Chapter 5 Sets

Exercise 5.1

Question 1.

State which of the following collections are sets :

(i) collection of odd natural numbers less than 50

(ii) collection of four colours of a rainbow.

(iii) collection of first three days of aweek

(iv) collection of all tall students of your class

(v) collection of clever students of your school

(vi) collection of all rich people of Bangalore

(vii) collection of some multiples of 5

(viii) collection of all prime numbers

(ix) collection of all even integers which lie between -5 and 15

(x) collection of all good cricket players of India

(xi) Collection of three youngest students of your class

(xii) Collection of three healthy students of your class

Solution :

(i) It is a set.

If we denote the given set by A, then $A = \{1, 3, 5, 7, ..., 47, 49\}$

(ii) It is not a set because the given collection is not well-defined-people may differ on four colours of a rainbow.

(iii) It is a set.

If we denote the given set by A, then $A = {Sunday, Monday, Tuesday}$

(iv) It is not a set because the given collection is not well-defined-people may differ on whether a student is tall or not.

(v) It is not a set because the given collection is not well-defined-people may differ on whether a student is clever or not.

(vi) It is not a set because the given collection is not well-defined-people may differ on whether a student is clever or not.

(vii) It is not a set because the given collection is not well definedpeople may differ on which are multiples of 5.

(viii) It is a set because the given collection is well defined.

(ix) It is a set. If we denote the given set by A, then A = { -4, -2, 0, 2, 4, 6, 8, 10, 12, 14} (x) It is not a set because the given collection is not well defined-people may differ on whether a cricket player of India is good or not.

(xi) It is a set because the given collection is well defined-people can choose three youngest students of their classes.

(xii) It is not a set because the given collection is not well definedpeople may differ on whether a student is healthy or not.

Question 2.

Let $E = \{ even integers \}$. Insert the appropriate symbol ϵ or \notin in the blanks.

- (i) 10..... E
- (ii) -8E
- (iii) 13.....E
- (iv) {6}E
- (v) a.....E
- (vi) -4, 12,....E

Solution:

 $E = \{ \text{ even numbers} \}$ $\Rightarrow E = \{ \dots, -6, -4, -2, 0, 2, 4, 6, 8, \dots \}$ (i) 10 \epsilon E (ii) -8 \epsilon E (iii) 13 ∉ E
(iv) {6} ∈ E
(v) a ∉ E
(vi) -4, 12 ∈ E

Question 3.

Let V = { vowels in English alphabet}. Write which of the following statements are true and which are false :

- (i) $c \in V$
- (ii) $\{a\} \in V$
- (iii) a, e, i \in V
- (iv) $a, b \in V$
- (v) $\{a, u\} \in V$
- (vi) $\{a, o, u\} \in V$

Solution:

V = { Vowels of English alphabet}

- (i) $c \in V$ Which is false.
- (ii) $\{a\} \in V$ Which is false.
- (iii) a, e, i \in V Which is True.
- (iv) a, $b \in V$ Which is false.
- (v) $\{a, u\} \in V$ Which is True.
- (vi) { a, o, u} \in V Which is True.

Question 4.

Write the following sets in roster form:

- (i) The set of first five odd counting numbers
- (ii) The set of all even natural numbers less than 101
- (iii) {months of year whose names begin with a vowel}
- (iv) {one digit natural numbers which are perfect squares}
- (v) { factor of 36}
- (vi) { prime factors of 360}
- (viii) The set of whole numbers which are multiples of 5

(ix) the set of all letters in the word ' CHENNAI'

- (x) The set of all vowels in the word 'MUSSOORIE'
- (xi) The set of all consonants in the word 'MATHEMATICS'

Solution:

- (i) The given set can be written as in roster form : $\{1, 3, 5, 7, 9\}$
- (ii) The given set can be written as in roster form : { 2, 4, 6, 8,....,98, 100}
- (iii) The given set can be written as in roster form : { April, August, October}
- (iv) The given set can be written as in roster form : $\{1, 4, 9\}$
- (v) The given set can be written as in roster form : $\{-14, -7, 0, 7, 14, 21\}$
- (vi) The given set can be written as in roster form : { 1, 2, 3, 4, 6, 9, 12, 18, 36}
- (vii) The given set can be written as in roster form : $\{2, 3, 5\}$

(viii) The given set can be written as in roster form : $\{0, 5, 10, 15\}$

(ix) The given set can be written as in rroster form : {C,H, E, N,A,I}

(x) The given set can be written as in roster from : $\{U, O, I, E\}$

(ix) The given set can be written as in roster form : { C, H, E, N, A, I}

(x) The given set can be written as in roster form : $\{U, O, I, E\}$

(xi) The given set can be written as in roster form : $\{M, T, H, C, S\}$

Question 5.

Write the following sets in tabular form :

- (i) $\{ x : \text{ is a natural number and } x < 7 \}$
- (ii) $\{ x : x \in W \text{ and } x \le 5 \}$
- (iii) { x : x is a month of a year having less than 31 days}
- (iv) { x | x is a letter in the word 'CIRCUMFERENCE'}
- (v) { x | x is a vowel in the word 'NOTATION'}
- (vi) (\mathbf{x} : x is a digit in the numeral 110526715}
- (vii) $\{ x : x \text{ is a factor of } 48 \}$
- (viii) (x : x is a multiple of 11 and $0 \le x \le 80$ }

(ix) [y : y is a two digit natural number divisible by 10}

Solution:

- (i) The given set can written as in Tabular form : $\{1, 2, 3, 4, 5, 6\}$
- (ii) The given set can be written as in Tabular form : $\{0, 1, 2, 3, 4, 5\}$
- (iii) The given set can be written as in Tabular form : { February, April, June, September, November}
- (iv) The given set can be written as in Tabular form: { C, I, R, U, M, F, E, N}
- (v) The given set can be written as in Tabular form : $\{ O, A, I \}$
- (vi) The given set can be written as in Tabular form : $\{1, 0, 5, 2, 6, 7\}$
- (vii) The given set can be written as in Tabular from : { 1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
- (viii) The given set can be written as in Tabular form : { 0, 11, 22, 33, 44, 55, 66, 77 }
- (ix) { y : y is a two digit natural number divisible by 10} = { 10, 20, 30, 40, 50, 60, 70, 80, 90}

Question 6.

Write the following sets in roster form and also in set builder form :

(i) the set of integers which lie bwteen -2 and 3 (both inclusive)

- (ii) the set of letters in the word 'ULTIMATUM'
- (iii) {months of a year whose names begin with J}
- (iv) The set of single digit whole mumbers which are perfect squares.

Solution:

(i) The given set can be written as $\{-2, -1, 0, 1, 2, 3\}$ (roster form) $\{x : x \in I, -2 \le x \le 3\}$ (set builder form)

- (ii) The given set can be written as { U, L,T, I, M,A} (roster form)
- { x : x is a letter in the word 'ULTIMATUM'} (set builder form)

(iii) The given set can be written as { january, june, July} (roster form)

 $\{ x \mid x \text{ is a month of a year whose names begin with J} \}$

(set builder form)

(iv) The given set can be written as $\{0, 1, 4, 9\}$ (roster form)

 $\{ x \mid x \text{ is a perfect square one digit number } \}$ (set builder form)

Question 7.

Write the following sets in tabular form and also in descriptive form :
(i) { x : x is a prime number less than 30 }
(ii) the set of whole numbers which are multiples of 8 and less than 50

(iii) { $x | x is a consonant in the word 'QUESTION PAPER'} Solution:$

(i) The given set can be written as { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

(Tabular form)
{ prime numbers less than 30} (descriptive form)

- (ii) The given set can be written as { 0, 8, 16, 24, 32, 40, 48} (Tabular form)
- { Whole numbers which are multiples of 8 and less than 50}
- (descriptive form)

(iii) The given set can be written as { Q, S, T, N, P, R} (Tabular form){consonants in the word "QUESTION PAPER'} (descriptive form)

Question 8.

Write the following sets in the set builder form :

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(i) A = \{ 0, 1, 2, ..., 11 \}

(ii) B = \{ 7, 14, 21, 28, ..., \}

(iii) C = \{ 1, 4, 9, 16, 25, 36, 49 \}

(iv) D = \{-12, -9, -b, -3, 0, 3, 6, 9, 12, 15, 18 \}

Solution:

(i) A = \{ 0, 1, 2, ..., 11 \}

= \{ x : x \in W, x \le 11 \}

(ii) B = \{ 7, 14, 21, 28, ..., \}

= \{ x : x = 1n, n \in N \}

(iii) C = \{ 1, 4, 9, 16, 25, 36, 49 \}

= \{ x : x = n^2, n \in N \text{ and } n \le 7 \}

(iv) D = \{-12, -9, -b, -3, 0, 3, 6, 9, 12, 15, 18 \}

= \{ x : x = 3n, n \in 1 \text{ and } -4 \le n \le 6 \}
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Exercise 5.2

Question 1.

State whether the following sets into empty, finite and infinite sets. In case of (non-empty) finite sets, mention the cardinal number.

- (i) {all colours of a rainbow}
- (ii) { x | x is a prime number between 7 and 11 }
- (iii) { multiples of 5}
- (iv) { all straight lines drawn in a plane}
- (v) { x | x is a digit in the numeral 550131527 }
- (vi) { x | x is a letter in the word 'SUFFICIENT'}
- (vii) { x | x is a vowel in the word MATHEMATICS}
- (viii) { x : x is an even whole number and $x \le 20$ }
- (ix) $\{ x : x \in I \text{ and } -2 \le x \le 5 \}$
- (x) { x : x is a prime number less than 25}
- (xi) { x : x is a prime factor of 180}.
- (xii) { x : x \in N and x is a composite number < 12}

Solution:

- (i) Let A = { all colours of a rainbow}
- \Rightarrow A = { Red, Oranges, Yellow, Green, Blue, Indigo, Violet}
- \therefore the given set is finite
- \therefore Cardinal number = 7

- (ii) Let $\{x \mid x \text{ is a prime number between 7 and } 11\}$
- \Rightarrow B = {Ø}
- \therefore The given set is empty
- (iii) Let $C = \{ \text{ multiples of } 5 \}$
- \Rightarrow C = { 5, 10, 15}
- \therefore the given set is infinite
- (iv) Let $C = \{ \text{ multiples of } 5 \}$
- \Rightarrow C = { 5, 10, 15}
- \therefore the given set is infinite
- (v) Let $D = \{ x \mid x \text{ is a digit in the numeral } 550131527 \}$ $\Rightarrow D = \{ 5, 0, 1, 3, 2, 7 \}$, the given set is finite \therefore the cardinal number = 6
- (vi) Let E = { x | x is a letter in the word ' SUFFICIENT'}
 ⇒ E = { S, U, F, I, C, E, N, T }, the given set is finite
 ∴ the cardinal number = 8

(vii) Let F = { x | x is a vowel in the word MATHEMATICS}
⇒ F = { A, E,I}, the given set is finite

∴ The cardinal number = 3
(viii) Let F = { x : x is an even whole number and x ≤ 20}
⇒ F = { 0, 2, 4,6, 8, 10, 12, 14, 16, 18, 20}, the given set is finite
∴ the cardinal number = 11

(ix) { $x : x \in I \text{ and } -2 \le x \le 5$ } = { -2, -1, 0, 1, 2, 3, 4}

It is a finite set as it has countable element which are 5

(x) G = { x : x is a prime number less than 25 }
⇒ G = { 2, 3, 5, 7, 11, 13, 17, 19, 23 }
∴ the given set is finite

 \therefore the cardinal number = 9

(xi) $H = \{ x : x \text{ is a prime factor of } 180 \}$ $\Rightarrow H = \{ 2, 3, 5 \}$

the given set is finite the cardinal number = 3

(xii) { x : x e N and x is a composite number < 12}{ 4, 6, 8, 9, 10} given set is finite cardinal number is = 5

Question 2.

State whether the following pairs of sets are equal or not:

(i) $A = \{ 2, 4, 6, 8, 10 \}, B = \{ even natural numbers \}$

(ii) $A = \{3, 5, 7, 9, 11, 13\}, B = \{ \text{ odd numbers between } 2 \text{ and } 14 \}$

(iii)
$$A = \{PUPPET\}, B = \{P, U, E, T\}$$

(iv) $A = \{ x \mid x \text{ is a letter in the word SOPHIA} \}$

 $B = \{ x \mid x \text{ is a letter in the word MAMAZ} \}$

(v) $A = \{ kids 5 meters tall \}, B = \{ x : x \in N and 2x = 3 \}.$

Solution:

(i) $A = \{ 2, 4, 6, 8, 10 \}$ B = { 0, 2, 4, 6, 8, 10, 12.....} $\therefore A \neq B$

(ii) $A = \{ 3, 5, 7, 9, 11, 13 \}$ B = { Odd numbers between 2 and 14} $\Rightarrow B = \{ 3, 5, 7, 9, 11, 13 \}$ $\therefore A = B$

(iii) $A = \{PUPPET\}, B = \{P, U, E, T\}$, then A = B because the elements in a set can be repeated or rearranged.

(iv) $A = \{x \mid x \text{ is a letter in the word SOPHIA}\}$

 $\Rightarrow A = S, O. P, H, I A$ B = { x | x is a letter in the word MUMTAZ} $\Rightarrow B = \{ M, U, T, A, Z \}$ $\therefore A \neq B$

(v) $A = \{ kids 5 metres tall \}$ $\Rightarrow A = \{ \} \Rightarrow A is empty set$ $B = \{ x : x \in N \text{ and } 2x = 3 \}$ $\therefore A = B$

Question 3.

Given that $A = \{ 2, 5, 7, 8, 10 \}$, $B = \{ 5, 7, 2, x, 10 \}$ and A = B, write the value of x.

Solution :

 $A = \{ 2, 5, 7, 8, 10 \}$ $B = \{ 5, 7, 2, x, 10 \}$ $\therefore A = B$ $\therefore x = 8$

Objective Types Questions

Mental maths

Question 1.

Fill in the blanks :

(i) A collection ofObjects is called a set.

(ii) If x is a member of the set A, we write is as.....

(iii) The Order of listing the elements of a set can be.....

(iv) If one or more elements are repeated, the set remains.....

(v) If X is the set of all letters in the word "MATHEMATICS'. then the cardinal number of the set X is

Solution:

(i) A collection of well defined objects is called a set.

(ii) If x is a member of the set A, we write it as x e A.

(iii) The Order of listing the elements of a set can be chaged.

(iv) If X is the set of all letters in the word 'MATHEMATICS' then the cardinal number of the set X is 8.

Question 2.

State whether the following statements are true (T) or false (F). Justify your answer.

- (i) A collection of stamps is a set.
- (ii) A collection of some fruits is a set.
- (iii) A group of boys platying cricket is a set.
- (iv) Collection of all students of your class taller than you is a set.
- (v) Collection of all students of your class taller than you is a set.
- (v) Collection of five rivers of India is a set.

Solution:

(i) A collection of stamps is a set. False

Correct :

It is not a set because it is not known which stamps are included in the collection.

(ii) A collection of some fruits is a set. False

Correct :

It is not a set because it is not known which fruits are included in the collection.

(iii) A group of boys playing cricket is a set. False

Correct :

it is not a set because it is not known which students are included in the group.

(iv) Collection of all students of your class taller than you is a set. True

It is a set because every students of your class can be compared with certainly in relation to your height, so it is very easy to select students of your class who are taller than you i.e. it is well defined collection.

(v) Collection of five rivers of India is a set. False Correct :

It is not a set because it is not known which five rivers of India are included in the collection.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 9):

Question 3.

Which of the following Collection is a set?

- (a) Collection of all tasty fruits
- (b) Collection of all good football players of your school
- (c) Collection of all months of a year
- (d) Collection of 5 most intelligent students of your class.

Solution:

Collection of all months of a year is a set,

If we denote thr givrn set, then A = { January, February, March, April,, December} (c)

Question 4.

The tabular form of the statement 'All months of a year whose names begin with the letters J' is

- (a) { January, June, July}
- (b) { months of a year whose names begin with the letter J}
- (c) $\{x \mid x \text{ is a month of a year whose name begins with the letter J}\}$
- (d) none of these

Solution:

The given set can be written as in Tabular form:

{ January, June, July} (a)

Question 5.

The method of representation used in the set $A = \{ x | x \text{ is an even natural number less than 15} \}$ is called

- (a) Description method
- (b) Rule method
- (c) Roster method
- (d) None of these

Solution:

Rule method (b)

Question 6.

The cardinal numbere of the empty set is

(a) 2

(b) 1

(c) 0

(d) none of these

Solution :

0 (c)

Question 7.

If $S = \{ x \mid x \text{ is a letter in the word AHMEDABAD} \}$, then the cardinal number of S is

- (a) 9
- (b) 8
- (c) 7
- (d) 6

Solution:

 $S = \{ Letters of AHMEDABAD \} = \{ A, H, M, E, D, B \}$ has 6 different elements, so n(S) = 6(d)

Questiom 8.

If $A = \{ x : x \in N \text{ and } x \text{ is an odd prime number less than 17} \}$, then the cardinal number of A is

(a) 8

(b) 6

(c) 5

(d) none of these

Solution:

Set $D = \{ x : x \in N \text{ and } x \text{ is an odd prime number less than } 17 \}$

 \Rightarrow D = { 3, 5, 7, 11, 13 }

 \therefore the cardinal number = 5(c)

Question9.

{months of a year whose names begin with the letter F} is

(a) an infinite set

(b) empty set

(c) singlton set

(d) none of these

Solution:

Let $A = \{ \text{ months of a year whose name begin with the letter } F \}$

 \Rightarrow A = { February}

 \therefore It is a singleton set. (c)

Check Your Progress

Question 1.

State which of the given collections are sets :

(i) Collection of all poor people of Dhanbad.

(ii) Collection of all difficult problems in your maths book.

(iii) Collection of all fools.

(iv) Collection of all countries of Asia.

(v) Collection of four countries of Asia.

(vi) Collection of three cities of India whose name start with the letter 'j'.

(vii) Collection of all people in thid world over 50 year of age.

Solution:

(i) It is not set because elemetns are not countable.

(ii) It is also not set because problem are different to different students

(iii) It is also not set.

(iv) It is set because the countries are countable.

(v) It is not set because the elements are countable but not defined.

(vi) It is also not set because cities are not defined.

(vii) It is a set.

Question 2.

If A = (3, 5, 7, 9, 11), then write which of the following statements are true. If a statements is not true, mention why.

- (i) $3 \in A$
- (ii) 5, $9 \in A$
- (iii) 8 ∉ A
- (iv) $7 \notin A$
- (v) $\{3\} \in A$
- (vi) $\{5, 9\} \in A$

Solution:

- (i) $3 \in A$ is ture. 3 is element of set.
- (ii) 5, 9 \in A is true because these are element of set.
- (iii) $8 \notin A$ is true because 8 is not element of set.
- (iv) It is false because 7 is element of set.
- (v) It is false because $\{3\}$ is different set and not element.
- (vi) It is not true because (5,9) is different set.

Question 3.

Write the following sets in the roster farm :

- (i) A = (x | x is a month of a year having 30days)
- (ii) $B = (x | x = 2n, n \in W \text{ and } n < 5)$
- (iii) C = (x | x ϵ N and $x^2 < 40$ }
- (iv) D = (all letters in the word PERMISSION}

- (v) $E = (x : x \in I \text{ and } x^2 < 10)$
- (vi) $F = (x : x \in N, 15 < x < 50 \text{ and } x \text{ is divisible by 6})$
- (vii) The set of whole numbers which are greater than 14 and divisible by 7.
- (viii) the set of signs of four fundamental operation of arithmetic.

Solution:

(i) A = (April, june, September, November}
(ii) B = (0, 2, 4, 6, 8}
(iii) C = (1, 2, 3, 4, 5, 6}
(iv) D = (P, E, R, M, I, S, O, N}
(v) E = { -3, -2, -1, 0, 1, 2, 3}
(vi) F = (18, 24, 30, 36, 42, 48}
(vii) (21, 28, 35, 42}
(viii) {x, -+, - }

Question 4.

Write the following sets in set builder form :

- (i) A = (2, 3, 5, 7, 11, 13, 17, 19)
- (ii) $B = (all months of a year \}$
- (iii) C = (Monday, Tuesday, Wednesday}

Solution:

(i) A = {x | x is a prime number x < 20}
(ii) B = {x : x is any month of a year}
(iii) C = {x | x is any of the first three days of a week}

Question 5.

Write the following sets in roster form and also in set builder form:

(i) $A = \{$ even whole numbers which are less than 50 $\}$

(ii) B = { two digit numbers which are perfect square}

(iii) The set of letters in the word MUSSOORIE

Solution:

(i) A = { 0, 2, 4,...,48 }
A = { x / x \epsilon W and x is an even number < 50 }

(ii) $B = \{ 16, 25, 36, 49, 64, 81 \}$

 $B = \{ x : x \text{ is perfect square and two digit number} \}$

- (iii) $\{M, U, S, O, R, I, E\}$
- $\{ x \mid x \text{ is a letter in the word MUSSORIE.} \}$

Question. 6

The sets on the left are in tabular form while the sets on the right are in set builder form. Match them.

(i) $\{2, 3\} - (a) \{ x / x \in N \text{ and } x < 6 \}$

(ii) { P, A, Y} – (b) {x / x is a prime factor of 6 }

(iii) $\{1, 3, 5\} - (c) \{x \mid x \text{ is an odd natural number less than } 6\}$

(iv) { 1, 2, 3, 4, 5} – (d) {x/x is a letter in word PAPAYA}

Solution:

(i) $\{2,3\}$ - (b) - (b) $\{x : x \text{ is a prime factor of } 6\}$

(ii) { P, A, Y} – (d) { x/x is a letter in word PAPAPYA}

(iii) $\{1, 3, 5\} - (c) \{x \mid x \text{ is an odd natural number less than } 6\}$

(iv) { 1, 2, 3, 4, 5} – (a) { $x / x \in N \text{ and } x < 6$ }

Question 7.

Classify the following sets as empty set, finite set or infinite set:

(i) The set of all even prime number > 2.

(ii) The set of even prime numbers.

(iii) The set of prime numbers less than one crore.

(iv) { All points on a line segment of length 3 cm}.

Solution:

(i) Empty set.

(ii) Finite set.

(iii) Finite set.

(iv) Infinite set.

Question 8.

Find the cardinal number of the following sets.

(i) $A = \{ x \mid x \text{ is a consonant in the word HUNDRED} \}$

(ii) $B = \{ x | x \text{ is a vowel in the word DEHRADOON} \}$

(iii) C = { x | x \in W and $x^2 < 50$ }

(iv) D = { Students of your school having 10 heads}

(v) $E = \{ x \mid x \text{ is a prime between 8 and 30} \}$

Solution:

(i) {HNDR} = 4
(ii) { E, A, O} = 3
(iii) { 0, 1, 2, 3, 4, 5, 6, 7} = 8
(iv) 0
(v) { 11, 13, 17, 19, 23, 29} = 6