MATHEMATICS PAPER IIB - MARCH 2010

COORDINATE GEOMETRY & CALCULUS

TIME : 3hrs

Max. Marks.75

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 = 20

Noe : Attempt all questions. Each question carries 2 marks.

- 1. Obtain the parametric equation of each of the following circle $x^{2} + y^{2} - 6x + 4y - 12 = 0$
- 2. Find the center and radius of the sphere $x^2 + y^2 + z^2 2x 4y 6z = 11$
- 3. Find the value of k if the lines 2x+3y+4=0 and x+y+k=0 are conjugate w.r.t $y^2 = 8x$
- 4. Find the value of k if the lines 2x+3y+4=0 and x+y+k=0 are conjugate w.r.t $y^2 = 8x$
- 5. Find the nth derivative of $\log(4-x^2)$

6. Evaluate
$$\int \left(\frac{1+x\log x}{x}\right) dx \text{ on } (0,\infty)$$

7. Evaluate
$$\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$$
 on $I \subset R \setminus \{x \in R : \cos(xe^x) = 0\}$

- 8. Evaluate $\int_{0}^{\frac{\pi}{2}} \cos^5 x \sin^4 x \, dx$
- 9. Find the area of the region enclosed by the given curves $x=4-y^2, x=0$

10. Find the order and degree of
$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{5/3} = 0$$

SECTION B

SHORT ANSWER TYPE QUESTIONS.

5X4 = 20

Note : Answer any FIVE questions. Each question carries 4 marks.

- 11. Find the equation of the circle whose center lies on X-axis and passing through the points (-2,3),(4,5).
- 12. Show that the equations of the common tangents to the circle $x^2+y^2=2a^2$ and the parabola $y^2=8ax$ are y=(x+2a).

- 13. Find the eccentricity, foci and the equations of directrices of the following ellipse $4x^2 + y^2 8x + 2y + 1 = 0$
- 14. Show that the polar equation of a conic in the standard form is $\frac{l}{r} = 1 + e \cos \theta$ ('*l*' is semilatus rectum, e is eccentricity)
- 15. Evaluate $\int \frac{dx}{5+4\cos x}$
- 16. Solve the differential equation $(x^2 + y^2)dy = 2xydx$

17. Solve the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{Tan^{-1}x}$

SECTION C

LONG ANSWER TYPE QUESTIONS.

5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

- 18. Show that the circles $x^2+y^2-6x-2y+1=0$; $x^2+y^2+2x-8y+13=0$ touch each other. Find the point of contact and the equation of common tangent at the point of contact.
- 19. Find the limiting points of the coaxial system determined by the circles

 $x^{2}+y^{2}+10x-4y-1=0, x^{2}+y^{2}+5x+y+4=0.$

- 20. If the polar of P with respect to the Parabola $y^2=4ax$ touches the circle $x^2+y^2=4a^2$, then show that P lies on the curve $x^2-y^2 = 4a^2$
- 21. If $y = \cos(m \log x), x > 0$, then show that $x^2 y_2 + x y_1 + m^2 y = 0$ and hence deduce that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (m^2 + n^2) y_n = 0$
- 22. Evaluate $\int \frac{x+1}{(x^2+3x+12)} dx$
- 23. Evaluate $\int_{0}^{1} \frac{\log(1+x)}{1+x^2} dx$
- 24. Find the approximate value of π from $\int_{0}^{1} \frac{1}{1+x^2} dx$ by using Simpson's rule by dividing [0, 1] into 4 equal parts.