CBSE Class 12 - Mathematics
Sample Paper 01

Maximum Marks:80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If A = [x y z], B= $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and C = $[xyz]^t$, then ABC is
   a. not defined
   b. 1×1 matrix
   c. 3×3 matrix
   d. none of these
2. If each element of a 3×3 matrix A is multiplied by 3, then the determinant of the newly formed matrix is
   a. 9 det A
   b. 3 det A

c. (det det A)^3

d. 27 det A

3. If both f and g are defined in a nhd of 0; f(0) = 0 = g(0) and f‘(0) = 8 = g’(0), then $\underset{x \to 0}{Lt} \frac{f(x)}{g(x)}$ is equal to

   a. None of these

   b. 0

   c. 1

   d. 16

4. The probability of obtaining an even prime number on each die , when a pair of dice is rolled, is given by :

   a. 0

   b. $\frac{1}{36}$

   c. $\frac{1}{3}$

   d. $\frac{1}{2}$

5. Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. Find P(A ∩ B)

   a. 0.15

   b. 0.10

   c. 0.14

   d. 0.12

6. In an LPP if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same

   a. Upper limit value

   b. Minimum value

   c. Maximum value

   d. Mean value

7. If $\cot^{-1}\left(\sqrt{\cos\alpha}\right) + \tan^{-1}\left(\sqrt{\cos\alpha}\right) = \mu$, then sinμ is equal to

   a. $\tan^2\alpha$

   b. $tan\ 2\alpha$

   c. $\cot^2\left(\frac{\alpha}{2}\right)$

   d. 1

8. $\int_{-\pi/2}^{\pi/2} \cos\, t\, dt$ is equal to

   a. 1
   b. 0
   c. – 1
   d. 2

9. Find the vector and cartesian equations of the planes that passes through the point (1 ,4 ,6) and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$

   a. $\left[\vec{r} - \left(\hat{i} + 5\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + 2z + 1 = 0$
   b. $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 7\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 5 = 0$
   c. $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 1 = 0$
   d. $\left[\vec{r} - \left(2\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 3y + z + 1 = 0$

10. If λ is a real number $\lambda\vec{a}$ is a

   a. vector
   b. unit vector
   c. scalar
   d. inner product

11. Fill in the blanks:

   The set of first elements of all ordered pairs in R, i.e., {x : (x, y) $\in$ R} is called the ________ of relation R.

12. Fill in the blanks:

   If f(x) = $x^2 \sin \frac{1}{x}$, where x $\neq$ 0, then the value of the function f at x = 0, so that the function is continuous at x = 0, is ________.

13. Fill in the blanks:

    If A and B are two skew-symmetric matrices of same order, then AB is symmetric matrix if ________.

14. Fill in the blanks:

    The vector equation of a plane which is at a distance p from the origin, where $\hat{n}$ is the unit vector normal to the plane is ________.

    OR

    Fill in the blanks:

    If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 =$ ________.

15. Fill in the blanks:

    The value of λ such that vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is ________.

    OR

    Fill in the blanks:

    The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 is ________.

16. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

17. Evaluate $\int_0^{\pi/2} e^x(\sin x - \cos x)dx.$

    OR

    Integrate $\left(\frac{2a}{\sqrt{x}} - \frac{b}{x^2} + 3c\sqrt[3]{x^2}\right)$ w.r.t. x

18. Evaluate $\int \frac{(x^2+2)}{x+1} dx$

19. If the line ax+by+c=0 is a tangent to the curve xy=4,then show that either a>0,b>0 or

a<0, b<0.

20. Find the differential equation representing the family of curves $V = \frac{A}{r} + B$ where A and B are arbitrary constants.

Section B

21. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

Let A = {x ∈R : 0 ≤ x ≤1]. If f : A →A is defined by

$$f(x) = \begin{cases} x, \text{ if } x \in Q \\ 1-x, \text{ if } x \notin Q \end{cases}$$

then prove that fof (x) = x for all x ∈A.

22. Find the points on the curve y = x^3 at which the slope of the tangent is equal to the y - coordinate of the point.

23. A function $f : R \rightarrow R$ satisfies the equation $f(x+y) = f(x) . f(y)$ for all $x, y \in R, f(x) \neq 0$ Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f'(x) = 2f(x)$

24. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$, $\vec{a}$ and $\vec{b}$

OR

Find the direction ratios and the direction cosines of the vector $\vec{r} = 2\hat{i} - 7\hat{j} - 3\hat{k}$.

25. Find the equation of the plane passing through (a,b,c) and parallel to the plane $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

26. A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

Section C

27. Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, add a minimum number of ordered pairs so that the enlarged relation is symmetric, transitive and reflexive.

28. If x = asin pt, y = b cospt. find the value of $\frac{d^2y}{dx^2}$ at t = 0

OR

Find the percentage error in calculating the volume of a cubical box if an error of 1% is made in measuring the length of edges of the cube.

29. Find the general solution: $\frac{dy}{dx} = \sin^{-1} x$

30. Evaluate $\int (x-3)\sqrt{x^2+3x-18dx}.$

31. Consider the probability distribution of a random variable X:

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15

Calculate:

i. $V\left(\frac{X}{2}\right)$
ii. Variance of X.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

32. Minimise Z = 13x - 15y, subject to the constraints:
$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0.$

Section D

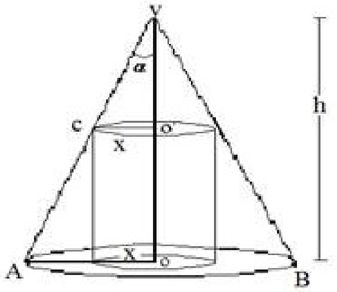
33. Obtain the inverse of the following matrix using elementary operations
$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

OR

If f(x) = ax^2 + bx + c is a quadratic function such that f(1) = 8, f(2) = 11 and f(-3) = 6, find f(x) by using determinants. Also, find f(0).

34. Find the area of the region bounded by the curves y = x^2 + 2, y = x, x = 0 and x = 3

35. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi^3 h \tan\alpha$



OR

A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs Rs. $5/cm^2$ and the material for the sides costs Rs. $2.50/cm^2$ Find the least cost of the box.

36. Find the position vector of the foot of perpendicular and the perpendicular distance, from the, point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$to the plane $\vec{r}\cdot(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$Also, find image of P in the plane.

Solution
Section A

1. (b) 1×1 matrix

   Explanation:

   Here $[A]_{1X3}$, $[B]_{1X3} \Rightarrow [AB]_{1X3}$, $[C]_{3X1} \Rightarrow [ABC]_{1X1}$

2. (d) 27 det A

   Explanation:

   $|3A| = 3^3|A| = 27|A|$

   if A is a square matrix of order n, then $|kA| = k^n |A|$ where n is the order of matrix

3. (c) 1

   Explanation:

   $\lim\limits_{x \to 0} \frac{f(x)}{g(x)} = \lim\limits_{x \to 0} \frac{f'(x)}{g'(x)} = \frac{f'(0)}{g'(0)} = 1$ (by using L'Hospital Rule)

4. (b) $\frac{1}{36}$

   Explanation:

   Clearly , n(s) =36. Favourable cases are { 2, 2 } Therefore required probability = $\frac{1}{36}$.

5. (d) 0.12

   Explanation:

   Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4

   $P(A \cap B) = P(A).P(B)$

   $\Rightarrow P(A \cap B) = 0.3 \times 0.4 = 0.12$

6. (c) Maximum value

   Explanation:

In an LPP if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same maximum value . If the problem has multiple optimal solutions at the corner points, then both the points will have the same (maximum or minimum)value.

7. (d) 1

   Explanation:

   $\cot^{-1}(\sqrt{\cos\alpha}) + \tan^{-1}(\sqrt{\cos\alpha}) = \mu$

   Let $\sqrt{\cos\alpha} = \theta$

   $\cot^{-1}\theta + \tan^{-1}\theta = \mu \implies \frac{\pi}{2} = \mu$

   $\therefore sin\mu = \sin\frac{\pi}{2} = 1.$

8. (d) 2

   Explanation:

   $= [\sin t]_{-\pi/2}^{\pi/2} = \sin\frac{\pi}{2} - \sin(\frac{-\pi}{2}) = 1 + 1 = 2$

9. (c) $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 1 = 0$

   Explanation:

   Let $\overrightarrow{a}$
   be the position vector of the point (1, 0, - 2)
   $\therefore \overrightarrow{a} = \hat{i} + 4\hat{j} + 6\hat{k}$, here,

   $\therefore \overrightarrow{n} = \hat{i} - 2\hat{j} + \hat{k}$
   Therefore, the required vector equation of the plane is:

   $\overrightarrow{r} . \overrightarrow{n} = \overrightarrow{a} . \overrightarrow{n}$

   $\Rightarrow \overrightarrow{r}(\hat{i} - 2\hat{j} + \hat{k}) = (\hat{i} + 4\hat{j} + 6\hat{k}).(\hat{i} - 2\hat{j} + \hat{k})$

   $\Rightarrow \overrightarrow{r}(\hat{i} - 2\hat{j} + \hat{k}) = -1$

On putting $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get:

$(x\hat{i} + y\hat{j} + z\hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = -1$

$\Rightarrow x - 2y + z = -1$

10. (a) vector

Explanation:

If a vector is multiplied by any scalar then, the result is always a vector.

11. domain

12. 0

13. AB = BA

14. $\vec{r}.\hat{n} = p$

OR

1

15. $-\frac{5}{2}$

OR

$\frac{4\vec{a}+\vec{b}}{3}$

16. $2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

RHS is $4|A| = 4 \times (2-8) = 4 \times (-6) = -24$

L.H.S = $|2A| = 8 - 32$

= - 24

Hence Proved

17. Let $I = \int_0^{\pi/2} e^x$ (sin x - cos x) dx

$\Rightarrow \quad I = -\int_0^{\pi/2} e^x (\cos x - \sin x)\, dx$

Now, consider, f(x) = cos x

then f'(x) = -sin x

Now, by using $\int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + C,$

we get, $I =$ -$\left[e^x \cos x\right]_0^{\pi/2}$

$= -e^{\pi/2} \cos \frac{\pi}{2} + e^0 \cos(0)$

= 0 + 1(1) = 1

OR

$$\int \left(\frac{2a}{\sqrt{x}} - \frac{b}{x^2} + 3c\sqrt[3]{x^2} \right) dx$$

$$= \int 2a(x)^{\frac{-1}{2}}\, dx - \int bx^{-2} dx + \int 3cx^{\frac{2}{3}}\, dx$$

$$= 4a\sqrt{x} + \frac{b}{x} + \frac{9cx^{\frac{5}{3}}}{5} + C$$

18. Let $I = \int \frac{(x^2+2)}{x+1} dx$

$$= \int \left(x - 1 + \frac{3}{x+1} \right) dx$$

$$= \int (x - 1)\, dx + 3 \int \frac{1}{x+1} dx$$

$$= \frac{x^2}{2} - x + 3 \log|(x+1)| + C$$

19. we have, xy=4

$\Rightarrow \; x.\frac{dy}{dx} + y.1 = 0$

$\Rightarrow \; \frac{dy}{dx} = -\frac{y}{x} = -\frac{4}{x^2} \; [\because xy = 4]$

$\therefore$ slope of tangent $= -\frac{a}{b}$

slope of the line ax + by + c = 0 is $-\frac{a}{b}$

Since the given line is a tangent to the given curve, therefore

$-\frac{4}{x^2} = -\frac{a}{b}$

$\Rightarrow \; \frac{a}{b} > 0$

It is possible only when a>0, b>0 or a<0, b<0

20. According to the question,the family of curves is given by,

$V=\frac{A}{r}+B$, where A and B are arbitrary constants.

On differentiating both sides w.r.t. r, we get

$\frac{dV}{dr}=\frac{-A}{r^2}+0 \Rightarrow \frac{dV}{dr}=\frac{-A}{r^2}$ ...(i)

Now, again differentiating both sides w.r.t. r, we get

$\frac{d^2V}{dr^2}=\frac{2A}{r^3}$

$\Rightarrow \quad \frac{d^2V}{dr^2}=\frac{2}{r^3}\times\left(-r^2\frac{dV}{dr}\right)$ [from Eq. (i)]

$\Rightarrow \quad \frac{d^2V}{dr^2}=-\frac{2}{r}\frac{dV}{dr}$

Thus, the required differential equation is

$\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}=0$.

Section B

21. We have, $\cos^{-1}\left(\frac{1}{2}\right)=\cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$=\frac{\pi}{3}\left[\because \frac{\pi}{3}\in[0,\pi]\right]$

Also $\sin^{-1}\left(-\frac{1}{2}\right)=\sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$=\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$=-\frac{\pi}{6}\left[\because -\frac{\pi}{6}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{3}-2\left(-\frac{\pi}{6}\right)$

$=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

OR

Let $x \in A$. Then, either x is rational or x is irrational. So two cases arise.

CASE I When $x \in Q$:

In this case, we have f(x) = x.

$\therefore$ fof (x) = f(f(x)) = f (x) = x [$\because$f(x) = x]

CASE II When $x \notin Q$:

In this case, we have f (x) = 1 - x

$\therefore$ fof (x) = f (f(x))

$\Rightarrow$ fof (x) = f (1 - x) [$\because x \notin Q \therefore$ f(x) = 1 - x]

$\Rightarrow$ fof (x) = 1 - (1 - x) = x [$\because$ x $\notin$Q $\Rightarrow$1 - x$\notin$Q $\Rightarrow$f(1 - x) = 1 -(1 - x)]

Thus, fof (x) = x whether x $\in$Q or, x $\notin$Q.

Hence, fof (x) = x for all x $\in$A.

22. Given: Equation of the curve y = x^3 ...(i)

$\therefore$ Slope of tangent at (x, y)

$= \frac{dy}{dx} = 3x^2$...(ii)

According to question, Slope of the tangent = y - coordinate of the point

$\therefore 3x^2 = x^3$

$\Rightarrow 3x^2 - x^3 = 0$

$\Rightarrow x^2(3 - x) = 0$

$\Rightarrow x^2 = 0$ or $3 - x = 0$

$\Rightarrow x = 0$ or x = 3

$\therefore$ From eq. (i), at x = 0, y = 0. The point is (0, 0).

And From eq. (i), at x = 3, y = 27 The point is (3, 27).

Therefore, the required points are (0, 0) and (3, 27).

23. Let $f : R \rightarrow R$ satisfies the equation $f(x + y) = f(x) . f(y), \forall x, y \in R, f(x) \neq 0$

Let f(x) is differentiable at x = 0 and f '(0) = 2

$\Rightarrow f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{0+h-0}$

since f(x+y)=f(x)f(y), therefore f(0+h)=f(0)f(h) and f'(0)=2, therefore,we get

$2 = \lim_{h \to 0} \frac{f(0).f(h) - f(0)}{h}$

$\Rightarrow 2 = \lim_{h \to 0} \frac{f(0)[f(h) - 1]}{h}$.....(1)

Also, $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\frac{f(x).f(h)-f(x)}{h}[\because f(x+y)=f(x).f(y)]$

$=\lim\limits_{h\to 0}\frac{f(x)[f(h)-1]}{h}$

$=f(x).\lim\limits_{h\to 0}\frac{[f(h)-1]}{h}$

= 2f(x) [using (1)]

$\therefore f'(x)=2f(x)$

24. L.H.S $=\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$

$=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$=0+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-0.[\overrightarrow{a}\times\overrightarrow{a}=\overrightarrow{b}\times\overrightarrow{b}=0]$

$=\overrightarrow{a}\times\overrightarrow{b}+\overrightarrow{a}\times\overrightarrow{b}\ [\overrightarrow{a}\times\overrightarrow{b}=-\overrightarrow{b}\times\overrightarrow{a}]$

$=2(\vec{a}\times\vec{b})$

OR

Direction Ratios of $\vec{r}$ and 2, -7, -3

$|\vec{r}|=\sqrt{4+49+9}=\sqrt{62}$

Direction Cosines of $\vec{r}$ are $\frac{2}{\sqrt{62}},\frac{-7}{\sqrt{62}},\frac{-3}{\sqrt{62}}$

25. Equation of any plane parallel to the plane $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=2$ is

$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda$...(i)

Plane (i) passes through (a,b,c)

$\therefore$ Putting $\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$ in eq. (i), we get

$\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda$

$\Rightarrow a(1)+b(1)+c(1)=\lambda \Rightarrow \lambda=a+b+c$

Putting the value of λ in eq. (i), to get the required plane is

$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$

26. Let X be a random variable denoting number of odd numbers ,then X is a random variable which takes values 0,1,2,3,4,5
Here, n = 5, $p=\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right)=\frac{1}{2}$ and q = 1 - p $=1-\frac{1}{2}=\frac{1}{2}$
Also, r = 3.Therefore,by binomial distribution, we have,
P(X = 3) = ${}^5C_3(\frac{1}{2})^3(\frac{1}{2})^{5-3}$
$=\frac{5!}{3!2!}.\frac{1}{8}.\frac{1}{4}=\frac{10}{32}=\frac{5}{16}$

Section C

27. We have,
A = {1, 2, 3} and R {(1, 2) (2, 3)}
Now,
To make R reflexive, we will add (1, 1) (2, 2) and (3, 3) to get
$\therefore$ R1 = {(1, 2), (2, 3), (1, 1), (2, 2), (3, 3)} is reflexive
Again to make R' symmetric we shall add (3, 2) and (2, 1)
$\therefore$ R" = {(1, 2), (2, 3), (1, 1), (2, 2), (3, 3), (3, 2), (2, 1)} is reflexive and symmetric
Now,
To R" transitive we shall add (1, 3) and (3, 1)
$\therefore$ R"' = {(1, 2), (2, 3), (1, 1), (2, 2), (3, 3), (3, 2), (2, 1)} is reflexive and symmetric
Now,
To make R" transitive we shall add (1, 3) and (3, 1)
$\therefore$ R"' {(1, 2), (2, 3), (1, 1) ,(2, 2), (3, 3), (3, 2), (2, 1), (1, 3) ,(3, 1)}
$\therefore$ R"' is reflexive, symetric and transitive.

28. x = a sin pt

$\frac{dx}{dt}=a\cos pt.p$ ...(1)

y = b cos pt

$\frac{dy}{dt} = -b \sin pt.p$ ...(2)

$\frac{dy}{dx} = \frac{-b}{a} \tan pt$ ...[(2) divide by (1)]

$\frac{d^2y}{dx^2} = \frac{-b}{a}.\frac{d}{dt}(\tan pt).\frac{dt}{dx}$

$= \frac{-b}{a}.\sec^2 pt.p.\frac{1}{a \cos pt.p}$

$= \frac{-b}{a^2} \sec^3 pt$

$\left[\frac{d^2y}{dx^2}\right]_{t=0} = \frac{-b}{a^2} \sec^3 (p.0)$

$= \frac{-b}{a^2}(1)$

$= \frac{-b}{a^2}$

OR

Let x be the length of an edge of the cube and y be its volume. Then, y = x. Let Δx be the error in x and Δy be the corresponding error in y. Then,

$\frac{\Delta x}{x} \times 100$ = 1 (given)

$\Rightarrow \frac{dx}{x} \times 100$ = 1 [$\because$ dx $\cong$ Δx]..(i)

We have to find $\frac{\Delta y}{y} \times 100$

Now, y = x^3

$\Rightarrow$ dy = $3x^2$ dx

Now dy = $\frac{dy}{dx} dx$

$\Rightarrow$ dy = $3x^2$ dx $\Rightarrow \frac{dy}{y}$ = $\frac{3x^2}{y}$ dx $\Rightarrow \frac{dy}{y}$ = $\frac{3x^2}{x^3} dx$ [$\because$ y = x^3]

$\Rightarrow \frac{dy}{y}$ = 3 $\frac{dx}{x}$

$\Rightarrow \frac{dy}{y} \times 100$ = 3 $\left(\frac{dx}{x} \times 100\right)$ = 3 [Using (i)]

$\Rightarrow \frac{\Delta y}{y} \times 100$ = 3 [$\because$ dy $\cong$ Δy]

So, there is a 3% error in calculating the volume of the cube.

29. Given: Differential equation $\frac{dy}{dx} = \sin^{-1} x$

$\Rightarrow dy = \sin^{-1} x dx$

Integrating both sides, $\int 1 dy = \int \sin^{-1} x dx$

$\Rightarrow y = \int \sin^{-1} x.1 dx$

Applying product rule,

$y = \left(\sin^{-1} x\right) \int 1 dx - \int \frac{d}{dx}\left(\sin^{-1} x\right) \int 1 dx dx$

$= x\sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} x dx$ ...(i)

To evaluate $\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx$

Putting, $1 - x^2 = t$, differentiate $-2x dx = dt$

$\Rightarrow \int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \int t^{-1/2} dt = \frac{1}{2} . \frac{t^{1/2}}{1/2} = -\sqrt{t} = -\sqrt{1-x^2}$

Putting this value in eq. (i), the required general solution is

$y = x\sin^{-1} x + \sqrt{1-x^2} + c$

30. According to the question, $I = \int (x-3)\sqrt{x^2+3x-18} dx$
$(x-3)$ can be written as
$x - 3 = A\frac{d}{dx}\left(x^2+3x-18\right) + B$
$\Rightarrow x - 3 = A(2x+3) + B$
comparing the coefficients of x and constant terms from both sides,
$\Rightarrow 2A = 1$
and $3A + B = -3$
$\Rightarrow \quad A = \frac{1}{2}$ and $3 \times \frac{1}{2} + B = -3$
$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{3}{2} - 3$
$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{9}{2}$
The given integral reduces in the following form :
$I = \int \left\{\frac{1}{2}(2x+3) - \frac{9}{2}\right\} \sqrt{x^2+3x-18} dx$
$\Rightarrow \quad I = \frac{1}{2} \int (2x+3)\sqrt{x^2+3x-18} dx - \frac{9}{2} \int \sqrt{x^2+3x-18} dx$
$let\ I = \frac{1}{2} I_1 - \frac{9}{2} I_2$ ...(i)
Consider $I_1 = \int (2x+3)\sqrt{x^2+3x-18} dx$

Put $x^2+3x-18=t$

$\Rightarrow (2x+3)dx=dt$

$\therefore \quad I_1=\int t^{1/2}dt=\frac{2}{3}t^{3/2}+C_1$

Put $x^2+3x-18=t$

$=\frac{2}{3}\left(x^2+3x-18\right)^{3/2}+C_1$

consider $I_2=\int\sqrt{x^2+3x-18dx}$

$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-18-\frac{9}{4}dx}$

$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{81}{4}dx}$

$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)^2}dx$

$=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x-18}-\frac{81}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x-18}\right|+C_2$

$\left[\because\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$=\frac{2x+3}{4}\sqrt{x^2+3x-18}-\frac{81}{8}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|+C_2$

Putting the values of I_1 and I_2 in Eq. (i),

$\Rightarrow I=\frac{1}{2}\left[\frac{2}{3}\left(x^2+3x-18\right)^{3/2}+C_1\right]-\frac{9}{2}\left[\frac{2x+3}{4}\sqrt{x^2+3x-18}\right.$

$\left.-\frac{81}{8}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|+C_2\right]$

$\Rightarrow \quad I=\frac{1}{3}\left(x^2+3x-18\right)^{3/2}-\frac{9}{8}(2x+3)\sqrt{x^2+3x-18}$

$+\frac{729}{16}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|+C\left[\because C=\frac{C_1}{2}-\frac{9C_2}{2}\right]$

31. We have,

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15
XP(X)	0	0.25	0.6	0.6	0.60
X^2P(X)	0	0.25	1.2	1.8	2.40

$Var(X)=E\left(X^2\right)-[E(X)]^2$

Where, $E(X)=\mu=\sum_{i=1}^{n}x_iP_i(x_i)$

And $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$

$\therefore$ E(X) = 0 + 0.25 + 0.6 + 0.6 + 0.60 = 2.05

$E(X^2)$ = 0 + 0.25 + 1.2 + 1.8 + 2.40 = 5.65

i. $V\left(\frac{X}{2}\right) = \frac{1}{4}V(X) = \frac{1}{4}[5.65 - (2.05)^2]$
$\frac{1}{4}[5.65 - 4.2025] = \frac{1}{4} \times 1.4475 = 0.361875$

ii. V(X) = 1.44475

OR

Here, n(S) = 6 $\times$ 6 = 36

Let E = Event of getting a total 10

= {(4, 6) , (5, 5) , (6, 4) }

$\therefore$ n(E) = 3

$\therefore$ P(getting a total of 10) = P(E) = $\frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

and P(not getting a total of 10) = P(E)

$= 1 - P(E) = 1 - \frac{1}{12} = \frac{11}{12}$

Thus, P(A getting 10) = P(B getting 10) = $\frac{1}{12}$

and P(A is not getting 10) = P(B is not getting 10)

$= \frac{11}{12}$

Now, P(A winning) =$P(A) + P(\overline{A} \cap \overline{B} \cap A)$

$+P(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A) + \ldots$

$= P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A})P(\overline{B})P(\overline{A})$

$P(\overline{B})P(A) + \ldots$

$= \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \ldots$

$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\right] = \frac{1}{12}\left[\frac{1}{1-\left(\frac{11}{12}\right)^2}\right]$

$\left[\because \text{ the sum of an infinite GP is } S_\infty = \frac{a}{1-r}\right]$

$= \frac{1}{12}\left[\frac{1}{\frac{144-121}{144}}\right] = \frac{12}{23}$

Now, P(B winning)= 1 - P(A winning)

$= 1 - \frac{12}{23} = \frac{11}{23}$

Hence, the probabilities of winning A and Bare respectively $\frac{12}{23}$ and $\frac{11}{23}$

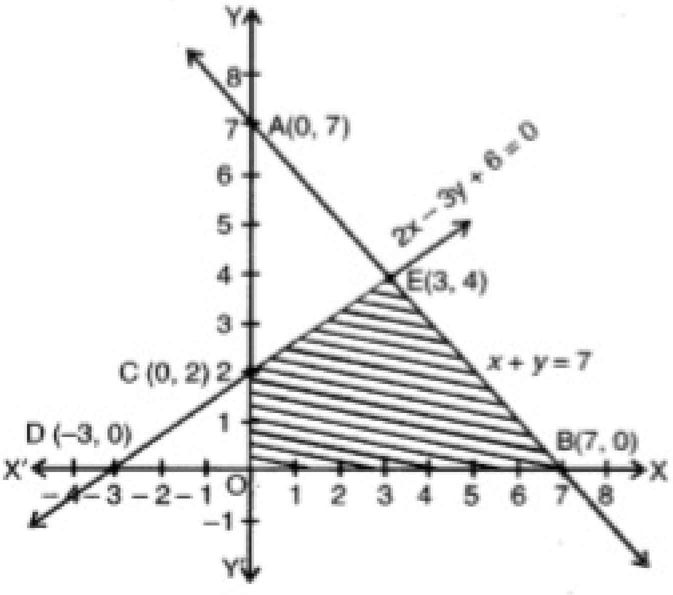
32. Consider$x + y = 7$

When $x = 0,\ then\ y = 7$ and

when $y = 0,\ then\ x = 7$

So, A(0, 7) and B(7, 0) are the points on line

$x + y = 7$



Consider $2x - 3y + 6 = 0$

When x = 0, then y = 2 and when y = 0, then x = - 3, So C(0, 2) and D(-3, 0) are the points on line $2x - 3y + 6 = 0$

Also, we have x > 0 and v > 0.

The feasible region OBEC is bounded, so, minimum value will obtain at a comer point of this feasible region.

Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2)

$Z = 13x - 15y$

At $O(0,0), Z = 0$

At $B(7,0), Z = 13(7) - 15(0) = 91$

At $E(3,4), Z = 13(3) - 15(4) = -21$

At $C(0,2), Z = 13(0) - 15(2)$

= -30 (minimum)

Hence , the minimum value is -30 at the point (0, 2).

Section D

33. A = IA

$$\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}.A$$

$$\begin{bmatrix}1&2&3\\0&1&2\\3&1&1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}.A\ R_1\Leftrightarrow R_2$$

$$\begin{bmatrix}1&2&3\\0&1&2\\0&-5&-8\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&-3&1\end{bmatrix}.A\ R_3\to R_3-3R_1$$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&-5&-8\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\0&-3&1\end{bmatrix}.A\ R_1\to R_1-2R_2$$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&2\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\5&-3&1\end{bmatrix}.A\ R_3\to R_3+5R_2$$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&1\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}.A\ R_3\to\frac{1}{2}R_3$$

$$\begin{bmatrix}1&0&0\\0&1&2\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\1&0&0\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}.A\ R_1\to R_1+R_3$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}.A\ [R_2\to R_2-2R_3]$$

$$A^{-1}=\begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}$$

OR

We have, f(x) = ax^2 + bx + c

$\therefore$ f(1) = 8 $\Rightarrow$ a + b + c = 8

f(2) = 11 $\Rightarrow$ 4a + 2b + c = 11 and, f(-3) = 6 $\Rightarrow$ 9a - 3b + c = 6

Thus, we obtain the following system of equations

a + b + c = 8

4a + 2b + c = 11

9a − 3b + c = 6

From this system of equations, we have

D = $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & -3 & 1 \end{vmatrix}$ = 1(2 + 3) − 1(4 − 9) + 1(− 12 − 18) = 5 + 5 − 30 = −20

D_1 = $\begin{vmatrix} 8 & 1 & 1 \\ 11 & 2 & 1 \\ 6 & -3 & 1 \end{vmatrix}$ = 8(2 + 3) - 1(11 - 6) + 1(- 33 - 12) = 40 - 5 - 45 = -10

D_2 = $\begin{vmatrix} 1 & 8 & 1 \\ 4 & 11 & 1 \\ 9 & 6 & 1 \end{vmatrix}$ = 1(11 - 6) - 8(4 - 9) + 1(24 - 99) = 5 + 40 - 75 = - 30

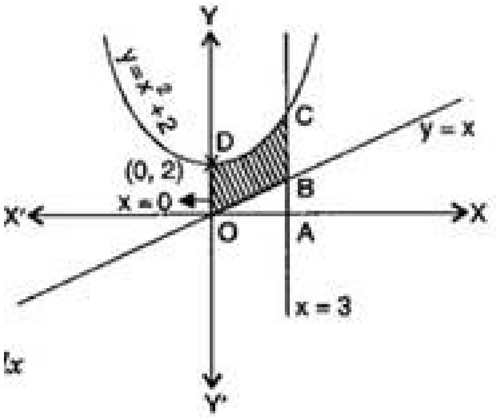
and, D_3 = $\begin{vmatrix} 1 & 1 & 8 \\ 4 & 2 & 11 \\ 9 & -3 & 6 \end{vmatrix}$ = 1(12 + 33) - 1(24 - 99) + 8(-12 -18) = 45 + 75 - 240 = -120

$\therefore$ a = $\frac{D_1}{D}$ = $\frac{-10}{-20}$ = $\frac{1}{2}$, b = $\frac{D_2}{D}$ = $\frac{-30}{-20}$ = $\frac{3}{2}$ and C = $\frac{D_3}{D}$ = $\frac{-120}{-20}$ = 6

Hence, f(x) = $\frac{1}{2}x^2 + \frac{3}{2}x + 6$

Consequently, f(0) = 6

34. Equation of the given curve is



$y = x^2 + 2$ ...(i)

$\Rightarrow x^2 = y - 2$

Here Vertex of the parabola is (0, 2).

Equation of the given line is y = x ...(ii)

Table of values for the line y = x

x	0	1	2
y	0	1	2

We know that it is a straight line passing through the origin and having slope 1 i.e., making an angle of 45^0 with x-axis.

Here also, Limits of integration area given to be x = 0 to x = 3

∴ Area bounded by parabola (i) namely y = x^2 + 2 the x - axis and the ordinates x = 0 to x = 3 is the area OACD and $\int_0^3 y dx = \int_0^3 \left(x^2+2\right) dx$

$= \left(\frac{x^3}{3}+2x\right)_0^3 = (9+6)-0 = 15$...(iii)

Again Area bounded by parabola (ii) namely y = x the x - axis and the ordinates x = 0 to x = 3 is the area OAB and $\int_0^3 y dx = \int_0^3 x dx$

$= \left(\frac{x^2}{2}\right)_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$ ...(iii)

∴ Required area = Area OBCD = Area OACD – Area OAB

= Area given by eq. (iii) – Area given by eq. (iv)

$= 15 - \frac{9}{2} = \frac{21}{2}$ sq. units

35. $\frac{vo'}{x} = \cot\alpha$

$vo' = x\cot\alpha$

$oo' = h - x\cot\alpha$

$V = \pi x^2.\left(h - x\cot\alpha\right)$

$V = \pi x^2 h - \pi x^3 \cot\alpha$

$\frac{dV}{dx} = 2\pi x h - 3\pi x^2 \cot\alpha$

for maximum/minimum

$\frac{dV}{dx} = 0$

$2\pi x h - 3\pi x^2 \cot\alpha = 0$

$x = \frac{2h}{3}\tan\alpha$

$\frac{d^2V}{dx^2} = 2\pi h - 6\pi x\cot\alpha$

$\left.\frac{d^2V}{dx^2}\right]_{x=\frac{2h}{3}\tan\alpha} = \pi\left(2h-4h\right)$=-2πh<0

Therefore, V is maximum

$V = \pi x^2 (h - x \cot \alpha)$

$= \pi \left(\frac{2h}{3} \tan \alpha \right)^2 \left[h - \frac{2h}{3} \tan \alpha \cot \alpha \right]$

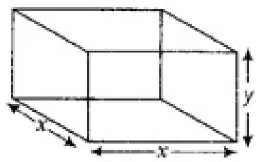
$= \pi \frac{4h}{9}^2 \tan^2 \alpha . \frac{h}{3}$

$V = \frac{4}{27} \pi h^3 \tan^2 \alpha$

OR

Since, volume of the box $= 1024\, cm^3$

Let length of the side of square base be x cm and height of the box be y cm.



$\therefore$ Volume of the box (V) $= x^2 . y = 1024$

Since, $x^2 y = 1024 \Rightarrow y = \frac{1024}{x^2}$

Let C denotes the cost of the box.

$\therefore C = 2x^2 \times 5 + 4xy \times 2.50$

$= 10x^2 + 10xy = 10x(x + y)$

$= 10x \left(x + \frac{1024}{x^2} \right)$

$= \frac{10x}{x^2} (x^3 + 1024)$

$\Rightarrow C = 10x^2 + \frac{1024}{x}$ ...(i)

On differentiating both sides w.r.t. x, we get

$\frac{dC}{dx} = 20x - 10240(x)^{-2}$

$= 20x - \frac{10240}{x^2}$ ...(ii)

Now, $\frac{dC}{dx} = 0$

$\Rightarrow 20x = \frac{10240}{x^2}$

$\Rightarrow 20x^3 = 10240$

$\Rightarrow x^3 = 512 = 8^3 \Rightarrow x = 8$

Again, differentiating Eq. (ii) w.r.t. x, we get

$\frac{d^2C}{dx^2} = 20 - 10240(-2) . \frac{1}{x^3}$

$= 20 + \frac{20480}{x^3}$

$\therefore \left(\frac{d^2C}{dx^2} \right)_{x=8} = 20 + \frac{20480}{512} = 60 > 0$

For x = 8, cost is minimum and the corresponding least cost of the box

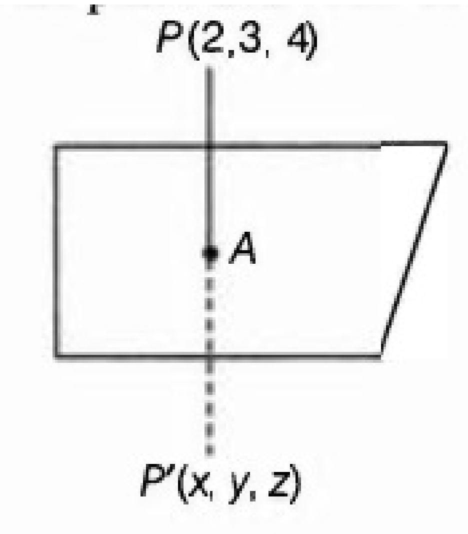
$C(8) = 10.8^2 + \frac{10240}{8}$

$\therefore$ Least cost = Rs. 1920

36. Given, a point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ and the plane

$\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$ or 2x + y + 3z = 26

Let A be the foot of perpendicular. Then, PA is the normal to the plane and so its Dr's are 2,1 and 3.



Now, the equation of perpendicular line PA is

$\frac{x-2}{2} = \frac{y-3}{1} = \frac{z-4}{3} = \lambda(\text{ say })$

$\Rightarrow \quad x = 2\lambda + 2, y = \lambda + 3 \text{ and } z = 3\lambda + 4$

Coordinates of any point on PA is of the form

$(2\lambda + 2, \lambda + 3, 3\lambda + 4)$

$\therefore$ Coordinates of A are $(2\lambda + 2, \lambda + 3, 3\lambda + 4)$ for some λ

Since, A lies on the plane, therefore we have

$2(2\lambda + 2) + (\lambda + 3) + 3(2\lambda + 4) = 26$

$4\lambda + 4 + \lambda + 3 + 9\lambda + 12 = 26$

$14\lambda + 19 = 26 \Rightarrow 14\lambda = 7 \Rightarrow \lambda = \frac{1}{2}$

So, the coordinates of foot of perpendicular are

$\left(2 \times \frac{1}{2} + 2\ ,\ \frac{1}{2} + 3, 3 \times \frac{1}{2} + 4\right)$ i.e. $\left(3, \frac{7}{2}, \frac{11}{2}\right)$

and therefore it's position vector is

$3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}$

Now, the required perpendicular distance

$= \sqrt{(3-2)^2 + \left(\frac{7}{2}-3\right)^2 + \left(\frac{11}{2}-4\right)^2}$

$= \sqrt{1+\frac{1}{4}+\frac{9}{4}} = \sqrt{\frac{7}{2}}$ units

Now, let P(x, y, z) be the image of point P in the plane.

Then, A will be mid-point of PP'

$\therefore \quad \left(3, \frac{7}{2}, \frac{11}{2}\right) = \left(\frac{2+x}{2}, \frac{3+y}{2}, \frac{4+z}{2}\right)$

$\Rightarrow 3 = \frac{2+x}{2}; \frac{7}{2} = \frac{3+y}{2}; \frac{11}{2} = \frac{4+z}{2}$

$\Rightarrow$ x = 4, y = 4 and z = 7

Thus, the coordinates of the image of the point P are (4, 4, 7).