16 PRACTICE PAPER

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper* – 1.

Section–A

Choose the correct option:

1. **If the matrix A is both symmetric and skew symmetric, then**
(*a*) A is diagonal matrix. (*b*) A is zero matrix. (*c*) A is square matrix. (*d*) None of these

2. **Let A be a square matrix of order 3×3, then $|kA|$ is equal to**
(*a*) $k|A|$ (*b*) $k^2|A|$ (*c*) $k^3|A|$ (*d*) $3k|A|$

3. **If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to**
(*a*) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (*b*) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (*c*) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (*d*) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

4. **If $x \cos y + y \cos x = \pi$ then $y''(0)$ is**
(*a*) π (*b*) $-\pi$ (*c*) 0 (*d*) 1

5. **The slope of the tangent of the locus $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$ is**
(*a*) 1 (*b*) 0 (*c*) 2 (*d*) -1

6. ***Read the following statements.***
Statement I : Integration of $\int_0^1 \frac{e^x}{1+e^{2x}}dx$ is $\tan^{-1}e - \frac{\pi}{4}$.
Statement II : $\int_a^b f(x)\,dx = \int_a^b f(a+b+x)\,dx$
Choose the correct option:
(*a*) Statement I is correct but statement II is not correct.
(*b*) Statement II is correct but statement I is not correct.
(*c*) Both statements I and II are correct.
(*d*) None of these.

7. **The value of $\int \tan(x-\alpha)\tan(x+\alpha)\tan(2x)\,dx$ is**
(*a*) $\log\left|\frac{\sqrt{\sec 2x}\sec(x+\alpha)}{\sec(x-\alpha)}\right| + C$ (*b*) $\log\left|\tan^{-1}(\sec x + \cos x)\right| + C$
(*c*) $\log\left|\frac{\sqrt{\sec 2x}\sec(x-\alpha)}{\sec(x+\alpha)}\right| + C$ (*d*) $\log\left|\frac{\sqrt{\sec 2x}}{\sec(x-\alpha)\sec(x+\alpha)}\right| + C$

8. If $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\,dx = f(x)$ then $f(x)$ equals

(a) $\frac{3}{2}\sin^{-1}(\cos^{3/2}x) + C$ (b) $-\frac{2}{3}\cos^{-1}(\cos^{3/2}x) + C$

(c) $-\frac{2}{3}\sin^{-1}(\cos^{3/2}x) + C$ (d) $\frac{2}{3}\sin^{-1}(\cos^{3/2}x) + C$

9. $\int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1}dx$, equals

(a) $\frac{x^m}{x^{m+n}+1} + C$ (b) $\frac{x^n}{x^{m+n}+1} + C$ (c) $\frac{x^{m+n}-1}{x^{m+n}+1} + C$ (d) $-\frac{x^n}{x^{m+n}+1} + C$

10. Area bounded by the curve $y = x^3$, the x-axis and the abscissae $x = -2$ and $x = 1$ is (in square units)

(a) -9 (b) $\frac{-15}{4}$ (c) $\frac{15}{4}$ (d) $\frac{17}{4}$

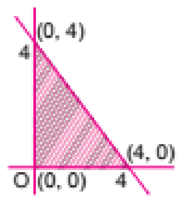
11. The differential equation of family of curves $y^2 = 4a\,(x+a)$ is

(a) $y^2 = 4\frac{dy}{dx}\left(x + \frac{dy}{dx}\right)$ (b) $2y\frac{dy}{dx} = 4a$ (c) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (d) $2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2 - y = 0$

12. If $\frac{dy}{dx} = \frac{x-y}{x+y}$, then its solution is

(a) $y^2 + 2xy - x^2 = C$ (b) $y^2 + 2xy + x^2 = C$ (c) $y^2 - 2xy - x^2 = C$ (d) $y^2 - 2xy + x^2 = C$

13. Feasible region shaded for a LPP is shown in figure. Maximum of $Z = 2x + 3y$ occurs at the point



(a) (0, 0) (b) (4, 0) (c) (0, 4) (d) none of these

14. The probability that a man can hit a target is $\frac{3}{4}$. He tries 5 times. The probability that he will hit the target at most one time is

(a) $\left(\frac{1}{4}\right)^3$ (b) $\left(\frac{3}{4}\right)^5$ (c) $\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$ (d) none of these

15. The probability distribution of a random variable is given below

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

then the variance of k is

(a) $\frac{8}{15}$ (b) $\frac{7}{15}$ (c) $\frac{4}{15}$ (d) None of these

Section–B (B1)

16. Identity relation R on a set A is

(a) Reflexive only (b) Symmetric only (c) Transitive only (d) Equivalence

17. Let $f : (-1, 1) \longrightarrow B$ where $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is one-one and onto, then B equals

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right)$

18. **The inverse of the function $f(x) = \frac{e^x - 2e^{-x}}{e^x + 2e^{-x}} + 1$ is**

(a) $\log_{10}\frac{2x}{2-x}$ (b) $\log_{10}\frac{x}{2-x}$ (c) $\log_e\left(\frac{2x}{2-x}\right)^{1/2}$ (d) $\log_e\left(\frac{1}{2-x}\right)^{\frac{1}{2}}$

19. **Two functions are defined as under**

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 \le x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} x^2, & -1 \le x \le 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

then $(fog)(x)$ equals

(a) $\begin{cases} x^2+1, & |x| \le 1 \\ 2x^2+1, & 1 < x \le \sqrt{2} \end{cases}$ (b) $f(x) = \begin{cases} x^2+1, & |x| \le 1 \\ 2x^2+1, & 1 \le x < \sqrt{2} \end{cases}$

(c) $\begin{cases} x^2+1, & |x| > 1 \\ 2x^2+1, & |x| \ge \sqrt{2} \end{cases}$ (d) None of these

20. **Let * be a binary operation on $\mathbb{N}$ such that $a * b = a + b$. Then $(3 * 2) * 4$ is equal to**

(a) 5 (b) 6 (c) 9 (d) 10

21. **If $f(x) = \sin^{-1} x$, then domain of $f(x)$ is**

(a) $x \ge 1$ or $x \le -1$ (b) $-1 \le x \le 1$ (c) $x \ge 1$ (d) None of these

22. **The solution of equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\frac{1}{\sqrt{3}}$ is**

(a) No solution (b) $x = \sqrt{3}$
(c) Infinite many solutions (d) None of these

23. **$3\cos^{-1} x - \pi x - \frac{\pi}{2} = 0$ has**

(a) two solutions (b) one and only one solution
(c) no solution (d) more than one solution.

24. **If $A = 2\tan^{-1}(2\sqrt{2} - 1)$ and $B = \sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$, then**

(a) $A = B$ (b) $A < B$ (c) $A > B$ (d) none of these

26. **If $A^3 = 0$ (zero matrix), then $I + A + A^2$ equals**

(a) $(I - A)^{-1}$ (b) $I - A^{-1}$ (c) $I - A$ (d) A

27. **If a, b, c are in AP then the determinant**

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is}$$

(a) 0 (b) 1 (c) x (d) $2x$

28. **If A is invertible matrix of order 3×3, then $|A^{-1}|$ is equal to**

(a) $\frac{1}{A}$ (b) $\frac{1}{|A|}$ (c) $\frac{-1}{3|A|}$ (d) $\frac{1}{9|A|}$

29. **If $y = \left(\frac{x}{n}\right)^{nx}\left(1 + \log\frac{x}{n}\right)$, $y'(n)$ is given by**

(a) $\frac{n^2+1}{n}$ (b) $\frac{1}{n}$

(c) $\left(\frac{1}{n}\right)^n$ (d) $\left(\frac{1}{n}\right)^n\left(\frac{n^2+1}{n}\right)$

30. **Let $f(x) = |\sin x|$; $0 \leq x \leq 2\pi$ then**

(*a*) $f(x)$ is differentiable function at infinite number of points.

(*b*) $f(x)$ is non-differentiable at 3 points and continuous everywhere.

(*c*) $f(x)$ is discontinuous everywhere.

(*d*) $f(x)$ is discontinuous at 3 points.

31. **Let** $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4}+x\right)\right]^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$

then the value of k such that $f(x)$ holds continuity at $x = 0$ is

(*a*) e (*b*) $\frac{1}{e^2}$ (*c*) e^2 (*d*) None of these

32. **Let $f(x) = x^2|x|$ then the set of values, where $f(x)$ is three times differentiable, is**

(*a*) Infinite (*b*) 2 (*c*) 3 (*d*) None of these

33. **Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?**

(*a*) $\tan 2x$ (*b*) $\cos x$ (*c*) $\cos 3x$ (*d*) none of these

34. ***Read the following statements.***

Statement I : If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$.

Statement II : $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx = \tan(xe^x) + C$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

35. **If $\int f(x)\sin x \cos x\, dx = \frac{1}{2(b^2-a^2)}\log(f(x)) + C$ then $\int f(x)\, dx$ equals**

(*a*) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + K$ (*b*) $ab\tan^{-1}\left(\frac{a\tan x}{b}\right) + K$

(*c*) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right) + K$ (*d*) $\frac{1}{ab}\tan^{-1}\left(\tan\left(\frac{bx}{a}\right)\right) + K$

36. **If $\int \frac{(x^m)^x \log_e(ex)}{1+(x^{2m})^x}dx = \left(1 - \frac{A}{B}\right)\tan^{-1}(x^m)^x + K$ then $A + B$ equals**

(*a*) $2m - 1$ (*b*) $m - 1$ (*c*) $2m$ (*d*) $m + 1$

37. **If $y = f(x)$ makes positive intercept of 2 and 0 unit on x and y axes and enclosed an area of $\frac{3}{4}$ square units with the axes then $\int_0^2 xf'(x)\, dx$ is**

(*a*) 0 (*b*) $\frac{1}{3}$ (*c*) $-\frac{3}{4}$ (*d*) $\frac{4}{3}$

38. **The general solution of $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is**

(*a*) $e^y = e^x - 1 + Ce^{-x}$ (*b*) $e^x = e^y + 1 + Ce^{-y}$ (*c*) $e^y = e^x - 1 + Ce^{-e^x}$ (*d*) None of these

39. The solution of the differential equation $y\cos\left(\frac{y}{x}\right)(xdy - ydx) + x\sin\left(\frac{y}{x}\right)(xdy + ydx) = 0$ which satisfy $y(1) = \frac{\pi}{2}$, is

(a) $y\sin\left(\frac{y}{x}\right) = \frac{\pi}{x}$ (b) $y\sin\left(\frac{y}{x}\right) = \frac{\pi}{3x}$ (c) $y\sin\left(\frac{y}{x}\right) = \frac{\pi}{2x}$ (d) None of these

40. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is

(a) $-\hat{i} + 12\hat{j} + 4\hat{k}$ (b) $5\hat{i} + 2\hat{j} - 4\hat{k}$ (c) $-5\hat{i} + 2\hat{j} + 4\hat{k}$ (d) $\hat{i} + \hat{j} + \hat{k}$

41. The value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $\frac{-5}{2}$

42. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}, \hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

(a) $\lambda = -2$ (b) $\lambda = 0$ (c) $\lambda = 1$ (d) $\lambda = -1$

43. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 2, |\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

(a) 0 (b) 1 (c) –19 (d) 38

44. *ABC* is a triangle such that the coordinate of *A*, *B*, *C* are (2, 3, 5), (–1, 3, 2) and $(\lambda, 5, \mu)$ respectively. If the median through *A* is equally inclined to the axis then

(a) $\lambda = 5 = \mu$ (b) $\lambda = 5, \mu = 7$ (c) $\lambda = 7, \mu = 10$ (d) $\lambda = 0 = \mu$

45. A line *OP* through origin *O* is inclined at 30° and 45° to *OX* and *OY* respectively. The angle at which it is inclined to *OZ* is

(a) 90° (b) $\cos^{-1}\left(\frac{-1}{4}\right)$ (c) 60° (d) Such line does not exist

46. The perpendicular distance of a corner of a unit cube from a diagonal not passing through it is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

47. The equation of plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, –7) is

(a) $x + y + z = 1$ (b) $x + y + z = 2$ (c) $x + y + z = 0$ (d) none of these

48. If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$ then $P(A'/B').\ P(B'/A')$ is equal to

(a) $\frac{15}{16}$ (b) $\frac{7}{5}$ (c) $\frac{25}{42}$ (d) 1

49. If *A* and *B* are two events such that $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A)$ is equals to

(a) $\frac{3}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

50. If $x \in [0, 8]$, the probability that $x^2 - 8x + 12 \geq 0$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) none of these