DIFFERENTIAL EQUATIONS AND THEIR FORMATION
(XII, R. S. AGGARWAL)

EXERCISE 18A (Pg. No.: 896)

Write order and degree (if defined) of each of the following differential equations

1. $\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$

Sol. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 1

$\therefore$ its order = 2 and degree = 1

2. $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$

Sol. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 2

$\therefore$ its order = 2 and degree = 2

3. $\left(\frac{d^2s}{dt^2}\right)^2 + \left(\frac{ds}{dt}\right)^3 + 4 = 0$

Sol. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 2

$\therefore$ its order = 2 and degree = 2

4. $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y^5 = 0$

Sol. In the given equation, the highest-order derivative is $\frac{d^3y}{dx^3}$ and its power is 2

$\therefore$ its order = 3 and degree = 2

5. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y = 0$

Sol. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 1

$\therefore$ its order = 2 and degree = 1

6. $\frac{dy}{dx} + y = e^x$

Sol. In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 1

$\therefore$ its order = 1 and degree = 1

7. $\frac{d^2y}{dx^2} + y^2 + e^{(dy/dx)} = 0$

Sol. In this equation, the highest order derivative is $\frac{d^2y}{dx^2}$, so its order is 2

It has a term $e^{(dy/dx)}$, so its degree is not defined

8. $\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)=0$

Sol. In this equation, the highest order derivative is $\frac{dy}{dx}$, so its order is 1

It has a term $\sin\left(\frac{dy}{dx}\right)$, so its degree is not defined.

9. $\frac{d^4y}{dx^4}-\cos\left(\frac{d^3y}{dx^3}\right)=0$

Sol. In this equation, the highest order derivative is $\frac{d^4y}{dx^4}$, so its order is 4

It has a term $\cos\left(\frac{d^3y}{dx^3}\right)$, so its degree is not defined.

10. $\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$

Sol. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 1

$\therefore$ its order = 2 and degree = 1

11. $\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$

Sol. In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 3

$\therefore$ its order = 1 and degree = 3

12. $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

Sol. In the given equation, the highest-order derivative is $\frac{d^3y}{dx^3}$ and its power is 1

$\therefore$ its order = 3 and degree = 1

13. $x\left(\frac{dy}{dx}\right)+\frac{2}{\left(\frac{dy}{dx}\right)}+9=y^2$

Sol. Given differential equation may be written as $x\left(\frac{dy}{dx}\right)^2+\left(9-y^2\right)\left(\frac{dy}{dx}\right)+2=0$

In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 2

$\therefore$ its order = 1 and degree = 2

14. $\sqrt{1-\left(\frac{dy}{dx}\right)^2}=\left(a\frac{d^2y}{dx^2}\right)^{1/3}$

Sol. Given differential equation may be written as $\left(1-\left(\frac{dy}{dx}\right)^2\right)^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 2

$\therefore$ its order = 2 and degree = 2

15. $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$

Sol. Given differential equation may be written as $\sqrt{1-y^2} + \sqrt{1-x^2}\frac{dy}{dx} = 0$

In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 1

$\therefore$ its order = 1 and degree = 1

16. $(y'')^3 + (y')^2 + \sin y' + 1 = 0$

Sol. In this equation, the highest order derivative is (y''), so its order is 2

It has a term $\sin y'$, so its degree is not defined

17. $(3x+5y)dy - 4x^2dx = 0$

Sol. Given differential equation may be written as $(3x+5y)\frac{dy}{dx} - 4x^2 = 0$

In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 1

$\therefore$ its order = 1 and degree = 1

18. $y = \frac{dy}{dx} + \frac{5}{\left(\frac{dy}{dx}\right)}$

Sol. Given differential equation may be written as $y\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right)^2 + 5$

In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 2

$\therefore$ its order = 1 and degree = 2

EXERCISE 18B (Pg. No.: 902)

1. Verify that $x^2 = 2y^2 \log y$ is a solution of the differential equation $\left(x^2+y^2\right)\frac{dy}{dx} - xy = 0$

Sol. Given equation is $x^2 = 2y^2 \log y$;

Differentiating both sides w.r.t. x we get $\frac{d}{dx}\left(x^2\right) = 2\frac{d}{dx}\left\{y^2 . \log y\right\}$

$\Rightarrow 2x = 2\left[y^2 \frac{d}{dx}(\log y) + \log y \frac{d}{dx}\left(y^2\right)\right] \Rightarrow 2x = 2\left[y^2 \frac{2}{y}\frac{dy}{dx} + 2y \log \frac{dy}{dx}\right]$

$\Rightarrow \frac{2x}{2} = y\frac{dy}{dx} + 2y \log y \frac{dy}{dx} \Rightarrow x = y\frac{dy}{dx} + 2y\left\{\frac{x^2}{2y^2}\right\}\frac{dy}{dx}$

$\Rightarrow x = y\frac{dy}{dx} + \frac{x^2}{y}\frac{dy}{dx} \Rightarrow xy = y^2\frac{dy}{dx} + x^2\frac{dy}{dx} \Rightarrow xy = \left(x^2+y^2\right)\frac{dy}{dx} \Rightarrow \left(x^2+y^2\right)\frac{dy}{dx} - xy = 0$

Hence $x_2 = 2y^2 \log y$ i.e. a solution of the differential equation $\left(x^2+y^2\right)\frac{dy}{dx} - xy = 0$

2. Verify that $y = e^x \cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Sol. Given equation $y = e^x \cos x$ .... (i)

Differentiating both sides w.r.t x we have $\frac{dy}{dx} = e^x \frac{d}{dx}(\cos bx) + \cos bx \frac{d}{dx}\left(e^x\right)$

$\Rightarrow \frac{dy}{dx} = -e^x \sin bx + e^x \cos x = e^x\left[-\sin bx + \cos x\right]$ .... (ii)

Differentiating both sides w.r.t x we get $\frac{d^2y}{dx^2} = -\frac{d}{dx}\left(e^x \sin x\right) + \frac{d}{dx}\left(e^x \cos x\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\left\{e^x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}e^x\right\} + e^x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}\left|e^x\right|$

$\Rightarrow \frac{d^2y}{dx^2} = -e^x \cos x - e^x \sin x - e^x \sin x + e^x \cos x$

$= e^x\left\{-\cos x - 2\sin x + \cos x\right\}$ .... (iii)

L.H.S. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

$= e^x\left[-\cos x - 2\sin x + \cos x\right] - e^x\left[-\sin x + \cos x\right] + 2e^x \cdot \cos bx$

$= e^x\left[-\cos x - 2\sin bx + \cos x + 2\sin x - 2\cos x + 2\cos x\right] = e^x \times 0 = 0 = R.H.S$

Hence given equation is a solution of given differential equation

3. Verify that $y = e^{m\cos^{-1}x}$ is a solution of the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

Sol. Given equation is $y = e^{m\cos^{-1}x}$ ..... (i)

Differencing both sides w.r.t x we have $\frac{dy}{dx} = e^{m\cos^{-1}x} \cdot \frac{d}{dx}\left(m\cos^{-1}x\right)$

$\Rightarrow \frac{dy}{dx} = \frac{-m \cdot e\cos^{-1}x}{\sqrt{1-x^2}}$ .... (ii)

Differentiating both sides w.r.t x we get $\frac{d^2y}{dx^2}=\frac{d}{dx}\left[\frac{-m\cdot e}{\sqrt{1-x^2}}\right]$

$$\Rightarrow \frac{d^2y}{dx^2}=-m\left[\frac{\sqrt{1-x^2}\frac{d}{dx}\left|e^{m\cos^{-1}x}\right|-e^{m\cos^{-1}x}\frac{d}{dx}\sqrt{1-x^2}}{\left(1-x^2\right)}\right]$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}=-m\left[\frac{-m\sqrt{1-x^2}\cdot e^{m\cos^{-1}x}}{\sqrt{1-x^2}}+\frac{x\cdot e^{m\cos^{-1}x}}{\sqrt{1-x^2}}\right]\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}=m^2\cdot e^{m\cos^{-1}x}-\frac{mx\cdot e^{m\cos^{-1}x}}{\sqrt{1-x^2}}$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}=m^2y+x\cdot\frac{dy}{dx}\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$$

Hence $y=e^{m\cos^{-1}x}$ is a solution of the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$

4. Verify that $y=\left(a+bx\right)e^{2x}$ is the general solution of the differential equation

Sol. Given equation is $y=\left(a+bx\right)\cdot e^{2x}$

$$\Rightarrow e^{-2x}y=a+bx$$

Differentiatiy both sides w.r.t x we have $e^{-2x}\frac{dy}{dx}+y\frac{d}{dx}\left(e^{-2x}\right)=b$

$$\Rightarrow e^{-2x}\frac{dy}{dx}+y\cdot e^{-2x}.\left(-2\right)=b\Rightarrow e^{-2x}\frac{dy}{dx}-2y\cdot e^{-2x}=b$$

Differentiating both sides w.r.t x we have $e^{-2x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{d}{dx}\left(e^{-2x}\right)-2\left\{y\cdot\frac{d}{dx}\left(e^{-2x}\right)+e^{-2x}\frac{dy}{dx}\right\}=0$

$$\Rightarrow e^{-2x}\frac{d^2y}{dx^2}-2\cdot e^{-2x}\frac{dy}{dx}+4\cdot e^{-2x}y-2.e^{-2x}\frac{dy}{dx}=0\Rightarrow e^{-2x}\left[\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y\right]=0\Rightarrow\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4=0$$

Hence the given equation is the solution of given differential equation

5. Verify that $y=e^x\left(A\cos x+B\sin x\right)$ is the general solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

Sol. Given equation is $y=e^x\left\{A\cdot\cos x+B\sin x\right\}$

$$\Rightarrow e^{-x}\cdot y=A\cdot\cos x+B\cdot\sin x$$

Differentiate both sides w.r.t x we get

$$\Rightarrow e^{-x}\cdot\frac{dy}{dx}+y\cdot\frac{d}{dx}\left(e^{-x}\right)=-A\sin x+B\cos x$$

Differentiate both sides w.r.t x we get

$$\left\{e^{-x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{d}{dx}\left(e^{-x}\right)\right\}-\left[y\cdot\frac{d}{dx}\left(e^{-x}\right)+e^{-x}\frac{dy}{dx}\right]=-A\cos x-B\sin x$$

$$\Rightarrow\left\{e^{-x}\cdot\frac{d^2y}{dx^2}-e^{-x}\cdot\frac{dy}{dx}\right\}-\left\{-y\cdot e^{-x}+e^{-x}\cdot\frac{dy}{dx}\right\}=-A\cos x-B\sin x$$

$$\Rightarrow e^{-x}\left\{\frac{d^2y}{dx^2}-\frac{dy}{dx}+y-\frac{dy}{dx}\right\}=-A\cos x-B\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=-e^x\left(A\cos x+B\sin x\right) \Rightarrow \frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=-y \Rightarrow \frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

Hence the given equation is a solution of differential equation of given differential equation

6. Verify that $y=A\cos 2x-B\sin 2x$ is the general solution of the differential equation $\frac{d^2y}{dx^2}+4y=0$

Sol. Given equation is $y=a\cos 2x-B\sin 2x$

Differentiatiy both sides w.r.t x we get

$$\Rightarrow \frac{dy}{dx}=-2A\cdot\sin 2x-2B\cos 2x$$

Diff. both sides w.r.t x we get $\frac{d^2y}{dx^2}=-4A\cos 2x+4B\sin 2x$

$$\Rightarrow \frac{d^2y}{dx^2}=-4\left\{A\cdot\cos 2x-B\sin 2x\right\} \Rightarrow \frac{d^2y}{dx^2}=-4y \Rightarrow \frac{d^2y}{dx^2}+4y=0$$

Hence $y=A\cdot\cos 2x-B\sin 2x$ is a solution of $\frac{d^2y}{dx^2}+4y=0$

7. Verify that $y=ae^{2x}+be^{-x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$

Sol. Given differential equation is $y=a\cdot e^{2x}+b\cdot e^{-x}$

Differentialiy both sides w.r.t x we get $\frac{dy}{dx}=2a\cdot e^{2x}=b\cdot e^{-x}$ .... (ii)

Again differentiatiy both sides w.r.t x, we get

$$\frac{d^2y}{dx^2}=4a.e^{2x}+b\cdot e^{-x} \quad \text{......(iii)}$$

Now, L.H.S. $=\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$

$=4a\cdot e^{2x}+b\cdot e^{-x}-2a\geq e^{2x}+b\cdot e^{-x}-2a\cdot e^{2x}-2b\cdot e^{-x}=0=$ R.H.S

Hence the given equation is the solution of given differential equation

8. Verify that $y=e^x\left(A\ \cos x+B\ \sin x\right)$ is a solution of the differential equations $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0.$

Sol. $y=e^x\left(A\ \cos x+B\ \sin x\right)$ ...(1)

Differentiating both sides of (1), we get, $\frac{dy}{dx}=e^x\left(-A\ \sin x+B\ \cos x\right)+\left(A\ \cos x+B\ \sin x\right)e^x$

$$\Rightarrow \frac{dy}{dx}=e^x\left(-A\ \sin x+B\ \cos x\right)+y \quad \text{...(2)}$$

Again differentiating both sides of (2), we get,

$$\frac{d^2y}{dx^2}=e^x\left(-A\ \cos x-B\ \sin x\right)+\left(-A\ \sin x+B\ \cos x\right)e^x+\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-e^x\left(A\ \cos x+B\ \sin x\right)+\left(-A\ \sin x+B\ \cos x\right)e^x+\frac{dy}{dx} \quad \text{...(3)}$$

From equation (2), $\frac{dy}{dx}-y=e^x\left(-A\ \sin x+B\ \cos x\right)$

Now, Putting the value of $e^x\left(-A\ \sin x+B\ \cos x\right)$ in equation (3),

$$\frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx} \quad \Rightarrow \frac{d^2y}{dx^2} = -2y + 2\frac{dy}{dx} \quad \therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

9. Verify that $y^2 = 4a(x+a)$ is a solution of the differential equation $y\left\{1-\left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$.

Sol. $y^2 = 4a(x+a)$ ...(1)

Differentiating both sides (1), we get, $2y\frac{dy}{dx} = 4a$

$\Rightarrow 2y\frac{dy}{dx} = 4a \quad \Rightarrow y\frac{dy}{dx} = 2a \quad \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$ ...(2)

$\therefore$ Putting the value of (2) in $y\left\{1-\left(\frac{dy}{dx}\right)^2\right\}$

$$\text{LHS} = y\left\{1-\left(\frac{dy}{dx}\right)^2\right\} = y\left\{1-\left(\frac{2a}{y}\right)^2\right\} = y\left(\frac{y^2-4a^2}{y^2}\right) = \frac{4a(x+a)-4a^2}{y}$$

$\Rightarrow \frac{4ax}{y} = 2x\left(\frac{2a}{y}\right) = 2x\frac{dy}{dx}$. Hence proved.

10. Verify that $y = ce^{\tan^{-1}x}$ is a solution of the differential equation $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$.

Sol. $y = ce^{\tan^{-1}x}$ ...(1)

Differentiating both sides of (1), we get, $\frac{dy}{dx} = ce^{\tan^{-1}x}.\frac{1}{(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{(1+x^2)} \quad \Rightarrow (1+x^2)\frac{dy}{dx} = y$ ...(2)

Again differentiating both sides of (2), we get, $(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{dy}{dx}$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2x\frac{dy}{dx} \quad \Rightarrow (1+x^2)\frac{d^2y}{dx^2} = \frac{dy}{dx}(1-2x) \quad \therefore (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

11. Verify that $y = e^{bx}$ is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

Sol. Given equation is $y = e^{bx}$ ..... (i)

Differentiating both sides w.r.t x we get $\frac{dy}{dx} = b \cdot e^{bx}$ .... (ii)

Differentiating both sides w.r.t. x we get $\frac{d^2y}{dx^2} = b^2 \cdot e^{bx}$ ..... (iii)

Now R.H.S. $= \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = \frac{1}{e^{bx}}\{b \cdot e^{bx}\}^2 = b^2 \cdot e^{bx} = \frac{d^2y}{dx^2} = L.H.S$

Hence $y = e^{bx}$ is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

12. Verify that $y = \frac{a}{x} + b$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

Sol. Given differential equation is $y=\frac{a}{x}+b$

Differentiating both sides w.r.t. x we get $\frac{dy}{dx}=\frac{-a}{x^2}$

$\Rightarrow x^2.\frac{dy}{dx}=-a$

Again differentiating both sides w.r..t x we get $\frac{d}{dx}\left\{x^2\cdot\frac{dy}{dx}\right\}=0$

$\Rightarrow x^2\cdot\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$

Dividing both sides by x^2 we have $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}=0$

Hence the given equation is the solution of given differential equation

13. Verify that $y=e^{-x}+Ax+B$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)=1$.

Sol. $y=e^{-x}+Ax+B$ ...(1)

Differentiating both sides of (1), we get, $\frac{dy}{dx}=e^{-x}(-1)+A \Rightarrow \frac{dy}{dx}=-e^{-x}+A$ ...(2)

Again differentiating both sides of (2), we get, $\frac{d^2y}{dx^2}=-e^{-x}(-1)$

$\Rightarrow \frac{d^2y}{dx^2}=e^{-x} \Rightarrow \frac{d^2y}{dx^2}=\frac{1}{e^x} \quad \therefore e^x\left(\frac{d^2y}{dx^2}\right)=1$

14. Verify that $Ax^2+By^2=1$ is a solution of the differential equation $x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$

Sol. Given equation is $Ax^2+By^2=1$ .... (i)

Diff. both sides w.r.t x we get $2Ax+2By\frac{dy}{dx}=0$

$\Rightarrow Ax+By\frac{dy}{dx}=0$

Multiplying x on both sides $Ax^2+Bxy\frac{dy}{dx}=0$ .... (ii)

Subtracting (i) from (ii) we have $Bxy\frac{dy}{dx}-By^2=-1$

$\Rightarrow xy\frac{dy}{dx}-y^2=-\frac{1}{B}$

Differentiating both sides w.r.t x

$\frac{d}{dx}\left\{x\cdot y\cdot\frac{dy}{dx}\right\}-\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}\left(-\frac{1}{B}\right)$

$\Rightarrow x\cdot y\cdot\frac{d^2y}{dx^2}+x\frac{dy}{dx}\frac{dy}{dx}+y\frac{dy}{dx}-2y\frac{dy}{dx}=0 \Rightarrow x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$

Hence given equation is the solution of given differential equation

15. Verify that $y=\dfrac{c-x}{1+cx}$ is a solution of the differential equation $\left(1+x^2\right)\dfrac{dy}{dx}+\left(1+y^2\right)=0$

Sol. Given equation is $y=\dfrac{c-x}{1+cx}$

Differentiating both sides w.r..t x we get $\dfrac{dy}{dx}=\dfrac{d}{dx}\left\{\dfrac{c-x}{1+cx}\right\}$

$$\Rightarrow \frac{dy}{dx}=\frac{(1+cx)\cdot\frac{d}{dx}(c-x)-(c-x)\frac{d}{dx}(1+cx)}{(1+cx)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(1+cx)-c(c-x)}{(1+cx)^2} \Rightarrow \frac{dy}{dx}=\frac{-1-c-c^2+cx}{(1+cx)^2} \Rightarrow \frac{dy}{dx}=\frac{-\left(1+c^2\right)}{(1+cx)^2}$$

Now L.H.S. $=\left(1+x^2\right)\dfrac{dy}{dx}+\left(1+y^2\right)$

$$=-\left(1+x^2\right)\frac{\left(1+c^2\right)}{(1+cx)^2}+\left\{1+\left(\frac{c-x}{1+cx}\right)^2\right\}=\frac{-\left(1+x^2\right)\left(1+c^2\right)+(1+cx)^2+(c-x)^2}{(1+cx)^2}$$

$$=\frac{-1-c^2-x^2-c^2x^2+1+c^2x^2+2cx+c^2+x^2-2cx}{(1+cx)^2}=\frac{0}{(1+cx)^2}=0=\text{R.H.S}$$

Hence given equation is the solution of given diff. equation

16. Verify that $y=\log\left(x+\sqrt{x^2+a^2}\right)$ satisfies the differential equation $\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=0$

Sol. Given equation is $y=\log\left(x+\sqrt{x^2+a^2}\right)$

Differenting both sides w.r.t x we get $\dfrac{dy}{dx}=\dfrac{d}{dx}\left\{\log\left(x+\sqrt{x^2+a^2}\right)\right\}$

$$=\frac{1}{x+\sqrt{x^2+a^2}}\frac{d}{dx}\left\{x+\sqrt{x^2+a^2}\right\}=\frac{1}{x+\sqrt{x^2+a^2}}\left\{1+\frac{2x}{2\sqrt{x^2+a^2}}\right\}=\frac{1}{x\sqrt{x^2+a^2}}\cdot\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$$

$$=\frac{1}{\sqrt{x^2+a^2}} \qquad \ldots\text{(i)}$$

Again differentiating both sides w.r.t. x, we get $\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left\{\dfrac{1}{\sqrt{x^2+a^2}}\right\}$

$$=\frac{-1}{\left(x^2+a^2\right)}\times\frac{2x}{2\sqrt{x^2+a^2}}=\frac{-x}{\left(x^2+a^2\right)\sqrt{x^2+a^2}}$$

Now. L.H.S. $=\left(x^2+a^2\right)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=\dfrac{-x}{\sqrt{x^2+a^2}}+\dfrac{x}{\sqrt{x^2+a^2}}$

$=0=\text{R.H.S}$

Hence given equation is the solution of given differential equation

17. Verify that $y=e^{-3x}$ is a solution of the differential equation $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-6y=0$

Sol. Given equation is $y=e^{-3x}$ ..... (i)

Differentiating both sides w.r..t x, we get $\frac{dy}{dx} = -3 \cdot e^{-3x}$ … (ii)

Differentiating both sides w.r..t x we get $\frac{d^2y}{dx^2} = 9 \cdot e^{-3x}$ …. (iii)

Now L.H.S. $= \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$= 9 \cdot e^{-3x} - 3 \cdot e^{-3x} - 6 \cdot e^{-3x}$

$= 0 =$ R.H.S

Hence $y = e^{-3x}$ is a solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

EXERCISE 18C (Pg.No.: 910)

1. From the differential equation of the family of straight liens $y = mx + c$, where m and c are arbitrary constants

Sol. Given equation is $y = mx + c$

$$\Rightarrow \frac{dy}{dx} = m \Rightarrow \frac{d^2y}{dx^2} = 0$$

This is the required differential equation

2. From the differential equation of the family of concentric circles $x^2 + y^2 = a^2$, where $a > 0$ and a is a parameter

Sol. Given equation is $x^2 + y^2 = a^2$

Differentiating both sides w.r.t x we have

$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) = 0$$

$$\Rightarrow 2x + 2y \cdot \frac{dy}{dx} = 0 \Rightarrow x + y \cdot \frac{dy}{dx} = 0$$

This is the required differential equation

3. Form the differential equation of the family of cruves $y = a\sin(bx + c)$ where a and c are parameters

Sol. Given equation is $y = a \cdot \sin(bx + c)$ .... (i)

Differentiating both sides w.r.t x we get

$$\frac{dy}{dx} = ab\cos(bx + c) \quad \text{.... (ii)}$$

Differentiating both sides w.r.t x we get

$$\frac{d^2y}{dx^2} = -ab^2 \cdot \sin(bx + c)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -b^2 y \Rightarrow \frac{d^2y}{dx^2} + b^2 y = 0$$ this is the required differential equation

4. Form the differential equation of the family of curves $x = A\cos nt + B\sin nt$, where A and B are arbitary constants

Sol. Given equation is $x = A \cdot \cos nt + B\sin nt$ ..... (i)

$$\Rightarrow \frac{dx}{dt} = -nA \cdot \sin nt + nB\cos nt \quad \text{..... (ii)}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -n^2 A\cos nt - n^2 B\sin nt \Rightarrow \frac{d^2x}{dt^2} = -n^2\left\{A\cos nt + B\sin nt\right\}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -n^2 x$$

$$\Rightarrow \frac{d^2x}{dt^2} + n^2 x = 0$$ this is the required differential equation

5. Form the differential equation of the family of curve $y = ae^{bx}$, where a and b are arbitary constants

Sol. Given equation is $y = a \cdot e^{bx}$ ... (i)

$$\Rightarrow \frac{dy}{dx} = abe^{bx} \Rightarrow \frac{dy}{dx} = by \Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = b$$

Diff both sides w.r.t x we get $\frac{dy}{dx}\left\{\frac{1}{y}\cdot\frac{dy}{dx}\right\}=0$

$$\Rightarrow \frac{1}{y}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{d}{dx}\left(\frac{1}{y}\right)=0 \Rightarrow \frac{1}{y}\cdot\frac{d^2y}{dx^2}-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2=0 \Rightarrow \cdot\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=0$$

This is the required differential equation

6. Form the differential equation of the family of curve $y^2=m\left(a^2-x^2\right)$ where a and m are parameters

Sol. Given equation is $y^2=m\left(a^2-x^2\right)$ ..... (i)

Differentiating both sides w.r.t x we get $2y\frac{dy}{dx}=-2mx$

$\Rightarrow y\cdot\frac{dy}{dx}=-mx$ ....(ii)

Differentiating both sides w.r..t x we get $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-m$

$$\Rightarrow y\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{y}{x}\frac{dy}{dx} \Rightarrow xy\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

This is the required differential equation

7. Form the differential equation of the family of curve given by $(x-a)^2+2y^2=a^2$, where a is an arbitary constant

Sol. Given equation is $(x-a)^2+2y^2=a^2$

$\Rightarrow x^2+a^2-2ax+2y^2=a^2 \Rightarrow x^2-2ax+2y^2=0$

$\Rightarrow x^2+2y^2=2ax \Rightarrow \frac{x^2+2y^2}{x}=2a$

Differentiating both sides w.r..t x we get

$\frac{d}{dx}\left\{\frac{x^2+2y^2}{x}\right\}=0$

$$\Rightarrow \frac{x\cdot\frac{d}{dx}\left(x^2+2y^2\right)-\left(x^2+2y^2\right)\frac{dy}{dx}}{x^2}=0 \Rightarrow x\cdot\frac{d}{dx}\left\{x^2+2y^2\right\}-\left(x^2+2y^2\right)=0$$

$$\Rightarrow x\left\{2x+4y\frac{dy}{dx}\right\}-\left(x^2+2y^2\right)=0 \Rightarrow 2x^2+4xy\frac{dy}{dx}-x^2-2y^2=0$$

$$\Rightarrow 4xy\frac{dy}{dx}+x^2-2y^2=0 \Rightarrow \frac{dy}{dx}=\frac{2y^2-x^2}{4xy}$$

This is the required differential equation of the equation

8. Form the differential equation of the family of curve given by $x^2+y^2-2ay=a^2$, where a is an arbitary constant

Sol. Given family at waves is $x^2+y^2-2ay=a^2$ ..... (i)

Differentiating (i) w.r.t. x we get $2x+2y\cdot\frac{dy}{dx}-2a\frac{dy}{dx}=0$

$$\Rightarrow x+y\frac{dy}{dx}-a\frac{dy}{dx}=0 \Rightarrow a=\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Putting this value of a in equation (i) we have $x^2+y^2-2y\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}=\left(\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2$

$$\Rightarrow \left(x^2+y^2\right)\left(\frac{dy}{dx}\right)^2-2y\frac{dy}{dx}\left\{x+y\frac{dy}{dx}\right\}=\left(x+y\frac{dy}{dx}\right)^2$$

$$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2+y^2\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)-2y^2\left(\frac{dy}{dx}\right)^2=x^2+y^2\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}$$

$$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2-2y^2\left(\frac{dy}{dx}\right)^2-4xy\frac{dy}{dx}-x^2=0$$

$$\Rightarrow \left(x^2-2y^2\right)\left(\frac{dy}{dx}\right)^2-4xy\frac{dy}{dx}-x^2=0$$

This is the required differential equation

9. Form the differential equation of the family of all circles touching the y-axis at the origin

Sol. Equation of the family of all circles touching the y-axis at the origin is given by

$$(x-a)^2+y^2=a^2$$

$$\Rightarrow x^2-2ax+a^2+y^2=a^2$$

$$\Rightarrow x^2+y^2-2ax=0$$

$$\Rightarrow x^2+y^2=2ax \Rightarrow \frac{x^2+y^2}{x}=2a$$

Differentiating both sides ..

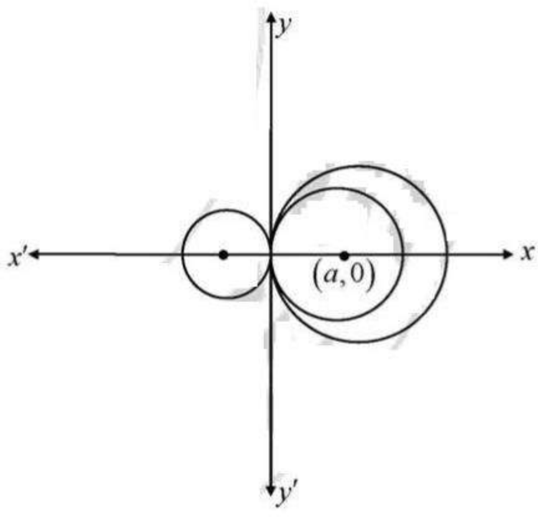
$$\frac{x\cdot\frac{d}{dx}\left(x^2+y^2\right)-\left(x^2+y^2\right)\frac{dy}{dx}}{x^2}=0$$

$$\Rightarrow x\left\{2x+2y\frac{dy}{dx}\right\}-x^2-y^2=0$$

$$\Rightarrow 2x^2+2xy\frac{dy}{dx}-x^2-y^2=0$$

$$\Rightarrow x^2-y^2+2xy\frac{dy}{dx}=0$$

This is the required differential equation



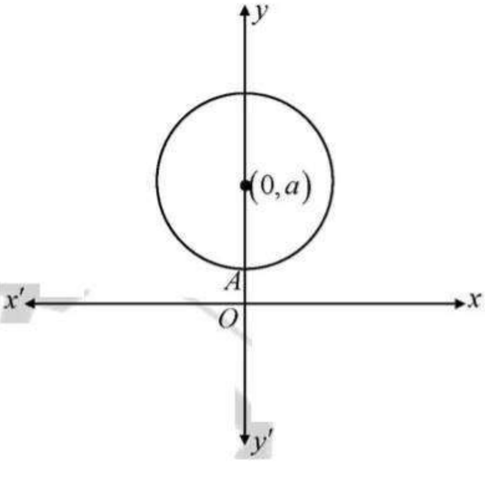
10. From the differential equation of the family of circles having centers on the y-axis and radius 2 units

Sol. Equation of family of circles having contres on the y axis and radius 2 units is given by

$$x^2+(y-a)^2=4 \qquad \text{..... (i)}$$

Differentiating both sides w.r.d. x we get

$$2x+2(y-a)\cdot\frac{dy}{dx}=0$$



$$\Rightarrow x+(y-a)\frac{dy}{dx}=0$$

$$\Rightarrow (y-a)=-\frac{x}{dy/dx} \quad \text{..... (ii)}$$

From (i) and (ii) we have $x^2+\left(\frac{-x}{dy/dx}\right)^2=4$

$$\Rightarrow x^2+\frac{x^2}{\left(\frac{dy}{dx}\right)^2}=4 \quad \Rightarrow x^2\left\{1+\left(\frac{dx}{dy}\right)^2\right\}=4$$

This is the required differential equation

11. Form the differential equation of the family of circles in second quadrant and touching the coordinate axes

Sol. The equation of the given family of circle is $(x+a)^2+(y-a)^2=a^2$ .... (i)

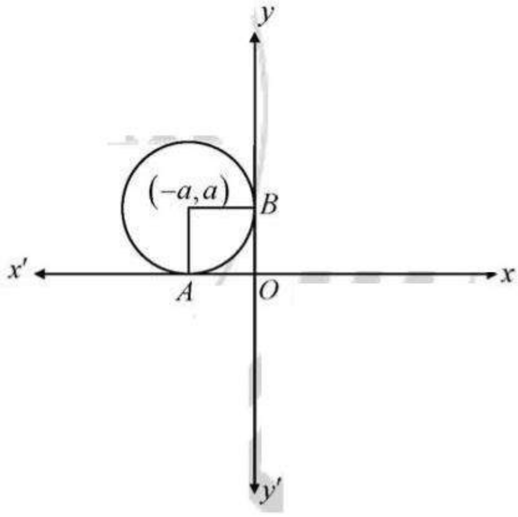
Differentiating both sides w.r.t x we get $2(x+a)+2(y-a)\cdot y_1=0$

$$\Rightarrow x+a+yy_1-ay_1=0$$

$$\Rightarrow x+yy_1=ay_1-a$$

$$\Rightarrow x+yy_1=a(y_1-1)$$

$$\Rightarrow a=\frac{x+yy_1}{y_1-1} \quad \text{... (ii)}$$



From (i) and (ii)

we have $\left[x+\frac{x+yy_1}{y_1-1}\right]^2+\left[y-\frac{x+yy_1}{y_1-1}\right]^2=\left[\frac{x+yy_1}{y_1-1}\right]^2$

$$\Rightarrow\left[\frac{x(y_1-1)+x+yy_1}{y_1-1}\right]^2+\left[\frac{y(y_1-1)-x-yy_1}{y_1-1}\right]^2$$

$$=\left[\frac{x+yy_1}{y_1-1}\right]^2$$

$$\Rightarrow (xy_1-x+x+yy_1)^2+(yy_1-y-x-yy_1)^2$$

$$\Rightarrow (x+y)^2\cdot y_1^2+(x-y)^2=(x+yy_1)^2$$

$$\Rightarrow (x+y)^2\cdot\{y_1^2+1\}=[x+yy_1)^2$$

$$\Rightarrow (x+y)^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}=\left\{x+y\frac{dy}{dx}\right\}^2$$

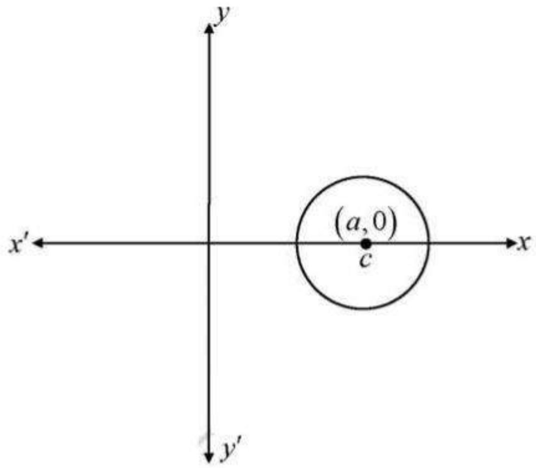
This is the required differential equation

12. Form the differential equation of the family of circles having centers on the x-axis and radius unity

Sol. The equation of given family of circle is $(x-a)^2+y^2=1$ ... (i)

Differentiating both sides w.r.t x, we get $2(x-a)+2yy_1=0$

$\Rightarrow x-a=0yy_1$ ..... (ii)



From (i) and (ii) we have $(-yy_1)^2+y^2=1$

$\Rightarrow y^2y_1^2+y^2=1 \Rightarrow y^2\{1+y_1^2\}=1 \Rightarrow y^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}=1$

This is the required differential equation

13. From the differential equation of the family of circles passing through the fixed points $(a,0)$ and $(-a,0)$ where a is the parameter

Sol. The general equation f a circle is $x^2+y^2+2gx+2fy+c=0$ .... (i)

If it passes through the points $A(-a,0)$ and $B(a,0)$ we have $a^2-2ga+c=0$ ... (ii)

And $a^2+2ga+c=0$

Adding (ii) and (iii), we get $2(a^2+c)=0 \Rightarrow a^2+c=0 \Rightarrow c=-a^2$ .... (iii)

Putting $c=-a^2$ in (iii), we get $2ga=0 \Rightarrow g=0$

Putting $g=0$ and $c=-a^2$ in (i), we get

$x^2+y^2+2fy-a^2=0$ .... (iv) where f is the parameter

Differentiating (iv) w.r.t. x we get $2x+2yy_1+2fy_1=0 \Rightarrow fy_1=-(x+yy_1)$

$\Rightarrow f=\frac{-(x+yy_1)}{y_1}$

Putting this value of f in (iv) we get $x^2+y^2-\frac{2y(x+yy_1)}{y_1}-a^2=0$

$\Rightarrow x^2y_1+y^2y_1-2xy-2y^2y_1-a^2y_1=0 \Rightarrow (x^2-y^2-a^2)y_1=2xy$

14. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis

Sol. The equation of the given family of parabolas is given by x^2-4ay .... (i)

Differentiating (i) both sides w.r.t x, we have

$2x=4a\frac{dy}{dx}$ .... (ii)

From (i) and (ii) we have $2x = \frac{x^2}{y} \cdot \frac{dy}{dx}$ $\left\{ \because x^2 = 4ay \Rightarrow \frac{x^2}{y} = 4a \right\}$

$\Rightarrow 2xy = x^2 \frac{dy}{dx} \quad \Rightarrow x\frac{dy}{dx} = 2y$

This is the required differtial equation

15. Form the differential equation of the family of ellipses having foci on the y-axis and centre at the origin

Sol. The equation of family of ellipses having centre at the origin and foci on the y-axis is given by

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .... (i)

Where $b > a$ and a, b are the parameters

Differentiating (i), w.r.t. x, we get $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{x}{a^2} + \frac{yy_1}{b^2} = 0$ .... (ii)

Differentiating (ii) w.r.t. x we get $\frac{1}{a^2} + \frac{yy_2}{b^2} + \frac{y_1^2}{b^2} = 0 \Rightarrow \frac{x}{a^2} + \frac{xyy_2}{b^2} + \frac{xy_1^2}{b^2} = 0$ .... (iii)

Subtracting (ii) from (iii) we get $\frac{1}{b^2}\left[xyy_2 + xy_1^2 - yy_1\right] = 0 \Rightarrow xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$

16. Form the differential equation of the family of hyperbolas having foci on the x-axis and centre at the origin

Sol. The equation of the family of hyperbolas having foci on the x-axis and centre at the origin is given by

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .... (i) , where a and b are the parameters

Differentiating (i), w.r..t x , we get

$\frac{2x}{a^2} - \frac{2yy_1}{b^2} = 0 \Rightarrow \frac{x}{a^2} - \frac{yy_1}{b^2} = 0$

Differentiating (ii) w.r.t. x we get $\frac{1}{a^2} - \frac{yy_2}{b^2} - \frac{y_1^2}{b^2} = 0$ .... (iii)

Multiplying (iii) by x and subtracting from (ii), we get $\frac{1}{b^2}\left\{xyy_2 + xy_1^2 - yy_1\right\} = 0$

$\Rightarrow xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$