance Level

64. To maximize the objective function $z = 2x + 3y$ under the constraints $x + y \le 30, x - y \ge 0, y \le 12, x \le 20, y \ge 3$ and $x, y \ge 0$, is at

(a) $x = 12, y = 18$ (b) $x = 18, y = 12$ (c) $x = 12, y = 12$ (d) $x = 20, y = 10$

65. The point at which the maximum value of $x + y$ subject to the constraints $2x + 5y \le 100, \frac{x}{25} + \frac{y}{49} \le 1, x, y \ge 0$ is obtained, is

(a) (10, 20) (b) (20, 10) (c) (15, 15) (d) $\left(\frac{50}{3}, \frac{40}{3}\right)$

66. The maximum value of $Z = 4x + 3y$ subject to the constraints $3x + 2y \ge 160, 5x + 2y \ge 200, x + 2y \ge 80; x, y \ge 0$ is **[MP PET 1998]**

(a) 320 (b) 300 (c) 230 (d) None of these

67. By graphical method, the solution of linear programming problem **[MP PET 1996]**

maximize $z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \le 18$,

$x_1 \le 4$,

$x_2 \le 6$

$x_1 \ge 0, x_2 \ge 0$ is

(a) $x_1 = 2, x_2 = 0, z = 6$ (b) $x_1 = 2, x_2 = 6, z = 36$ (c) $x_1 = 4, x_2 = 3, z = 27$ (d) $x_1 = 4, x_2 = 6, z = 42$

68. For the L.P. problem *Max* $z = 3x_1 + 2x_2$, such that $2x_1 - x_2 \ge 2, x_1 + 2x_2 \le 8$ and $x_1, x_2 \ge 0$, $z =$

(a) 12 (b) 24 (c) 36 (d) 40

69. The maximum value of $P = 2x + 5y$ subject to the constraints **[AMU 1994]**

$x + 4y \le 24$, $3x + y \le 21$, $x + y \le 9$ and $x \ge 0, y \ge 0$ is

(a) 33 (b) 35 (c) 20 (d) 105

70. The maximum value of $P = 5x + 7y$ subject to the constraints

$x + y \le 4$, $3x + 8y \le 24$, $10x + 7y \le 35$ and $x \ge 0, y \ge 0$ is

(a) 14.8 (b) 24.8 (c) 34.8 (d) None of these

71. The point which provides the solution to the linear programming problem , Max. $(2x + 3y)$, subject to constraints : **[MP PET 2000]**

$x \ge 0, y \ge 0, \quad 2x + 2y \le 9, \quad 2x + y \le 7, \quad x + 2y \le 8$ is

(a) (3, 2.5) (b) (2, 3.5) (c) (2, 2.5) (d) (1, 3.5)

72. For maximum value of $Z = 5x + 2y$, subject to the constraints

$2x + 3y \ge 6$, $x - 2y \le 2$, $6x + 4y \le 24$, $-3x + 2y \le 3$ and $x \ge 0, y \ge 0$ the values of x and y are

(a) 18/7, 2/7 (b) 7/2, 3/4 (c) 3/2, 15/4 (d) None of these

73. For the following linear programming problem : **[MP PET 2003]**

'Minimize $Z = 4x + 6y$ ', subject to the constraints $2x + 3y \geq 6$, $x + y \leq 8$, $y \geq 1, x \geq 0$, the solution is

(a) (0, 2) and (1, 1) (b) (0, 2) and $\left(\frac{3}{2}, 1\right)$ (c) (0, 2) and (1, 6) (d) (0, 2) and (1, 5)

74. The minimum value of $Z = 2x_1 + 3x_2$ subject to the constraints $2x_1 + 7x_2 \geq 22$, $x_1 + x_2 \geq 6$, $5x_1 + x_2 \geq 10$ and $x_1, x_2 \geq 0$ is **[MP PET 2003]**

(a) 14 (b) 20 (c) 10 (d) 16

75. For the L.P. problem Min. $z = x_1 + x_2$, such that $5x_1 + 10x_2 \leq 0, x_1 + x_2 \geq 1, x_2 \leq 4$ and $x_1, x_2 \geq 0$

(a) There is a bounded solution (b) There is no solution

(c) There are infinite solution s (d) None of these

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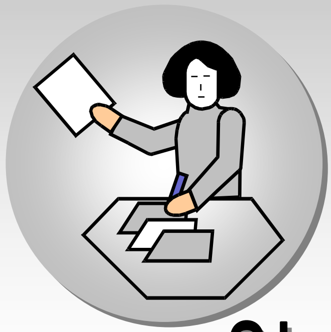

Answer Sheet

Linear Programming **_Assignment (Basic and Advance Level)_**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	a	b	a	c	b	a	c	a	d	a	a	d	c	b	b	b	b	d	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	d	a	d	b	d	d	b	d	c	b	a	b	a	a	a	b	c	a	d
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	a	c	b	c	b	c	a	b	c	d	a	b	c	c	c	a	d	b	a
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**					
a	d	b	b	d	d	b	b	a	b	d	b	b	a	c					