**CBSE Test Paper 01**

**Chapter 5 Continuity and Differentiability**

1. Let f (x + y) = f(x) + f(y) $\forall$ x, y $\in \mathbf{R}$. Suppose that f (6) = 5 and f ‘ (0) = 1, then f ‘ (6) is equal to
   a. 1
   b. 30
   c. None of these
   d. 25
2. Derivative of log|x| w.r.t. |x| is
   a. None of these
   b. $\frac{1}{x}$
   c. $\pm\frac{1}{x}$
   d. $\frac{1}{|x|}$
3. The function f (x) = 1 + |sin x| is
   a. differentiable everywhere
   b. continuous everywhere
   c. differentiable nowhere
   d. continuous nowhere
4. $\underset{x\to 0}{Lt} \frac{1-\cos x}{x\sin x}$ is equal to
   a. 1
   b. 2
   c. 0
   d. $\frac{1}{2}$
5. $\underset{x\to \pi/4}{Lt} \frac{\cos x-\sin x}{x-\frac{\pi}{4}}$ is equal to
   a. $-\frac{2}{\sqrt{2}}$

b. -1

c. $-\frac{1}{\sqrt{2}}$

d. $\frac{2}{\sqrt{2}}$

6. The value of c in Mean value theorem for the function f(x) = x(x - 2), x $\in$[1, 2] is ________.

7. The set of points where the function f given by f(x) = |2x - 1| sin x is differentiable is ________.

8. Differential coefficient of sec ($\tan^{-1}$x) w.r.t. x is ________.

9. Discuss the continuity of the function $f(x) = \sin x . \cos x$.

10. Determine the value of 'k' for which the following function is continuous at x = 3 : f(x) = $\begin{cases} \frac{(x+3)^2-36}{x-3}, x \neq 3 \\ k \quad , x = 3 \end{cases}$.

11. Determine the value of the constant 'k' so that the function f(x) = $\begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at x= 0.

12. Find $\frac{dy}{dx}$, $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

13. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

14. Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, if\ x \neq 0 \\ 0, if\ x = 0 \end{cases}$ is a continuous function.

15. Find the value of k so that the following function is continuous at x = 2.

f(x) = $\begin{Bmatrix} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{Bmatrix}$

16. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

17. If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

18. Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$.

## Solution

1. a. 1

   **Explanation:** $f'(6) = \lim_{h \to 0} \frac{f(6+h) - f(6)}{h}$ = $\lim_{h \to 0} \frac{f(6+h) - f(6+0)}{h}$
   $= \lim_{h \to 0} \frac{f(6) + f(h) - \{f(6) + f(0)\}}{h}$
   $= \lim_{h \to 0} \frac{f(h) - f(0)}{h} = f'(0) = 1$

2. d. $\frac{1}{|x|}$

   **Explanation:** $\frac{d}{d|x|}(\log|x|) = \frac{1}{|x|}$

3. b. continuous everywhere

   **Explanation:** f(x) = 1 + |sinx| is not derivable at those x for which x for which sinx = 0, however, 1 + |sinx| is continuous everywhere (being the sum of two continuous functions)

4. d. $\frac{1}{2}$

   **Explanation:** $\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$
   $= \lim_{x \to 0} \frac{1 - \cos^2 x}{x \sin x (1 + \cos x)} \lim_{x \to 0} \frac{\sin x}{x} . \frac{1}{1 + \cos x} = 1 . \frac{1}{1+1} = \frac{1}{2}$

5. a. $-\frac{2}{\sqrt{2}}$

   **Explanation:** $\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}}$
   $= \lim_{x \to \frac{\pi}{4}} \frac{-\sin x - \cos x}{1} = -\sin \frac{\pi}{4} - \cos \frac{\pi}{4} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

6. $\frac{3}{2}$
7. R - $\left\{\frac{1}{2}\right\}$
8. $\frac{x}{\sqrt{1+x^2}}$
9. Since sin x and cos x are continuous functions and product of two continuous function is a continuous function, therefore $f(x) = \sin x . \cos x$ is a continuous function.

10. Given, f(x) = $\begin{cases} \frac{(x+3)^2-36}{x-3}, & x \neq 3 \\ x, & x = 3 \end{cases}$

We shall use definition of continuity to find the value of k.

If f(x) is continuous at x = 3,

Then, we have $\lim_{x\to 3} f(x) = f(3)$

$\Rightarrow \quad \lim_{x\to 3} \frac{(x+3)^2-36}{x-3} = k$

$\Rightarrow \quad \lim_{x\to 3} \frac{(x+3)^2-6^2}{x-3} = k$

$\Rightarrow \lim_{x\to 3} \frac{(x+3-6)(x+3+6)}{x-3} = k$ [∵ $a^2$ - $b^2$ = (a - b)(a + b)]

$\Rightarrow \quad \lim_{x\to 3} \frac{(x-3)(x+9)}{(x-3)} = k$

$\Rightarrow \quad \lim_{x\to 3}(x+9) = k$

⇒ 3 + 9 = k ⇒ k = 12

11. Let f(x) = $\begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ be continuous at x = 0

Then, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

$\Rightarrow \quad \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) = f(0)$

$\Rightarrow \quad 3 = \lim_{h\to 0} \frac{k(-h)}{|-h|} = 3$

$\Rightarrow \quad \lim_{h\to 0}\left(\frac{-kh}{h}\right) = 3$

$\lim_{h\to 0}(-k) = 3$

∴ k = - 3

12. Given: $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Putting $x = \tan\theta$

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$

13. Let $f(x) = x^2$ and $g(x) = \cos x$, then

$(gof)(x) = g[f(x)] = g(x^2) = \cos x^2$

Now f and g being continuous it follows that their composite (gof) is continuous.

Hence $\cos x^2$ is continuous function.

14. Here, $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x^2 \sin\frac{1}{x} = 0$ x a finite quantity = 0

$\left[\because \sin\frac{1}{x} \text{lies between - 1 and 1}\right]$

Also f(0) = 0

Since, $\lim\limits_{x \to 0} f(x) = f(0)$ therefore, the function f is continuous at x = 0.

Also,when $x \neq 0$ ,then f(x) is the product of two continuous functions and hence Continuous.Hence,f(x) is continuous everywhere.

15. According to the question, f(x) = $\left\{\begin{array}{cc} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{array}\right\}$ is continuous at $x = 2$.

Now, we have f(2) = k

$\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \frac{x^3+x^2-16x+20}{(x-2)^2}$

$= \lim\limits_{x \to 2} \frac{(x-2)(x^2+3x-10)}{(x-2)^2}$

$= \lim\limits_{x \to 2} \frac{(x-2)(x+5)(x-2)}{(x-2)^2}$

$= \lim\limits_{x \to 2}(x+5)$ = 2+ 5 = 7

f(x) is continuous at x = 2.

$\therefore \lim\limits_{x \to 2} f(x)$= f(2) $\Rightarrow$ 7 = k$\Rightarrow$k = 7

16. We have, $x^y + y^x = a^b$.........(i)

Let $x^y = v$ and $y^x = u$......(ii)

Therefore,on putting these values in Eq. (i), we get,

$v + u = a^b$

Therefore,on differentiating both sides w.r.t. x, we get,

$\frac{dv}{dx} + \frac{du}{dx} = 0$........(iii)

Now consider, $x^y = v$ [ from Eq.(ii)]

Therefore,on taking log both sides, we get,

$\log x^y = \log v$

$\Rightarrow$ y log x = log v

Therefore,on differentiating both sides w.r.t. x, we get,

$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = \frac{1}{v}\frac{dv}{dx}$

$\Rightarrow v\left(\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right) = \frac{dv}{dx}$

$\Rightarrow \quad \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$.........(iv) [ From Eq.(ii)]

Also, $y^x = u$ [From Eq(ii)]

Therefore,on taking log both sides, we get,

$\log y^x = \log u \Rightarrow x \log y = \log u$

Therefore,on differentiating both sides w.r.t. 'x', we get,

$$x \cdot \frac{1}{y}\frac{dy}{dx} + 1 \cdot \log y = \frac{1}{u}\frac{du}{dx}$$
$$\Rightarrow \quad \frac{x}{y}\frac{dy}{dx} + \log y = \frac{1}{u}\frac{du}{dx}$$
$$\Rightarrow \quad u\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = \frac{dy}{dx}$$
$$\Rightarrow \quad y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = \frac{du}{dx} \text{ ........(v) [ From Eq(ii)]}$$

Therefore,on substituting the values of $\frac{dv}{dx}$ and $\frac{du}{dx}$ from Eqs. (iv) and (v) respectively in Eq. (iii), we get

$$x^y\left(\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$
$$\Rightarrow x^y\frac{y}{x} + x^y \log x \cdot \frac{dy}{dx} + y^x \cdot \frac{x}{y}\frac{dy}{dx} + y^x \log y = 0$$
$$\Rightarrow x^y \log x \cdot \frac{dy}{dx} + y^x\frac{x}{y} \cdot \frac{dy}{dx} = -x^y\frac{y}{x} - y^x \log y$$
$$\Rightarrow \quad \frac{dy}{dx}\left[x^y \log x + y^x \cdot \frac{x}{y}\right] = -x^y \cdot \frac{y}{x} - y^x \log y$$
$$\therefore \quad \frac{dy}{dx} = \frac{-x^{y-1} \cdot y - y^x \log y}{x^y \log x + y^{x-1} \cdot x}$$

17. According to the question, $e^y(x+1) = 1$

Taking log both sides,

$$\Rightarrow log[e^y(x+1)] = log1$$
$$\Rightarrow loge^y + log(x+1) = log1$$
$$\Rightarrow y + log(x+1) = log1 \text{ [} \because \log e^y = y\text{]}$$

differentiating both sides w.r.t. x,

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x+1} = 0 \text{.........(i)}$$

Differentiating both sides w.r.t. 'x',

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{(x+1)^2} = 0$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} - \left(-\frac{dy}{dx}\right)^2 = 0 \text{ [ From Equation(i)]}$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

18. Let $u = y^x, v = x^y, w = x^x$

$u+v+w=a^b$

Therefore $\frac{du}{dx}+\frac{dw}{dx}+\frac{dv}{dx}=0$ ....(1)

$u=y^x$

Taking log both side

$\log u=\log y^x$

$\log u=x.\log y$

Differentiate both side w.r.t. to x

$\frac{1}{u}.\frac{du}{dx}=x.\frac{1}{y}.\frac{dy}{dx}+\log y.1$

$\frac{du}{dx}=u\left[\frac{x}{y}.\frac{dy}{dx}+\log y\right]$

$\frac{du}{dx}=y^x\left[\frac{x}{y}.\frac{dy}{dx}+\log y\right]$.... (2)

$v=x^y$

Taking log both side

$\log v=\log x^y$

$\log v=y.\log x$

$\frac{1}{v}.\frac{dv}{dx}=y.\frac{1}{x}+\log x.\frac{dy}{dx}$

$\frac{dv}{dx}=v\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right]$

$\frac{dv}{dx}=x^y\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right]$.... (3)

$w=x^x$

Taking log both side

$\log w=\log x^x$

$\log w=x\log x$

$\frac{1}{w}.\frac{dw}{dx}=x.\frac{1}{x}+\log x.1$

$\frac{1}{w}.\frac{dw}{dx}=1+\log x$

$\frac{dw}{dx}=w(1+\log x)$

$\frac{dw}{dx}=x^x(1+\log x)$.... (4)

$\frac{dy}{dx}=\frac{-x^x(1+\log x)-y.x^{y-1}-y^x\log y}{x.y^{x-1}+x^y\log x.}$ (by putting 2,3 and 4 in 1)