## Diffenential Equations

Differentiation: An equation involving the independent variable x (say), dependent variable y (say) and the differential cofficients of dependent variable with

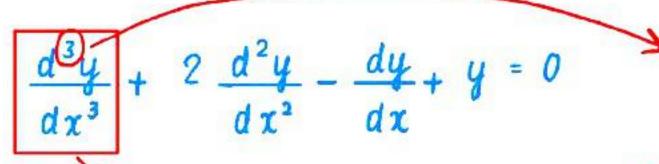
respect to independent vaniable i.e.  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$ ,  $\frac{d^3y}{dx^3}$ ,..., etc.

Example:  $\frac{dy}{dx} + 4y = x$ ,  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = x^2$  and differential equations.

onden and Degnee of a differential equation:

The Onder of a differential equation is the highest order derivative occumning in the differential equation

The degree of a differential equation is the degree of the highest order derivative occumning in the equation, when the differential cofficients are made free from nadicals, fractions and it is written as a polynomial in differential co-efficient.



highest order denivative = 3 => onder = 3

the degree of the highest order derivative occurring in the equation degnee = 1

$$\left(\frac{d^2y}{dx^2}\right)^3 + \sin\frac{dy}{dx} = 0$$
 onder = 2 degree =

degnee = not defined (because this differential eqn cannot be written in the form of polynomial in diff" co-efficient)

- Note: Onden and degree of a differential eqn are always positive integers.
- General Solution: The solution which contains ambitnary constants is called the general solution (primitive) of the differential equation.
- Panticular solution: The solution obtained from the general solution by giving panticular values to the antitrary constants is called a panticular solution of the differential equation.
- Equations in variable separable form:

Consider the equation dy = X.Y where X is a function of x only and Y is a function dx of y only.

- (1) Put the equation in the form 1 dy = X.dx
- (ii) Integrating both the sides, we get  $\int \frac{dy}{v} = \int X dx + C$  where c is an arbitrary constant. Thus the required soln is obtained.
- Equations Reducible to variables Sepenable form:
- (i) Write the given equation in form  $\frac{dy}{dx} = f(ax + by + c)$
- (ii) Put ax + by + c = z, so that  $\frac{dy}{dx} = \frac{1}{b} \left( \frac{dz}{dx} a \right)$

- (iii) Putting this dy in the given equation, we get  $\int_{b}^{a} \left(\frac{dz}{dx} a\right) = f(z)$ . This eq<sup>n</sup> is neduced in the form: dz = dx. After integrating, we get the nequired a + bf(z)nesult.
- Momogenous differential Equation A differential equation of the form  $\frac{dy}{dx} = F(x,y)$ is said to be homogenous differential equation if F(x,y) is a homogenous function of degree zero.
- (i) Suppose y = vx and so  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ .
- (ii) The value y = vx and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  is substituted in given eq<sup>n</sup>. The vanuable sepanable form, which can be solved by integrating both sides.
- (IV) finally v is neplaced by y to get the nequined solution.
- Tote: If the homogenous differential equation is in the form dx = F(x)Substitute x = vy and so  $\frac{dx}{dy} = v + y \frac{dv}{dy}$  and proceed as above.
- Finst onden linean differential equation

$$\frac{dy}{dx} + Py = Q - (i)$$

$$I \cdot F \cdot = e^{\int P dx} \quad \text{(I.F. = Integnating factor)} \qquad I \cdot F \cdot = e^{\int P dy} \quad \text{(I.F. = Integnating factor)}$$

$$Solution \quad of (i) \quad is \quad ;$$

$$y \cdot (I \cdot F \cdot) = \int Q \times (I \cdot F \cdot) dx + C$$

$$x \cdot (I \cdot F \cdot) = \int Q \times (I \cdot F \cdot) dy + C$$

$$\frac{dy}{dx} + Py = Q \qquad (i)$$

$$\frac{dx}{dy} + Px = Q \qquad (i)$$
where P and Q are constants on function of x only, where P and Q are constants on function of y only.

$$T \cdot F = e^{\int Pdy}$$
 (I.f. = Integnating factor)

$$x \cdot (I \cdot F.) = \int Q \times (I \cdot F.) dy + c$$