Differential Equations

✓ **Differentiation** : An equation involving the independent variable x (say), dependent variable y (say) and the differential cofficients of dependent variable with respect to independent variable i.e. $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots,$ etc.

Example : $\frac{dy}{dx} + 4y = x$, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = x^2$ are differential equations.

✓ **Order and Degree of a differential equation** :

The Order of a differential equation is the highest order derivative occurring in the differential equation

The degree of a differential equation is the degree of the highest order derivative occurring in the equation, when the differential cofficients are made free from radicals, fractions and it is written as a polynomial in differential co-efficient.

Example : $\boxed{\frac{d^3y}{dx^3}} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$

highest order derivative = 3 ⇒ order = 3

the degree of the highest order derivative occuring in the equation

degree = 1

$\left(\frac{d^2y}{dx^2}\right)^3 + \sin\frac{dy}{dx} = 0$

order = 2

degree = not defined (because this differential eq^n cannot be written in the form of polynomial in diff^n co-efficient)

📍 Note : Order and degree of a differential eq^n are always positive integers.

✓ **General Solution** : The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.

✓ **Particular solution** : The solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation.

✓ **Equations in variable separable form** :

Consider the equation $\frac{dy}{dx} = X.Y$ where X is a function of x only and Y is a function of y only.

(i) Put the equation in the form $\frac{1}{Y}dy = X.dx$

(ii) Integrating both the sides, we get $\int\frac{dy}{Y} = \int X dx + C$ where c is an arbitrary constant.

Thus the required sol^n is obtained.

✓ **Equations Reducible to variables Seperable form** :

(i) Write the given equation in form $\frac{dy}{dx} = f(ax + by + c)$

(ii) Put $ax + by + c = z$, so that $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dz}{dx} - a\right)$

(iii) Putting this $\frac{dy}{dx}$ in the given equation, we get $\frac{1}{b}\left(\frac{dz}{dx} - a\right) = f(z)$. This eq^n is reduced in the form : $\frac{dz}{a + bf(z)} = dx$. After integrating, we get the required result.

Homogenous differential Equation A differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogenous differential equation if $F(x,y)$ is a homogenous function of degree zero.

(i) Suppose $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

(ii) The value $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ is substituted in given eq^n. The variable separable form, which can be solved by integrating both sides.

(iv) Finally v is replaced by $\frac{y}{x}$ to get the required solution.

Note : If the homogenous differential equation is in the form $\frac{dx}{dy} = F(x,$ substitute $x = vy$ and so $\frac{dx}{dy} = v + y\frac{dv}{dy}$ and proceed as above.

First order linear differential equation

$$\frac{dy}{dx} + Py = Q \quad \text{———} (i)$$

where P and Q are constants or function of x only.

$$I.F. = e^{\int P dx}$$ (I.F. = Integrating factor)

Solution of (i) is ;

$$\boxed{y.(I.F.) = \int Q \times (I.F.)\, dx + c}$$

$$\frac{dx}{dy} + Px = Q \quad \text{———} (i)$$

where P and Q are constants or function of y only.

$$I.F. = e^{\int P dy}$$ (I.F. = Integrating factor)

Solution of (i) is ;

$$\boxed{x.(I.F.) = \int Q \times (I.F.)\, dy + c}$$