Exercise 10.6

Q1E

Given ellipse with eccentricity $\frac{1}{2}$, directrix $x = 4$

$$\Rightarrow e = \frac{1}{2}, d = 4$$

we know $r = \dfrac{ed}{1 \pm e\cos\theta}$

$$\Rightarrow r = \frac{\frac{1}{2}(4)}{1 \pm \frac{1}{2}\cos\theta}$$

$$= \frac{2}{1 \pm \frac{1}{2}\cos\theta}$$

$$= \boxed{\frac{4}{2 \pm \cos\theta}}$$

Q2E

Given parabola with directrix $x = -3$

$\Rightarrow e = 1, d = -3$

we know $r = \dfrac{ed}{1 \pm e\cos\theta}$

$$\Rightarrow \boxed{r = \frac{-3}{1 \pm \cos\theta}}$$

Q3E

Consider the hyperbola with eccentricity 1.5 and directrix $y = 2$

That is $e = 1.5, d = 2$

Always, a polar equation of conic with eccentricity e and the directrix $y = d$ is

$$r = \frac{ed}{1 + e\sin\theta}$$

Therefore, the require conic (Hyperbola) is

$$r = \frac{(1.5)2}{1+(1.5)\sin\theta}$$

$$r = \frac{3}{1+\frac{3}{2}\sin\theta}$$

$$\boxed{r = \frac{6}{2+3\sin\theta}}$$

Q4E

Given hyperbola with eccentricity 3 and directrix $x = 3$
$\Rightarrow e = 3, d = 3$

we know $r = \dfrac{ed}{1 \pm e\cos\theta}$

$$\Rightarrow r = \frac{(3)3}{1 \pm (3)\cos\theta}$$

$$= \boxed{\frac{9}{1 \pm 3\cos\theta}}$$

Q5E

Vertex of the parabola is given as $(4, 3\pi/2)$
Since the focus is at origin (0, 0) so the vertex is 4 units below the focus
Therefore the directrix will be 8 units below the focus so we have $d = 8$
Since the eccentricity of the parabola is $e = 1$
Therefore the polar equation of the parabola is given as

$$r = \frac{ed}{1 - e\sin\theta}$$

Then $\boxed{r = \dfrac{8}{1 - \sin\theta}}$ is the equation of given conic

Q6E

Vertex of an ellipse is given as $(1, \pi/2)$, and eccentricity $e = 0.8$
Since the focus is at the origin so the vertex is 1 unit above the focus
For an ellipse we have $e = \dfrac{1}{d-1}$ $\quad \Rightarrow 0.8 = \dfrac{1}{d-1}$

$$\Rightarrow 0.8d = 1.8$$

$$\Rightarrow d = 2.25$$

So the equation of an ellipse is $r = \dfrac{ed}{1+e\sin\theta}$

$$= \frac{0.8(2.25)}{1+0.8\sin\theta}$$

$$= \frac{1.8}{1+0.8\sin\theta} \times \frac{5}{5}$$

$$\Rightarrow \boxed{r = \frac{9}{5+4\sin\theta}}$$

Q7E

We have the eccentricity $e = \dfrac{1}{2}$,

And directrix $r = 4\sec\theta$

$$\Rightarrow r = \frac{4}{\cos\theta}$$

$$\Rightarrow r\cos\theta = 4$$

$$\Rightarrow x = 4$$

Since the directrix is right to the focus that is at the origin
Then polar equation of the ellipse with $d = 4$ is

$$r = \frac{ed}{1+e\cos\theta}$$

$$= \frac{(1/2)(4)}{1+(1/2)\cos\theta}$$

$$= \frac{4}{2\left(\dfrac{2+\cos\theta}{2}\right)}$$

$$\Rightarrow \boxed{r = \frac{4}{2+\cos\theta}}$$

Q8E

We have the eccentricity $e = 3.$ and directrix is $r = -6\csc\theta$

$$\Rightarrow r = \frac{-6}{\sin\theta} \qquad [\csc\theta = 1/\sin\theta]$$

$$\Rightarrow r\sin\theta = -6$$

$$\Rightarrow y = -6$$

Since the directrix is below the focus that is at the origin
Then polar equation of the hyperbola with $d = 6$ is

$$r = \frac{ed}{1-e\sin\theta}$$

$$= \frac{3(6)}{1-3\sin\theta}$$

Or $\boxed{r = \dfrac{18}{1-3\sin\theta}}$

Q9E

Consider the polar equation

$$r = \frac{4}{5 - 4\sin\theta}$$

divide numerator and denominator by 5, write the equation as

$$r = \frac{\left(\frac{4}{5}\right)}{1 - \left(\frac{4}{5}\right)\sin\theta}$$

A polar equation of the form

$r = \frac{ed}{1 \pm e\cos\theta}$ Or $r = \frac{ed}{1 \pm e\sin\theta}$

Represents a conic section with eccentricity e.

The conic is an ellipse if $e < 1$, a parabola if $e = 1$, or a hyperbola if $e > 1$.

Compare this equation with $r = \frac{\left(\frac{4}{5}\right)}{1 - \left(\frac{4}{5}\right)\sin\theta}$

$$e = \frac{4}{5}, ed = \frac{4}{5}$$

(a)

Therefore, the eccentricity $e = \boxed{\frac{4}{5}}$.

(b)

the eccentricity

$$\begin{aligned} e &= \frac{4}{5} \\ &= 0.8 < 1 \end{aligned}$$

Therefore, the conic is $\boxed{\text{an ellipse}}$.

(c)

Need to find the equation of the directrix

Take $ed = \frac{4}{5}$

$\left(\frac{4}{5}\right)d = \frac{4}{5}$ substitute $e = \frac{4}{5}$

$d = 1$ Multiply on both sides by $\frac{5}{4}$

Recollect the polar equation $r = \frac{ed}{1 - e\sin\theta}$ form then the directrix is chosen to be parallel to the polar axis as $y = -d.$

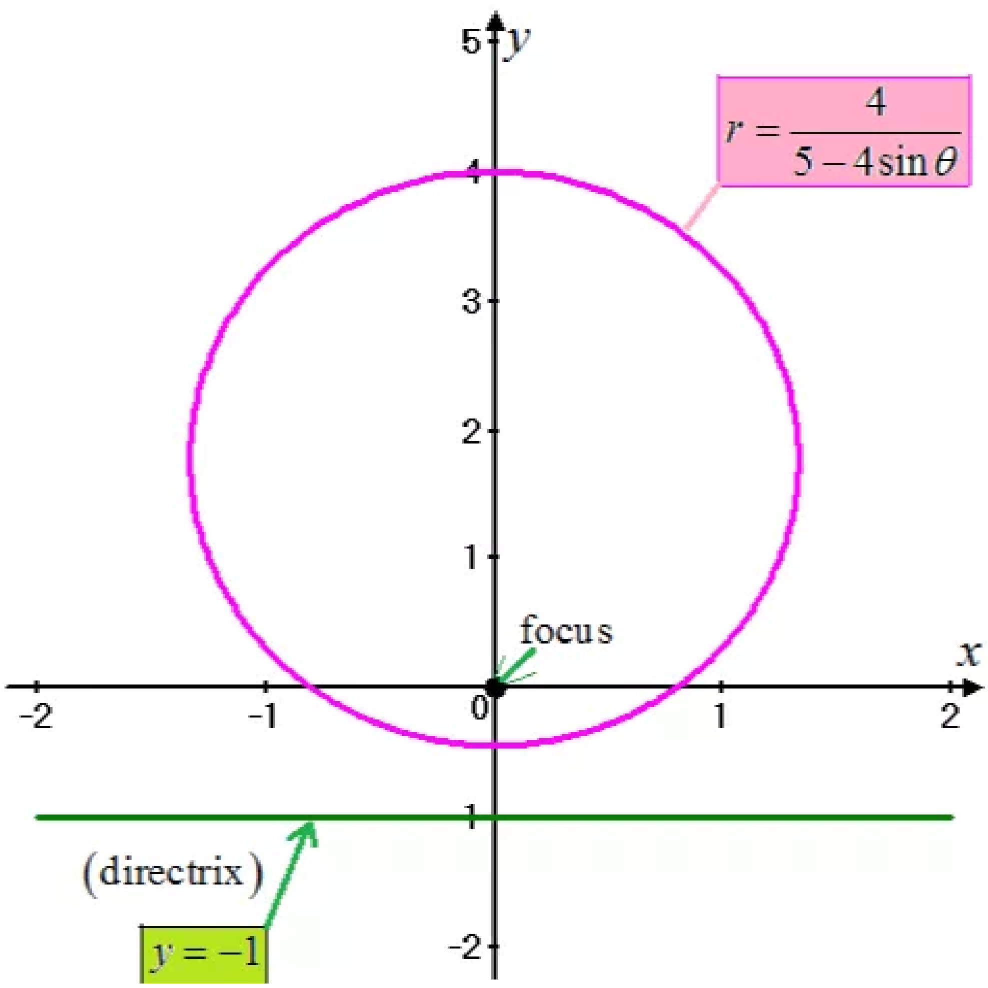
Substitute $d = 1$ in $y = -d$, get

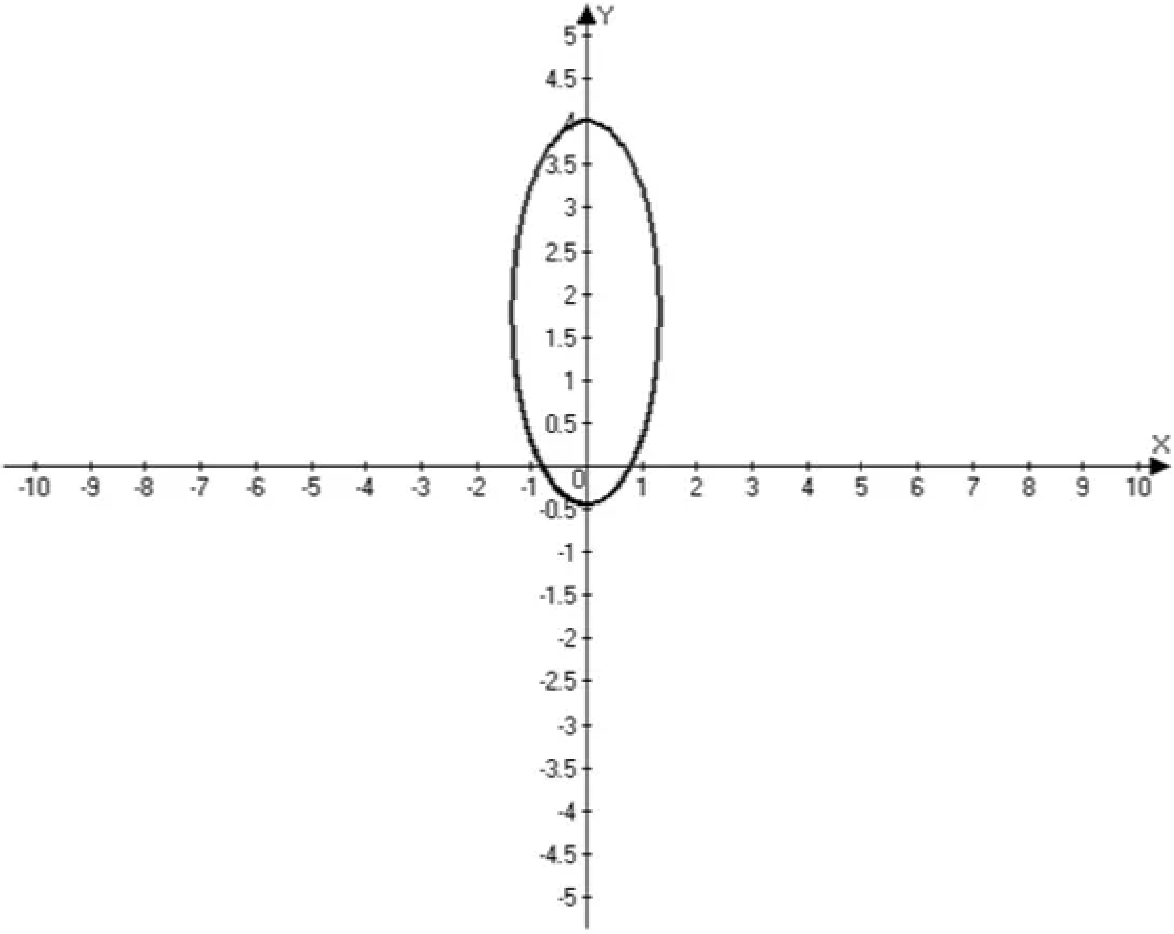
$y = -1$

Therefore, the equation of the directrix $\boxed{y = -1}$.

(d)

Sketch the conic





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Q10E

Theorem: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. the conic is an ellipse if $e < 1,$ a parabola if $e = 1,$ or a hyperbola if $e > 1$.

Consider the polar equation,

$$r = \frac{12}{3 - 10\cos\theta}$$

Dividing the numerator and denominator by 3, write the equation as

$$\begin{aligned} r &= \frac{12}{3 - 10\cos\theta} \\ &= \frac{12/3}{(3/3) - (10/3)\cos\theta} \\ &= \frac{4}{1 - (10/3)\cos\theta} \end{aligned}$$

(a) To find the eccentricity:

If $r=\dfrac{4}{1-(10/3)\cos\theta}$ comparing with $r=\dfrac{ed}{1\pm e\cos\theta}$, then $e=\dfrac{10}{3}$.

So, the eccentricity is $\boxed{10/3}$.

(b) To identify the conic:

Since $e\left(=\dfrac{10}{3}\right)>1$, so given conic is a $\boxed{\text{Hyperbola}}$.

(c) To find an equation of the directrix:

If $r=\dfrac{4}{1-(10/3)\cos\theta}$ comparing with $r=\dfrac{ed}{1\pm e\cos\theta}$, then $ed=4$.

Since $e=10/3$, to get

$$\begin{aligned} d&=\frac{4}{e}\\ &=\frac{4}{(10/3)}\\ &=4\times\frac{3}{10}\\ &=\frac{6}{5} \end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\cos\theta$ is negative

So an equation of the directrix is $x=-d$ that is

$\boxed{x=-6/5}$.

When $\theta = \pi/2$:

$$r = \frac{12}{3-10\cos(\pi/2)}$$
$$= \frac{12}{3-0}$$
$$= 4$$

When $\theta = 3\pi/2$:

$$r = \frac{12}{3-10\cos(3\pi/2)}$$
$$= \frac{12}{3+0}$$
$$= 4$$

So, the vertices have polar coordinates are $(4,\pi/2),(4,3\pi/2)$.

When $\theta = 0$:

$$r = \frac{12}{3-10\cos 0}$$
$$= \frac{12}{3-10}$$
$$= -\frac{12}{7}$$

When $\theta = \pi$:

$$r = \frac{12}{3-10\cos\pi}$$
$$= \frac{12}{3+10}$$
$$= \frac{12}{13}$$

So, the *x*- intercepts are $\left(-\frac{12}{7},0\right),\left(\frac{12}{13},\pi\right)$.

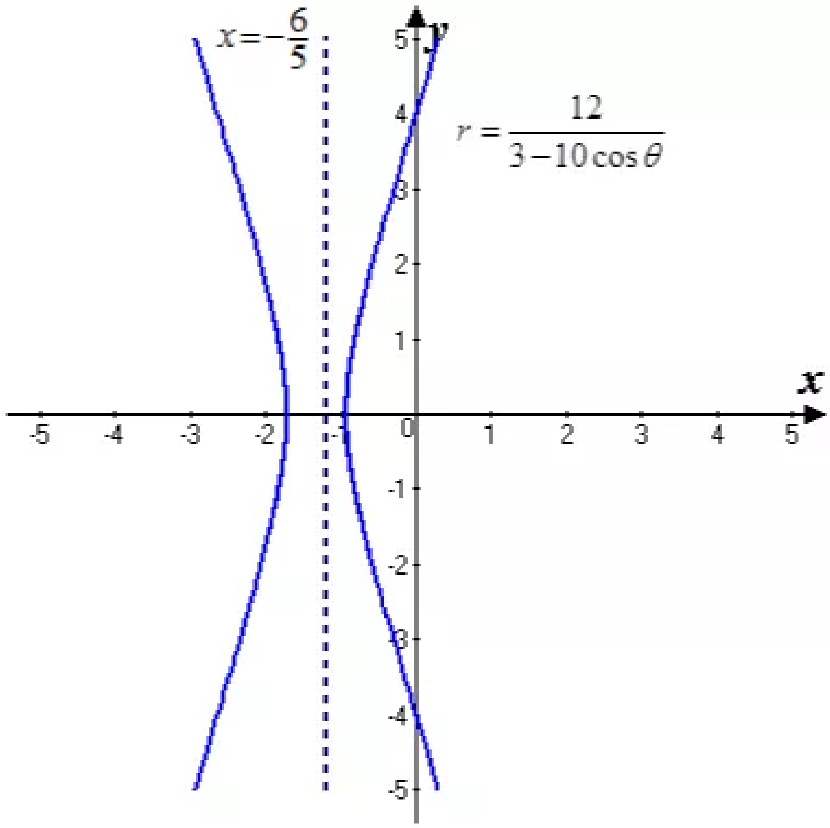
Note that $r \to \pm\infty$ when $1-(10/3)\cos\theta \to 0^+$ or 0^-

And $1-(10/3)\cos\theta = 0$ when $\cos\theta = 3/10$

Thus the asymptotes are parallel to the rays $\theta = \cos^{-1}(3/10)$.

(d)

Sketch of the graph of the hyperbola $r = \frac{12}{3-10\cos\theta}$ as shown below:



Q11E

Given

$$r=\frac{2}{3+3\sin\theta}$$

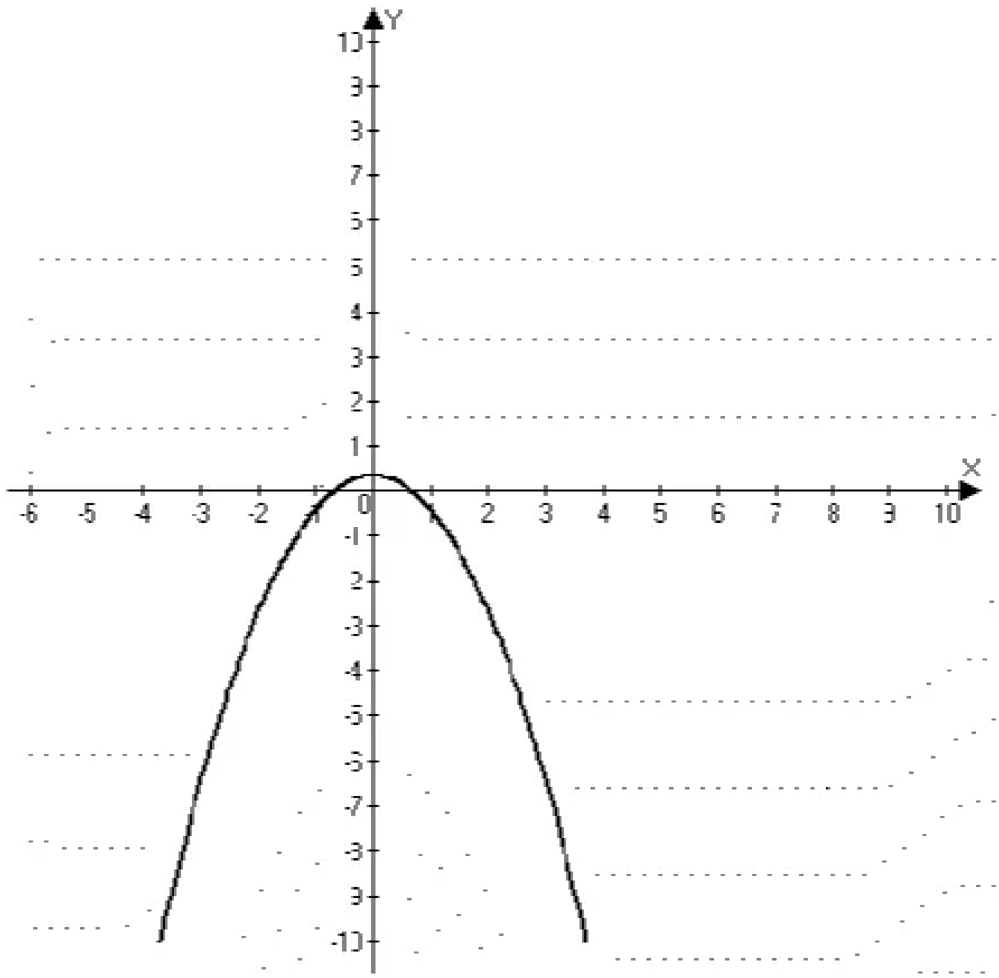
$$=\frac{2/3}{1+\sin\theta}$$

$$\Rightarrow e=1,\ ed=\frac{2}{3}$$

$$\Rightarrow e=1,\ d=\frac{2}{3}$$

The conic is a parabola.

Graph



Q12E

Theorem: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. the conic is an ellipse if $e < 1$, a parabola if $e = 1$, or a hyperbola if $e > 1$.

Consider the polar equation,

$$r = \frac{3}{2 + 2\cos\theta}$$

Dividing the numerator and denominator by 2, write the equation as

$$\begin{aligned} r &= \frac{3}{2 + 2\cos\theta} \\ &= \frac{3/2}{(2/2) + (2/2)\cos\theta} \\ &= \frac{3/2}{1 + \cos\theta} \end{aligned}$$

(a) To find the eccentricity:

If $r=\frac{3/2}{1+\cos\theta}$ comparing with $r=\frac{ed}{1\pm e\cos\theta}$, then $e=1$.

So, the eccentricity is $\boxed{1}$.

(b) To identify the conic:

Since $e=1$, so given conic is a $\boxed{\text{Parabola}}$.

(c) To find an equation of the directrix:

If $r=\frac{3/2}{1+\cos\theta}$ comparing with $r=\frac{ed}{1\pm e\cos\theta}$, then $ed=\frac{3}{2}$.

Since $e=1$, to get

$$\begin{aligned} d &= \frac{3/2}{e} \\ &= \frac{3/2}{1} \\ &= \frac{3}{2} \end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\cos\theta$ is positive

So an equation of the directrix is $x=d$ that is

$$\boxed{x=\frac{3}{2}}.$$

When $\theta=0$:

$$\begin{aligned} r &= \frac{3}{2+2\cos\theta} \\ &= \frac{3}{2+2\cos 0} \\ &= \frac{3}{4} \end{aligned}$$

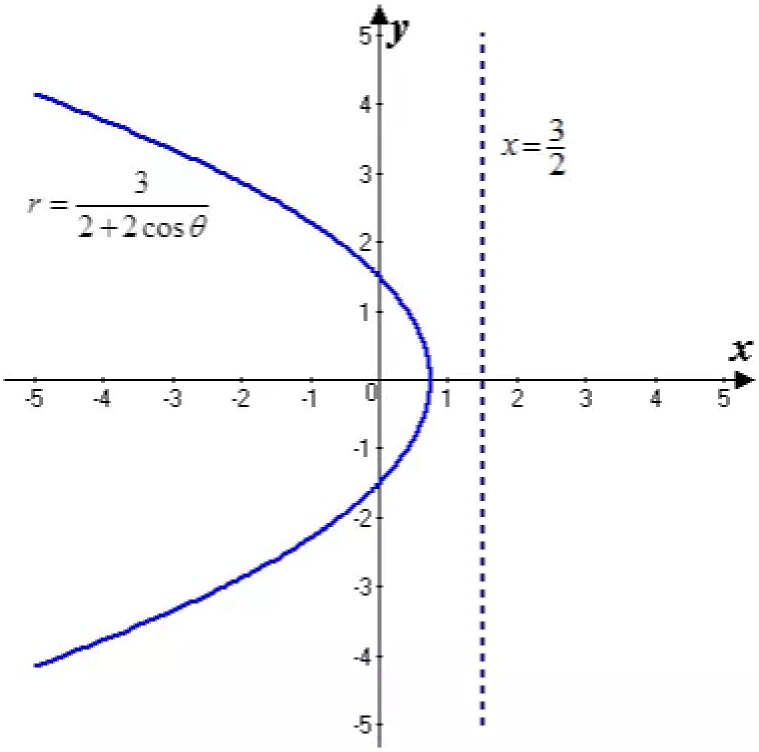
When $\theta=\pi/2$:

$$\begin{aligned} r &= \frac{3}{2+2\cos\theta} \\ &= \frac{3}{2+2\cos(\pi/2)} \\ &= \frac{3}{2} \end{aligned}$$

So, the vertices have polar coordinates are $(0,3/4),(3/2,\pi/2)$.

(d)

Sketch of the graph of the parabola $r = \dfrac{3}{2+2\cos\theta}$ as shown below:



Q13E

Given equation is $r = \dfrac{9}{6+2\cos\theta}$

Dividing the numerator and denominator by 6

$$\Rightarrow r = \frac{9/6}{1+\frac{1}{3}\cos\theta} \qquad \Rightarrow r = \frac{3/2}{1+\frac{1}{3}\cos\theta}$$

Comparing with $r = \dfrac{ed}{1 \pm e\cos\theta}$

(A) We have $\boxed{e = 1/3}$

(B) Since $e = 1/3 < 1$, so given conic is an $\boxed{ellipse}$

(C) We have

$$ed = 3/2 \qquad \Rightarrow d = 3/(2e)$$

$$\Rightarrow d = \frac{2}{2/3} \Rightarrow d = 9/2$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\cos\theta$ is positive.

So equation of the directrix is $\boxed{x = 9/2}$

(D) Now we sketch the conic

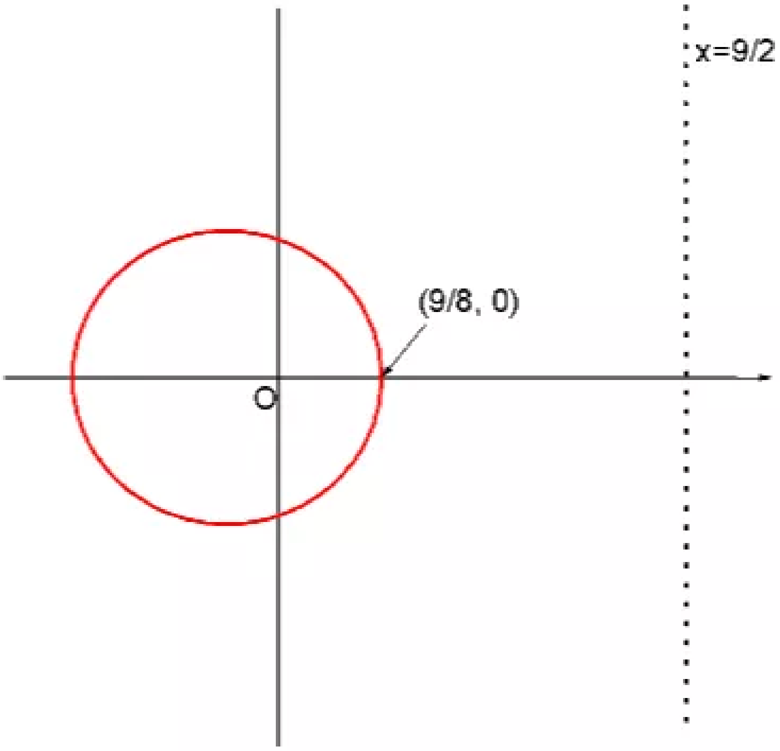


Fig.1

Q14E

Theorem: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. the conic is an ellipse if $e < 1$, a parabola if $e = 1$, or a hyperbola if $e > 1$.

Consider the polar equation,

$$r = \frac{8}{4 + 5\sin\theta}$$

Dividing the numerator and denominator by 4, write the equation as

$$\begin{aligned} r &= \frac{8}{4 + 5\sin\theta} \\ &= \frac{8/4}{(4/4) + (5/4)\sin\theta} \\ &= \frac{2}{1 + (5/4)\sin\theta} \end{aligned}$$

(a) To find the eccentricity:

If $r=\dfrac{2}{1+(5/4)\sin\theta}$ comparing with $r=\dfrac{ed}{1\pm e\sin\theta}$, then $e=\dfrac{5}{4}$.

So, the eccentricity is $\boxed{5/4}$.

(b) To identify the conic:

Since $e\left(=\dfrac{5}{4}\right)>1$, so given conic is a $\boxed{\text{Hyperbola}}$.

(c) To find an equation of the directrix:

If $r=\dfrac{2}{1+(5/4)\sin\theta}$ comparing with $r=\dfrac{ed}{1\pm e\sin\theta}$, then $ed=2$.

Since $e=5/4$, to get

$$\begin{aligned} d &= \frac{2}{e} \\ &= \frac{2}{(5/4)} \\ &= 2\times\frac{4}{5} \\ &= \frac{8}{5} \end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\sin\theta$ is positive

So an equation of the directrix is $y=d$ that is

$\boxed{y=8/5}$.

When $\theta = \pi/2$:

$$r = \frac{8}{4+5\sin(\pi/2)}$$
$$= \frac{8}{4+5}$$
$$= \frac{8}{9}$$

When $\theta = 3\pi/2$:

$$r = \frac{8}{4+5\sin(3\pi/2)}$$
$$= \frac{8}{4-5}$$
$$= -8$$

So, the vertices have polar coordinates are $\left(\frac{8}{9}, \frac{\pi}{2}\right), \left(-8, \frac{3\pi}{2}\right)$.

When $\theta = 0$:

$$r = \frac{8}{4+5\sin 0}$$
$$= \frac{8}{4}$$
$$= 2$$

When $\theta = \pi$:

$$r = \frac{8}{4+5\sin\pi}$$
$$= \frac{8}{4}$$
$$= 2$$

So, the *x*- intercepts are $(2,0),(2,\pi)$.

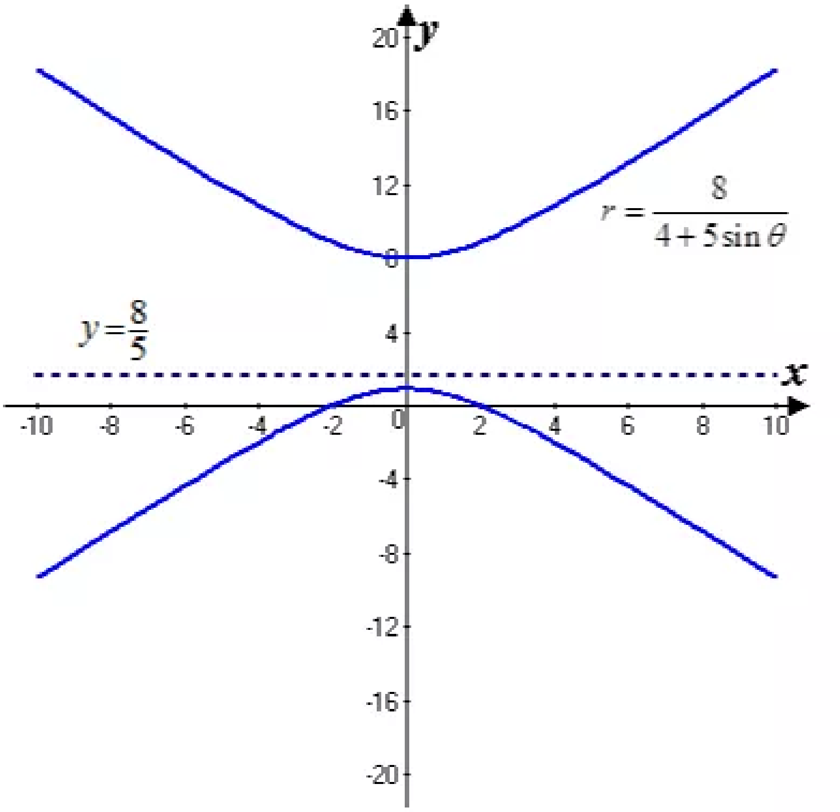
Note that $r \to \pm\infty$ when $1+(5/4)\sin\theta \to 0^+$ or 0^-

And $1+(5/4)\sin\theta = 0$ when $\sin\theta = -5/4$

Thus the asymptotes are parallel to the rays $\theta = \pi - \sin^{-1}(5/4)$.

(d)

Sketch of the graph of the hyperbola $r = \dfrac{8}{4+5\sin\theta}$ as shown below:



Q15E

Given equation is $r = \frac{3}{4-8\cos\theta}$

Dividing the numerator and denominator by 4

$$\Rightarrow r = \frac{3/4}{1-2\cos\theta}$$

Comparing with $r = \frac{ed}{1 \pm e\cos\theta}$

(A) We have $\boxed{e=2}$

(B) Since $e = 2 > 1$, so given conic is a $\boxed{hyperbola}$

(C) We have $ed = 3/4 \quad \Rightarrow d = 3/(4e)$

$\Rightarrow d = 3/8$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\cos\theta$ is negative

So equation of directrix is $\boxed{x = -3/8}$

(D) Now we sketch the conic

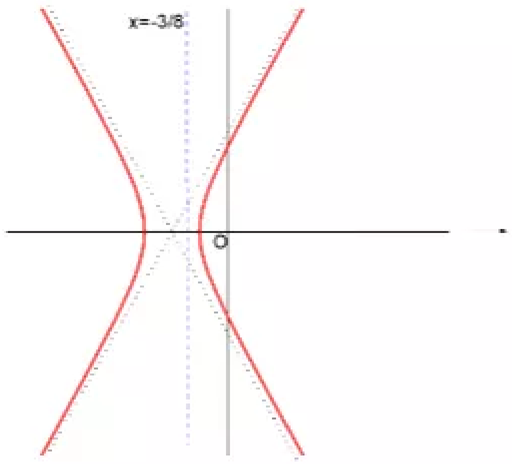


Fig.1

Q16E

Theorem: A polar equation of the form

$$r=\frac{ed}{1\pm e\cos\theta} \qquad \text{or} \qquad r=\frac{ed}{1\pm e\sin\theta}$$

represents a conic section with eccentricity *e*. the conic is an ellipse if $e<1$, a parabola if $e=1$, or a hyperbola if $e>1$.

Consider the polar equation,

$$r=\frac{10}{5-6\sin\theta}$$

Dividing the numerator and denominator by 5, write the equation as

$$\begin{aligned} r&=\frac{10}{5-6\sin\theta} \\ &=\frac{10/5}{(5/5)-(6/5)\sin\theta} \\ &=\frac{2}{1-(6/5)\sin\theta} \end{aligned}$$

(a) To find the eccentricity:

If $r=\dfrac{2}{1-(6/5)\sin\theta}$ comparing with $r=\dfrac{ed}{1\pm e\sin\theta}$, then $e=\dfrac{6}{5}$.

So, the eccentricity is $\boxed{6/5}$.

(b) To identify the conic:

Since $e\left(=\dfrac{6}{5}\right)>1$, so given conic is a $\boxed{\text{Hyperbola}}$.

(c) To find an equation of the directrix:

If $r=\frac{2}{1-(6/5)\sin\theta}$ comparing with $r=\frac{ed}{1\pm e\sin\theta}$, then $ed=2$.

Since $e=5/4$, to get

$$\begin{aligned}d&=\frac{2}{e}\\&=\frac{2}{(6/5)}\\&=2\times\frac{5}{6}\\&=\frac{5}{3}\end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\sin\theta$ is negative

So an equation of the directrix is $y=-d$ that is

$\boxed{y=-5/3}$.

When $\theta=\pi/2$:

$$\begin{aligned}r&=\frac{10}{5-6\sin(\pi/2)}\\&=\frac{10}{5-6}\\&=-10\end{aligned}$$

When $\theta=3\pi/2$:

$$\begin{aligned}r&=\frac{10}{5-6\sin(3\pi/2)}\\&=\frac{10}{5+6}\\&=\frac{10}{11}\end{aligned}$$

So, the vertices have polar coordinates are $\left(-10,\frac{\pi}{2}\right),\left(\frac{10}{11},\frac{3\pi}{2}\right)$.

When $\theta = 0$:

$$r = \frac{10}{5 - 6\sin 0}$$
$$= \frac{10}{5}$$
$$= 2$$

When $\theta = \pi$:

$$r = \frac{10}{5 - 6\sin \pi}$$
$$= \frac{10}{5}$$
$$= 2$$

So, the *x*- intercepts are $(2,0),(2,\pi)$.

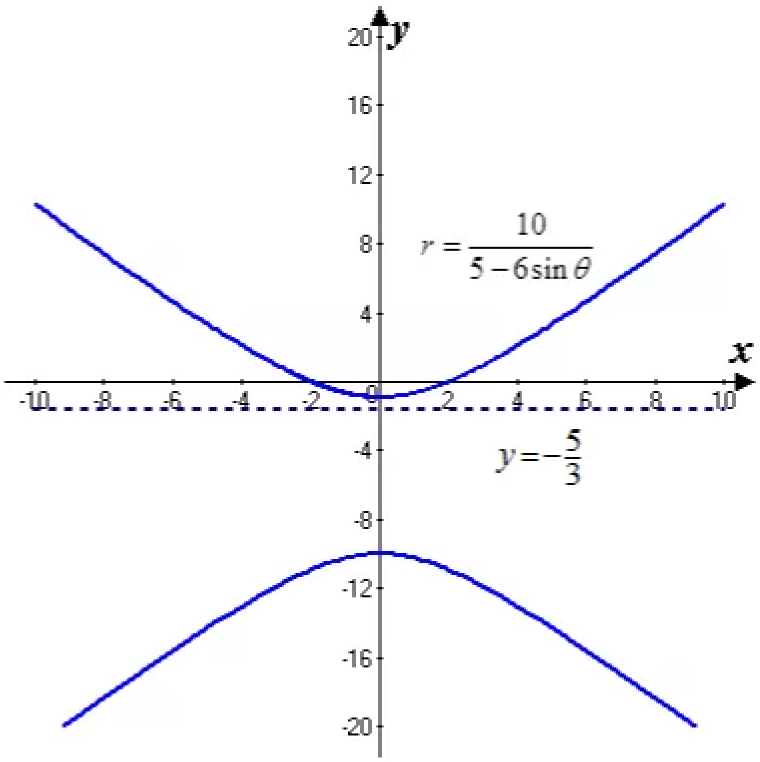
Note that $r \to \pm\infty$ when $1-(6/5)\sin\theta \to 0^+$ or 0^-

And $1-(6/5)\sin\theta = 0$ when $\sin\theta = 5/6$

Thus the asymptotes are parallel to the rays $\theta = \sin^{-1}(5/6)$.

(d)

Sketch of the graph of the hyperbola $r = \dfrac{10}{5-6\sin\theta}$ as shown below:



Q17E

Consider the polar equation,

$$r = \frac{1}{1-2\sin\theta}$$

Theorem: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. the conic is an ellipse if $e<1$, a parabola if $e=1$, or a hyperbola if $e>1$.

(a)

To find the eccentricity:

If $r = \dfrac{1}{1-2\sin\theta}$ comparing with $r = \dfrac{ed}{1 \pm e\sin\theta}$, then $e=2$.

So, the eccentricity is $\boxed{2}$.

Since $e(=2)>1$, so given conic is a hyperbola.

To find an equation of the directrix:

If $r = \dfrac{1}{1-2\sin\theta}$ comparing with $r = \dfrac{ed}{1 \pm e\sin\theta}$, then $ed=1$.

Since $e=2$, to get

$$\begin{aligned} d &= \frac{1}{e} \\ &= \frac{1}{2} \end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\sin\theta$ is negative

So an equation of the directrix is $y=-d$ that is

$\boxed{y=-1/2}$.

When $\theta = \pi/2$:

$$r = \frac{1}{1-2\sin(\pi/2)}$$
$$= -1$$

When $\theta = 3\pi/2$:

$$r = \frac{1}{1-2\sin(3\pi/2)}$$
$$= \frac{1}{3}$$

So, the vertices have polar coordinates are $\left(-1, \frac{\pi}{2}\right), \left(\frac{1}{3}, \frac{3\pi}{2}\right)$.

When $\theta = 0$:

$$r = \frac{1}{1-2\sin 0}$$
$$= 1$$

When $\theta = \pi$:

$$r = \frac{1}{1-2\sin \pi}$$
$$= 1$$

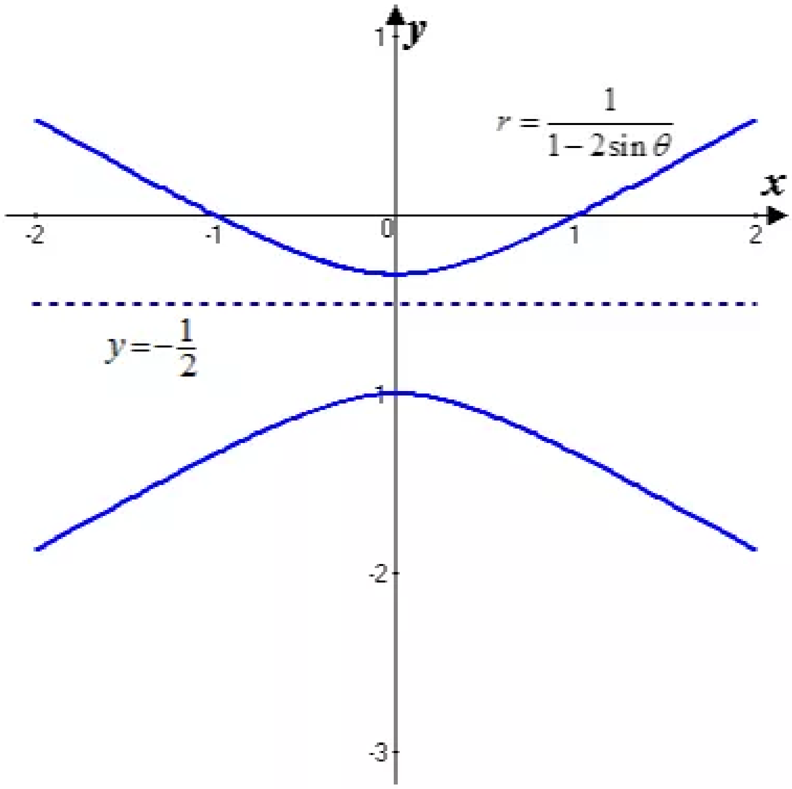
So, the *x*- intercepts are $(1,0), (1,\pi)$.

Note that $r \to \pm\infty$ when $1-2\sin\theta \to 0^+$ or 0^-

And $1-2\sin\theta = 0$ when $\sin\theta = 1/2$

Thus the asymptotes are parallel to the rays $\theta = \pi/6$ and $\theta = 5\pi/6$.

Sketch of the graph of the hyperbola $r = \dfrac{1}{1-2\sin\theta}$ as shown below:



(b)

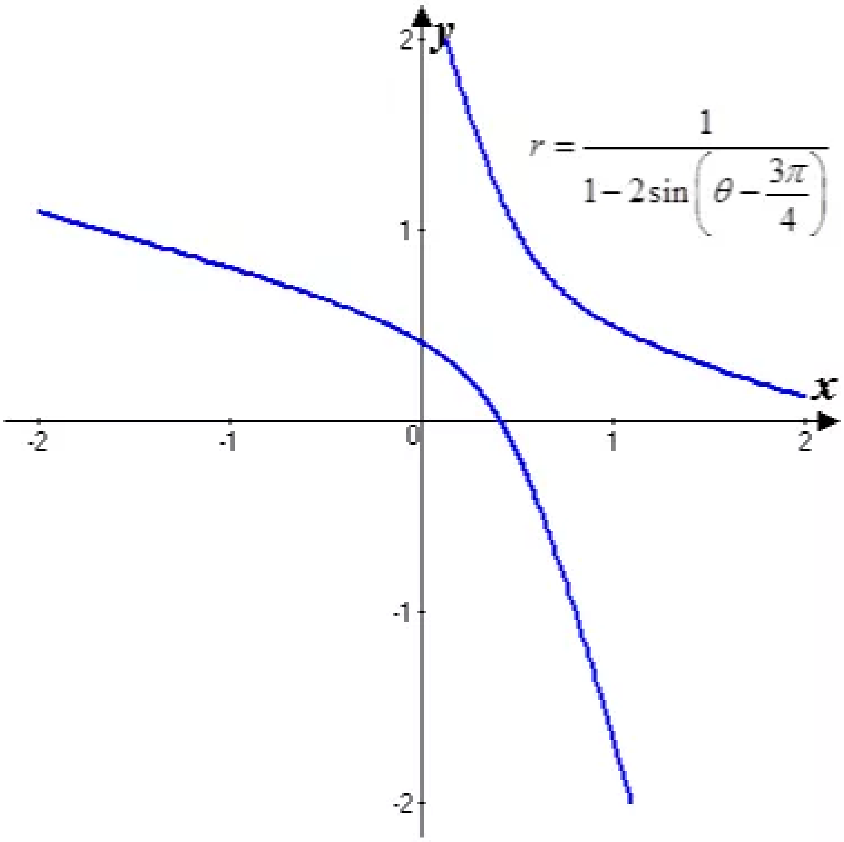
Since this conic is rotated counterclockwise about the origin through an angle $\dfrac{3\pi}{4}$, the equation of the rotated hyperbola by replacing θ with $\theta - \dfrac{3\pi}{4}$ in the equation given in

$r = \dfrac{1}{1-2\sin\theta}$.

So, the new equation is

$$\boxed{r = \frac{1}{1-2\sin\left(\theta - \frac{3\pi}{4}\right)}}.$$

Sketch of the graph of the hyperbola $r = \dfrac{1}{1-2\sin\left(\theta - \dfrac{3\pi}{4}\right)}$ as shown below:



Q18E

Theorem: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. the conic is an ellipse if $e < 1$, a parabola if $e = 1$, or a hyperbola if $e > 1$.

Consider the polar equation,

$$r = \frac{4}{5 + 6\cos\theta}$$

Dividing the numerator and denominator by 5, write the equation as

$$\begin{aligned} r &= \frac{4}{5+6\cos\theta} \\ &= \frac{4/5}{(5/5)+(6/5)\cos\theta} \\ &= \frac{4/5}{1+(6/5)\cos\theta} \end{aligned}$$

(a)

If $r=\dfrac{4/5}{1+(6/5)\cos\theta}$ comparing with $r=\dfrac{ed}{1\pm e\cos\theta}$, then $e=\dfrac{6}{5}$.

So, the eccentricity is $\dfrac{6}{5}$.

Since $e(=6/5)>1$, so given conic is a hyperbola.

To find an equation of the directrix:

If $r=\dfrac{4/5}{1+(6/5)\cos\theta}$ comparing with $r=\dfrac{ed}{1\pm e\cos\theta}$, then $ed=\dfrac{4}{5}$.

Since $e=\dfrac{6}{5}$, to get

$$\begin{aligned} d&=\frac{4/5}{e}\\ &=\frac{4/5}{6/5}\\ &=\frac{4}{6}\\ &=\frac{2}{3}\end{aligned}$$

Since in the denominator of the given equation of the conic, the sign of coefficient of $\cos\theta$ is positive

So an equation of the directrix is $x=d$ that is

$$\boxed{x=\frac{2}{3}}.$$

When $\theta=\pi/2$:

$$\begin{aligned} r&=\frac{4}{5+6\cos(\pi/2)}\\ &=\frac{4}{5}\end{aligned}$$

When $\theta=3\pi/2$:

$$\begin{aligned} r&=\frac{4}{5+6\cos(3\pi/2)}\\ &=\frac{4}{5}\end{aligned}$$

So, the vertices have polar coordinates are

$$\left(\frac{4}{5},\frac{\pi}{2}\right),\left(\frac{4}{5},\frac{3\pi}{2}\right).$$

When $\theta = 0$:

$$r = \frac{4}{5+6\cos 0}$$
$$= \frac{4}{5+6}$$
$$= \frac{4}{11}$$

When $\theta = \pi$:

$$r = \frac{4}{5+6\cos\pi}$$
$$= \frac{4}{5-6}$$
$$= -4$$

So, the *x*- intercepts are

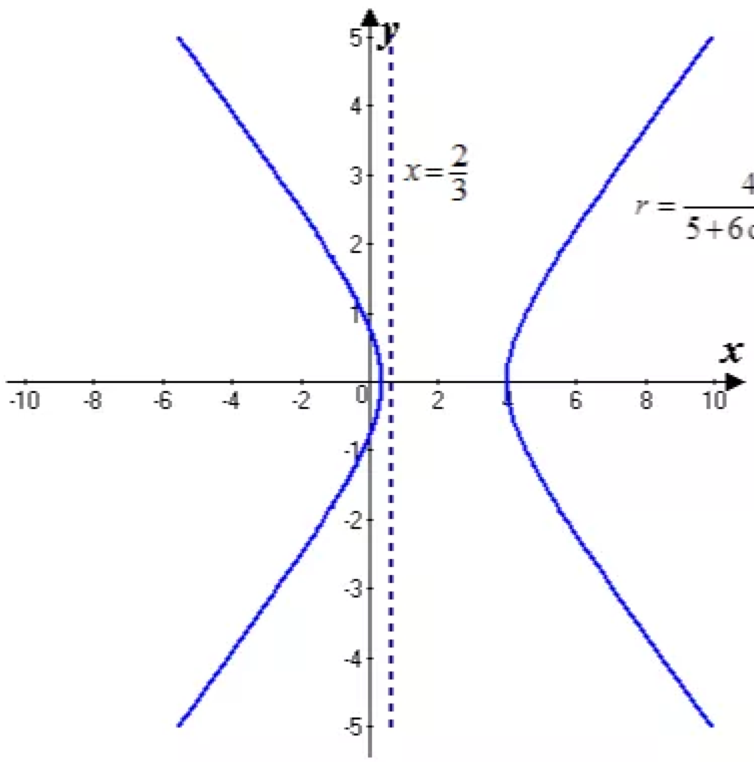
$\left(\frac{4}{11}, 0\right), (-4, \pi)$.

Note that $r \to \pm\infty$ when $1+(6/5)\cos\theta \to 0^+$ or 0^-

And $1+(6/5)\cos\theta = 0$ when $\cos\theta = -5/6$

Thus the asymptotes are parallel to the rays $\theta = \cos^{-1}(-5/6)$.

Sketch of the graph of the hyperbola $r = \frac{4}{5+6\cos\theta}$ as shown below:



(b)

Since this conic is rotated counterclockwise about the origin through an angle $\frac{\pi}{3}$, the equation

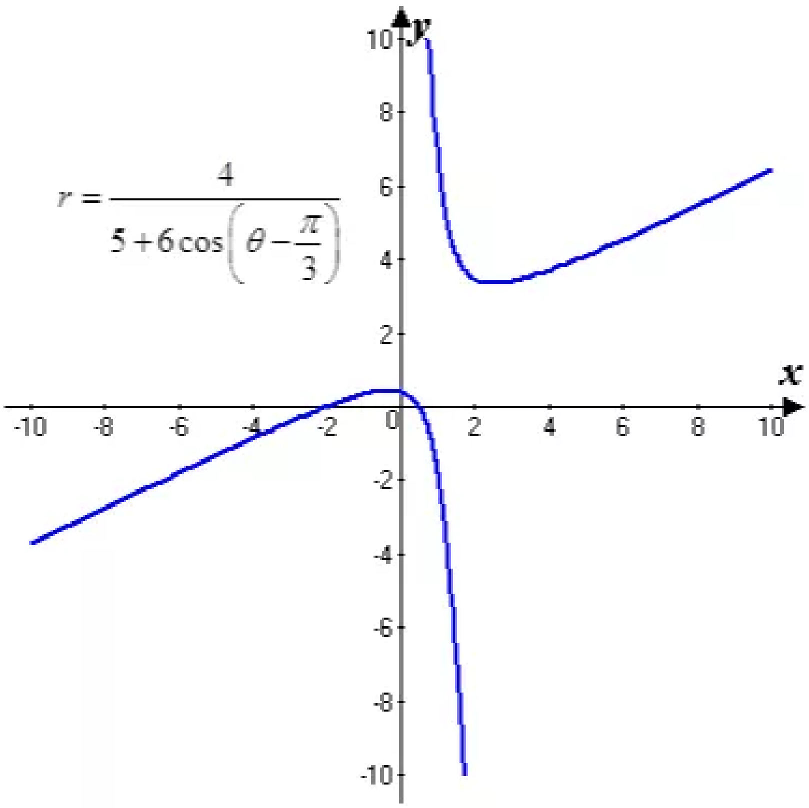
of the rotated hyperbola by replacing θ with $\theta-\frac{\pi}{3}$ in the equation given in

$$r=\frac{4}{5+6\cos\theta}.$$

So, the new equation is

$$\boxed{r=\frac{4}{5+6\cos\left(\theta-\frac{\pi}{3}\right)}}.$$

Sketch of the graph of the hyperbola $r=\frac{4}{5+6\cos\left(\theta-\frac{\pi}{3}\right)}$ as shown below:



Q19E

Given equation is $r = \frac{e}{(1 - e\cos\theta)}$

We sketch the conic with $e = 0.4$, 0.6, 0.8 and 1.0 on a common screen.

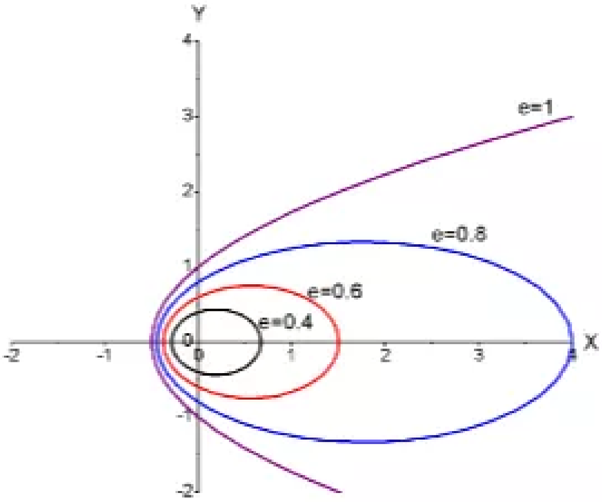


Fig.1

We see that ellipse is nearly circular when e is close to 0 and becomes more elongated as e reaches close to 1 but less than1 and for $e = 1$, the curve becomes a parabola.

Q20E

(A) Given equation is $r = \frac{ed}{(1 + e\sin\theta)}$

We sketch the conics for e = 1 and d = 0.2, 0.4, 0.6, 0.8, 1.0
Respectively on the common screen

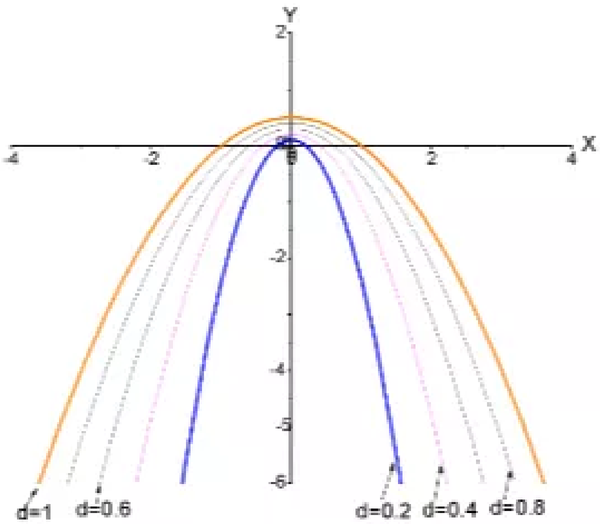


Fig.1

We see that as d increases the opening of the parabola increases.

(B) Now we sketch the conics for $d = 1$ and $e = 0.2, 0.4, 0.6, 0.8, 1$.

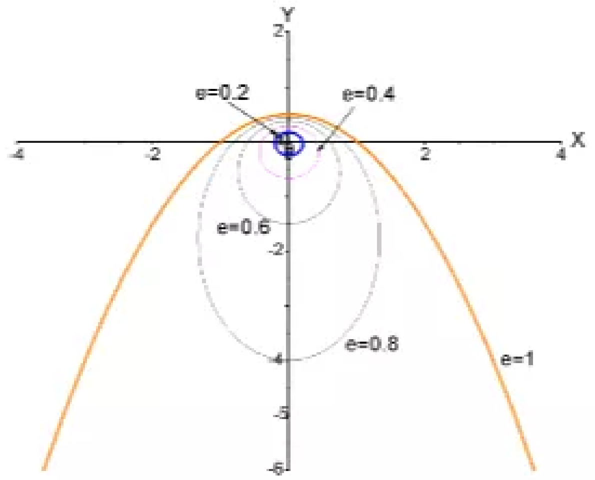
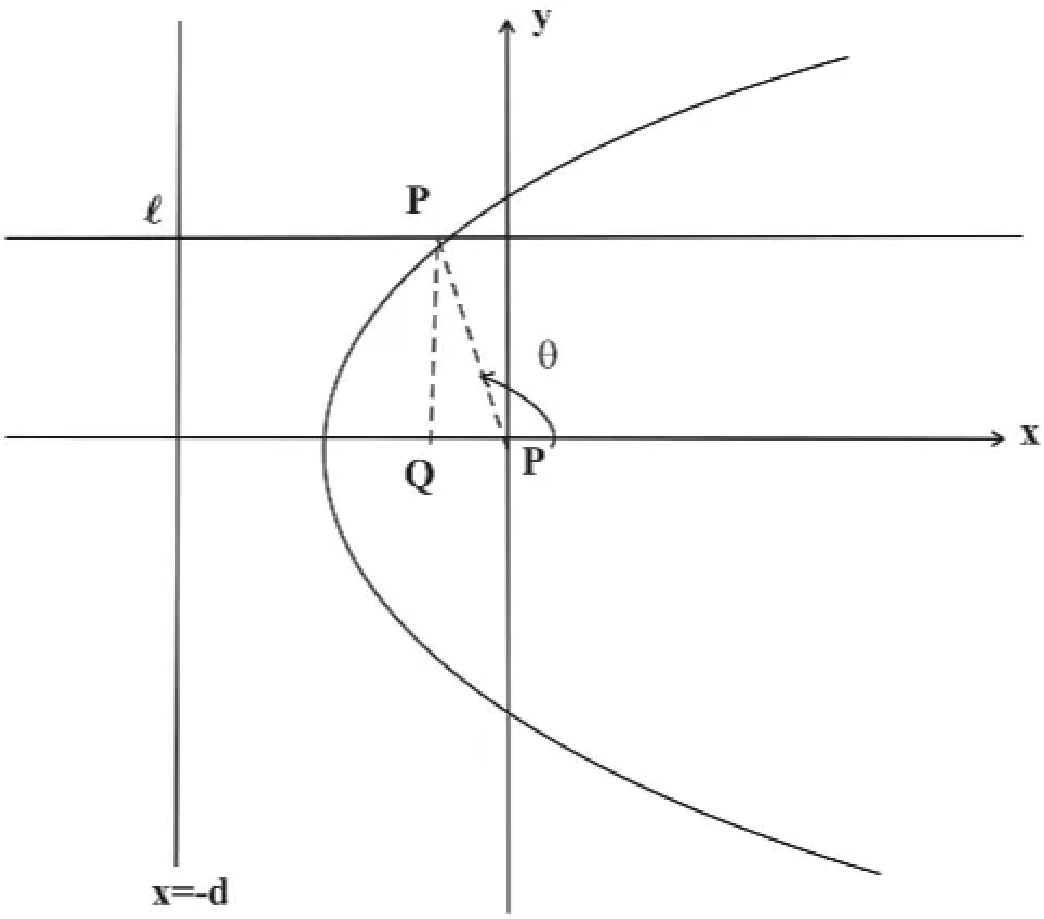


Fig.2

We see that as $e \to o^{+}$, the curve becomes more circular and as $e \to 1^{-}$ the curve becomes more elongated and at $e = 1$ curve becomes a parabola.

Q21E



In figure there is a conic with eccentricity e, directrix x=-d and focus at the origin. Let P be any point on the conic then ratio of distance of P from the directrix to the distance of P from the focus will be constant

$$\frac{|PF|}{|Pl|} = e$$

$$\Rightarrow |PF| = e|Pl|$$

$$\left[\begin{array}{l} \text{In } \Delta PQF \\ \angle PFQ = \pi - \theta \\ \text{then } |QF| = r\cos(\pi - \theta) \end{array}\right.$$

We have $|PF| = r$

And $\quad |Pl| = d - r\cos(\pi - \theta) \qquad [|QF| = r\cos(\pi - \theta)$

Then

$$\Rightarrow r = e\left[d - r\cos(\pi - \theta)\right]$$

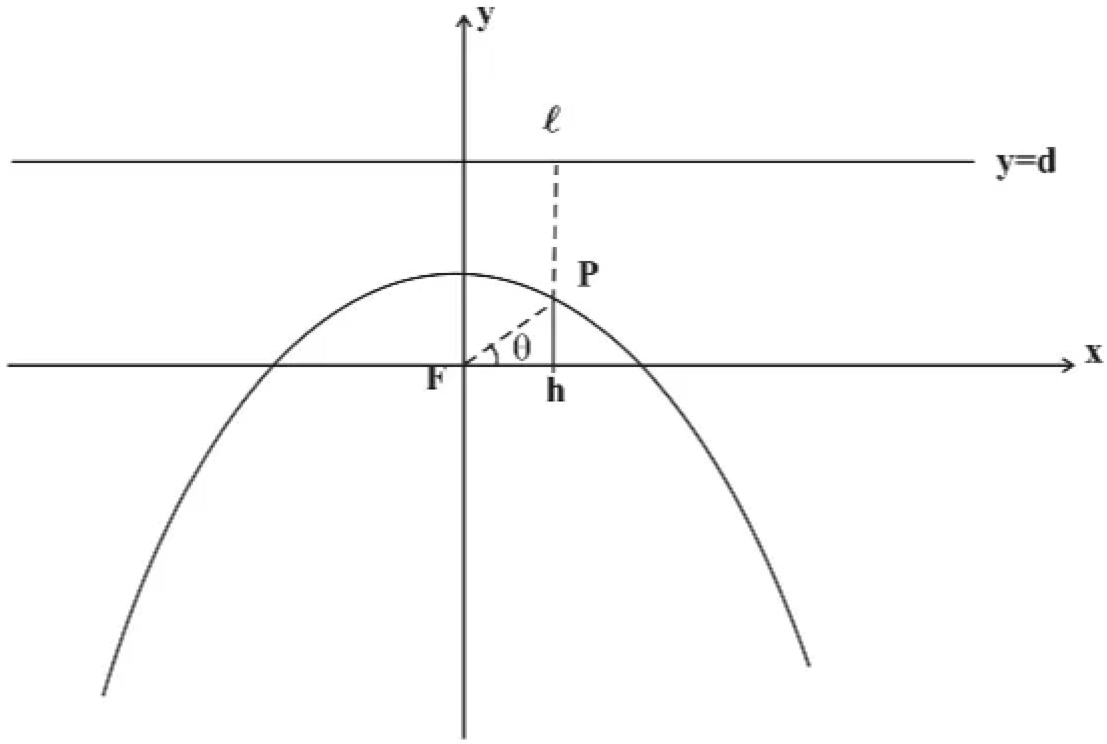
$$= ed + er\cos\theta$$

$$\Rightarrow r(1 - e\cos\theta) = ed$$

$$\Rightarrow \boxed{r = \frac{ed}{1 - e\cos\theta}}$$

This is the equation of conic.

Q22E



In figure there is a conic with eccentricity e, directrix y =d and focus at the origin

Let P be any point on the conic such that $|PF| = r$

Then $\quad \frac{|PF|}{|Pl|} = e$

$\Rightarrow r = e|Pl| \qquad$ ....(1)

In triangle PFQ,

$$\angle PFQ = \theta$$

$$|PF| = r$$

Then $|PQ| = r\sin\theta$

And so $|Pl| = d - r\sin\theta$

Then from (1) we have

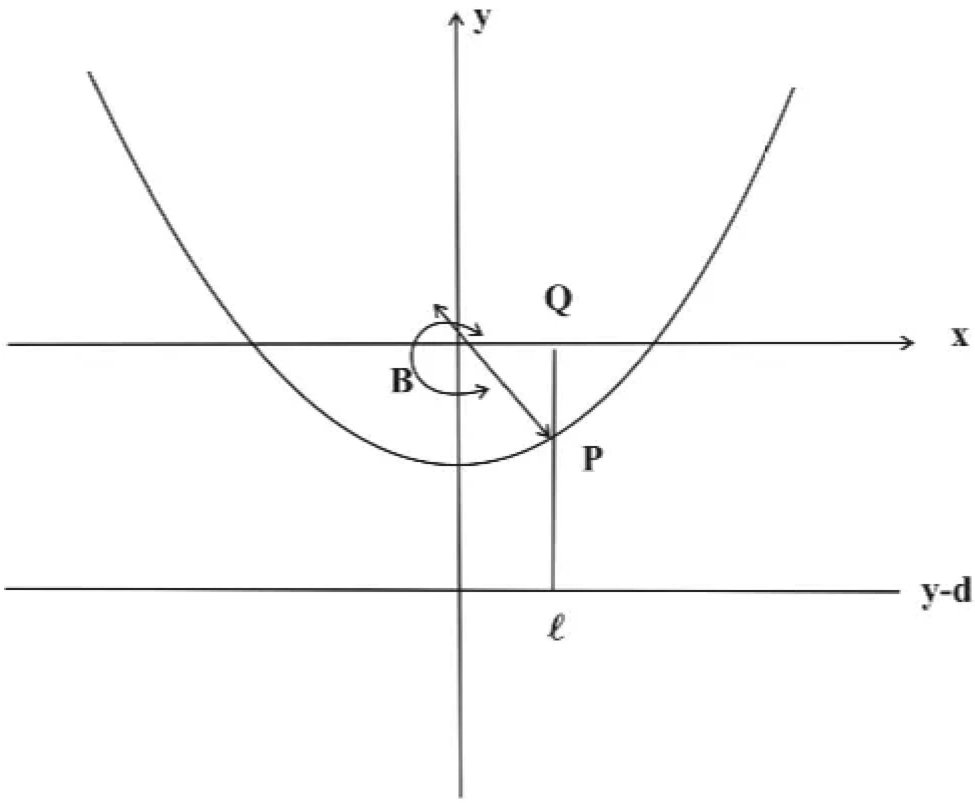
$$r = e(d - r\sin\theta)$$

$$\Rightarrow r(1 + e\sin\theta) = ed$$

$$\Rightarrow r = ed/(1 + e\sin\theta)$$

This is the equation of conic.

Q23E



In figure there is a conic with eccentricity e, directrix y = -d and focus at the origin.

Let P be any point on the conic and $|PF| = r$

Then In triangle PFQ, $|PQ| = r\sin(2\pi - \theta)$

And then $|Pl| = d - r\sin(2\pi - \theta)$

$= d + r\sin\theta$ Since $(\sin(2\pi - \theta) = -\sin\theta)$

We have $\dfrac{|PF|}{|Pl|} = e$

$$\Rightarrow r = e|Pl|$$

$$= e\,(d + r\sin\ \theta)$$

$$\Rightarrow r(1 - e\sin\theta) = ed.$$

$$\Rightarrow \boxed{r = \frac{ed}{(1 - e\sin\theta)}}$$

This is the equation of conic.

Q24E

Given equations of parabolas are $r = c/(1+\cos\theta)$ ....(1)

And $r = d/(1-\cos\theta)$ .....(2)

First we find the slope of the tangents to both the curves

For the first curve $r = c/(1+\cos\theta)$

$$\Rightarrow \frac{dr}{d\theta} = \frac{-c\sin\theta}{(1+\cos\theta)^2}$$

Then slope of the tangent to the curve $r = c/(1+\cos\theta)$ at any point is.

$$\text{Let } (m_1) = \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{-\frac{c\sin\theta}{(1+\cos\theta)^2}\sin\theta + \frac{c}{(1+\cos\theta)}\cos\theta}{-\frac{c\sin\theta\cos\theta}{(1+\cos\theta)^2} - \frac{c\sin\theta}{(1+\cos\theta)}}$$

$$= \frac{-c\sin^2\theta + c\cos\theta(1+\cos\theta)}{-c\sin\theta\cos\theta - c\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos^2\theta - \sin^2\theta + \cos\theta}{-2\sin\theta\cos\theta - \sin\theta} = \frac{\cos 2\theta + \cos\theta}{-(\sin 2\theta + \sin\theta)}$$

$$\Rightarrow m_1 = \frac{\cos 2\theta + \cos\theta}{-(\sin 2\theta - \sin\theta)} \qquad ---(3)$$

Now the second curve is $r = \dfrac{d}{1-\cos\theta}$

$$\Rightarrow \frac{dr}{d\theta} = \frac{d\sin\theta}{(1-\cos\theta)^2}$$

Then slope of the tangent to the curve at any point is

$$(Let) \quad m_2 = \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{\frac{d\sin^2\theta}{(1-\cos\theta)^2} + \frac{d\cos\theta}{(1-\cos\theta)}}{\frac{d\sin\theta}{(1-\cos\theta)^2}\cos\theta - \frac{d}{(1-\cos\theta)}\sin\theta}$$

$$= \frac{d\sin^2\theta + d\cos\theta(1-\cos\theta)}{d\sin\theta\cos\theta - d\sin\theta(1-\cos\theta)}$$

$$= \frac{d\left(\sin^2\theta - \cos^2\theta + \cos\theta\right)}{d\sin\theta\left(\cos\theta - 1 + \cos\theta\right)}$$

$$= \frac{\cos\theta - \cos 2\theta}{2\sin\theta\cos\theta - \sin\theta}$$

$$\Rightarrow m_2 = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta} \qquad ---(4)$$

Both the curves will intersect at right angle when $m_1 m_2 = -1$

So from (3) and (4) we have

$$m_1 m_2 = \frac{\cos 2\theta + \cos\theta}{-(\sin 2\theta + \sin\theta)} \cdot \frac{\cos\theta - \cos 2\theta}{(\sin 2\theta - \sin\theta)}$$

$$= \frac{\cos^2 2\theta - \cos^2\theta}{\sin^2 2\theta - \sin^2\theta}$$

$$= \frac{\left(1-\sin^2 2\theta\right) - \left(1-\sin^2\theta\right)}{\sin^2 2\theta - \sin^2\theta}$$

$$= \frac{1-\sin^2 2\theta - 1 + \sin^2\theta}{\left(\sin^2 2\theta - \sin^2\theta\right)}$$

$$= \frac{-\left(\sin^2 2\theta - \sin^2\theta\right)}{\left(\sin^2 2\theta - \sin^2\theta\right)}$$

$$\Rightarrow \boxed{m_1 m_2 = -1}$$

Thus both the curves intersect at right angle.

Q25E

Recollect the polar equation of an ellipse with focus at the origin, semimajor axis a, eccentricity e, and directrix $x = d$ can be written in the form

$$r = \frac{a\left(1-e^2\right)}{1 + e\cos\theta}$$

The orbit of Mars around the sun is an ellipse with eccentricity *e*=0.093 and semimajor axis *a*= 2.28×10^8 km.

Find the polar equation for the orbit.

$$r=\frac{a\left(1-e^{2}\right)}{1+e\cos\theta}$$
$$=\frac{\left(2.28\times10^{8}\right)\left[1-(0.093)^{2}\right]}{1+0.093\cos\theta}$$
$$=\frac{\left(2.28\times10^{8}\right)(0.991351)}{1+0.093\cos\theta}$$
$$=\frac{2.26028028\times10^{8}}{1+0.093\cos\theta}$$
$$=\frac{2.26\times10^{8}}{1+0.093\cos\theta}$$ Approximately

So, the polar equation for the orbit is

$$\boxed{r=\frac{2.26\times10^{8}}{1+0.093\cos\theta}}.$$

Q26E

Recollect the polar equation of an ellipse with focus at the origin, semimajor axis *a*, eccentricity *e*, and directrix $x=d$ can be written in the form

$$r=\frac{a\left(1-e^{2}\right)}{1+e\cos\theta}$$

The orbit of Jupiter around the sun is an ellipse with eccentricity *e*=0.048 and the length of major axis is 1.56×10^{9} km.

The length of the major axis is $2a=1.56\times10^{9}$, $a=\frac{1.56\times10^{9}}{2}$ Divide by 2 So the length of semimajor axis is

$a=0.78\times10^{9}$ km.

Find the polar equation for the orbit.

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

$$= \frac{(0.78\times10^9)\left[1-(0.048)^2\right]}{1+0.048\cos\theta}$$

$$= \frac{(0.78\times10^9)(0.997696)}{1+0.048\cos\theta}$$

$$= \frac{0.77820288\times10^9}{1+0.048\cos\theta}$$

$$= \frac{7.78\times10^8}{1+0.048\cos\theta} \text{ Approximately}$$

So, the polar equation for the orbit is

$$\boxed{r = \frac{7.78\times10^8}{1+0.048\cos\theta}}.$$

Q27E

Given the length of major axis $= 2a = 36.18AU$

$\Rightarrow a = 18.09AU$

And $e = 0.97$

Equation of orbit is $r = \frac{a(1-e^2)}{1-e\cos\theta}$

$$\Rightarrow r = \frac{18.09\left[1-(0.97)^2\right]}{1-(0.97)\cos\theta}$$

$$\boxed{r \approx \frac{1.07}{1-0.97\cos\theta}}$$

Maximum distance from the comet to the Sun

$= a(1+e)$

$= (18.09)(1+0.97)$

$= 18.09\times1.97$

$= \boxed{35.64AU}$.

Q28E

Given $e = 0.9951$

Length of major axis $2a = 356.5$ AU

$\Rightarrow a = 178.25$ AU

Equation of the orbit is $r = \dfrac{a\left(1-e^2\right)}{1-e\cos\theta}$

$$\Rightarrow r = \frac{(178.25)\left(1-(0.9951)^2\right)}{1-(0.9951).\cos\theta}$$

$$\Rightarrow \boxed{r \approx \frac{1.7426}{1-(0.9951)\cos\theta}}$$

Maximum distance form comet to Sun = a (1-e)

$$= (178.25)(1-0.9951)$$

$$\boxed{\approx 0.873 \text{ AU}}$$

Q29E

Given $e = 0.206$

And maximum distance from the Sun = $4.6\times10^7\ km$

=Perihelion distance from planet to Sun = a (1-e)

$$\Rightarrow a(1-e) = 4.6\times10^7$$

$$\Rightarrow a(1-0.206) = 4.6\times10^7$$

$$\Rightarrow \boxed{a = \frac{4.6\times10^7}{0.794}}$$

So maximum distance from the sun = a (1+e)

$$= \frac{4.6\times10^7}{0.794}(1+0.206)$$

$$\boxed{\approx 7.0\times10^7\ km}$$

Q30E

Given that

Perihelion distance from the Planet to Sun $= 4.43\times10^9$km

$$\Rightarrow a(1-e) = 4.43\times10^9$$

$$\Rightarrow a - ae = 4.43\times10^9 \qquad ---(1)$$

Aphelion distance from the Planet to Sun = 7.37×10^9km

$$\Rightarrow a(1+e) = 7.37\times10^9 \text{ km}$$

$$\Rightarrow a + ae = 7.37\times10^9 \text{ km} \qquad ---(2)$$

Adding equation (1) and (2), we have

$$2a = 11.8\times10^9 \text{ km}$$

$$\Rightarrow a = 5.90\times10^9 \text{ km}$$

Putting this value in (1)

$$a(1-e) = 4.43\times10^9$$

$$5.9\times10^9(1-e) = 4.43\times10^9$$

$$\Rightarrow 1-e = \frac{4.43}{5.9}$$

$$\Rightarrow e = 1-\frac{4.43}{5.9}$$

Eccentricity $\Rightarrow \boxed{e \approx 0.249}$

Q31E

We have $e = 0.206$

And $a(1-e) = 4.6\times10^7$

$$\Rightarrow a(1-0.206) = 4.6\times10^7$$

$$\Rightarrow a = \frac{4.6\times10^7}{0.794}$$

Equation of the orbit is $r = \dfrac{a(1-e^2)}{1-e\cos\theta}$

Then $\dfrac{dr}{d\theta} = \dfrac{-a(1-e^2)e\sin\theta}{(1-e\cos\theta)^2}$

$$\Rightarrow r^2 + \left(\frac{dr}{d\theta}\right)^2 = \frac{a^2(1-e^2)^2}{(1-e\cos\theta)^2} + \frac{a^2(1-e)^2 e^2\sin^2\theta}{(1-e\cos\theta)^4}$$

$$= \frac{a^2(1-e^2)^2}{(1-e\cos\theta)^4}\left(1+e^2\cos^2\theta - 2e\cos\theta + e^2\sin^2\theta\right)$$

$$= \frac{a^2(1-e^2)^2}{(1-e\cos\theta)^4}\left(1-2e\cos\theta+e^2\right) \qquad \left[\sin^2\theta+\cos^2\theta=1\right]$$

So length of the orbit is $L=\int_0^{2\pi}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

$$\Rightarrow L=\int_0^{2\pi} a\left(1-e^2\right)\sqrt{\frac{\left(1-2e\cos\theta+e^2\right)}{\left(1-e\cos\theta\right)^4}}\,d\theta$$

$$\Rightarrow L=a\left(1-e^2\right)\int_0^{2\pi}\frac{\sqrt{\left(1-2e\cos\theta+e^2\right)}}{\left(1-e\cos\theta\right)^2}d\theta$$

$$\Rightarrow L=a\left(1-e^2\right)\int_0^{2\pi}\frac{\sqrt{\left(1-2e\cos\theta+e^2\right)}}{\left(1-e\cos\theta\right)^2}d\theta$$

$$\Rightarrow L\approx 4.4\times10^7\left(1-\left(0.206\right)^2\right)\int_0^{2\pi}\frac{\sqrt{1-2\times0.206\cos\theta+\left(0.206\right)^2}}{\left(1-0.206\cos\theta\right)^2}d\theta$$

$$\Rightarrow L\approx 4.28\times10^7\int_0^{2\pi}\frac{\sqrt{1.04-0.412\cos\theta}}{\left(1-0.206\cos\theta\right)^2}d\theta$$

Let $F(\theta)=\frac{\sqrt{1.04-0.412\cos\theta}}{\left(1-0.206\cos\theta\right)^2}$

For evaluating the length of orbit we use Simpson's rule with $n=20$.

Then $\Delta\theta=\frac{2\pi}{20}=\frac{\pi}{10}\Rightarrow\frac{\Delta\theta}{3}=\frac{\pi}{30}$

Subintervals are $[0,\ \pi/10],[\pi/10,\ \pi/5],[\pi/5,\ 3\pi/10],\ldots,[19\pi/10,\ 2\pi]$

Then by Simpson's rule the length of the curve is

$$L\approx 4.28\times10^7\,\frac{\pi}{30}\left[f(0)+4f\left(\frac{\pi}{10}\right)+2f\left(\frac{\pi}{5}\right)+4f\left(\frac{3\pi}{10}\right)+\ldots\ldots+2f\left(\frac{18\pi}{10}\right)+4f\left(\frac{19\pi}{10}\right)+f(2\pi)\right]$$

$$\Rightarrow \boxed{L\approx 3.6\times10^8\text{ km}}$$