CBSE Test Paper 01

Chapter 11 Three Dimensional Geometry

1. Write the vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$.

   a. $\vec{r}=\vec{a}-\lambda\vec{b},\lambda\in R$
   b. $\vec{r}=\vec{a}+\lambda\vec{b},\ \lambda\in R$
   c. $\vec{r}=-\vec{a}+\lambda\vec{b},\lambda\in R$
   d. $\vec{r}=-\vec{a}-\lambda\vec{b},\ \lambda\in R$

2. If a line has the direction ratios – 18, 12, – 4, then what are its direction cosines ?

   a. $\frac{9}{11},\frac{6}{11},\frac{-2}{11}$
   b. $\frac{-9}{11},\frac{6}{11},\frac{-2}{11}$
   c. $\frac{-9}{11},\frac{6}{11},\frac{2}{11}$
   d. $\frac{-7}{11},\frac{6}{11},\frac{-3}{11}$

3. In the Cartesian form two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar if

   a. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ -a_2 & b_2 & c_2 \end{vmatrix}=0$
   b. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & -c_2 \end{vmatrix}=0$
   c. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$
   d. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & -b_2 & c_2 \end{vmatrix}=0$

4. Express the Cartesian equation of a line that passes through two points$(x_1,\ y_1,\ z_1)$ and

$(x_2,\ y_2,\ z_2)$.

a. $\frac{x+x_1}{x_2-x_1} = \frac{y-y_1}{y_2+y_1} = \frac{z-z_1}{z_2-z_1}$
b. $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z+z_1}{z_2-z_1}$
c. $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2+y_1} = \frac{z-z_1}{z_2-z_1}$
d. $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

5. Two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}\ and\ \vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are coplanar if

a. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(-\overrightarrow{b_1} \times \overrightarrow{-b_2}\right) = 0$
b. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$
c. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(-\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$
d. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times -\overrightarrow{b_2}\right) = 0$

6. Direction ratios of two ________ lines are proportional.
7. If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2$ = ________.
8. The distance of a point P(a, b, c) from x-axis is ________.

9. Find the vector equation for the line passing through the points (-1,0,2) and (3,4,6).

10. Write the vector equation of the plane passing through the point (a, b, c) and parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.

11. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

12. Find angle between lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}, \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

13. The x - coordinate of a point on the line joining the points Q(2, 2, 1) and R(5, 1, -2) is 4. Find its z - coordinate.

14. Find the vector and Cartesian equation of the line through the point (5, 2,-4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

15. Write the vector equations of following lines and hence find the distance between them.
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}, \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

16. The points A(4, 5,10), B(2, 3,4) and C(1, 2, -1) are three vertices of parallelogram ABCD. Find the vector equations of sides A and BC and also find coordinates of point D.

17. Find the shortest distance between the lines whose vector equations are
$\overrightarrow{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$
$\overrightarrow{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

18. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

Solution

1. b. $\vec{r}=\vec{a}+\lambda\vec{b}, \lambda \in R$

   Explanation: The vector equation of a line that passes through the given point whose position vector is$\vec{a}$ and parallel to a given vector $\vec{b}$ is given by : $\vec{r}=\vec{a}+\lambda\vec{b}$

   $\lambda \in R$

   Where, $\overrightarrow{r} = x\hat{i}+y\hat{j}+z\hat{k}$

   $\overrightarrow{a} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$

   $\overrightarrow{b} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$

2. b. $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

   Explanation: If a line has the direction ratios -18, 12, -4, then its direction cosines are given by:

   $l=\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

   $\frac{-18}{\sqrt{324+144+16}}=\frac{-18}{\sqrt{484}}$

   $=\frac{-18}{22}=\frac{-9}{11}$

   $m=\frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

   $=\frac{12}{\sqrt{324+144+16}}=\frac{12}{\sqrt{484}}$

   $=\frac{12}{22}=\frac{6}{11}$

   $n=\frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

   $=\frac{-4}{\sqrt{324+144+16}}=\frac{-4}{\sqrt{484}}$

   $=\frac{-4}{22}=\frac{-2}{11}$

3. c. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0.$

   Explanation: In the Cartesian form two lines

   $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

   and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

4. d. $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

   Explanation: The Cartesian equation of a line that passes through two points $(x_1,\ y_1,\ z_1)$ and $(x_2,\ y_2,\ z_2)$ is given by : $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

5. b. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

   Explanation: In vector form: Two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}\ and\ \vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are coplanar if

6. Parallel
7. 1
8. $\sqrt{b^2 + c^2}$

9. Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be the p.v of the points A (-1,0,2) and B (3, 4, 6)

   $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$

   $= \left(-\hat{i} + 2\hat{k}\right) + \lambda\left(4\hat{i} + 4\hat{j} + 4\hat{k}\right)$

10. According to the question, The required plane is passing through the point (a, b, c) whose position vector is $\vec{p} = a\hat{i} + b\hat{j} + c\hat{k}$ and is parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

    $\therefore$ it is normal to the vector

    $\vec{n} = \hat{i} + \hat{j} + \hat{k}$

    Required equation of plane is

    $(\vec{r} - \vec{p}).\vec{n} = 0 \Rightarrow \vec{r}.\vec{n} = \vec{p}.\vec{n}$

    $\Rightarrow \vec{r}.(\hat{i} + \hat{j} + \hat{k}) = (a\hat{i} + \hat{b} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

    $\therefore \quad \vec{r}.(\hat{i} + \hat{j} + \hat{k}) = a + b + c$

11. According to the question, the normal to the plane is equally inclined with coordinates axes, and the distance of the plane from origin is $5\sqrt{3}$ units

    $\therefore$ the direction cosines are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$

    The required equation of plane is

$\frac{1}{\sqrt{3}} \cdot x + \frac{1}{\sqrt{3}} \cdot y + \frac{1}{\sqrt{3}} \cdot z = 5\sqrt{3}$

$\Rightarrow x + y + z = 5 \times 3$

$\Rightarrow \quad x + y + z = 15$

[∵ If l, m and n are direction cosines of normal to the plane and P is a distance of a plane from origin, then the equation of plane is given by $lx + my + nz = p$]

12. $\frac{x-0}{2} = \frac{y-0}{2} = \frac{z-0}{1}$

$\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

$a_1 = 2, b_1 = 2, c_1 = 1$

$a_2 = 4, b_2 = 1, c_2 = 8$

$\cos\theta = \frac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}$

$= \left|\frac{2(4)+2(1)+1(8)}{\sqrt{2^2+2^2+1}\sqrt{4^2+1^2+8^2}}\right|$

$= \left|\frac{8+2+8}{\sqrt{9}\sqrt{81}}\right|$

$= \frac{18}{27}$

$= \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

13. Let the point P divide QR in the ratio $\lambda : 1$, then the co-ordinate of P are

$\left(\frac{5\lambda+2}{\lambda+1}, \frac{\lambda+2}{\lambda+1}, \frac{-2\lambda+1}{\lambda+1}\right)$

But x - coordinate of P is 4. Therefore,

$\frac{5\lambda+2}{\lambda+1} = 4 \Rightarrow \lambda = 2$

Hence, the z - coordinate of P is $\frac{-2\lambda+1}{\lambda+1} = -1$.

14. $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$

Vector equation of line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

$= 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

Cartesian equation is

$x\hat{i} + y\hat{j} + z\hat{k} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$

$\Rightarrow x = 5 + 3\lambda, y = 2 + 2\lambda, z = -4 - 8\lambda$

$\Rightarrow \frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8} = \lambda$

Therefore, required equation is,

$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$

15. The given equations of lines are

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

and $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

Now, the vector equation of given lines are

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$......(i)

[$\because$ vector form of equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$]

and $\vec{r} = (3i + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$...................(ii)

Here, $\overrightarrow{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}, \overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\overrightarrow{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}, \overrightarrow{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$

Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k})$

$= 2\hat{i} + \hat{j} - \hat{k}$.................(iii)

and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 4 & 6 & 12 \end{vmatrix}$

$= \hat{i}(36 - 36) - \hat{j}(24 - 24) + \hat{k}(12 - 12)$

$= 0\hat{i} - \hat{0}\hat{j} + 0\hat{k} = \vec{0}$

$\Rightarrow \quad \vec{b}_1 \times \vec{b}_2 = \vec{0},$

i.e. Vector b_1 is parallel to $\vec{b}_2$

[$\because$ if $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a} || \vec{b}$]

Thus, two lines are parallel.

$\therefore \quad \vec{b} = (2\hat{i} + 3\hat{j} + 6\hat{k})$...................(iv)

[since, DR's of given lines are proportional]

Since, the two lines are parallel, we use the formula for shortest distance between two parallel lines

$d = \left| \frac{\vec{b} \times \left(\vec{a}_2 - \overrightarrow{a_1}\right)}{|\vec{b}|} \right|$

$\Rightarrow \quad d = \left| \frac{(2\hat{i}+3\hat{j}+6\hat{k}) \times (2\hat{i}+\hat{j}-\hat{k})}{\sqrt{(2)^2+(3)^2+(6)^2}} \right|$..............(v)

[from Eqs. (iii) and (iv)]

Now, $(\hat{2i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$

$= \hat{i}(-3-6) - \hat{j}(-2-12) + \hat{k}(2-6)$

$= -9\hat{i} + 14\hat{j} - 4\hat{k}$

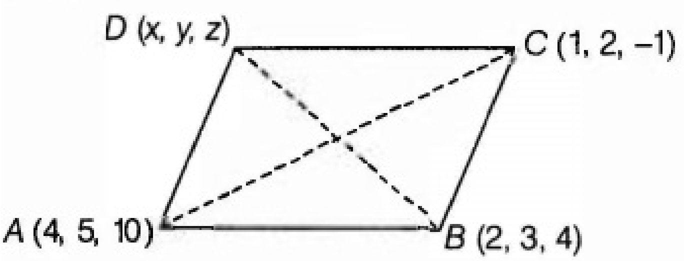
From Eq, (v), we get

$d = \left| \frac{-9\hat{i}+14\hat{j}-4\hat{k}}{\sqrt{49}} \right| = \frac{\sqrt{(-9)^2+(14)^2+(-4)^2}}{7}$

$\therefore d = \frac{\sqrt{81+196+16}}{7} = \frac{\sqrt{293}}{7}$ units

16. The vector equation of a side of a parallelogram, when two points are given, is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$. Also, the diagonals of a parallelogram intersect each other at mid-point.

Given points are A (4,5,10), B (2, 3,4) and C(1,2,-1).



We know that, two point vector form of line is given by

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$......................... ......(i)

where, $\vec{a}$ and $\vec{b}$ are the position vector of points through which the line is passing through. Here, for line AB, position vectors are

$\vec{a} = \overrightarrow{OA} = 4\hat{i} + 5\hat{j} + 10\hat{k}$

and $\vec{b} = \overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Using Equation. (i), the required equation of line AB is

$\vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda[(2\hat{i} + 3\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + 10\hat{k})]$

$\Rightarrow \quad \vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda(-2\hat{i} - 2\hat{j} - 6\hat{k})$

Similarly, vector equation of line BC, where B(2,3,4) and C (1, 2, -1) is

$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(\hat{i} + 2\hat{j} - \hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})]$

$\Rightarrow \quad \vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(-\hat{i} - \hat{j} - 5\hat{k})$

We know that, mid-point of diagonal BD

= Mid-point of diagonal AC

[∵ diagonal of a parallelogram bisect each other]

$\therefore \left(\frac{x+2}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right) = \left(\frac{4+1}{2}, \frac{5+2}{2}, \frac{10-1}{2}\right)$

Therefore, on comparing corresponding coordinates, we get

$\frac{x+2}{2} = \frac{5}{2}, \frac{y+3}{2} = \frac{7}{2}$ and $\frac{z+4}{2} = \frac{9}{2}$

$\Rightarrow \quad x = 3, y = 4$ and $z = 5$

Therefore, coordinates of point D (x, y, z) is (3,4,5) and vector equations of sides AB and BC are

$\vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) - \lambda(2\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) - \mu(\hat{i} + \hat{j} + 5\hat{k})$, respectively.

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k})$

$\overrightarrow{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\overrightarrow{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$

$\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \overrightarrow{a_1} = \hat{j} - 4\hat{k}$

$$\overrightarrow{b_1} \times \hat{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

17. $2\hat{i} - 4\hat{j} - 3\hat{k}$

$(\vec{a}_2 - \vec{a}_1) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) = (0\vec{i} + \vec{j} - 4\vec{k}) \cdot (2\vec{i} - 4\vec{j} - 3\vec{k}) = 0 - 4 + 12 = 8$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$

$= \sqrt{29}$

$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{8}{\sqrt{29}}$

18. $\vec{r}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$

$\Rightarrow \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda$ ...(1)

Any point on line (1) is,

$P(3\lambda+2, 4\lambda-1, 2\lambda+2)$

Now, $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}-\hat{j}+\hat{k})=5$

$x-y+z=5$ ...(2)

Since point P lies on (2), therefore, from (2), we have,

$(3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$

$\Rightarrow \lambda+5=5$

$\Rightarrow \lambda=0$

We get (2, -1, 2)

as the coordinate of the point of intersection of the given line and the plane

Now distance between the points (-1, -5, -10) and (2, -1, 2)

req. distance $=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

= $\sqrt{9+16+144}$ =13