8. Height and Distance

Miscellaneous Exercise 8

1. Question

The shadow of a vertical pillar is equal to the height of the pillar, then the angle of elevation of the Sun will be

A. 45°

B. 30°

C. 60°

D. 50°

Answer



Let AB is a pillar, AC is the shadow of the pillar, and angle of elevation of the pillar from point C is $\angle ACB = \theta$.

Given, AC = AB

The side with angle of elevation is called base and side opposite to angle of elevation is called height of triangle.

We know that, $\tan \theta = \frac{\text{perpendicular}}{\text{Base}}$

 $\tan \theta = \frac{AB}{AC}$ $\tan \theta = 1 \text{ (Given: AC = AB)}$ $\theta = 45^{\circ}$

2. Question

From a distance of 100 m from the foot point of a tower the angle of elevation of its top is 60°. Then height of the tower is

A.
$$100\sqrt{3}$$
 m
B. $\frac{100}{\sqrt{3}}$ m
C. $50\sqrt{3}$ m
D. $\frac{200}{\sqrt{3}}$ m

Answer



Let AB is a Tower, AC is the distance from A to point C, and angle of elevation of the Tower from point C is $\angle ACB = 60^{\circ}$. AB = h.

We know that, $\tan \theta = \frac{\text{perpendicular}}{\text{Base}} \tan 60 = \frac{h}{100}$ $\sqrt{3} = \frac{h}{100}$

 $h = 100\sqrt{3}$

3. Question

A ladder 15 m long reaches the top of a vertical wall. If this ladder makes an angle of 60° with the wall, then height of the wall is

A.
$$15\sqrt{3}$$
 m
B. $\frac{15\sqrt{3}}{2}$ m

C.
$$\frac{15}{2}$$
 m

D. 15 m

Answer



Let AC is a Ladder. Length of the ladder is 15m. The angle of elevation is 60° at point A.

The height of the wall is BC = h.

We know that, $\cos \theta = \frac{Base}{hypetenuse}$ $\cos \theta = \frac{AB}{AC}$ $\cos 60^\circ = \frac{b}{15}$ $\frac{1}{2} = \frac{b}{15}$ $b = \frac{15}{2}m$ 4. Question

From the top of a 100 m high tower the angle of depression of a point on the earth is 30° . The distance of the point from the base of the tower is

A.
$$10\sqrt{3}$$
 m
B. $\frac{10}{\sqrt{3}}$ m

C. 10 m

D. 5√3 m

Answer



Let BC is Height of the Tower. Height of the Tower is 100m. Angle of Depression is 30° from point C. The Length of the Base is BC = b.

We know that $\angle BAC = 30^{\circ}$ (interior angles)



5. Question

A bridge over a river makes an angle of 45° with the bank of the river. If the length of the bridge over the river is 150 m then the width of the river will be

A. 75 m

B. $50\sqrt{2}$ m

C. 150 m

D. $75\sqrt{2}$ m



Let AC is a width of the river, Angle of elevation from point B is **45**°. AB is a length of the bridge.

We know that, $\sin \theta = \frac{\text{perpendicular}}{\text{hypetenuse}}$ $\sin \theta = \frac{h}{150}$ $\frac{1}{\sqrt{2}} = \frac{h}{150}$ $h = \frac{150}{\sqrt{2}}$ $h = \frac{150}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ (Rationalization) $h = \frac{150\sqrt{2}}{2}$ $h = 75\sqrt{2}$

6. Question

The tops of two pillars whose heights are 20 m and 14 m are joined by a wire. If the wire makes an angle of 30° with the horizontal line, then the length of the wire is

A. 12 m B. 10 m C. 8 m D. 6 m



Let AD is Pillar 1 and BC is Pillar 2. Angle of depression is 30^o from D top of the Pillar1.

Wire length is DC = L.

AE = BC (ABCE is a rectangle and rectangle's opposite sides are equal)

 $\angle DCE = \angle 30^{\circ}$ $\sin \theta = \frac{DE}{DC}$ $\sin \theta = \frac{20 - 14}{L}$ $\sin \theta = \frac{6}{L}$ $L = \frac{6}{\sin 30}$ $L = \frac{L}{2}$ $L = 2 \times 6$ L = 12m

7. Question

The angles of elevation of the top of a tower from two points at a distance of a m and b m (a > b) from the base of the tower and in the same straight line with it are respectively 30° and 60° . Then height of the tower is

A.
$$\sqrt{a+b}$$

B. $\sqrt{a-b}$



Answer



Let height of the tower AC = g.

Given, AD = b, AB = a. $\angle ABC = 30^{\circ} \angle ADC = 60$

We know that, $\tan \theta$

in ΔABC ,

Multiplying Equation (1) and (2)

 $g^2 = ab$ $g = \sqrt{(ab)}$

8. Question

From the top of a 25m high pillar the angle of elevation of the top of a tower and the angle of depression of the foot of the tower are equal then the height of the tower is

A. 25 m

B. 100 m

C. 75 m

D. 50 m

Answer



Let AD is a Pillar, BC is a Tower, Angle of depression of the base is b^0 , Angle of elevation is a^0 .

Given, a = b in \triangle CDE, tan a = $\frac{CE}{DE}$ (1) \angle DBA = b (interior angle) in \triangle DBE, tan b = $\frac{h}{AB}$ (2) In rectangle, ABED, h = EB, AB = DE $\frac{h}{AB} = \frac{CE}{DE}$ h = CE

Height of The Tower = CE + BE

= 50

9. Question

The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$. Then the angle of elevation of the Sun is

A. 30°

B. 45°

C. 60°

D. 90°

Answer



Let Length of the rod = g, length of this shadow = f, g:f = $1:\sqrt{3}$

in <mark>∆</mark>ABC,

 $\tan \theta = \frac{g}{f}$ $\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = 30^{\circ}$

10. Question

The slope of a hill makes an angle of 60° with the horizontal of 500 m has to be walked to reach the top, then the height of the hill is

A.
$$500\sqrt{3}$$
 m
B. $\frac{500}{\sqrt{3}}$ m

D.
$$\frac{250}{\sqrt{3}}$$
 m

Answer



Let Slope of the hill is h, Height of the Hill is **g**. Angle of elevation is $\angle BAC = 60^{\circ}$.

Given, AB = 500m $\sin \theta = \frac{BC}{AC}$ $\sin 60 = \frac{BC}{AC}$ $\frac{\sqrt{3}}{2} = \frac{BC}{500}$ $BC = \frac{500 \times \sqrt{3}}{2}$ $BC = 250\sqrt{3}$

11. Question

A tower is standing vertically on a horizontal plane. If the angle of elevation of the Sun is 30° and the length of the shadow of the tower is 45 m then find the height of the tower.



Let Height of the tower is BC = g, Angle of elevation is 30° . Length of shadow of tower = AB.

Given AB = 45m

 $\tan \theta = \frac{BC}{AB}$ $\tan \theta = \frac{g}{45}$ $\tan 30 = \frac{g}{45}$ $g = \frac{45\sqrt{3}}{3} = 15\sqrt{3}$

12. Question

A tree breaks due to storm and the broken upper part of the tree makes an angle of 60° with the horizontal plane. The top of the tree meets the horizontal plane at a distance of 10 m from the root of the tree. Find the height of the tree before breaking. ($\sqrt{3} = 1.732$)



Let AB is the Height of the Tree before broken the upper part, DE is the Height of the tree after broken.

Given, $\angle DCE = 60^{\circ}$, DC = 10m $\tan \theta = \frac{ED}{CD}$ $\sqrt{3} = \frac{ED}{10}$ $ED = 10 \times 1.732$ ED = 17.32 $\sin \theta = \frac{ED}{EC}$ $\sin 60 = \frac{17.32}{EC}$ $EC = \frac{17.32}{2}$ EC = 2

Total Height of the tree = EC + ED = 17.32 + 20

= 37.32

13. Question

From a point at a distance of 120 m from the base of an incomplete tower the angle of elevation of the top of the tower is 30° . Find how much more height the tower be made so that its angle of elevation at the same point becomes 60° ?



Let total height is g, Incomplete Tower Height is DB.

Given Angle of Elevation of the top is 60° , Angle of Elevation of the Incomplete Tower is 30° . AB = 120.

in <mark>∆</mark>ABD,



$$\sqrt{3} = \frac{BC}{120}$$
$$BC = 120\sqrt{3}$$

Increase in Height To change the angle of Elevation = $120\sqrt{3} - 40\sqrt{3} = 80\sqrt{3}$

14. Question

From a point situated at a distance of 100 m from the base of a tower. The angle of elevation of the top is 30°. Then find the height of the tower.

Answer



Let Height of the tower g. Angle of elevation is 30° .

Given, $\angle BAC = 30^{\circ}$, AB = 100.

$$\tan \theta = \frac{BC}{AB}$$
$$\tan 30 = \frac{BC}{AB}$$
$$\frac{1}{\sqrt{3}} = \frac{g}{100}$$
$$g = \frac{100}{\sqrt{3}}$$
$$g = \frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$g = \frac{100 \times \sqrt{3}}{3}$$
$$g = 57.733$$
15. Question

The angle of elevation of the top of a pillar from a point situated on a plane in 15°. On walking 100 m towards the pillar the angle of elevation becomes 30°. Then find the height of the pillar. (where, tan $15^\circ = 2\sqrt{3}$)

Answer



Let Height of the tower is BC = g.

Given,
$$\angle CAB = 15^{\circ} \angle CDB = 30^{\circ} AD = 100$$
.

in ∆CAB,

$\tan \theta = \frac{BC}{AB}$
$\tan 15 = \frac{g}{AB}$
$AB = \frac{g}{\tan 15} \dots \dots$
in ΔCDB,
$\tan \theta = \frac{BC}{BD}$
$\tan 30 = \frac{BC}{BD}$
$BD = \frac{BC}{\tan 30} \dots (2)$
Equation (1) – Equation(2)
$AB - BD = \frac{g}{\tan 15} - \frac{g}{\tan 30}$
$100 = \frac{g}{2 - \sqrt{3}} - \frac{g}{\frac{1}{\sqrt{3}}}$
$100 = \frac{g}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} - g\sqrt{3}$

$$100 = \frac{g(2 + \sqrt{3})}{4 - 3} - g\sqrt{3}$$
$$100 = 2g + g\sqrt{3} - g\sqrt{3}$$
$$100 = 2g$$
$$g = 50$$

16. Question

The shadow of a tower standing on a plane ground becomes 40 m longer when the angle of elevation of the Sun becomes 30° after reduction from 60°. Find the height of the tower.

Answer



Let Height of the tower is BC = g.

Given, $\angle CAB = 30^{\circ} \angle CDB = 60^{\circ} AD = 40$.

in ∆CAB,





17. Question

On observing from the top of a lighthouse 60 m high from the sea level, the angles of depression of two ships are 30° and 45°. If one ship is just behind the other ship on the same side of the lighthouse then find the distance between the ships.

Answer



Let BC = g is the height of the lighthose.

Given, Angle of Depression for ship A is 45⁰, for ship B 30^o.Height of the lighthouse is 60.

in <mark>∆</mark>ABC,

$$\angle CAB = 45^{\circ}$$
$$\tan \theta = \frac{BC}{AB}$$
$$\tan 45 = \frac{BC}{AB}$$
$$AB = 60$$
$$in \triangle CDB,$$
$$\angle CDB = 30^{\circ}$$
$$\tan 30 = \frac{BC}{BD}$$
$$\frac{1}{\sqrt{3}} = \frac{60}{BD}$$
$$BD = 60\sqrt{3}$$

Distance between the ships = BD – AB

$$= 60\sqrt{3} - 60$$

$$= 60(\sqrt{3} - 1)$$

18. Question

A 1.5 m long boy is standing at a certain distance from a 30 m high building when he goes towards the high building then the angle of elevation of the top of the building from his eye becomes 60° from 30°. Find by how much distance he has walked towards the building.

Answer



Let Height of the boy is DE, Height of the Tower is GC.

Given DE = 1.5, GC = 30, \angle CDB = 30° \angle CAB = 60°. BC = 28.5, GC = 30.

in <mark>∆</mark>CDB,

$$\tan 30 = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{BD}$$

$$BD = 28.5\sqrt{3}$$
in CAB,
$$\tan 60 = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{BC}{AB}$$

$$AB = \frac{BC}{\sqrt{3}}$$

$$AB = \frac{28.5\sqrt{3}}{3}$$

Distance Travelled by boy = BD – AB = $28.5\sqrt{3}$ – $9.5\sqrt{3}$

= 19**√3**

19. Question

From the top of a 7m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45° . Find the height of the tower.

Answer



Let Height of the building is AD, Height of the tower is BC.

Given, $\angle CDE = 60^{\circ}$, $\angle EDB = 45^{\circ}$. AD = 7.

in <mark>∆</mark>BDA,

$$\angle DBA = 45^{\circ}$$

$$\tan 45 = \frac{AD}{AB}$$

$$AB = AD$$

$$AB = 7$$

$$in \triangle CDE,$$

$$\angle EDC = 60^{\circ}$$

$$\tan 60 = \frac{EC}{DE}$$

$$\sqrt{3} = \frac{EC}{7}$$

$$EC = 7\sqrt{3}$$

Height of the tower = $7\sqrt{3} + 7$

20. Question

From the peak of a mountain the angles of elevation of the peak from two points situated towards the east are 30° and 45°. If the distance between the points is 1 km then find the height of the mountain.

Answer



Let Height of the Tower is BC = g.

Given, $\angle CAB = 30^{\circ} \angle CDB = 45^{\circ} AD = 1 \text{km} = 1000 \text{m.}$

in ΔCAB ,

$$\tan \theta = \frac{BC}{AB}$$
$$\tan 30 = \frac{g}{AB}$$

$AB = \frac{g}{\tan 30} \dots \dots$
in ΔCDB,
$\tan \theta = \frac{BC}{BD}$
$\tan 60 = \frac{BC}{BD}$
$BD = \frac{BC}{\tan 45}$ (2)
Equation (1) – Equation(2)
$AB - BD = \frac{g}{\tan 30} - \frac{g}{\tan 45}$
$1000 = \frac{g}{\frac{1}{\sqrt{3}}} - \frac{g}{1}$
$1000 = g\sqrt{3} - g$
$1000 = g\sqrt{3} - g$
$1000 = g(\sqrt{3} - 1)$
$\frac{1000}{\sqrt{3}-1} = g$
$g = \frac{1000(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ (Rationalization)
$g = 500(\sqrt{3} + 1)$

21. Question

From a point A 20m high above the water level in a lake the angle of elevation of a cloud is 30° . If the angle of depression of the reflection of the cloud in the lake from point A is 60° then find the distance of the cloud from the point A.



Let Height of the Cloud is g from the surface of the lake and CE = g.

Given angle of elevation of the cloud $\angle CED = 30^{\circ}$, Angle of Depression of the reflection of the Cloud is $\angle EDF = 60^{\circ}$. AC = AF (reflections)(It doesn't seem in the image.)

in <mark>∆</mark>DEF,

 $\tan 60 = \frac{EF}{DE}$ $DE = \frac{EF\sqrt{3}}{3}$ $in \triangle CDE,$ $\tan 30 = \frac{CE}{DE}$ $DE = CE\sqrt{3}$ $(20 + g)\sqrt{3} = CE\sqrt{3}$ $\frac{(20 + g)\sqrt{3}}{3} = (g - 20)\sqrt{3}$ 20 + g = 3g - 60 80 = 2g g = 40

22. Question

From a point of the bridge of a river the angles of depression of the opposite banks of the river are 30° and 45° . If the bridge is at a height of 4m from the banks, then find the width of the river.

Answer



Let AB is the width of the river, Height between the river and the bridge is CF.

Given, \angle DCA = 30°, \angle ECB = 45°, CF = 4.

 $\angle DCA = \angle CAB = 30^{\circ}$

 $\angle ABC = \angle ECB = 45^{\circ}$

in ΔCAF

 $\tan 30 = \frac{CF}{AF}$ $AF = CF\sqrt{3} \dots \dots \dots (1)$ in \(\Delta BCF\), $\tan 45 = \frac{CF}{FB}$

FB = CF.....(2)

Width of the river = CF + CF $\sqrt{3}$

$$= 4(1 + \sqrt{3})$$

23. Question

A man is standing on the deck of a ship, which is 10 m above the water level. If he observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°, then find the distance of the hill from the ship and the height of the hill.



Let height of the deck is AB, Height of the Hill is CD.

Given, AB = 10, \angle CBE = 60⁰, \angle EBD = 30^o.

AB = DE (ABED is a rectangle)

in ∆BED,

 $\tan 30 = \frac{DE}{BE}$ $\frac{1}{\sqrt{3}} = \frac{10}{BE}$ $BE = 10\sqrt{3}$ $in \Delta CBE,$ $\tan 60 = \frac{CE}{BE}$ $CE = 10\sqrt{3} \times \sqrt{3}$ CE = 30

Total height of the tower = 40

24. Question

A 12 m high tree breaks due to wind such that its top touches the ground and makes an angle of 60° with the ground. Find at what height from the earth has the tree broken by the wind ($\sqrt{3} = 1.732$)?



Let AB is the Height of the tree, ED = h is the Height of the broken tree, CE is the Length of broken part.

Given, $\angle ECD = 60^{\circ}$, AB = 12.

 $\sin 60 = \frac{ED}{CE}$ $\frac{\sqrt{3}}{2} = \frac{ED}{CE}$ $CE\sqrt{3} = 2ED$ CE + DE = 12 $CE + \frac{CE\sqrt{3}}{2} = 12$ $\frac{2CE + CE\sqrt{3}}{2} = 12$ $2CE + CE\sqrt{3} = 24$ $CE = \frac{24}{2 + \sqrt{3}}$ $CE = 24(2 - \sqrt{3})$ $DE = 12 - 48 + 24\sqrt{3}$ $DE = 12(-3 + 2\sqrt{3})$

25. Question

A highway passes through the foot of a tower. A man observes a car at an angle of depression of 30° from the top of the tower. That car is approaching the foot of the tower. Six seconds later, the angle of depression of the car becomes 60°. In what time will the car pass through the foot of the tower?

Answer



Let The speed of the car is constant and it is v, AB is the Height of the tower is AB.

Given, \angle BCA = 30°, \angle BDA = 30°,

In ΔBCA,

 $\tan 30 = \frac{AB}{AC}$ $AB = \frac{AC\sqrt{3}}{3}$ $In \Delta BDA,$ $\tan 30 = \frac{AB}{AD}$ $AB = AD\sqrt{3}$ $AB = AD\sqrt{3}$ (2)Equation(1) = Equation(2) $AD\sqrt{3} = \frac{AC\sqrt{3}}{3}$ 3AD = (CD + AD) 2AD = CD 2vt = v6 t = 3s26. Question

The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are supplementary. Prove that the height of the tower is 6m.

Answer



Let Height of the tower is BC

Given, AB = 9, AD = 4. a + b = 90.

In ∆ABC,

 $\tan a = \frac{BC}{AB}$(1)

In ΔBCD ,

$$\tan b = \frac{BC}{BD}$$
....(2)

Multiplying Equation (1) and (2)

$$\tan(a) \times \tan(b) = \frac{BC \times BC}{AB \times BD}$$
$$\tan(a) \times \tan(90 - a) = \frac{BC \times BC}{AB \times BD}$$
$$\tan(a) \times \cot(a) = BC \times \frac{BC}{36}$$
$$1 = \frac{BC \times BC}{36}$$
$$36 = BC^{2}$$
$$BC = 6$$

27. Question

On one side of a road a tower is situated and on the other side a house in situated. The angles of depression of the root of the house and base of the house from the top of the tower are respectively 45° and 60°. If the height of the house is 12 m then find the height of the tower. ($\sqrt{3} = 1.732$)

Answer



Let Height of the House CD, Height of the Hill is AB.

Given, CD = 12, AE = 12 $\angle BCE = 30^{\circ}$ $\angle BDA = 60^{\circ}$ In ΔABD, $\tan 60 = \frac{\text{BE} + 12}{\text{AD}}$ $AD\sqrt{3} = BE + 12$ EC = $\frac{(BE + 12)\sqrt{3}}{3}$(1) In ΔBCE, $\tan 45 = \frac{BE}{EC}$ EC = BE.....(2) $\frac{(BE + 12)\sqrt{3}}{3} = BE$ $BE + 12 = \frac{BE3\sqrt{3}}{3}$ BE + 12 = BE $\sqrt{3}$

$$12 = BE(\sqrt{3} - 1)$$
$$\frac{12(\sqrt{3} + 1)}{2} = BE$$
$$BE = 6(\sqrt{3} + 1)$$

Height of the tower = BE + AE = $6\sqrt{3}$ + 6 + 12

= 28.39

28. Question

If the angle of elevation of the Sun changes from 30° to 60°. Then find the difference in the length of the shadows of a 15 m high pillar at both these angles of elevation.

Answer



Let Height of the pillar is AB,

Given, $\angle ADB = 30^{\circ}$, $\angle ACB = 60^{\circ}$, CD = 12

In ∆ABC,

 $\tan 30 = \frac{AB}{AC}$ $AC = AB\sqrt{3}$ $AC = AB\sqrt{3}....(1)$ $In \Delta ABD,$ $\tan 60 = \frac{AB}{AD}$ $AD = \frac{AB\sqrt{3}}{3}....(2)$ Eqt.(1) - Eqt.(2)

$$AC - AD = AB\sqrt{3} - \frac{AB\sqrt{3}}{3}$$
$$12 = \frac{AB2\sqrt{3}}{3}$$
$$12 \times \frac{3}{2} = AB\sqrt{3}$$
$$AB = \frac{18}{\sqrt{3}}$$
$$AB = \frac{18\sqrt{3}}{3}$$
$$AB = 6\sqrt{3}$$