Transformation & Symmetry in Geometrical Shapes

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At the house of Salma's friend a large table was needed for the birthday party.

Salma said, "I have a large table at home. We can bring that here."

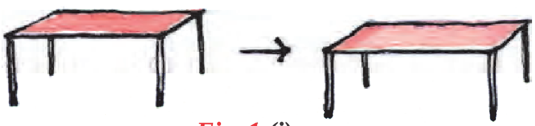


Fig. 1 (i)

At Salma's house, they first brought the table from the corner of the room near the door.

Then they began thinking about how to take the large table out of the door?

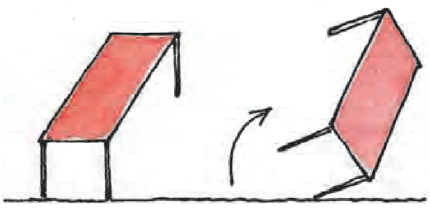


Fig. 1 (ii)

For this, they first tilted the table so that it could come out of the door and then inverted to place it on the van that would transport it. In this process, the orientation of the table changed many times.

In the first step table was displaced from one place to another. In the next step the table was rotated and then it was turned over.

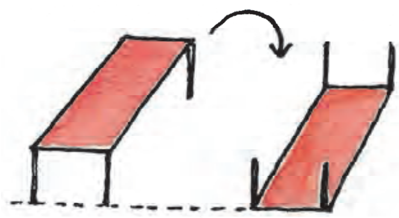


Fig. 1 (iii)

We note that in this entire transportation the orientation of the table changed many times. There was however, no change in its shape and size. Thus, by moving, turning or tilting does not affect the shape or size of a thing.

Let us now suppose that Salma wants to make pictures of this process of moving the table in her notebook.

Would the size of the picture of the table be different from the actual size of the table? *Fig.* 1(iv)

Discuss with your friends.

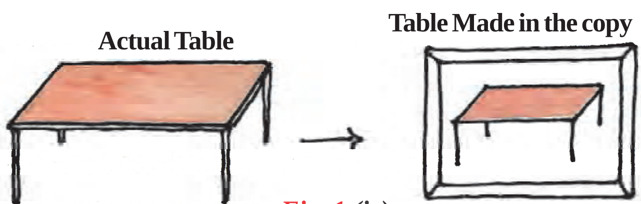


Fig. 1 (iv)

Around us we see shapes that are triangular, circular, spherical, and rectangular in our daily life. In actual life all things are three dimensional but if we look at the face of the three dimensional objects from the front, or from the top or from either the right or the left side then, we see only the face as a two dimensional shape.

Do This

We have given examples of some concrete materials here. Observe these objects from the top, front, right or the left and complete the following table accordingly:

S. No.	Name of the Object	Three dimensional shape	Seeing it from various perspective		
			Top	In front	Left/Right
1.	Dice	Cube	Square	....	....
2.	Toothpaste box	Cuboid	Rectangle	Rectangle	....
3.	Battery of a torch	Cylinder	....	....	....
4.	Ball	Sphere	....	....	....

Transformation

We have just seen that on inverting, rotating or inclining objects and observing them gives different appearances. In many situations, the shape of the objects changes, while for many others it stays the same. Such observations are usually made in our daily life activities. Deepti gave this example; "When I organize the furniture in my room then I rotate, shift, change positions of the furniture items like, sofas, tables, chairs and beds in many different ways". In order to change the place of a picture on the wall we shift the picture from one place to the other.

Ashwin said, "We keep the used utensils facing up and after washing they are placed facing down". The position of the utensils as they were originally and after turning over is not the same. The utensil seems different in these two positions.

Akanksha says, "When I make a picture of my school building, the picture has the structure and shape of the building but the size is smaller".

When you get a passport size picture enlarged into a bigger picture, is this also a transformation?

Dipti thought for a while and said, "I have found that in some situations the transformed shape appears to be the same as the original and is also congruent but in other situations the size changes after transformation, i.e. They are not congruent. That means any action on a shape that changes its position, shape or size can be called a transformation."

Consider the operations performed in *Fig.*2(i) and (ii)

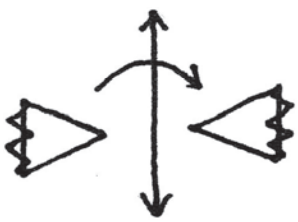
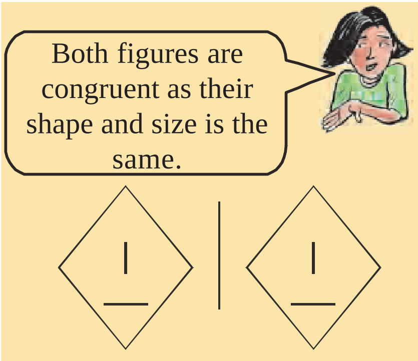


Fig. 2 (i)

Fig. 2 (ii)

Look at the *Fig.*2(i), if we rotate the initial shape about the line *l*, then we will get the second shape. In *Fig.*2 (ii), the shape has been moved from one position to another.



In these situations if we place the original shape on the transformed shape, then would both cover each other?

We know that similar shapes that are of the same size are also congruent. This means the transformed and the original shapes in *Fig.*2(i) and (ii) are congruent.

(These figures are congruent because they are of same shape and size)

Fig. 2 (iii)

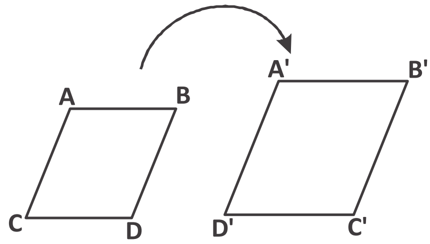


Fig. 2 (iv)

Now say, are the figures obtained after rotation and scaling up respectively in *Fig.*2(iii) and 2(iv) congruent to the original ?

Think and Discuss

Write two examples from your daily life in which you change the size, position or the shape of objects.

Think about these transformations, discuss with your friends and complete the table below:

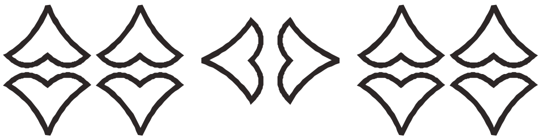
S.No.	What is happening	Are the figures same in size	Are the figures same in shape	Are the figures Congruent
(i)	Overturning	Yes		
(ii)	Shifting		Yes	
(iii)	Rotating			Yes
(iv)	Scaling up	No	Yes	No

What conclusions can you draw from the above table?

Maria immediately said, "In *Fig.*2(i) over turning and 2(ii) shifting and in 2(iii) rotating the initial shape is congruent to the shape after the transformation. However, in *Fig.*2(iv) there is no congruence due to the scaling up."

Playing with Geometrical Shapes

This is a design of a border on the wall, please extend it:-



The motif of this design is , we can get the entire border by rotating, inverting or sliding the above motif. Let us see how this can be done?

We get the first figure of the border  by inverting the motif. The second figure namely  of the border is got by sliding the motif and then inverting it. Finally rotating the motif  in an anticlockwise direction by 90°, we get the shape . By inverting this shape we get . Extend the border in this manner.

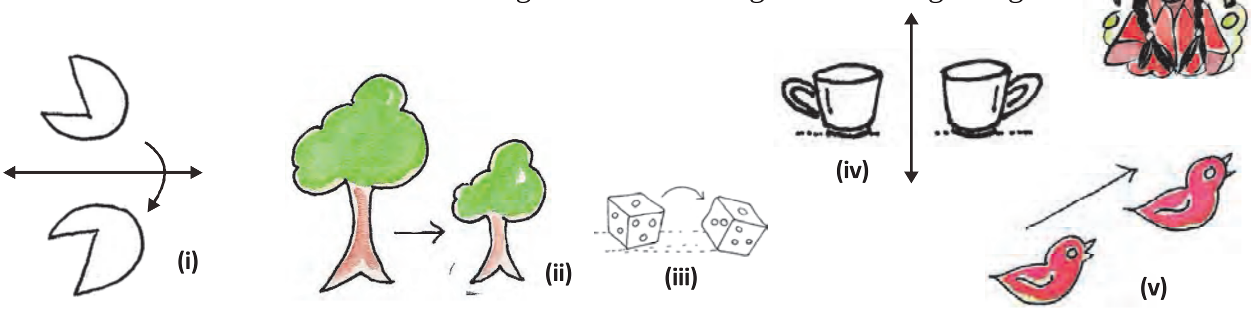
Can you make some more borders using the same motif? Think about it and make such borders.

Try This

Choose motif and using transformations make new designs and borders.

Exercise 12.1

1- What particular operation is happening in each of these figures? Observe, think and decide in which transformation the figure obtained is congruent to the original figure?



Types of Transformation

We can see two types of transformation:

1. **Rigid transformation**: Operations under which the transformed figure is congruent with the original figure are called rigid transformation.

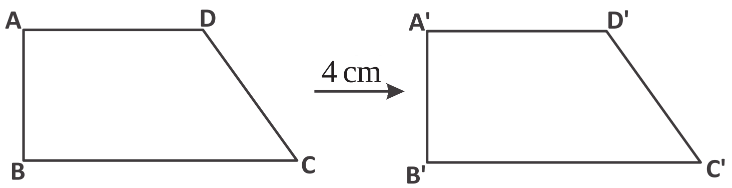


Fig. 3 (i)

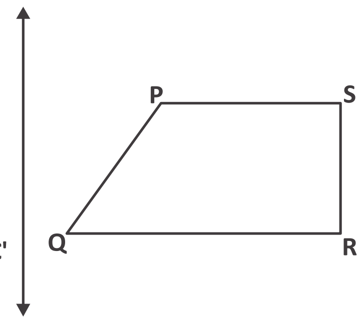


Fig. 3 (ii)

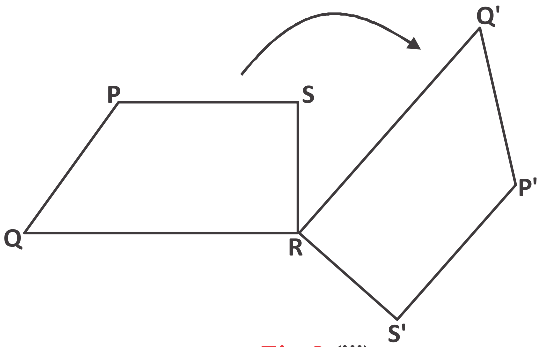


Fig. 3 (iii)

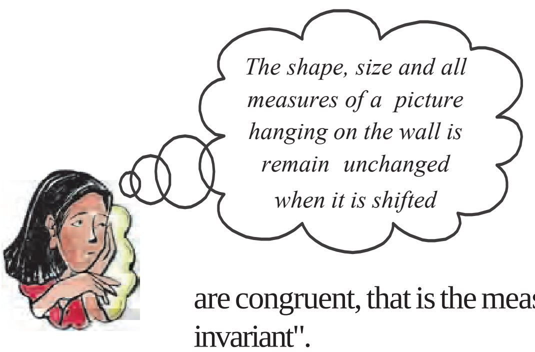
In the above set of diagrams quadrilateral ABCD is slid in *Fig.*3(i), inverted in *Fig.*3(ii) and then rotated in *Fig.*3(iii). Are all of these diagrams of the same quadrilateral?

In this we have performed three different operations on the same diagram. We can see that in these the position of the transformed diagram appears to be different from the original.

We know that in rigid transformations the shape and size of the figure does not change.

(i) Translation

Let us consider the first operation. In *Fig.*3(i) quadrilateral ABCD is moved horizontally by 4 *cm*. Will the size of all the sides remain invariant? Will the measure of all the angles remain invariant? Yes they will. (Why?)



Remember the properties of congruent shapes.

Dipti said: "Yes, in transformation (i) both the quadrilaterals are congruent, that is the measures of corresponding sides and corresponding angles remains invariant".

$\therefore$ AB = A'B', BC = B'C', CD= C'D' and DA= D'A'

Similarly the congruent angles would be equal

$\angle$ABC = $\angle$A'B'C', $\angle$BCD = $\angle$B'C'D', $\angle$CDA = C'D'A'

and DAB = $\angle$D'A'B'

Think and discuss

Would the corresponding sides and angles of a quadrilateral remain invariant under rotation and inversion as well? Think about the reasons, discuss and write.

The shape and size of the quadrilateral remains invariant under translation, therefore this is a rigid transformation. Any operation in which a shape or an object is shifted by a particular distance in a particular direction to a different location is called a translation.

(ii) Reflection

Look at the operation in *Fig.*4 -

If you place a mirror on line ℓ, then how will the image of quadrilateral ABCD appear?

Would it look like quadrilateral A'B'C'D'?

Ravi says if I invert (rotate by 180) quadrilateral ABCD about the line ℓ, then I get quadrilateral A'B'C'D'.

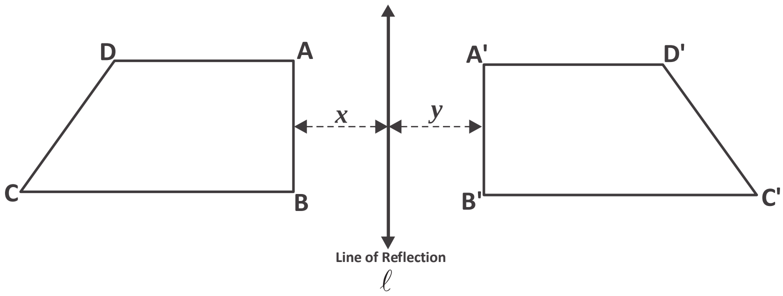
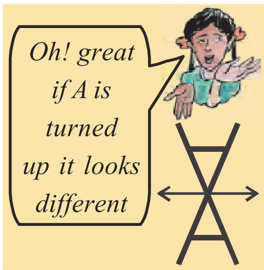


Fig. 4

Do you agree with Ravi's method? Take another shape and rotate it by 180, around a particular line and see what kind of shape you get? Is this picture the same as what you would get when you reflect the shape on a plane mirror placed on the same line?



This operation is called reflection and the line around which the shape is reflected is called the line of reflection. In this shape is inverted or rotated around a specific line by 180 to obtain the transform shape.

If we consider the distance of line 'ℓ' from the quadrilateral ABCD to be x and distance of the quadrilateral A' B' C' D' from 'ℓ' to be y, then would x be equal to y i.e is $x = y$? Yes, the original shape and the transform shape would be equidistant from the line 'ℓ'.

Notice that this is a particular property of reflections. Let us try to understand this with an example-

In *Fig.*5 here, square MNOP is transformed to M'N'O'P'. Join the corresponding vertices of the two squares with line segments. The line segments PP', OO' and NN', intersect with the line 'ℓ' at points A, B and C respectively.

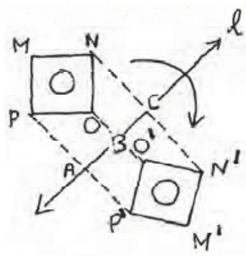


Fig. 5

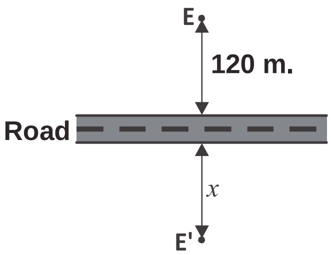
Can we say PA= P'A

Sakshi says: "The distance of point A from the vertex P is the same as a distance of point A from the vertex P'. This means PA= P'A.

Would the statements OB = OB' and NC=NC' be similarly true? (why)

From the above discussion we can conclude that the distance of the two squares from the line 'ℓ' is the same.

Try This



1. Reflection is a rigid transformation. Why? Discuss in your group and write with reason.
2. On one side of the road an electric pole E, is fixed. The distance of the road from the pole is 120 m. Taking the road as the line of reflection, reflect the pole. After reflection what would the perpendicular distance (x) of the image of the electric pole (E'), formed on the other side of the road be from the road?

(iii) Rotation

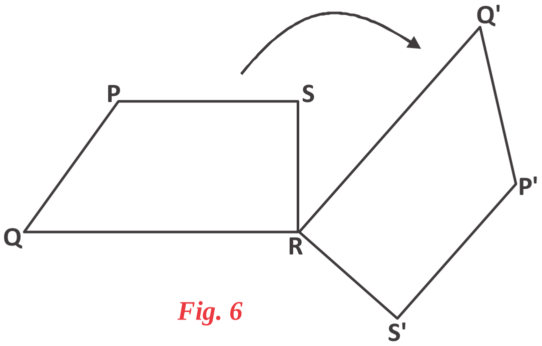


Fig. 6

Now we discuss the third kind of transformation.

Here the quadrilateral PQRS has been rotated about a point 'R' at the centre in a clockwise direction. This transformation is called rotation.

Look at the *Fig.*6 and say

Is quadrilateral PQRS ≅ quadrilateral P' Q' R S' ?

(≅ is the sign of congruence)

This rotation is a rigid transformation. Why?

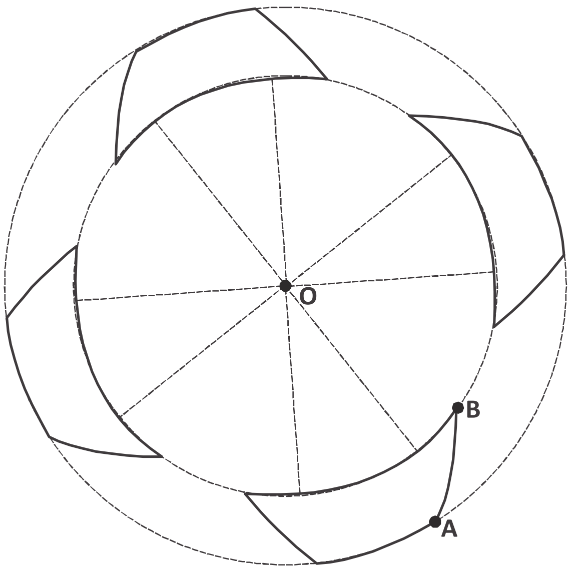


Fig. 7

You would have seen many rides in fairs that go round in big circles. One such ride is shown in *Fig.*7. The ride is revolving around the point 'O'. That is the point 'O' is the point of rotation. This point of rotation is located outside the body. If we see a moving ride going around in a circle we will notice that it revolves in a circular path around the point 'O'. (*Fig.*7) If we look carefully we realise any point A or B on the ride would move in a circular trajectory.

Let us discuss this process using a different example-

Look at *Fig.*8: in this the flagpole PQ is rotated clockwise by 30° with 'O' as the point of rotation. This point of rotation 'O' is also located outside the body.

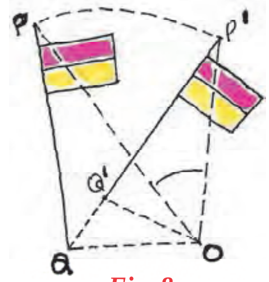


Fig. 8

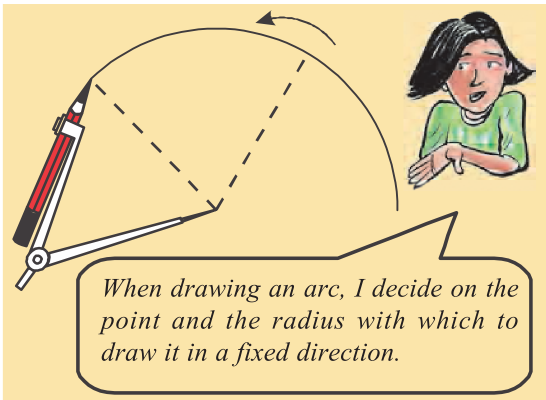
In this case, to rotate the flafpole by an angle of 30° we take any two points P and Q on it. We draw lines joining point P and Q respectively to the point 'O'. Keeping the lengths of the lines invariant, rotate each of them clockwise by 30°. The flagpole has rotated by 30° and the points P and Q have also rotated by 30°.

For the process of rotation we need to pay attention to the following three things-

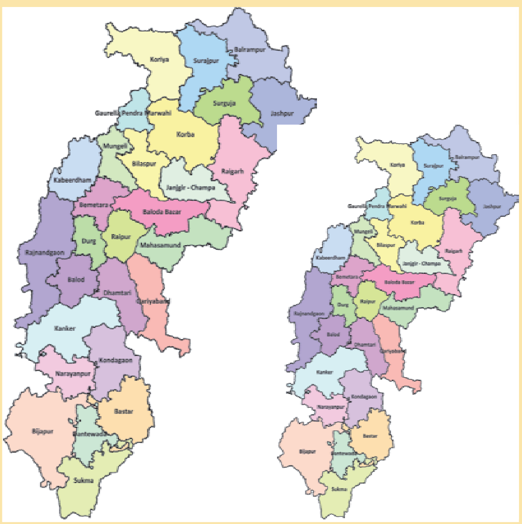
First- **Centre of rotation**: The fixed point around which the body will be rotated. This point can be on the body or outside it.

Second- **the direction of rotation**, which can be clockwise or anticlockwise.

Third- **the measure of angle of rotation**, apart from deciding the centre of rotation and the direction of rotation we need to know the angle of rotation.

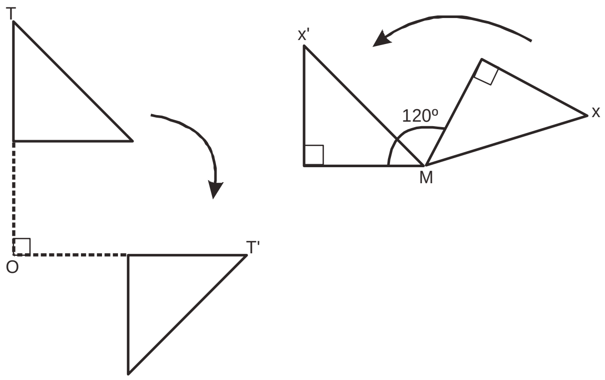


2. Non-rigid Transformation : Transformations under which the transformed object and original object are not congruent, for example scaling (magnifying or reducing) of the original object, are called non rigid transformations.



Look at the two maps of Chhattisgarh drawn above. The shape of both the maps is the same but the sizes are different.

Try This

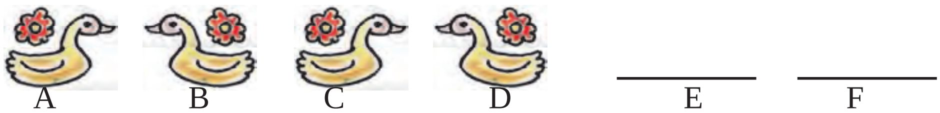


Look at the rotations in the two figures and answer the questions below separately for each-

1. What is the point of rotation?
2. Does the point lie inside or outside?
3. Direction of rotation
4. The angle of rotation?

Exercise 12.2

1. You have to make a border for the wall of a room.
   Complete the following border.



Look at the pattern and answer the following questions.

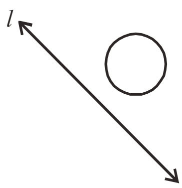
(i) If you only had the figure 'A' as above, would you be able to make this complete pattern by translating, rotating or reflecting A?

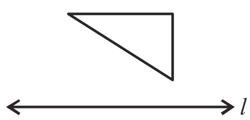
(ii) Which figures can be obtained by transforming A? Which transformation will you use for this purpose?

(iii) Which transformation will you use to get D from B?

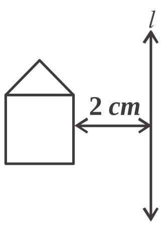
2. Taking 'l' to be the line of reflection complete the pictures-

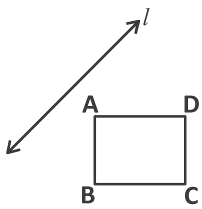
(i)


(ii)


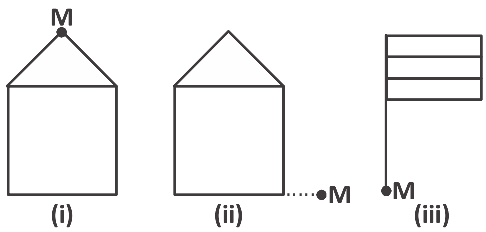
(iii)


(iv)


(v)


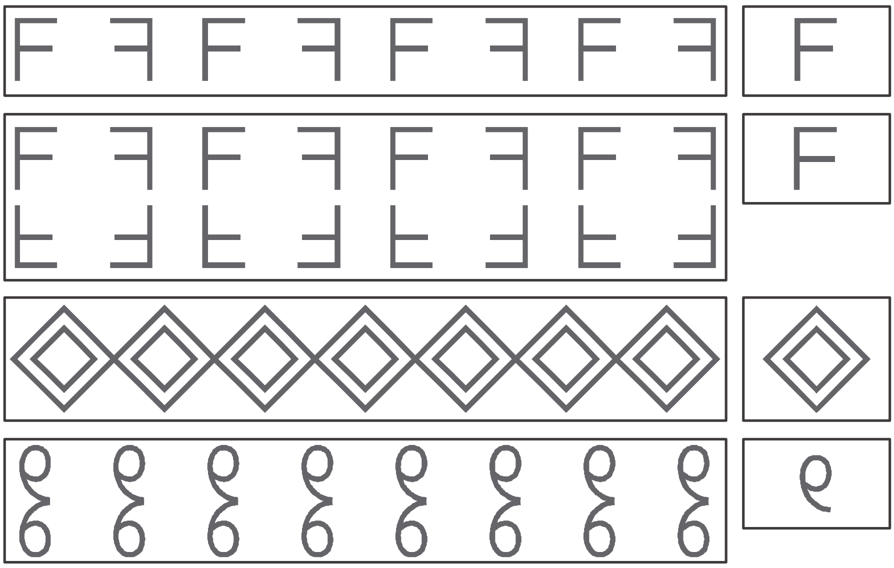
(vi)


3. Taking point 'M' to be the point of rotation, rotate the following figures as directed-
   (i) Clockwise by 90°
   (ii) Anticlockwise by 30°
   (iii) Clockwise by 60°



4. Choose any shape of your choice. Using translation, rotation, reflection design a border of the table cover.

5. Look carefully at each of the basic pattern (motif) given on the right. Draw the next figure for each pattern. Which transformation did you used?



Symmetry

Look at that figures drawn here. If we fold them right in their middle we will get exactly the same shapes on the two sides of the fold.

What do we call such figures? What do we call the line that divides the figure into two identical parts?

Fig. 9

These figures are called symmetrical shapes and the line that divides the figure into two identical parts is called the line of symmetry.

(i) Linear Symmetry

In many natural objects, buildings, geometrical shapes and other things, we can see symmetry.

Look at *Fig.*10. Draw a line on this that will divide it in two identical parts. The shapes on both sides of the line must be identical.

Fig. 10

Fig. 11

You can think of some more figures like this.

Look at the *Fig.*11. The line of symmetry is horizontal here and the picture of the bird on both sides is identical. This is called as linear symmetry.

Can any other line be drawn in these which divides them in identical parts?

(ii) Rotational Symmetry

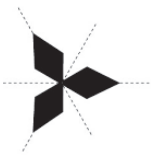
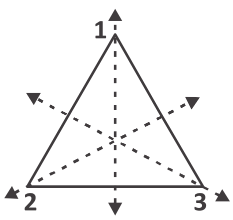


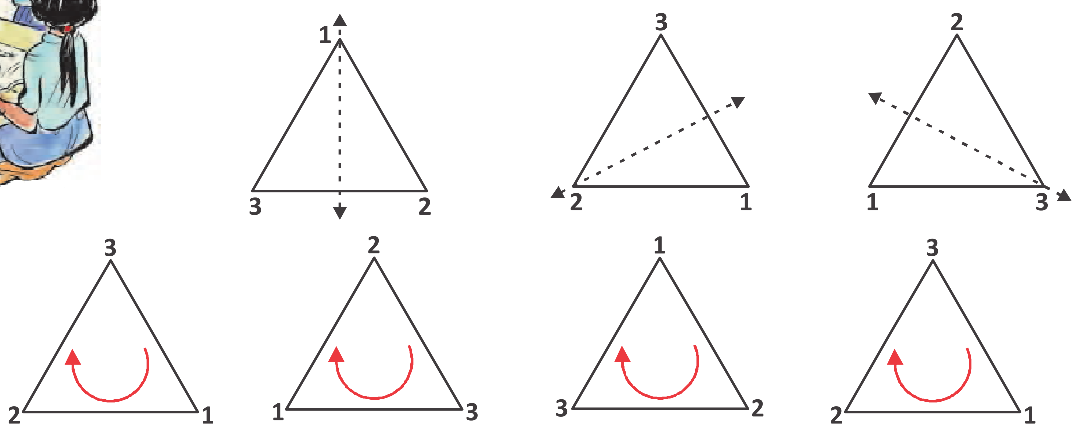
Fig. 12 (i)

Fig. 12 (ii)

Look at *Fig.*12 (i) and 12 (ii). How many lines of symmetry do these have? You would find that on in one rotation they will look like their initial state at least once.

Let us consider the rotational symmetry of an equilateral triangle. There are three lines of symmetry in a equilateral triangle. In a complete rotation an equilateral triangle is identical to the initial state at three positions. This number is the order of rotation. In the same manner find out the order of rotation for the other two shapes in *Fig.*12.





Initial state　After rotating by 120°　After rotating by 240°　After rotating by 360° (the initial state)

Fig. 13

On the basis of the above discussion complete the following table-

Figure	How many lines of symmetry?	In one complete rotation how many times the same as original	Order of rotation
Regular Pentagon			
Equilateral Triangle	3	3	3
Rectangle			
Square			
The letter U			
The letter M			

Think and Discuss

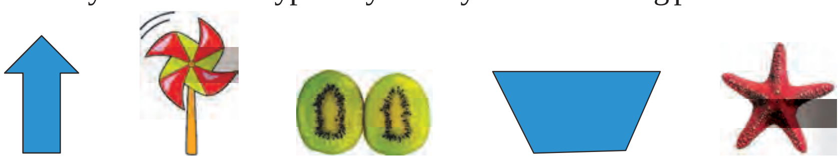
How many lines of symmetry are there in regular polygon? Is there a relation in the number of sides and order of rotations in such a polygon? What is it?

Try This

1. What symmetry can be seen in the following letters? Identify the point of symmetry and write it.

   I F N H G A O

2. Identify and write the type of symmetry in the following pictures?



Applications of Symmetry and Transformations

Many designs and patterns can be seen on floors, walls, wallpapers, saries, clothes etc. These have many symmetries and often are made with transformations on one motif. For example in *Fig.*14(i) the motif can be recognised at many places. It is the same in *Fig.*14(ii).

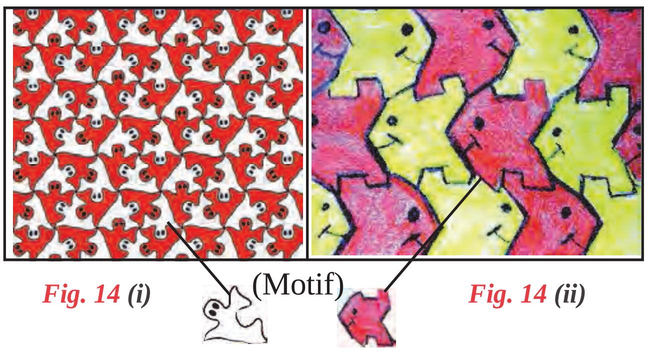


Fig. 14 (i) *Fig. 14 (ii)*

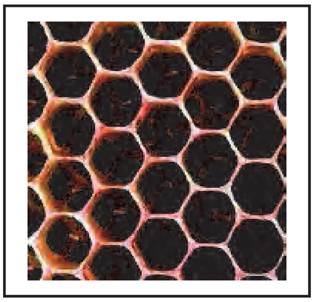


Fig. 15 (i)

Fig. 15 (ii)

Look at *Fig.*15(i) and *Fig.*15(ii) carefully. Identify the motif and draw.

Exercise - 12.3

1. Draw pictures of some objects that show-
   (i) linear symmetry
   (ii) rotational symmetry
2. Take a shape of your choice, using it as the motif, create a pattern.
3. Identify English alphabet, that have-
   (i) two lines of symmetry (ii) No lines of symmetry
   (iii) rotational symmetry

What Have We Learnt

1. If congruent shapes are inverted, rotated or translated then they remain congruent.
2. Transformations under which the transformed shape is congruent to the original shape are called rigid transformations.
3. Rigid transformations include the three processes translation, reflection and rotation.
4. For translation the distance and direction need to be specified.
5. For rotation, the point of rotation, the angle of rotation and direction of rotation needs to be known.
6. The original shape and the transformed shape are equidistant from the line of reflection.
7. We have learned about two symmetries- linear symmetry and rotational symmetry.
8. Repeating a motif and placing it on a plane in an organised manner without any gaps or overlaps can produce patterns.