Class: XII
SESSION : 2022-2023
SUBJECT: Mathematics SAMPLE
QUESTION PAPER - 17
with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. $\int \tan^2 \frac{x}{2} dx = ?$ **[1]**

a) $2\tan\frac{x}{2} - x + C$

b) $\tan\frac{x}{2} + x + C$

c) $\tan\frac{x}{2} - x + C$

d) $2\tan\frac{x}{2} + x + c$

2. Area of the region bounded by the curve y = cos x between x = 0 and x = π is **[1]**

a) 2 sq units
b) 3 sq units
c) 4 sq units
d) 1 sq units

3. The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is **[1]**

a) $\cos^{-1}\left(\frac{3}{8}\right)$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$

4. Area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$ is **[1]**

a) 32π sq units
b) 4π sq units
c) 16π sq units
d) 24 sq units

5. If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other then $\lambda = ?$ **[1]**

a) -6
b) -3
c) -1
d) -9

6. If A and B are two events such that $P(A) = \frac{4}{5}$, and $P(A \cap B) = \frac{7}{10}$, then P (B / A) = **[1]**

a) $\frac{1}{10}$
b) $\frac{17}{20}$
c) $\frac{7}{8}$
d) $\frac{1}{8}$

7. In a LPP, the linear inequalities or restrictions on the variables are called [1]

a) Limits
b) Inequalities
c) Linear constraints
d) Constraints

8. It is given that the probability that A can solve a given problem is $\frac{3}{5}$ and the probability that B can solve the same problem is $\frac{2}{3}$. The probability that at least one of A and B can solve a problem is [1]

a) $\frac{13}{15}$
b) $\frac{2}{15}$
c) $\frac{1}{15}$
d) $\frac{2}{5}$

9. Which of the following transformations reduce the differential equation $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ into the form $\frac{du}{dx}+P(x)u=Q(x)$ [1]

a) $u = e^x$
b) u = log x
c) $u = (\log z)^2$
d) $u = (\log z)^{-1}$

10. If $|\vec{a}|=\sqrt{2}$, $|\vec{b}|=\sqrt{3}$ and |a+ b| = $\sqrt{6}$, then what is $|\vec{a}+\vec{b}|$ equal to? [1]

a) 2
b) 4
c) 3
d) 1

11. $\int\frac{(1+\tan x)}{(1-\tan x)}dx=?$ [1]

a) log |cos x – sin x| + C
b) log |cos x + sin x | + C
c) none of these
d) - log |cos x – sin x| + C

12. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$ is [1]

a) 2
b) 1
c) 3
d) 4

13. The solution of the DE $\frac{dy}{dx}$ = 1 - x + y - xy is [1]

a) $e^y=x-\frac{x^2}{2}+C$
b) $\log(1+y)=x-\frac{x^2}{2}+C$
c) none of these
d) $e^{(1+y)}=x-\frac{x^2}{2}+C$

14. For any 2 × 2 matrix, If A(adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to **[1]**

a) 20
b) 10
c) 0
d) 100

15. The value of k for which f(x) = $\begin{cases} \frac{\sin 5x}{3x}, \text{ if } x \neq 0 \\ k, \text{ if } x = 0 \end{cases}$ is continuous at x = 0 is **[1]**

a) $\frac{5}{3}$
b) $\frac{3}{5}$
c) 0
d) $\frac{1}{3}$

16. Find a particular solution of $x\left(x^2 - 1\right)\frac{dx}{dy} = 1$; y = 0 when x = 2. **[1]**

a) $y = \frac{1}{2}\log\left(\frac{x^3-1}{x^2}\right) - \frac{1}{2}log\frac{3}{4}$
b) $y = \frac{1}{2}\log\left(\frac{x^2-1}{x^3}\right) + \frac{1}{2}log\frac{3}{4}$
c) $y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}log\frac{3}{7}$
d) $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

17. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is **[1]**

a) perpendicular to z-axis
b) parallel to z-axis
c) parallel to y-axis
d) parallel to x-axis

18. The domain of the function defined by f (x) = $\sin^{-1}$x + cosx is **[1]**

a) [–1, 1]
b) ϕ
c) $(-\infty, \infty)$
d) $[-1, \pi + 1]$

19. Let the feasible region of the linear programming problem with the objective function Z = ax + by is unbounded and let M and m be the maximum and minimum value of Z, respectively. **[1]**
Now, consider the following statements
Assertion (A): M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.
Reason (R): m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Every differentiable function is continuous but converse is not true. **[1]**
Reason (R): Function f(x) = |x| is continuous.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the solution of $\frac{dy}{dx} = 2^{y-x}$ **[2]**

22. Find which of the functions is continuous or discontinuous at the indicated points: **[2]**
$f(x) = |x| + |x-1|$ at x = 1

23. Cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$ write the direction ratios of a line parallel to AB. **[2]**

OR

Write the direction cosines of the line whose cartesian equation is 2x = 3y = -z.

24. A company has estimated that the probabilities of success for three products introduced in the market are $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{2}{3}$ respectively. Assuming independence, find the probability that none of the products is successful. **[2]**

25. Find the principal value of $\cot^{-1}(\sqrt{3})$. **[2]**

Section C

26. Minimize Z=400x + 200y subject to **[3]**
$5x+2y \geq 30$
$2x+y \geq 15$
$x \leq y$, $x \geq 0, y \geq 0$

27. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines x = 0 and x = 1. **[3]**

OR

If S be the area of the region enclosed by $y = e^{-x^2}$, y = 0 and x = 1, then show that
$1 - \frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

28. Find the acute angle between the lines whose direction ratios are proportional to 2 : 3 : 6 and 1 : 2 : 2 **[3]**

OR

Find the vector and Cartesian equations of the line passing through the points A(2, -1, 4) and B(1, 1, -2).

29. Evaluate $\int_1^5 (|x-1|+|x-2|+|x-4|)dx$: **[3]**

OR

Evaluate $\int_1^4 f(x)dx$, where $f(x)=\begin{cases} 2x+8, & 1\leq x\leq 2 \\ 6x & ,2\leq x\leq 4 \end{cases}$

30. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line y = x in the first quadrant. **[3]**

31. If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$. Prove that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$ **[3]**

Section D

32. Given $A=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17. **[5]**

OR

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third number we get double of the second number. Find the numbers using matrix method.

33. Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC. **[5]**

34. Let A = R – {3} and B = R- {1}. Consider the function of f: A → B defined by f(x) = $\frac{x-2}{x-3}$ is one – one and onto. **[5]**

OR

Each of the following defines a relation on N:

i. x is greater than y, x, y ∈ N
ii. x + y = 10, x, y ∈ N
iii. xy is square of an integer x, y ∈ N
iv. x + 4y = 10x, y ∈ N.

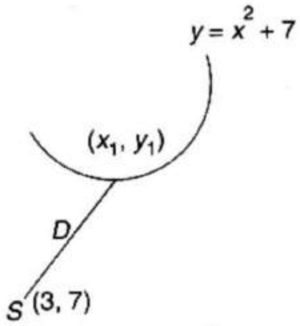
Determine which of the above relations are reflexive, symmetric and transitive.

35. Evaluate: $\int \frac{\sin x\cos x}{(\cos^2 x-\cos x-2)}dx$. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) If P (x_1, y_1) be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7).

(ii) Find the critical point such that distance is minimum.

(iii) Verify by second derivative test that distance is minimum at (1, 8).

OR

Find the minimum distance between soldier and helicopter?

37. **Read the text carefully and answer the questions:** **[4]**

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

(i) Represent the requirement of calories and proteins for each person in matrix form.

(ii) Find the requirement of calories of family A and requirement of proteins of family B.

(iii) Represent the requirement of calories and proteins If each person increases the protein intake by 5% and decrease the calories by 5% in matrix form.

OR

If A and B are two matrices such that AB = B and BA = A, then find $A^2 + B^2$ in terms of A and B.

38. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) Find the probability that both of them are selected.

(ii) The probability that none of them is selected.

SOLUTION

Section A

1. **(a)** $2\tan\frac{x}{2} - x + C$

Explanation: Given integral is $\int \tan^2\frac{x}{2}$

Let, $\frac{x}{2} = z$

$\Rightarrow$ dx = 2dz

So,

$\int \tan^2\frac{x}{2}dx$

$= 2\int \tan^2 z dz$

$= 2\int \frac{\sin^2 z}{\cos^2 z}dz$

$= 2\int \frac{1-\cos^2 z}{\cos^2 z}dz$ where c is the integrating constant.

$= 2\int \left(\sec^2 z - 1\right)dz$

= 2 [tan z - z] + c

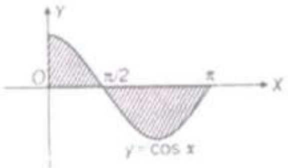
$= 2\left[\tan\frac{x}{2} - \frac{x}{2}\right] + c$

Hence, $\int \tan^2\frac{x}{2} = 2\left[\tan\frac{x}{2} - \frac{x}{2}\right] + c$

2. **(a)** 2 sq units

Explanation:

Required area enclosed by the curve y = cos x and x = π



$$A = \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{\pi} \cos x dx \right|$$

$$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\sin\frac{\pi}{2} - \sin\pi\right|$$

= 1 + 1 = 2 sq. units

3. **(d)** $\frac{\pi}{2}$

Explanation: Let's consider the first parallel vector to be $\vec{a} = 2\hat{i} + 7\hat{j} - 3\hat{k}$ and second parallel vector be $\vec{b} = -\hat{i} + 2\hat{j} + 4\hat{k}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$

$= \sqrt{62}$

$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$

$= \sqrt{21}$

$$\Rightarrow \cos\alpha = \frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-\hat{i}+2\hat{j}+4\hat{k})}{\sqrt{21}\times\sqrt{62}}$$

$$\Rightarrow \cos\alpha = \frac{-2+14-12}{\sqrt{21}\times\sqrt{62}}$$

$$\Rightarrow \cos\alpha = \frac{0}{\sqrt{21}\times\sqrt{62}}$$

$$\Rightarrow \alpha = \cos^{-1}0$$

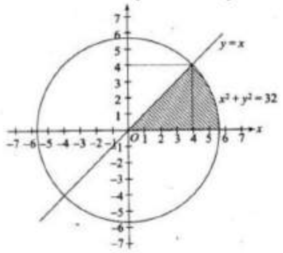
The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant

$$\therefore \alpha = \frac{\pi}{2}$$

4. **(b)** 4π sq units

Explanation:

We have, y = 0, y = x and the circle $x^2 + y^2 = 3$ in the first quadrant



Solving y = x with the circle

$x^2 + x^2 = 32$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$

When x = 4, y = 4

For point of intersection of circle with the x-axis,

Put y = 0

$\therefore x^2 + 0 = 32$

$\Rightarrow x = \pm 4\sqrt{2}$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2},0)$

From the figure, area of shaded region

$$A = \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2} dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_0^{4\sqrt{2}}$$

$$= \frac{16}{2} + \left[0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$$

$$= 8 + \left[16 \cdot \frac{\pi}{2} - 2 \cdot \sqrt{16} - 16 \cdot \frac{\pi}{4}\right]$$

$= 8 + [9\pi - 8 - 4\pi] = 4\pi$ sq. units

5. **(d)** -9

Explanation: Here, $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$

Since $\vec{a} \perp \vec{b}$

$\therefore\ \vec{a} \cdot \vec{b} = 0$

$\Rightarrow\ (3\hat{i} + \hat{j} + 2\hat{k}) \cdot \left(\hat{i} + \lambda\hat{j} - 3\hat{k}\right) = 0$

$\Rightarrow\ 3 + \lambda + 6 = 0$

$\Rightarrow\ \lambda = -9$

6. **(c)** $\frac{7}{8}$

Explanation: $P(A) = \frac{4}{5}$, $P(A \cap B) = \frac{7}{10}$ (Given)

Now we know,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{7}{10}}{\frac{4}{5}}$$

$$= \frac{7}{10} \times \frac{5}{4}$$

$$= \frac{7}{8}$$

7. **(c)** Linear constraints

Explanation: In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

8. **(a)** $\frac{13}{15}$

Explanation: P(A) = probability that A can solve the problem
= 3/5
And P(B) = probability that B can solve the problem = 2/3
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
Since the events are independent
$\Rightarrow P(A \cap B) = P(A).P(B)$

Thus,

$\Rightarrow P(A \cup B) = 3/5 + 2/3 - 2/5 = 13/15$

9. **(d)** u = $(\log z)^{-1}$

Explanation: We have ,

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$$

$$\frac{dz}{dx} = \frac{z}{x^2}(\log z)^2 - \frac{z}{x}\log z \text{ ....(i)}$$

Put v = $(\log z)^{-1}$

$$\frac{dv}{dx} = \frac{-1}{(\log z)^2}\frac{1}{z}\frac{dz}{dx}$$

$$\frac{dz}{dx} = -z(\log z)^2\frac{dv}{dx} \text{ ....(ii)}$$

From (i) and (ii)

$$-z(\log z)^2\frac{dv}{dx} = \frac{z}{x^2}(\log z)^2 - \frac{z}{x}\log z$$

$$(\log z)\frac{dv}{dx} = -\frac{1}{x^2}(\log z) + \frac{1}{x}$$

$$\frac{dv}{dx} = -\frac{1}{x^2} + \frac{u}{x}$$

$$\frac{dv}{dx} - \frac{u}{x} = -\frac{1}{x^2}$$

$$P(x) = \frac{-1}{x},\ q(x) = \frac{-1}{x^2}$$

Given differential equation can be reduced.

using v = $(\log z)^{-1}$

10. **(a)** 2

Explanation: 2

11. **(d) -** log |cos x – sin x| + C

Explanation:

The integral is $\int \frac{(1+\tan x)}{(1-\tan x)}dx$

since we know that, $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

sin (a + b) = sin a cos b + cos a sin b

$\int$cot x = log (sin x) + c

Therefore,

$$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx \text{ (Rationalizing the denominator)}$$

$$\Rightarrow \int \frac{\cos x+\sin x}{\cos x-\sin x} dx$$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$$\int \frac{-dt}{t} = -\log t + c$$

$\Rightarrow$ -log |cos x - sin x| + c

12. **(a)** 2

Explanation: We have $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} = \frac{d^2y}{dx^2}$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

13. **(b)** $\log(1+y) = x - \frac{x^2}{2} + C$

Explanation: Here, $\frac{dy}{dx} = 1 - x + y - xy$

$$\frac{dy}{dx} = 1 - x + y(1-x)$$

$$\frac{dy}{dx} = (1+y)(1-x)$$

$$\frac{\mathrm{dy}}{1+\mathrm{y}} = (1-\mathrm{x})\mathrm{dx}$$

On integrating on both sides, we obtain

$$\log(1+y) = x - \frac{x^2}{2} + c$$

14. **(b)** 10

Explanation: We know that

A $\times$ adjA = |A| I_{nxn} , where I is the unit matrix of order nxn.--------------------[1]

$A(\text{adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ Using the above property of matrices (1), we get

$A(\text{adj}A) = 10\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

A(adj A) = (10) I_{2x2}

|A| I_{2x2} = 10 I_{2x2}

|A| = 10

15. **(a)** $\frac{5}{3}$

Explanation: Since f(x) is continuous on 0,then we

$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{3x} = f(0)$

$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{3x} \times \frac{5x}{5x} = f(0)$

$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{5x} \times \frac{5x}{3x} = f(0)$

$\Rightarrow$ f(0) = $\frac{5}{3}$

$\Rightarrow$ k = $\frac{5}{3}$

16. **(d)** $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

Explanation: $x(x^2 - 1)dx = dy$

$\int(x^3 - x)dx = \int dy$

$\frac{x^4}{4} - \frac{x^2}{2} = y + c$

Here y = 0 when x = 2

$\frac{2^4}{4} - \frac{2^2}{2} = 0 + c$

$4 - 2 = c$

$\therefore c = 2$

Therefore,the required solution is $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

17. **(a)** perpendicular to z-axis

Explanation: We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

$3x + 4y + 0z = 0$

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

18. **(a)** [–1, 1]

Explanation: The domain of cos is R and the domain of $\sin^{-1}$ is [–1, 1]. Therefore, the domain of $\cos x + \sin^{-1} x$ is $R \cap [-1,1]$, i.e., [–1, 1].

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: In case, the feasible region is unbounded, we have

Assertion: M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason: Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value. Hence, Assertion is true and Reason is true but Reason is not the correct explanation of Assertion.

20. **(c)** A is true but R is false.

Explanation: **Assertion:** It is a true statement.

Reason: We have, $f(x) = |x|$

At $x = 0$,

$$\text{LHL} = \lim_{h \to 0^-} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0^-} \frac{|0-h| - 0}{-h}$$

$$= \lim_{h \to 0^-} \frac{h}{-h} = -1$$

$$\text{and RHL} = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{|0+h| - 0}{h} = \lim_{h \to 0^+} \frac{h}{h} = 1$$

Here, LHD $\neq$ RHD, hence f(x) is not continuous at x = 0.

Section B

21. Given that, $\frac{dy}{dx} = 2^{y-x}$

$\Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x}$

$\Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

$\Rightarrow \int 2^{-y} dy = \int 2^{-x} dx$ (Integrating both sides)

$\Rightarrow \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$

$\Rightarrow -2^{-y} + 2^{-x} = C \log 2$

$\Rightarrow 2^{-x} - 2^{-y} = C \log 2$

$\Rightarrow 2^{-x} - 2^{-y} = K$ [Where, $K = C \log 2$]

22. We have, $f(x) = |x| + |x-1|$ at x = 1

At x = 1, $LHL = \lim_{x \to 1^-} [|x| + |x-1|]$

$= \lim_{x \to 0} [|1-h| + |1-h-1|] = 1 + 0 = 1$

And $RHL = \lim_{x \to 1^-} [|x| + |x-1|]$

$= \lim_{n \to 0} [|1+h| + |1+h-1|] = 1 + 0 = 1$

and $f(1) = |1| + |0| = 1$

$\therefore$ LHL = RHL = f(1)

Hence, f(x) is continuous at x = 1

23. Given equation of a line can be written is

$$\frac{x-\frac{1}{2}}{1} = \frac{y-4}{-7} = \frac{z+1}{2}$$

The direction ratios of a line parallel to AB are (1, -7, 2)

OR

According to the question we are given that

2x = 3y = -z

The equation of the given line can be re-written as

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$$

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$$

The direction ratios of the line 3, 2, -6.

Hence, the direction cosines of the line

$$\frac{3}{\sqrt{3^2+2^2+(-6)^2}}, \frac{2}{\sqrt{3^2+2^2+(-6)^2}}, \frac{-6}{\sqrt{3^2+2^2+(-6)^2}}$$

$$= \frac{3}{7}, \frac{2}{7}, -\frac{6}{7}$$

24. Consider the following events:

A = First product is successful,

B = Second product is successful,

C = Third product is successful

We have,

$P(A) = \frac{1}{3}$, $P(B) = \frac{2}{5}$ and $P(C) = \frac{2}{3}$

Therefore, Required probability is given by,

P(None of the products is successful)

$= P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= P(\bar{A})P(\bar{B})P(\bar{C})$ [$\because$ A, B, C are independent events]

$= (1 - \frac{1}{3}) \times (1 - \frac{2}{5}) \times (1 - \frac{2}{3})$

$= \frac{2}{3} \times \frac{3}{5} \times \frac{1}{3} = \frac{2}{15}$

25. $\cot^{-1}x$ represents an angle in $(0, \pi)$ whose cotangent is x.

Let $x = \cot^{-1}(\sqrt{3})$

$$\Rightarrow \cot x = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

$\therefore$ Principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

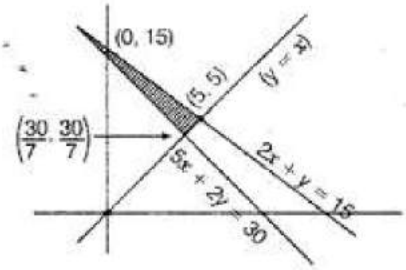
Section C

26. we have minimise Z=400x + 200y subject to 5x + 2y $\geqslant$ 30.

$2x + y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0$

On solving x-y=0 and 5x+2y=30, we get

$y = \frac{30}{7}, x = \frac{30}{7}$



On solving x - y = 0 and 2x + y = 15 we get x = 5, y = 5

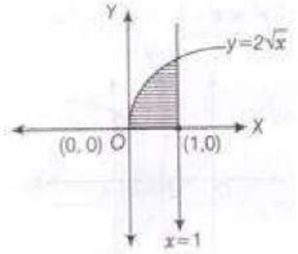
So, from the shaded feasible region it is clear that coordinates of corner points are

(0,15) , (5,5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	**Corresponding value of X = 400x + 200y**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7}$
	= 2571.43 (minimum)

Hence, the minimum is Rs 2571.43.

27. We have, $y = 2\sqrt{x}$. x = 0 and x = 1.



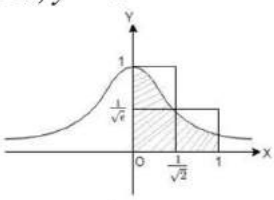
$\therefore$ Area of shaded region, $A = \int_0^1 \left(2\sqrt{x}\right)dx$

$$= 2.\left[\frac{x^{3/2}}{3}.2\right]_0^1$$

$$= 2\left(\frac{2}{3}.1 - 0\right) = \frac{4}{3} \text{ sq units}$$

OR

As, $y = e^{-x^2}$



since, $x^2 \leq x$ when x $\in$ [0, 1]

$\Rightarrow \; -x^2 \geq -x$

$\Rightarrow \; e^{-x^2} \geq e^{-x}$

$\therefore \; \int_0^1 e^{-x^2}dx \geq \int_0^1 e^{-x}dx$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} S \geq -(e^{-x})_0^1 = 1 - \frac{1}{e} \ldots\ldots (1)$$

Also, $\int_0^1 e^{-x^2} dx \leq$ Area of the two rectangles

$$\leq \left(1 \times \frac{1}{\sqrt{2}}\right) + \left(1 - \frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{e}}$$

$$\leq \left(1 \times \frac{1}{\sqrt{2}}\right) + \left(1 - \frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{e}}$$

$$\leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right) \ldots\ldots\ldots (2)$$

From (1) and (2), we conclude that

$$1 - \frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

If S be the area of the region enclosed by $y = e^{-x^2}$, y = 0 and x = 1, then

$$1 - \frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

28. Given that the direction ratios of the lines are proportional to 2:3:6 and 1:2:2.

Let us denote the lines in the form of vectors as $\vec{A}$ and $\vec{B}$.

Suppose we write the vectors:

$\Rightarrow \vec{A} = 2\vec{i} + 3\vec{j} + 6\vec{k}$

$\Rightarrow \vec{B} = 1\vec{i} + 2\vec{j} + 2\vec{k}$

We know that the angle between the vectors $a_1\vec{i} + b_1\vec{j} + c_1\vec{k}$ and $a_2\vec{i} + b_2\vec{j} + c_2\vec{k}$ is given by:

$$\alpha = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{(2\times 1)+(3\times 2)+(6\times 2)}{\sqrt{2^2+3^2+6^2}\sqrt{1^2+2^2+2^2}}\right)$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{2+6+12}{\sqrt{4+9+36}\sqrt{1+4+4}}\right)$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{20}{\sqrt{49}\sqrt{9}}\right)$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{20}{7\times 3}\right)$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{20}{21}\right)$$

Hence the required acute angle between the two vectors is given by $\cos^{-1}\left(\frac{20}{21}\right)$.

OR

Vector equation of the given line:

Let the position vectors of A and B be $\vec{r}_1$ and $\vec{r_2}$ respectively. Then

$\vec{r}_1 = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{r_2} = \hat{i} + \hat{j} - 2\hat{k}$

$\therefore \quad \left(\vec{r}_2 - \vec{r}_1\right) = (\hat{i} + \hat{j} - 2\hat{k}) - (2\hat{i} - \hat{j} + 4\hat{k}) = (-\hat{i} + 2\hat{j} - 6\hat{k})$

$\therefore$ the vector equation of the line AB is

$\vec{r} = \vec{r}_1 + \lambda\left(\vec{r}_2 - \vec{r}_1\right)$ for some scalar, λ,

i.e.,$\vec{r} = (2\vec{i} - \vec{j} + 4\vec{k}) + \lambda(-\vec{i} + 2\vec{j} - 6\vec{k})$ ...(i)

Cartesian equations of the given line:

Taking $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ equation (i) becomes

$(x\hat{i} + y\hat{j} + z\hat{k}) = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(-\hat{i} + 2\hat{j} - 6\hat{k})$

$\Leftrightarrow (x\hat{i} + y\hat{j} + z\hat{k}) = (2-\lambda)\hat{i} + (2\lambda - 1)\hat{j} + (4 - 6\lambda)\hat{k}$

$\Leftrightarrow x = 2 - \lambda, y = 2\lambda - 1$ and $z = 4 - 6\lambda$

$$\Leftrightarrow \frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6} = \lambda$$

Hence, $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$ are the Cartesian equations of the given line.

29. $\int_1^5 |x - 11 + 1x - 21 + 1x - 4|\, dx$

$= \int_1^2 (5 - x)dx + \int_2^4 (x + 1)dx + \int_4^5 (3x - 7)dx$

$= \left[5x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4 + \left[\frac{3x^2}{2} - 7x\right]_4^5$

$= \frac{7}{2} + 8 + \frac{13}{2} = 18$

OR

Using additivity property of integration, we can write integral as,

$\int_1^4 f(x)dx = \int_1^2 f(x)dx + \int_2^4 f(x)dx$

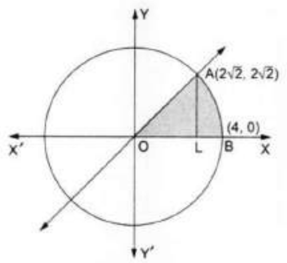
$\Rightarrow \int_1^4 f(x)dx = \int_1^2 (2x + 8)dx + \int_2^4 6xdx$ [Using the definition of f(x)]

$\Rightarrow \int_1^4 f(x)dx = \left[x^2 + 8x\right]_1^2 + \left[3x^2\right]_2^4$

$= [(4 + 16) - (1 + 8)] + [48 - 12] = 47$

30. The given circle is $x^2 + y^2 = 16$ ...(i)

The given line is $y = x$ ...(ii)



Putting y = x from (ii) into (i), we get

$2x^2 = 16 \Leftrightarrow x^2 = 8 \Leftrightarrow x = 2\sqrt{2}$ [$\because$ x is +ve in the first quad.}

Thus, the point of intersection of (i) and (ii) in the first quadrant is $A(2\sqrt{2}, 2\sqrt{2})$

Draw AL perpendicular on the x-axis

Therefore required area of region = (area of region OLA) + area of region(LBAL).

$= \int_0^{2\sqrt{2}} x dx + \int_{2\sqrt{2}}^4 \sqrt{16 - x^2} dx$

$$= \left[\frac{x^2}{2}\right]_0^{2\sqrt{2}} + \left[\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{2\sqrt{2}}^{4}$$

$$= \frac{1}{2}\left[(2\sqrt{2})^2 - 0\right] + \left[\left(0 + 8\sin^{-1}1\right) - \left(4 + 8\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= \left[4 + \left(8 \times \frac{\pi}{2}\right) - 4 - \left(8 \times \frac{\pi}{4}\right)\right] = (2\pi) \text{ sq units.}$$

31. We have

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$$

Differentiating both sides, we get $y.\ \frac{1}{2\sqrt{x^2+1}}(2x) + \sqrt{x^2+1}.\ \frac{dy}{dx}$

$$= \frac{1}{\sqrt{x^2+1}-x}\left[\frac{1\,(2x)}{2\sqrt{x^2+1}} - 1\right]$$

$$\Rightarrow \frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}.\ \frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x}\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]$$

$$\Rightarrow \frac{xy + \left(x^2+1\right)\frac{dy}{dx}}{\sqrt{x^2+1}} = \frac{-\left(\sqrt{x^2+1}-x\right)}{\left(\sqrt{x^2+1}-x\right)\sqrt{x^2+1}}$$

$$\Rightarrow xy + \left(x^2+1\right)\frac{dy}{dx} = -1$$

$$\Rightarrow \left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$$

Section D

32. We have, $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$$\therefore \ BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\therefore \ B^{-1} = \frac{A}{6} = \frac{1}{6} A = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Also, x - y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \text{[using Eq. (i)]}$$

$$= \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ x = 2, y = -1 and z = 4

OR

Let the three numbers be x,y and z.Then,
x + y + z = 6
y + 3z = 11
x + z = 2y
This system can be written as AX = B whose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$|A| = 9 \neq 0$

$A_{11} = 7, A_{12} = 3, A_{13} = -1$

$A_{21} = -3, A_{22} = 0, A_{23} = 3$

$A_{31} = 2, A_{32} = -3, A_{33} = 1$

$$adjA = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x=1, y=2, z=3

33. A (1, -2, -8), B (5, 0, -2), C (11, 3, 7)

$\vec{OA} = 1\hat{i} - 2\hat{j} - 8\hat{k}$

$\vec{OB} = 5\hat{i} - 0\hat{j} - 2\hat{k}$

$\vec{OC} = 11\hat{i} + 3\hat{j} + 7\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{BC} = \vec{OC} - \vec{OB}$

$= \left(6\hat{i} + 3\hat{j} + 9\hat{k}\right)$

$= 3\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$

$= \frac{3}{2}\left(4\hat{i} + 2\hat{j} + 6\hat{k}\right)$

$\vec{BC} = \frac{3}{2}\vec{AB}$

Thus $\vec{BC} \,||\, \vec{AB}$and one point B is common therefore A, B, C are collinear and B divides AC in 2:3.

34. For all $x_1\ x_2 \in A$

if $f(x_1) = f(x_2)$ implies $x_1 = x_2$ then f is one one

Now $f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

Cross multiplying and solving, we get

$x_1 = x_2$

f is one – one

$y = \frac{(x-2)}{(x-3)}$

$(x - 3)y = x - 2$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x = \frac{(3y-2)}{(y-1)}$

$f\left(\frac{3y-2}{y-1}\right) = y$

Hence f is onto.

OR

i. x is greater than y, x, y $\in$ N

For xRx $x > x$ is not true for any x $\in$ N.

Therefore, R is not reflexive.

Let (x, y) $\in$ R $\Rightarrow$ xRy

$x > y$

but $y > x$ is not true for any x, y $\in$ N

Thus, R is not symmetric.

Let xRy and yRz

$x > y$ and $y > z \Rightarrow x > z$

$\Rightarrow$ xRz

So, R is transitive.

ii. $x + y = 10$, x, y $\in$ N

R={(x,y) : x+y = 10 ,x,y $\in$ R}

R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)} (1, 1) $\notin$ R

So, R is not reflexive.

(x, y) $\in$ R $\Rightarrow$ (y, x) $\in$ R

Therefore, R is symmetric.

(1, 9) $\in$ R, (9, 1) $\in$ R $\Rightarrow$ (1, 1) $\notin$ R

Hence, R is not transitive.

iii. Given xy, is square of an integer x, y $\in$ N

$\Rightarrow$ R = {(x, y) : xy is a square of an integer x, y $\in$ N}

(x, x) $\in$ R, $\forall$x $\in$ N

As x^2 is square of an integer for any x $\in$ N

Hence, R is reflexive.

If (x, y) $\in$ R $\Rightarrow$ (y, x) $\in$ R

Therefore, R is symmetric.

If (x, y) $\in$ R (y, z) $\in$ R

So, xy is square of an integer and yz is square of an integer.

Let $xy = m^2$ and $yz = n^2$ for some m, n $\in$ Z

$x = \frac{m^2}{y}$ and $z = \frac{x^2}{y}$

$xz = \frac{m^2 n^2}{y^2}$, Which is square of an integer.

So, R is transitive.

iv. $x + 4y = 10$, x, y $\in$ N

R = {(x, y) : x + 4y = 10, x, y $\in$ N}

R{(2, 2), (6, 1)}

(1, 1), (3, 3) ..... $\notin$ R

Thus, R is not reflexive.

(6, 1) $\in$ R but (1, 6) $\notin$ R

Hence, R is not symmetric.

(x, y) $\in$ R $\Rightarrow$ x + 4y = 10 but (y, z) $\in$ R

y + 4z = 10 $\Rightarrow$ (x, z) $\in$ R

So, R is transitive.

35. Let, $I = \int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx$

Put, t = cos x

dt = -sin x dx

$I = \int \frac{(-dt)t}{t^2 - t - 2} = , -\int \frac{tdt}{(t+1)(t-2)}$

Using partial fractions,

$\frac{-t}{(t+1)(t-2)} = \frac{A}{t+1} + \frac{B}{t-2}$ .... (1)

A(t - 2) + B(t + 1) = -t

Now put, t - 2 = 0

Therefore, t = 2

A(0) + B(2 + 1) = -2

$B = \frac{-2}{3}$

Now put t + 1 = 0

Therefore, t = -1

A (-1 - 2) + B (0) = 1

$A = \frac{-1}{3}$

Now From equation (1) we get,

$$\frac{-t}{(t+1)(t-2)} = \frac{-1}{3} \times \frac{1}{t+1} - \frac{2}{3} \times \frac{1}{t-2}$$

$$\int \frac{-t}{(t+1)(t-2)} dt = \frac{-1}{3} \int \frac{1}{t+1} - \frac{2}{3} \int \frac{1}{t-2}$$

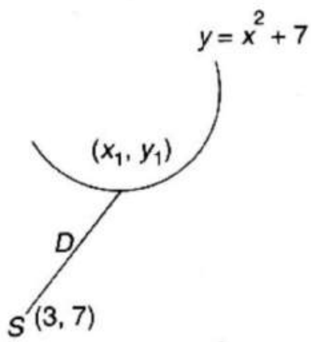
So,

$$I = \int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx = \frac{-1}{3} \log|\cos x + 1| - \frac{2}{3} \log|\cos x - 2| + c$$

Section E

36. **Read the text carefully and answer the questions:**

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) $P(x_1, y_1)$ is on the curve $y = x^2 + 7 \Rightarrow y_1 = x_1^2 + 7$

Distance from $p(x_1, x_1^2 + 7)$ and (3, 7)

$$D = \sqrt{(x_1 - 3)^2 + (x_1^2 + 7 - 7)^2}$$

$$\Rightarrow \sqrt{(x_1 - 3)^2 + (x_1^2)^2}$$

$$\Rightarrow D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$$

(ii) $D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

$D' = x_1^4 + x_1^2 - 6x_1 + 9$

$$\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6 = 0$$

$$\frac{dD'}{dx} = 2x_1^3 + x_1 - 3 = 0$$

$$\Rightarrow (x_1 - 1)\left(2x_1^2 + 2x_1 + 3\right) = 0$$

$x_1 = 1$ and $2x_1^2 + 2x_1 + 3 = 0$ gives no real roots

The critical point is (1, 8).

(iii) $$\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6$$

$$\frac{d^2D'}{dx^2} = 12x_1^2 + 2$$

$$\left.\frac{d^2D'}{dx^2}\right]_{x_1=1} = 12 + 2 = 14 > 0$$

Hence distance is minimum at (1, 8).

OR

$$D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$$

$$D = \sqrt{1 + 1 - 6 + 9} = \sqrt{5} \text{ units}$$

37. **Read the text carefully and answer the questions:**

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.


(i) Let F be the matrix representing the number of family members and R be the matrix representing the requirement of calories and proteins for each person. Then

$$F = \begin{matrix} \\ \text{Family A} \\ \text{Family B} \end{matrix} \begin{matrix} \text{Men Women Children} \\ \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \end{matrix}$$

$$R = \begin{matrix} \\ \text{Man} \\ \text{woman} \\ \text{Children} \end{matrix} \begin{matrix} \text{Calories Proteins} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{matrix}$$

(ii) The requirement of calories and proteins for each of the two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400 + 1900 + 1800) & 4(45 + 55 + 33) \\ 2(2400 + 1900 + 1800) & 2(45 + 55 + 33) \end{bmatrix}$$

$$FR = \begin{matrix} \text{Calories Proteins} \\ \begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix} \end{matrix} \begin{matrix} \\ \text{Family A} \\ \text{Family B} \end{matrix}$$

(iii)

$$R' = \begin{bmatrix} 2400 - 2400 \times 5\% & 45 + 45 \times 5\% \\ 1900 - 1900 \times 5\% & 55 + 55 \times 5\% \\ 1800 - 1800 \times 5\% & 33 + 33 \times 5\% \end{bmatrix}$$

$$\Rightarrow R' = \begin{bmatrix} 2400 - 120 & 45 + 2.25 \\ 1900 - 95 & 55 + 2.75 \\ 1800 - 90 & 33 + 1.65 \end{bmatrix}$$

$$\Rightarrow R' = \begin{matrix} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{matrix} \begin{matrix} \text{Calories Proteins} \\ \begin{bmatrix} 2280 & 45.25 \\ 1805 & 55.75 \\ 1710 & 34.65 \end{bmatrix} \end{matrix}$$

OR

Since, AB = B ...(i) and BA = A ..(ii)

$\therefore A^2 + B^2 = A \cdot A + B \cdot B$

= A(BA) + B(AB) [using (i) and (ii)]

- (AB)A + (BA)B [Associative law]

$= BA + AB$ [using (i) and (ii)]
$= A + B$

38. **Read the text carefully and answer the questions:**
Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) $P(A) = \frac{1}{3}$, $P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B) = \frac{1}{2}$, $P(b') = 1 - \frac{1}{3} = \frac{1}{2}$

P(Both are selected) $= P(A \cap B) = P(A) \cdot P(B) = \frac{1}{3} \cdot \frac{1}{2}$

P(Both are selected) $= \frac{1}{6}$

(ii) $P(A) = \frac{1}{3}$, $P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B) = \frac{1}{2}$, $P(b') = 1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) $= P(A' \cap B') = P(A') \cdot P(B') = \frac{2}{3} \cdot \frac{1}{2}$

P(Both are selected) $= \frac{1}{3}$