# Chapter 15. Circles

Formulae

Theorems based on chord properties:

1. **Theorem:** A straight line drawn from the centre of the circle to bisect a chord, which is not a diameter, is at right angles to the chord.
   Conversely, the perpendicular to a chord, from the centre of the circle, bisects the chord.
2. **Theorem:** There is one circle, and only one, which passes through three given points not in a straight line.
3. **Theorem:** Equal chords of a circle are equidistant from the centre.
   Conversely, chords of a circle, equidistant from the centre of the circle, are equal.
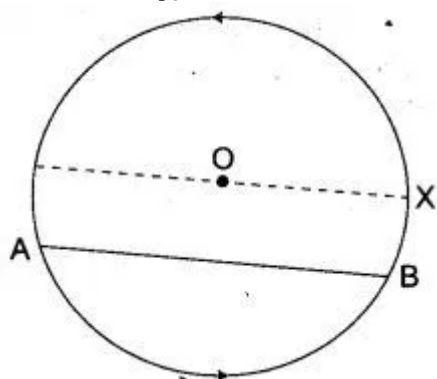

Theorems based on Arc and Chord properties:

1. **Theorem:** The angle which an arc of a circle subtends at the centre is double, that which it subtends at any point on the remaining part of the circumference.
2. **Theorem:** Angles in the same segment of a circle are equal.
3. **Theorem:** The angle in a semicircle is a right angle.
4. **Theorem:** In equal circles (or, in the same circle), if two arcs subtends equal angles at the centre, they are equal.
   Conversely, in equal circles (or, in the same circle), if two arcs are equal, they subtend equal angles at the centre.
5. **Theorem:** In equal circles (or, in the same circle), if two chord are equal, they cut off equal arcs.
   Conversely, in equal circles (or, in the same circle, if two arcs are equal the chords of the arcs are also equal.

Theorems based on Cyclic properties: ABCD is a cyclic quadrilateral.
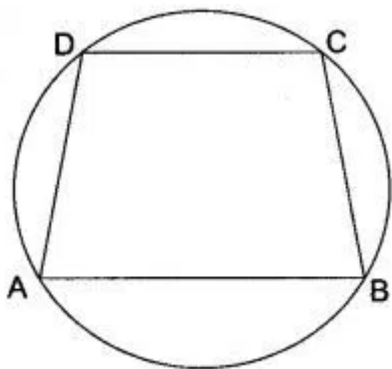
1. **Theorem:** The opposite angles of a cyclic quadrilateral (quadrilateral inscribed in a circle) are supplementary.
2. **Theorem:** The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

**Theorems based on Tangent Properties:**
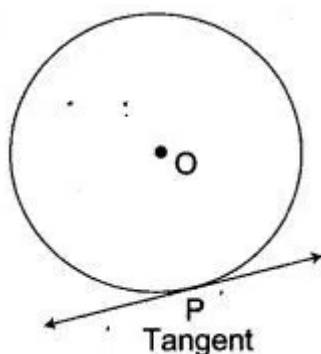
1. **Theorem:** The tangent at any point of a circle and the radius through this point are perpendicular to each other.
2. **Theorem:** If two circles touch each other, the point of contact lies on the straight line through the centres.
3. **Theorem:** From any point outside a circle two tangents can be drawn and they are equal in length.
4. **Theorem:** If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.
5. **Theorem:** If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Prove the Following

Question 1. If a diameter of a circle bisect each of the two chords of a circle, prove that the chords are parallel.

Solution : Let O be the centre of a circle and AB, CD be the two chords. Let PQ be the diameter bisecting chord AB and CD at L and M respectively.
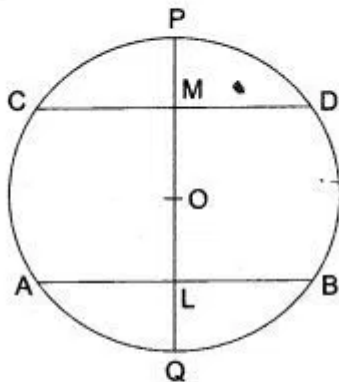
L is the mid-point of AB

So $\qquad$ OL $\perp$ AB $\Rightarrow \angle$ ALO = 90°

Similarly, $\angle$ CMO = 90°

$\qquad \angle$ ALO = $\angle$ CMO

but these are alternate angles

So $\qquad$ AB ‖ CD. $\qquad$ Hence proved.

Question 2. If two chords of a circle are equally inclined to the diameter through their point of intersection, prove that the chords are equal.

Solution : Let AB and AC be two chords and AOD be a diameter such that
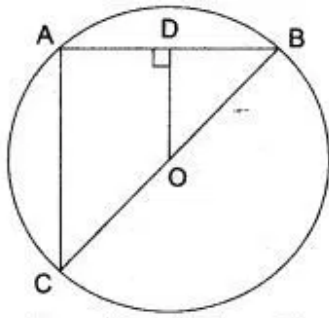
$\qquad \angle$ BAO = $\angle$ CAO

Draw OL $\perp$ AB and OM $\perp$ AC

Now prove, $\triangle$ OLA = $\triangle$ OMA

Then $\qquad$ OL = OM $\Rightarrow$ AB = CD

(Chords which are equidistant from the centre are equal) $\qquad$ Hence proved.

Question 3. In the given figure, OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2 OD.

Solution : Since, OD ⊥ AB and the perpendicular drawn from the centre to a chord bisects the chord.

∴ D is the mid-point of AB.
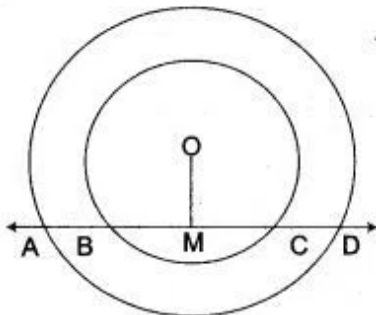
Also, O being the centre, is the mid-point of BC.

Thus, in ΔABC, D and O are mid-point of AB and BC respectively. Therefore, OD ∥ AC

and $\qquad$ $OD = \dfrac{1}{2} CA$

[∵ Segment joining the mid-points of two sides of a triangle is half of the third side]

⇒ $\qquad$ CA = 2OD. $\qquad$ Hence proved.

Question 4. In Fig., l is a line intersecting the two concentric circles, whose common centre is O, at the points A, B, C and D. Show that AB = CD.

Solution : Let OM be perpendicular from O on line l. We know that the perpendicular from the centre of a circle to as chord; bisects the chord. Since BC is a chord of the smaller circle and OM ⊥ BC.

∴ $\qquad$ BM = CM $\qquad$ ...(i)

Again, AD is a chord of the larger circle and OM ⊥ AD.

∴ $\qquad$ AM = DM $\qquad$ ...(ii)

Subtracting (i) from (ii), we get

$\qquad$ AM − BM = DM − CM ⇒ AB = CD.

$\qquad\qquad$ Hence proved.

Question 5. In the given below figure, O is the centre of the circle and $\angle AOC = 160°$. Prove that $3 \angle y - 2 \angle x = 140°$.
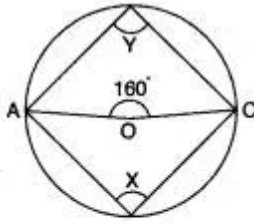
Solution : We know that angle by same arc at circle i.e., on circumference is half of the angle by same arc at centre.

$$\therefore \quad \angle x = \frac{1}{2} \times 160° = 80°$$

(Opposite triangles of a cyclic quadrilateral supplementary)

$$\therefore \qquad \angle x + \angle y = 180°$$

$$\therefore \qquad \angle y = 100°$$

$$\therefore \qquad 3\angle y - 2\angle x = 3 \times 100° - 2 \times 80°$$

$$= 300° - 160°$$

$$= 140° \quad \text{Hence proved.}$$

Question 6. ABCD is a cyclic quadrilateral AB and DC are produced to meet in E. Prove that $\triangle EBC \sim \triangle EDA$.
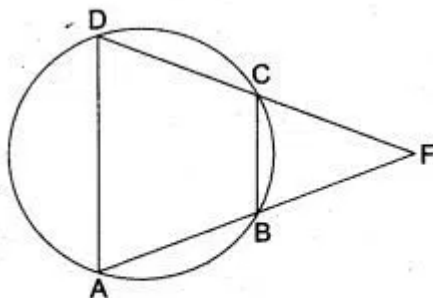
Solution : In triangle EBC and EDA, we have

$$\angle EBC = \angle EDA$$

[∵ Exterior or angle in a cyclic quad. is equal to opposite interior angel]

$$\angle ECB = \angle EAD$$

[∵ Exterior angle in a cyclic quad. is equal to opposite interior angle]

and $\qquad \angle E = \angle E$
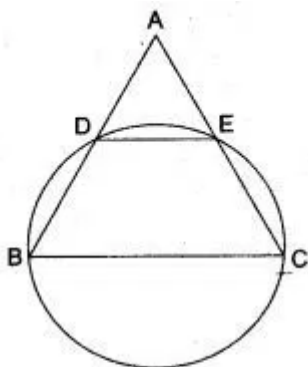

So, by AAA enterior of similarly, we get

$$\triangle EBC \sim \triangle EDA. \quad \text{Hence Proved.}$$

Question 7. In an isosceles triangle ABC with AB = AC, a circle passing through B and C intersects the sides. AB, and AC at D and E respectively. Prove that DE || BC.

Solution : In order to prove that DE || BC, it is sufficient to show that $\angle B = \angle ADE$.

In $\triangle ABC$, we have

$$AB = AC \Rightarrow \angle B = \angle C \qquad \text{...(i)}$$

In the cyclic quadrilateral CBDE, side BD is produced to A.

$$\therefore \quad \angle ADE = \angle C \qquad \text{...(ii)}$$

[∵ Exterior angle = Opposite interior angle]

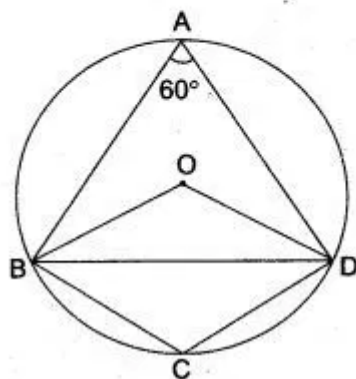From (i) and (ii), we get $\angle B = \angle ADE$.

Hence, DE || BC. Hence proved.

Question 8. ABCD is quadrilateral inscribed in circle, having $\angle A = 60°$, O is the centre of the circle, show that

$$\angle OBD + \angle ODB = \angle CBD + \angle CDB$$

Solution : Here, $\angle BOD = 2 \times \angle BAD$

$$= 2 \times 60°$$
$$= 120°$$

Now in $\triangle BOD$,

$$\angle OBD + \angle ODB = 180° - 120°$$
$$= 60° \qquad \text{...(1)}$$

Also ∵ $\angle DAB + \angle DCB = 180°$

(ABCD is a cyclic quadrilateral)

$$\Rightarrow \qquad \angle DCB = 180° - 60° = 120°$$

∴ In $\triangle BCD$,

$$\angle CBD + \angle CDB = 180° - \angle DCB$$
$$= 180° - 120°$$
$$= 60° \qquad \text{... (2)}$$
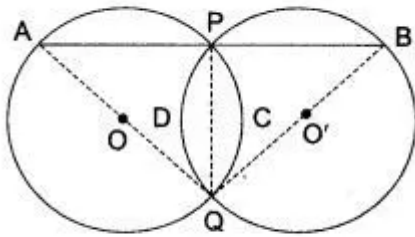
From (1) and (2) we get the required result.

Hence proved

Question 9. Two equal circles intersect in P and Q. A straight line through P meets the circles in A and B. Prove that QA = QB

  Solution : Let C(O, r) and C(O′, r) be two equal circles. Clearly C(O, r) ≅ C(O′, r).

  Since, PQ is a common chord of two congruent circles. Therefore,

$$\text{arc PCQ} = \text{arc PDQ}$$

⇒ $$\angle QAP = \angle QBP$$

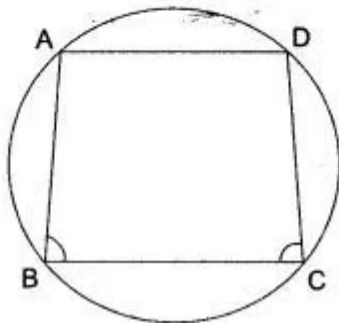Thus, in ΔQAB , we have

$$\angle QAP = \angle QBP$$

⇒ $$QA = QB. \qquad \text{Hence proved.}$$

Question 10. If ABCD is a cyclic quadrilateral in which AD ∥ BC. Prove that ∠ B = ∠ C.

Solution : ABCD is a cyclic quadrilateral

So $\angle A + \angle C = 180°$ ...(i)

Since   AD ∥ BC

So $\angle B + \angle A = 180°$ ...(ii)

From (i) & (ii)

$$\angle A + \angle C = \angle B + \angle A$$

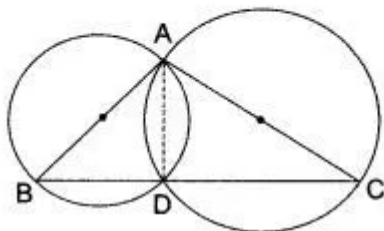⇒ $$\angle C = \angle B$$

or $$\angle B = \angle C. \qquad \text{Hence proved.}$$

Question 11. Two circles are drawn with sides AB, AC of a triangle ABC as diameters. The circles intersect at a point D. Prove that D lies on BC.

Solution : Join AD.

Since, angle in a semi-circle is a right angle.

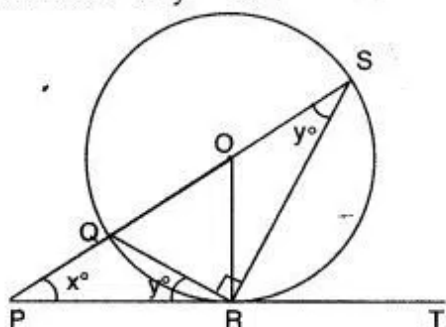Therefore,

$$\angle ADB = 90° \text{ and } \angle ADC = 90°$$

⇒ $\angle ADB + \angle ADC = 90° + 90°$

⇒ $\angle ADB + \angle ADC = 180°$

BDC is a straight line ⇒ D lies on BC.

  Hence proved.

Question 12. In the given figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If $\angle SPR = x°$ and $\angle QRP = y°$. show that $x° + 2y° = 90°$.

Solution : PRT is tangent at R and QR is chord.

∴ $\angle QRP = \angle QSR$

(Angle in alternate segment)

    $= y°$

and $\angle QRS = 90°$

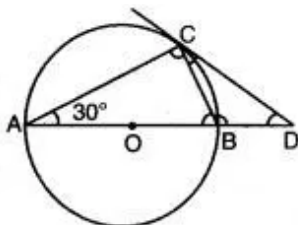(∵ QS is diameter and angle in semicircle is rt. angle)

Now in ΔPRS,

$\angle SPR + \angle PRS + \angle RSP = 180°$

$x° + y° + 90 + y° = 180°$

$x° + 2y° = 180° - 90°$

$x° + 2y° = 90°$. Hence proved.

Question 13. In the figure AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30°$. The tangent at C intersect AB produced at D. Prove that BC = BD.

Solution : Join OC.

$\angle ACB = 90°$ (Angle of the semicircle).

$\angle ABC = 60°$ (Angle sum property)

$\angle CBD = 120°$ (adj to angle CBA 30°)

$\angle OCD = 90°$ tangent

$\angle COB = 60°$

(Angle at the centre is equal to twice that of the circumference.

$\angle OCB = 60°$ (Angle sum Property)

$\angle BCD = \angle OCD - \angle OCB = 30°$

∴ $\angle BDC = \angle BDC = 30°$
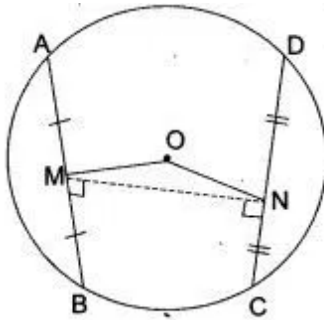
BD = BC   Hence proved.

Question 14. Prove that the line segment joining the midpoints of two equal chords of a circle substends equal angles with the chord.

Solution : Here, M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. We have to prove that

$$\angle BMN = \angle CNM$$
$$\angle AMN = \angle DNM$$

Join OM, ON and MN.

$$\therefore \quad \angle OMA = \angle OMB = 90° \atop \angle OND = \angle ONC = 90°\} \dots (1)$$

$$\left[\begin{matrix}\text{Line joining the centre}\\ \text{and mid-point of a chord is}\\ \text{perpendicular to the chord}\end{matrix}\right]$$

Since, $\qquad AB = CD \Rightarrow OM = ON$

$\therefore$ In $\triangle$ OMN, $\angle OMN = \angle ONM \qquad \dots (2)$

(i) $\qquad \angle OMB = \angle ONC$

[Using (1) and (2)]

$$\angle OMN = \angle ONM$$

$\Rightarrow \quad \angle OMB - \angle OMN = \angle ONC - \angle ONM$

$\Rightarrow \qquad\qquad \angle BMN = \angle CNM$

(ii) $\qquad\qquad \angle OMA = \angle OND$

$\qquad\qquad \angle OMN = \angle OND$

[Using (1) and (2)]

$\Rightarrow \quad \angle OMA + \angle OMN = \angle OND + \angle ONM$

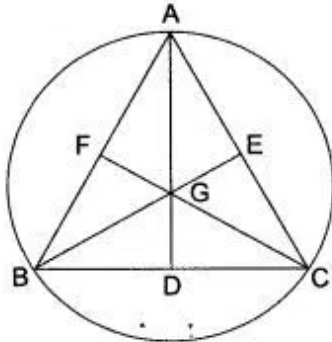$\Rightarrow \qquad\qquad \angle AMN = \angle DNM$

Hence proved

Question 15. In an equilateral triangle, prove that the centroid and centre of the circum-circle (circumcentre) coincide.

Solution : Given : An equilateral triangle ABC in which D, E and F are the mid-points of sides BC, CA and AB respectively.

To prove : The centroid and circumcentre are coincident.

Construction : Draw medians AD, BE and CF.

Proof : Let G be the centroid of $\triangle ABC$ i.e., the point of intersection of AD, BE and CF. In triangles BEC and BFC, we have

$$\angle B = \angle C = 60°$$

$$BC = BC$$

and $$BF = CE$$

$$\left[\because AB = AC \Rightarrow \frac{1}{2}AB = \frac{1}{2}AC \Rightarrow BF = CE \right]$$

$\therefore$ $$\triangle BEC \cong \triangle BFC$$

$\Rightarrow$ $$BE = CF \qquad \ldots(i)$$

similarly, $$\triangle CAF \cong \triangle CAD$$

$$CF = AD \qquad \ldots(ii)$$

From (i) & (ii) $AD = BE = CF$

$\Rightarrow$ $$\frac{2}{3}AD = \frac{2}{3}, BE = \frac{2}{3}CF$$

$$CG = \frac{2}{3}CF$$

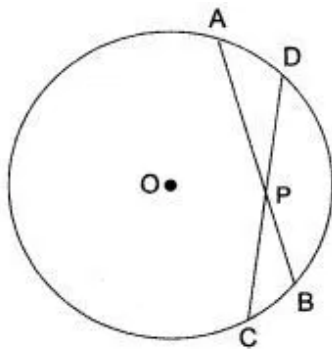$$GA = \frac{2}{3}AD,$$

$$GB = \frac{2}{3}BE$$

$\Rightarrow$ $$GA = GB = GC$$

$\Rightarrow$ G is equidistant from the vertices

$\Rightarrow$ G is the circumcentre of $\triangle ABC$

Hence, the centroid and circum centre are coincident.

Question 16. In Fig. AB and CD are two chords of a circle intersecting each other at P such that AP = CP. Show that AB = CD.

Solution : In order to prove the desired result, we shall first prove that Δ PAD ~ Δ PCB.

In triangles PAD and PCB, we have :

$$\angle PAD = \angle PCB$$

[Angles in the same segment of arc BD]

$$\angle APD = \angle CPB$$

[Vertically opposite angles]

So, by AAA criterion of similarity, we have

$$\Delta PAD \sim \Delta PCB$$

$$\Rightarrow \quad \frac{PA}{PC} = \frac{PD}{PB}$$

[∵ Corresponding sides of similar triangles are in the same ratio]

$$\Rightarrow \quad \frac{AP}{CP} = \frac{PD}{PB}$$

$$\Rightarrow \quad 1 = \frac{PD}{PB}$$

$$\left[\because AP = CP, \therefore \frac{AP}{CP} = 1 \right]$$

$$\Rightarrow \quad PB = PD$$

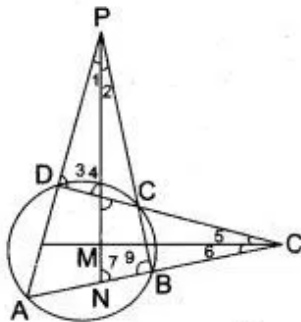$$\Rightarrow \quad AP + PB = AP + PD$$

[Adding AP on both sides]

$$\Rightarrow \quad AP + PB = CP + PD \quad [\because AP = CP]$$

$$\Rightarrow \quad AB = CD. \quad \text{Hence proved.}$$

Question 17. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (Provided they are not parallel) intersect at right angle.

Solution : Here, ABCD is a cyclic quadrilateral. PM is bisector of ∠ APB and QM is bisector of ∠ AQD.

In Δ PDL and Δ PBN,

$$\angle 1 = \angle 2$$

(PM is the bisector of ∠ P)

∠ 3 = ∠ 9 (Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle)

∴ $$\angle 4 = \angle 7$$

But ∠ 4 = ∠ 8 (Vertically opposite angles)

∴ $$\angle 7 = \angle 8$$

Now in Δ QMN and Δ QML,

$$\angle 7 = \angle 8 \quad \text{(Proved above)}$$

$$\angle 5 = \angle 6$$

(QM is bisector of Q)

$$\Delta\ QMN \sim \Delta\ QML$$

∴ $$\angle QMN = \angle QML$$

But ∠ QMN + ∠ QML = 180°

∴ $$\angle QMN = \angle QML = 90°$$

Hence, $$\angle PMQ = 90°$$

(∵ ∠ PMQ = ∠ QML)

Hence proved.

Question 18. In Fig. P is any point on the chord BC of a circle such that AB = AP. Prove that CP = CQ.
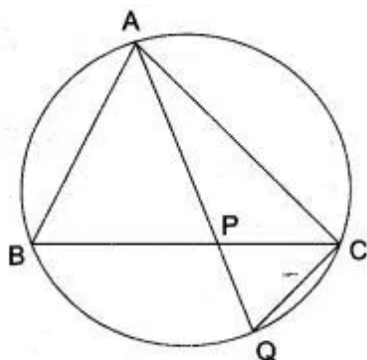
Solution : We have to prove that CP = CQ i.e., ΔCPQ is an isosceles triangle. For this it is sufficient to prove that ∠CPQ = ∠CQP.

In ΔABP, we have

$$AB = AP$$

$$\Rightarrow \quad \angle APB = \angle ABP$$

$$\Rightarrow \quad \angle CPQ = \angle ABP \qquad \text{...(i)}$$

[∵ ∠APB and ∠CPQ are vertically opposite angles ∴ ∠APB = ∠CPQ]

Now, consider arc AC. Clearly, it subtends ∠ABC and ∠AQC at points B and Q.

$$\therefore \quad \angle ABC = \angle AQC$$

[∵ Angles in the same segment]

$$\Rightarrow \quad \angle ABP = \angle PQC$$

[∵ ∠ABC = ∠ABP and ∠AQC = ∠PQC]

$$\Rightarrow \quad \angle ABP = \angle CQP \qquad \text{...(ii)}$$

[∵ ∠PQC = ∠CQP]

From (i) and (ii), we get

$$\angle CPQ = \angle CQP$$

$$\Rightarrow \quad CQ = CP. \qquad \text{Hence proved.}$$

Question 19. The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection on any side when produced backward bisects the opposite side.

Solution : Let ABCD be a cyclic quadrilateral such that its diagonals AC and BD intersect in P at right angles. Let PL ⊥ AB such that LP produced to meet CD in M. We have to prove that M bisects CD i.e.,

$$CM = MD.$$
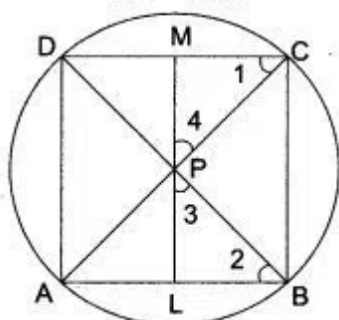
Consider arc AD. Clearly, it makes angles $\angle 1$ and $\angle 2$ in the same segment.

$$\angle 1 = \angle 2 \qquad \qquad ...(i)$$

In right angled triangle PLB, we have

$$\angle 2 + \angle 3 + \angle PLB = 180°$$
$$\Rightarrow \quad \angle 2 + \angle 3 + 90° = 180°$$
$$\Rightarrow \quad \angle 2 + \angle 3 = 90° \qquad \qquad ...(ii)$$

Since, LPM is a straight line.

$$\therefore \angle 3 + \angle BPD + \angle 4 = 180°$$
$$\Rightarrow \quad \angle 3 + 90° + \angle 4 = 180°$$
$$\Rightarrow \quad \angle 3 + \angle 4 = 90° \qquad \qquad ...(iii)$$

From (ii) and (iii), we get

$$\angle 2 + \angle 3 = \angle 3 + \angle 4$$
$$\Rightarrow \quad \angle 2 = \angle 4 \qquad \qquad ...(iv)$$

From (i) and (iv), we get

$$\angle 1 = \angle 4 \qquad \qquad ...(v)$$
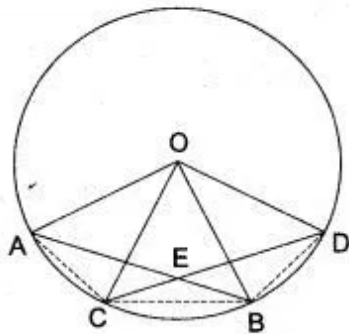$$\Rightarrow \quad PM = CM$$

Similarly, $\quad PM = DM.$

Hence, $\quad CM = MD.$ Hence proved.

Question 20. In a circle with centre O, chords AB and CD intersect inside the circumference at E. Prove that ∠AOC + ∠BOD = 2∠AEC.

Solution : Consider arc AC of the circle with centre at O.

Clearly, arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \qquad \angle AOC = 2\angle ABC \qquad \qquad ...(i)$$

Similarly, arc BD subtends $\angle BOD$ at the centre and $\angle BCD$ at the remaining part of the circle.

$$\therefore \qquad \angle BOD = 2\angle BCD \qquad \qquad ...(ii)$$

Adding (i) and (ii), we get

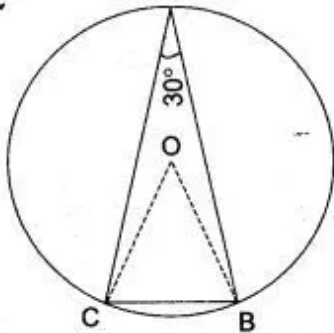$$\angle AOC + \angle BOD = 2(\angle ABC + \angle BCD)$$
$$\Rightarrow \angle AOC + \angle BOD = 2 \angle AEC$$

[∵ ∠AEC is the exterior angle and ∠ABC and ∠BCD are other interior angles of △ BEC

∴ ∠ABC + ∠BCD = ∠ AEC]

Hence proved.

Question 21. In Fig. ABC is a triangle in which ∠BAC = 30°. Show that BC is the radius of the circum circle of A ABC, whose centre is O.

Solution : Join OB and OC. Since, the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

$\therefore$ $\angle BOC = 2\angle BAC$

$\Rightarrow$ $\angle BOC = 2 \times 30° = 60°$

Now, in $\triangle BOC$, we have

$$OB = OC$$

[Each equal to radius of the circle]

$\Rightarrow$ $\angle OBC = \angle OCB$

[∵ Angles opposite to equal sides of a triangle are equal]

But $\angle OBC + \angle OCB + \angle BOC = 180°$

$\therefore$ $2\angle OBC + 60° = 180°$

$\Rightarrow$ $2\angle OBC = 120° \Rightarrow \angle OBC = 60°$

Thus, $\angle OBC = \angle OCB$

$$= \angle BOC = 60°$$
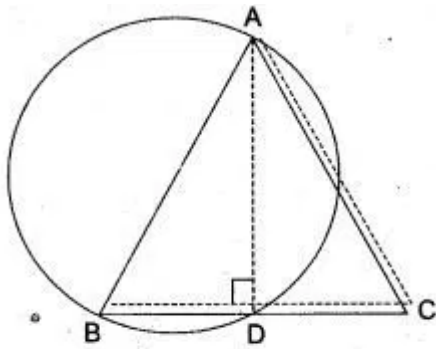
$\Rightarrow$ Triangle OBC is an equilateral

$\Rightarrow$ $OB = BC$ showed

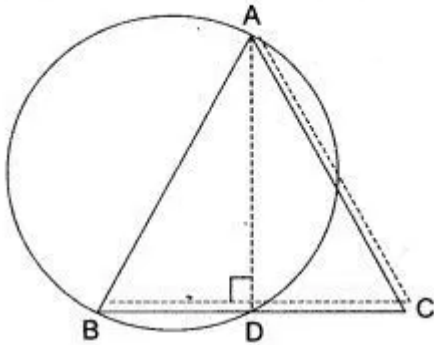$\Rightarrow$ BC is the radius of the circum circle of $\triangle$ ABC.

Hence proved.

Question 22. Prove that the circle drawn on any one of the equal sides of an isosceles triangles as diameter bisects the base.

Solution :

Given : In Isosceles Δ ABC. A circle is drawn taken AB as diameter which intersect BC at D.

To prove : BD = DC.

Construction : Join AD.

Proof :   ∠ ADB = 90° (Angle of semi-circle)

In Δ ABD & Δ ACD,

$$AB = AC \qquad \text{(Given)}$$
$$\angle ADB = \angle ADC \qquad (90°)$$
$$AD = AD \qquad \text{(Common)}$$

∴   Δ ABD ≅ Δ ADC

Hence   BD = DC.

Question 23. In Fig. ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in the points E and F respectively. Prove that EF || DC.
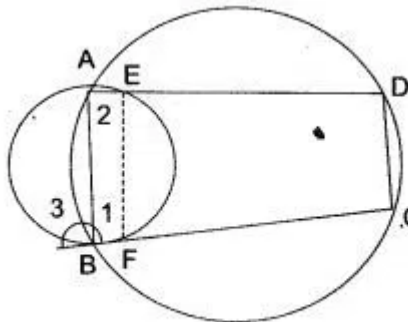
Solution : In order to prove that EF || DC. It is sufficient to show that ∠2 = ∠3.

Since, ABCD is a cyclic quadrilateral.

∴   ∠1 + ∠3 = 180°   ...(i)

Similarly, in the cyclic quadrilateral ABFE, we have

∠1 + ∠2 = 180°   ...(ii)

From (i) and (ii), we get

∠1 + ∠3 = ∠1 + ∠2 ⇒ ∠2 = ∠3.

Hence,   EF ‖ DC.

Question 24. If O is the circumcentre of a △ ABC and OD ⊥ BC, prove that ∠ BOD = ∠ A.

Solution : Join OB and OC

In △ OBD and △ OCD, we have

$$OB = OC$$

[Each equal to the radius of circumcircle]

$$\angle ODB = \angle ODC$$

[Each equal to 90°]

and $$OD = OD$$ [Common]
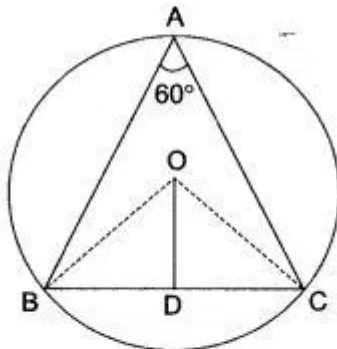
∴   △ OBD ≅ △ OCD

⇒   ∠ BOD = ∠ COD

⇒   ∠ BOC = 2∠ BOD = 2∠ COD

Now, arc BC subtends ∠ BOC at the centre and ∠ BAC = ∠ A at a point in the remaining part of the circle.

∴   ∠ BOC = 2∠ A

⇒   2∠ BOD = 2∠ A

[∵ ∠ BOC = 2∠ BOD]

⇒   ∠ BOD = ∠ A.   Hence proved.

Question 25. If PA and PB are two tangent drawn from a point P to a circle with centre C touching it A and B, prove that CP is the perpendicular bisector of AB.
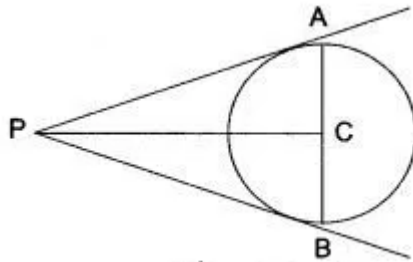
Solution : We shall prove that

$$\angle ACP = \angle BCP = 90°$$

and

$$AC = BC$$

Now, $\angle APC = \angle BPC$

Since O lies on the bisector of $\angle APB$.

Δs ACP and BCP are congruent traingles by SAS congruence criterion.

$\therefore$ $AC = BC$

and $\angle ACP = \angle BCP$

Since $\angle ACP + \angle BCP = 180°$

$$2\angle ACP = 180°$$

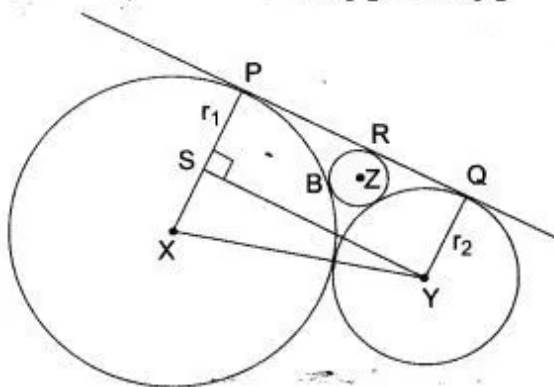$$\angle ACP = 90°$$

$\therefore$ $\angle ACP = \angle BCP = 90°$

Hence proved.

Question 26. Two circle with radii r_1 and r_2 touch each other externally. Let r be the radius of a circle which touches these two circle as well as a common tangent to the two circles, Prove that:

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$$

Solution : From the adjoining figure,

$$PQ = SY = \sqrt{XY^2 - XS^2}$$
$$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$
$$= \sqrt{4r_1r_2} = 2\sqrt{r_1r_2}$$

Similarly, $PR = 2\sqrt{r_1 r}$ and $RQ = 2\sqrt{rr_2}$

Now $PQ = PR + RQ$

$\therefore$ $2\sqrt{r_1r_2} = 2\sqrt{rr_1} + 2\sqrt{rr_2}$

$\Rightarrow$ $\sqrt{r_1r_2} = \sqrt{rr_1} + \sqrt{rr_2}$

Dividing by $\sqrt{r\,r_1r_2}$ on both sides,

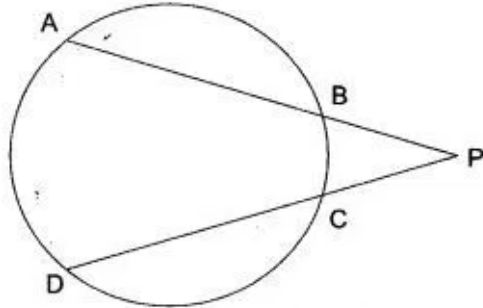$\Rightarrow$ $\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$ Hence proved.

Question 27. If AB and CD are two chords which when produced meet at P and if AP = CP, show that AB = CD.

Solution : Here, chords AB and CD of the circle intersect at P.

$\therefore$ $\qquad$ $PA \times PB = PC \times PD$

$\Rightarrow$ $\qquad$ $PB = \dfrac{PC \times PD}{PA}$

$\Rightarrow$ $\qquad$ $PB = \dfrac{AP \times PD}{AP}$

$\qquad\qquad\qquad$ [$\because$ PC = AP (given)]

$\Rightarrow$ $\qquad$ $PB = PD$ $\qquad$ ...(i)

Now, $\qquad$ $AB = AP - BP$

$\Rightarrow$ $\qquad$ $AB = CP - PD$

$\qquad$ [$\because$ AP = CP (given), BP = PD (From (i)]

$\Rightarrow$ $\qquad$ $AB = CD$

Hence, $\qquad$ $AB = CD.$ $\qquad$ Hence proved.

Question 28. In the figure, PM is a tangent to the circle and PA = AM. Prove that:
(i) ΔPMB is isosceles.
(ii) $PA \times PB = MB^2$.
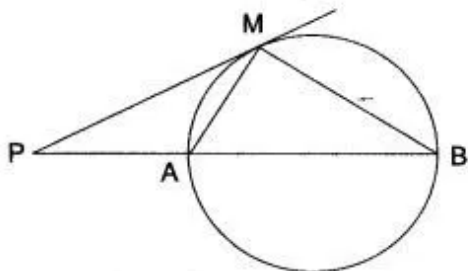
Solution : (i) In Δ PAM

$\qquad$ $\angle APM = \angle AMP$ $\qquad$ ... (i)

$\qquad$ $PA = AM$ $\qquad$ (Given)

by alternate segment property of tangent

$\qquad$ $\angle ABM = \angle AMP$

$\therefore$ $\qquad$ $\angle APM = \angle ABM$ from (i) and (ii)

$\therefore$ $\qquad$ $PM = MB$

i.e. Δ PMB is an isosceles $\qquad$ Proved.

(ii) By rectangle property of tangent and chord

$\qquad$ $PM^2 = PA \times PB$

$\therefore$ $\qquad$ $MB^2 = PA \times PB$ $\qquad$ [$\because$ PM = MB]

$\qquad\qquad\qquad$ Hence proved.

Question 29. In Fig. the incircle of ΔABC, touches the sides BC, CA and AB at D, E respectively. Show that:

$$AF + BD + CE = AE + BF + CD$$

$$= \frac{1}{2} \text{ (Perimeter of } \Delta ABC).$$
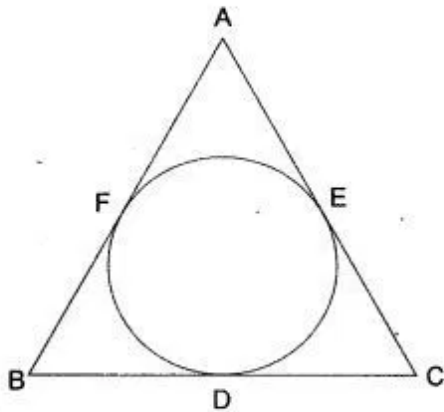
Solution : Since, lengths of the tangents drawn from an exterior point to a circle are equal.

∴ $\quad\quad\quad$ AF = AE $\quad\quad\quad$ ...(i)

$\quad\quad\quad\quad$ BD = BF $\quad\quad\quad$ ...(ii)

and $\quad\quad\quad$ CE = CD $\quad\quad\quad$ ...(iii)

Adding (i), (ii) and (iii), we get

$\quad$ AF + BD + CE = AE + BF + CD

Now, Perimeter of

$\quad$ Δ ABC = AB + BC + AC

$\quad\quad\quad\quad$ = (AF + FB) + (BD + CD) + (AE + EC)

$\quad\quad\quad\quad$ = (AF + AE) + (BF + BD) + (CD + CE)

$\quad\quad\quad\quad$ = 2AF + 2BD + 2CE

$\quad\quad\quad\quad$ = 2(AF + BD + CE)

$\quad\quad\quad\quad\quad$ [From (i), (ii) and (iii), we get

$\quad\quad\quad$ AE = AF, BD = BF and CD = CE]

∴ AF + BD + CE $= \frac{1}{2}$ (Perimeter of ΔABC)

Hence, AF + BD + CE = AE + BF + CD $= \frac{1}{2}$ (Perimeter of Δ ABC). $\quad\quad\quad\quad$ Hence proved.

Question 30. In Fig. TA is a tangent to a circle from the point T and TBC is a secant to the circle. If AD is the bisector of ∠BAC, prove that ΔADT is isosceles.
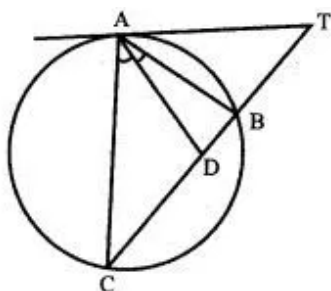
Solution : In order to prove that Δ ADT is isosceles i.e., TA = TD, it is sufficient to show that ∠ TAD = ∠ TDA.

Since ∠ TAB and ∠ BCA are angles in the alternate segments of chord AB.

∴ ∠ TAB = ∠ BCA ...(i)

It is given that AD is the bisector of ∠ BAC.

∴ ∠ BAD = ∠ CAD ...(ii)

Now, ∠ TAD = ∠ TAB + ∠ BAD

⇒ ∠ TAD = ∠ BCA + ∠ CAD

[Using (i) and (ii)]

⇒ ∠ TAD = ∠ DCA + ∠CAD

[∵ ∠ BCA = ∠ DCA]

⇒ ∠ TAD = 180° − ∠ CDA

[In ΔCAD, ∠ CAD + ∠ DCA + ∠ CDA = 180°

∴ ∠ CAD + ∠ BCA = 180° − ∠ CDA]

⇒ ∠ TAD = ∠ TDA

[∵ ∠ CDA + ∠ TDA = 180°]

⇒ TD = TA

Hence, ΔADT is isosceles. Hence proved.

Question 31. In Fig. AP is a tangent to the circle at P, ABC is a secant and PD is the bisector of ∠BPC. Prove that $\angle BPD = \frac{1}{2}(\angle ABP - \angle APB)$.
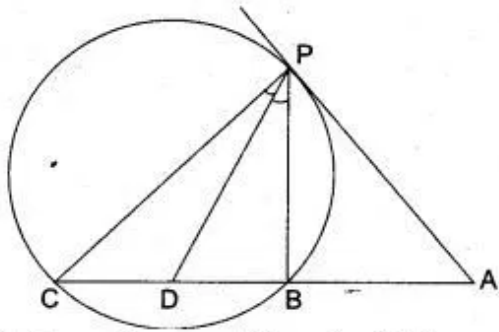
Solution : Since, ∠ APB and ∠ BCP are angles in alternate segments of chord PB.

∴   ∠ APB = ∠ BCP   ...(i)

Since, PD is bisector of ∠ BPC

∴   ∠ CPB = 2∠ BPD   ...(ii)

In ΔPCB, side CB has been produced to A, forming exterior angle ∠ ABP.

∴   ∠ ABP = ∠ BCP + ∠ CPB

⟹   ∠ ABP = ∠ APB + 2∠ BPD

                      [Using (i) and (ii)]

⟹   2∠ BPD = ∠ ABP – ∠ APB

⟹   ∠ BPD = $\frac{1}{2}$(∠ ABP – ∠ APB).

                            Hence proved.

Question 32. In Fig. l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between n and m. Prove that ∠ DFE = 90^0
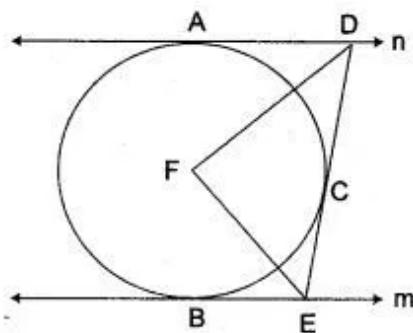
Solution : In triangles ADF and DFC, we have

          DA = DC

                  [Tangents drawn from an external
                      point are equal in length]

          DF = DF   [Common]

          AF = CF[Radii of the same circle]

So, by SSS-criterion of congruence, we get

          Δ ADF ≅ Δ DFC

⟹   ∠ ADF = ∠ CDF

⟹   ∠ ADC = 2 ∠ CDF   ...(i)

Similarly, we can prove that

          ∠ BEF = ∠ CEF

⟹   ∠ CEB = 2∠ CEF   ...(ii)

Now, $\angle ADC + \angle CEB = 180°$

[Sum of the interior angles on the same side of transversal is 180°]

$\Rightarrow \quad 2\,(\angle CDF + \angle CDF) = 180°$

$\Rightarrow \quad \angle CDF + \angle CEF = \dfrac{180°}{2}$

$\Rightarrow \quad \angle CDF + \angle CEF = 90°$

Now, in $\angle DEF$;

$\angle DFE + \angle CDF + \angle CEF = 180°$

$\Rightarrow \qquad \angle DFE + 90° = 180°$

$\Rightarrow \qquad \angle DFE = 180° - 90°$

$\Rightarrow \qquad \angle DFE = 90°$

Hence proved.

Question 33. A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Solution : Given : A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral $ABCD$ at the points P, Q, R and S respectively.

To prove :

$$\angle AOB + \angle COD = 180°$$

and $\qquad \angle AOD + \angle BOC = 180°$

Construction : Join OP, OQ, OR and OS.
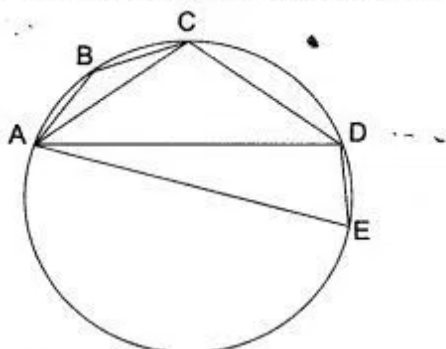
Proof : Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$

...(i)

Now,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

[Sum of all the angles subtended at a point is 360°]

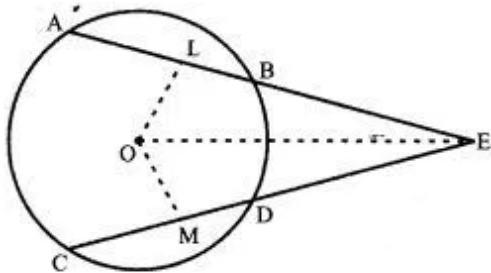$\Rightarrow \qquad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

and $\quad 2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$

$\Rightarrow \qquad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$

and $\quad (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

$$\Rightarrow \qquad \angle AOB + \angle COD = 180°$$

$$[\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD,$$
$$\angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC]$$

and $\qquad \angle AOD + \angle BOC = 180°.$

Hence proved.

Question 34. Two equal chords AB and CD of a circle with centre O, when produced meet at a point E, as shown in Fig. Prove that BE = DE and AE = CE.

Solution : Given : Two equal chords AB and CD intersecting at a point E.

To prove : BE = DE and AE = CE

Construction : Join OE. Draw OL ⊥ AB and OM ⊥ CD.

Proof : We have

$$AB = CD$$
$$\Rightarrow \qquad OL = OM$$

[∵ Equal chords are equidistant from the centre]

In triangles OLE and OME, we have

$$OL = OM$$
$$\angle OLE = \angle OME$$

[Each equal to 90°]

and $\qquad OE = OE \qquad$ [Common]

So, by SAS-criterion of congruences

$$\triangle OLE \cong \triangle OME$$
$$\Rightarrow \qquad LE = ME \qquad \qquad ...(i)$$

Now, $\qquad AB = CD$

$$\Rightarrow \qquad \frac{1}{2}AB = \frac{1}{2}CD \Rightarrow BL = DM \quad ...(ii)$$

Subtracting (ii) from (i), we get

$$LE - BL = ME - DM$$
$$\Rightarrow \qquad BE = DE$$

Again, $\qquad AB = CD$ and $BE = DE$

$$\Rightarrow \qquad AB + BE = CD + DE$$
$$\Rightarrow \qquad AE = CE$$

Hence, $\qquad BE = DE$ and $AE = CE.$

Hence proved.

Question 35. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral ABCD is also cyclic.

Solution : Given : In cyclic `ABCD` the bisectors formed a quadrilateral ABCD.

To prove : PQRS is a cyclic quadrilateral.

Proof : In cyclic quadrilateral ABCD, AR & BS be the bisectors of $\angle$ A and $\angle$ B.

So $\angle 1 = \angle A/2$ and $\angle 2 = \angle B/2$

In Δ ASB, $\angle$ RSP is the exterior angle

so $\angle RSP = \angle 1 + \angle 2$
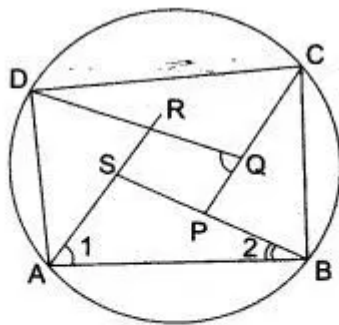
$\angle RSP = \dfrac{\angle A}{2} + \dfrac{\angle B}{2}$ ...(i)

similarly $\angle PQR = \dfrac{\angle C}{2} + \dfrac{\angle D}{2}$ ...(ii)

Adding (i) & (ii)

$\angle PQR + \angle RSP = \dfrac{1}{2}(\angle A + \angle B + \angle C + \angle D)$

$= \dfrac{1}{2} \times 360° = 180°$

$\Rightarrow \angle PQR + \angle RSP = 180°$

But these are the opposite angles of quadrilateral PQRS

Hence PQRS is a cyclic quadrilateral.

Hence proved.

Figure Based Questions

Question 1. Two concentric circles with centre 0 have A, B, C, D as the points of intersection with the lines L shown in figure. If AD = 12 cm and BC s = 8 cm, find the lengths of AB, CD, AC and BD.

Solution. Since, OM ⊥ BC

$$BM = CM = \frac{1}{2}BC = 4 \text{ cm}$$
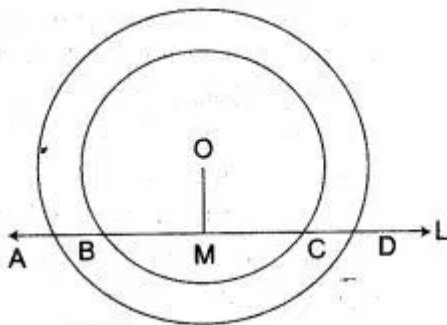
also   OM ⊥ AD

so $AM = DM = \frac{1}{2}AD = 6 \text{ cm}$
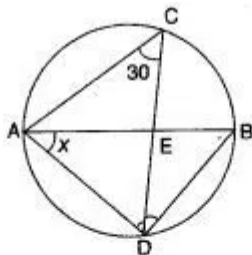
now $AB = AM - BM = (6 - 4) \text{ cm} = 2 \text{ cm}$

         $CD = DM - CM = (6 - 4) \text{ cm} = 2 \text{ cm}$

∴ $AC = AB + BC = (2 + 8) \text{ cm} = 10 \text{ cm}$

and $BD = BC + CD = (8 + 2) \text{ cm} = 10 \text{ cm}.$

Question 2. In the given circle with diameter AB, find the value of x.

Solution.   ∠ABD = ∠ACD = 30°

                  (∠s of same segment)

       ∠ADB = 90°

                  (∠ in the semi circle)

In Δ ADB,

    $x° + 90° + 30° = 180$

              (Sum of all ∠s of triangle)

       $x = 180 - 120$

         $= 60°$ Ans.

Question 3. In the given figure, the area enclosed between the two concentric circles is 770 cm². If the radius of outer circle is 21 cm, calculate the radius of the inner circle.
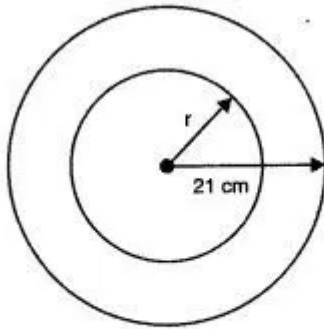
Solution. Let the radius of inner circle be r.

Area enclosed between two concentric circles

$\Rightarrow \quad \pi[(21)^2 - (r)^2] = 770$

$\Rightarrow \quad (21)^2 - (r)^2 = \dfrac{770}{\pi} = \dfrac{770 \times 7}{22}$

$\qquad\qquad\qquad = 35 \times 7$

$\Rightarrow \quad (21)^2 - (r)^2 = 245$

$\Rightarrow \quad\quad 441 - r^2 = 245$

$\Rightarrow \quad\quad 441 - 245 = r^2$

$\Rightarrow \quad\qquad\quad 196 = r^2$

$\Rightarrow \quad\qquad\quad\; 14 = r$

The radius of inner circle = 14 cm.

Question 4. Two chords AB and CD of a circle are parallel and a line L is the perpendicular bisector of AB. Show that L bisects CD.

Solution. We know that the perpendicular bisector of any chord of a circle always passes through the centre of the circle.
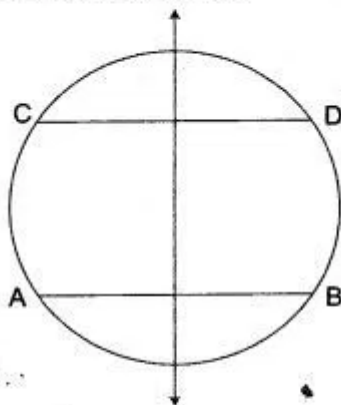
Since, L is the perpendicular bisector of AB. Therefore L passes through the centre of the circle.

But   L ⊥ AB and AB || CD ⇒ L ⊥ CD

Thus, L ⊥ CD and passes through the centre of the circle.

So, L is perpendicular bisector of CD.   Ans.

Question 5. In the adjoining figure, AB is the diameter of the circle with centre O. If ∠BCD = 120°, calculate:

(i) ∠BAD   (ii) ∠DBA
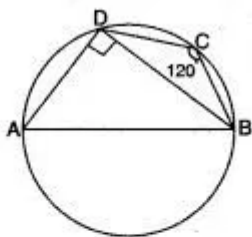
Solution : (i) Since AOB is a diameter

∴   ∠ ADB = 90° (C is a semi circle)

Also, ABCD is a cyclic quadrilateral.

∴   ∠ BCD + ∠ BAD = 180°

                    ∠ BAD = 180° – 120°

⇒   ∠ BAD = 60°

(ii) Now, In Δ BAD.

        ∠ BAD + ∠ BDA + ∠ DBA = 180°

                60° + 90° + ∠ DBA = 180°

                              ∠ DBA = 180° – 150°

                              ∠ DBA = 30°   Ans.

Question 6. Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 13 cm.

    Solution : Let AB be a chord of a circle with centre O and radius 13 cm. Draw OL ⊥ AB.

    Join OA. Clearly, OL = 5 cm and OA = 13 cm.

    In the right triangle OLA, we have

            $OA^2 = OL^2 + AL^2$
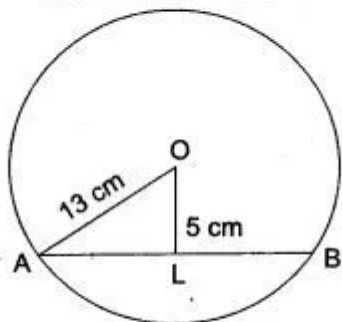
⇒ $13^2 = 5^2 + AL^2$

⇒ $AL^2 = 144 \text{ cm}^2$

⇒ $AL = 12$ cm

    Since, the perpendicular from the centre to the chord bisects the chord. Therefore,

            AB = 2AL = (2 × 12) cm

                  = 24 cm.   Ans.

Question 7. The radius of a circle is 13 cm and the length of one of its chord is 10 cm. Find the distance of the chord from the centre.
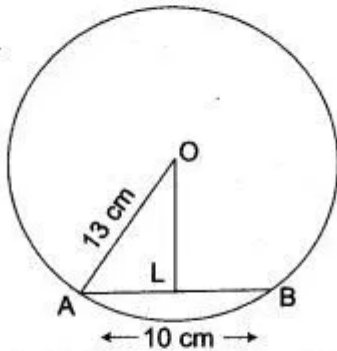
Solution : Let AB be a chord of a circle with centre O and radius 13 cm such that AB = 10 cm.

From O, draw OL ⊥ AB. Join OA.

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore \qquad AL = LB = \frac{1}{2}AB = 5 \text{ cm}$$

Now, in right triangle OLA, we have

$$OA^2 = OL^2 + AL^2$$
$$\Rightarrow \qquad 13^2 = OL^2 + 5^2$$
$$\Rightarrow \qquad 13^2 - 5^2 = OL^2$$
$$\Rightarrow \qquad OL^2 = 144$$
$$\Rightarrow \qquad OL = 12 \text{ cm.}$$

Hence, the distance of the chord from the centre is 12 cm. **Ans.**

Question 8. C is a point on the minor arc AB of the circle, with centre O. Given ∠ ACB = x° and ∠ AOB = y° express y in terms of x. Calculate x, if ACBO is a parallelogram.

Solution. Clearly, major arc AB subtends x° at a point on the remaining part of the circle.

$$\therefore \quad \text{reflex} \angle AOB = 2x°$$
$$\Rightarrow \qquad 360° - y° = 2x°$$
$$y° = 360° - 2x°$$
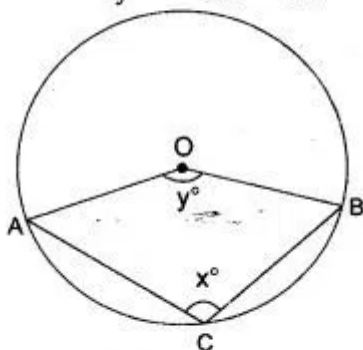
Thus, $\qquad y = 360° - 2x°$

If ACBO is a parallelogram, then

$$x° = y° \text{ i.e., } x = y$$
$$\Rightarrow \qquad x = 360° - 2x \Rightarrow 3x = 360°$$
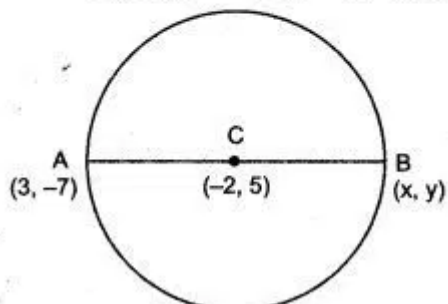$$\Rightarrow \qquad x = 120°. \qquad \textbf{Ans.}$$

Question 9. AB is a diameter of a circle with centre C = (- 2, 5). If A = (3, – 7). Find
(i) the length of radius AC

(ii) the coordinates of B.

Solution. (i) $AC = \sqrt{(3+2)^2 + (-7-5)^2}$

$(\because$ Distance formula)

$$= \sqrt{25 + 144}$$

$\therefore$ Radius $= \sqrt{169} = 13$ units

(ii)   As 'C' is mid point of AB

$$-2 = \frac{3+x}{2}$$

$[\because$ mid point formula]

or $-4 = 3 + x$
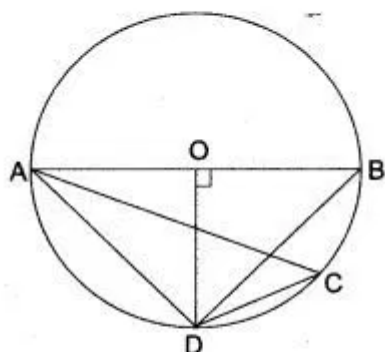
$$x = -7$$

and $5 = \frac{-7+y}{2}$

and $10 = -7 + y$

and $y = 17$

$\therefore$ B (– 7, 17).   Ans.

Question 10. AB is a diameter of a circle with centre O and radius OD is perpendicular to AB. If C is any point on arc DB, find ∠ BAD and ∠ ACD.

Solution. Since, chord BD makes ∠ BOD at the centre and ∠ BAD at A.

$\therefore$ $\angle BAD = \frac{1}{2} \angle BOD = \frac{1}{2} \times (90°) = 45°$

Similarly, chord AD makes ∠ AOD at the centre and ∠ ACD at C.

$\therefore$ $\angle ACD = \frac{1}{2} \angle AOD$

$$= \frac{1}{2} \times (90°) = 45°$$

Thus, ∠ BAD = ∠ ACD = 45°.   Ans.

Question 11. In the given below figure,

$$\angle BAD = 65°$$
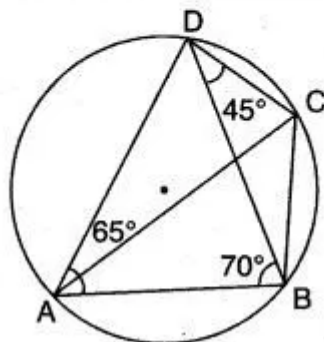$$\angle ABD = 70°$$

and $\quad \angle BDC = 45°$. Find :

(i) $\angle BCD,$ (ii) $\angle ADB.$

Hence show that AC is a diameter.

Solution. Given : $\quad \angle BAD = 65°,$

$$\angle ABD = 70°$$

and $\quad\quad\quad\quad \angle BDC = 45°.$

(i) Quadrilateral ABCD is cyclic quadrilateral.

$\therefore \angle DAB + \angle BCD = 180°$

$65° + \angle BCD = 180°$

$\therefore \quad\quad \angle BCD = 180° - 65°$

$= 115°$. **Ans.**

(ii) In Δ ADB,

$\angle DAB + \angle ABD + \angle ADB = 180°$

$65° + 70° + \angle ADB = 180°$

$\angle ADB = 180° - 135°$

$\angle ADB = 45°$. **Ans.**

Question 12. If Fig. PT is a tangent to a circle. If $m (\angle BTA) = 45°$ and $m (\angle PTB) = 70°$, find $m \angle (ABT)$.
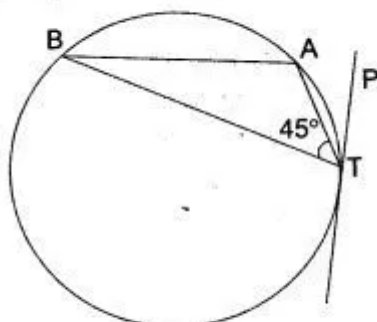
Solution : $\angle ATP = \angle PTB - \angle BTA$

$= 70° - 45° = 25°$

$\therefore \quad\quad \angle ABT = \angle ATP$
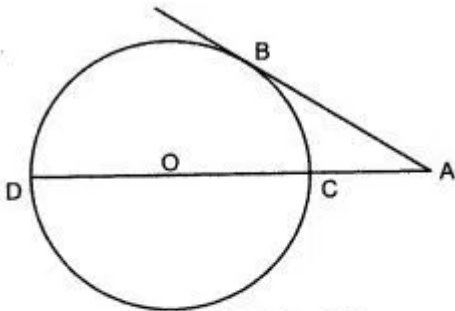
(Angles are in alternate segments)

$\Rightarrow \quad\quad \angle ABT = 25°.$ **Ans.**

Question 13. In Fig. AT is a tangent to the circle. If m∠ABC = 50°, AC = BC, find ∠BAT.
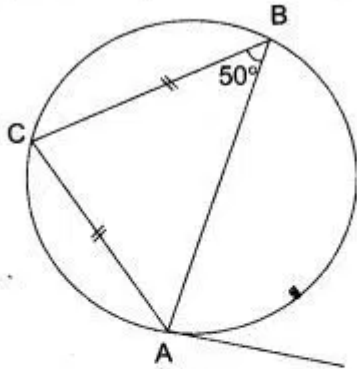
Solution :   AC = BC

⇒   ∠CBA = ∠CAB

⇒   ∠CAB = 50°

∴   ∠ACB = 180° – (50° + 50°) = 80°

Now,   ∠BAT = ∠BCA

            [Angles are in alternate segments]

⇒   ∠BAT = 80°.   Ans.

Question 14. In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.

Solution : $AB^2 = AC \times AD$

                    $(PT^2 = PA \times PB)$

⇒ $15^2 = 7{\cdot}5 \times AD$

⇒ $AD = \dfrac{225}{7{\cdot}5} = 30$

⇒ $CD = AD - AC$

                    $= 30 - 7{\cdot}5 = 22{\cdot}5$

∴ $Radius = \dfrac{1}{2} \times CD$

            $Radius = \dfrac{1}{2} \times 22{\cdot}5$

            $Radius = 11{\cdot}25$ cm.

Question 15. In Fig., chords AB and CD of the circle intersect at O. AO = 5 cm, BO = 3 cm and CO = 2.5 cm. Determine the length of DO.
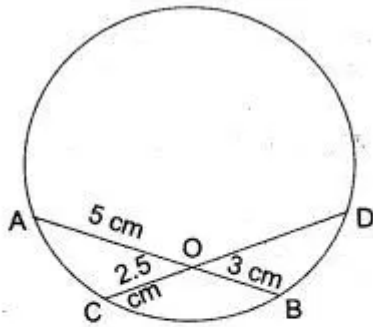
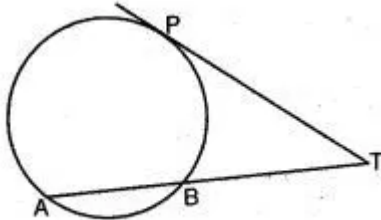Solution : Clearly, chords AB and CD intersect at O.

$\therefore \quad OA \times OB = OC \times OD$

$\Rightarrow \quad 5 \times 3 = 2.5 \times OD$

$\Rightarrow \quad OD = \left(\dfrac{5 \times 3}{2.5}\right) = 6$ cm.   Ans.

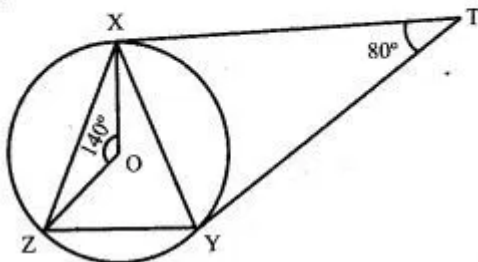**Question 16.** In the figure given below, PT is a tangent to the circle. Find PT if AT = 16 cm and AB = 12 cm.

Solution : PT is tangent.
Hence by theorem,

$$PT^2 = AT \times BT$$
$$= 16 \times (AT - AB)$$
$$= 16 \times (16 - 12)$$
$$= 16 \times 4 = 64$$
$$\therefore \quad PT = 8 \text{ cm.}$$

Question 17. In the alongside, figure, O is the centre of the circumcircle of triangle XYZ. Tangents at X and Y intersect at T. Given ∠XTY = 80° and ∠ XOZ = 140° Calculate the value of ∠ ZXY.

Solution :

$$\angle TXY = \angle TYX = 50° \quad \text{(since XT = YT)}$$
$$\angle OXZ = \angle OZX = 20° \quad \text{(since OX = OZ)}$$
$$[\text{In } \triangle XOZ, 140° + \angle OXZ + \angle OZX = 180°]$$
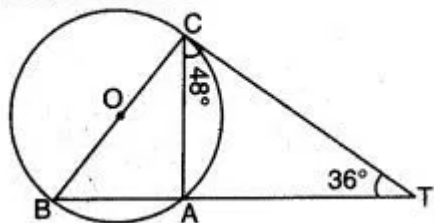$$\Rightarrow \quad \angle OXZ = 20°$$
$$\angle OXY = 40°, \quad \text{(since } \angle OXT = 90°)$$
$$\angle ZXY = \angle OXZ + \angle OXY$$
$$= 20° + 40° = 60°. \qquad \text{Ans.}$$

Question 18. A, B and C are three points on a circle. The tangent at C meets BN produced at T. Given that ∠ ATC = 36° and ∠ ACT = 48°, calculate the angle subtended by AB at the centre of the circle.

Solution : Join BC. Let O be the centre of the circle. Join OA and OB.

In △ BCT, △ ACT,
$$\angle BTC = \angle ATC = 36°$$
$$\angle ACT = \angle ABC = 48°$$
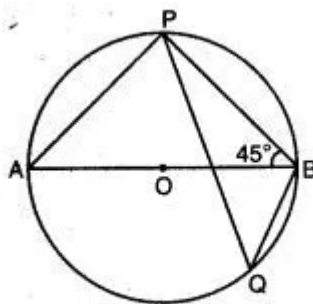$$\angle BAC = \angle ACT + \angle ATC$$
$$= 48° + 36° = 84°$$
$$\therefore \quad \angle BCA = 180° - (\angle ABC + \angle BAC)$$
$$= 180° - (48° + 84°) = 48°$$
$$\therefore \quad \angle BOA = 2 \angle BCA$$
$$= 2 \times 48° = 96° \qquad \text{Ans.}$$

Question 19. In the given figure, O is the centre of the circle and ∠PBA = 45°. Calculate the value of ∠PQB.

Solution : Given
$$\angle PBA = 45°$$
AOB is diameter of circle.
$$\therefore \qquad \angle APB = 90°$$
$$\text{(Angle in semi circle)}$$
So in △ APB, $\angle PAB = 180° - (90° + 45°)$
$$= 45°$$
$$\angle PAB = \angle PQB$$
$$\text{(Angle in same segment)}$$
$$\therefore \qquad \angle PQB = 45°. \qquad \text{Ans.}$$

Question 20. Two chords AB, CD of lengths 16 cm and 30 cm, are parallel. If the distance between AB and CD is 23 cm. Find the radius of the circle.

Solution : Let AB and CD be the two parallel chords of a circle with centre O and radius r cm.

$$OM \perp AB \text{ and } ON \perp CD$$

$$AM = \frac{1}{2}AB = \frac{1}{2} \times 16 = 8 \text{ cm}$$

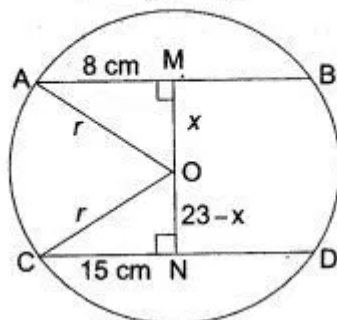$$CN = \frac{1}{2}CD = \frac{1}{2} \times 30 \text{ cm} = 15 \text{ cm}$$

let $\quad OM = x \text{ cm}, MN = 23 \text{ cm}$

so $\quad ON = (23 - x) \text{ cm}$

$$OA = OC = r \text{ cm}$$

InΔ OAM, $OA^2 = AM^2 + OM^2$

$\Rightarrow \qquad r^2 = (8)^2 + x^2 \qquad\qquad ...(i)$

In Δ OCN,$OC^2 = CN^2 + ON^2$

$\Rightarrow \qquad r^2 = (15)^2 + (23 - x)^2 \qquad ...(ii)$

From (i) & (ii),

$$x^2 + 64 = 225 + (23 - x)^2$$

$\Rightarrow \qquad x^2 + 64 = 225 + 529 - 46x + x^2$

$\Rightarrow \qquad\quad 46x = 225 + 529 - 64$

$\Rightarrow \qquad\quad 46x = 690 \Rightarrow x = 15 \text{ cm}$

From (i), $\quad r^2 = (8)^2 + (15)^2 = 64 + 225 = 289$

$\Rightarrow \qquad\qquad r = 17 \text{ cm}$
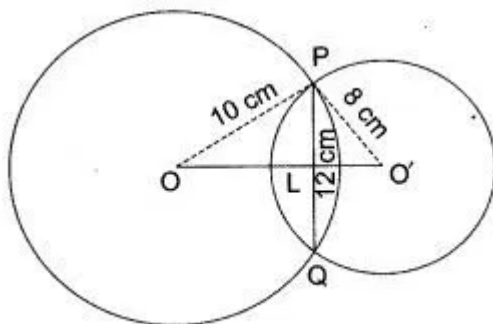
Hence, the radius of a circle is 17 cm. $\qquad$ Ans.

Question 21. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.

Solution : Let O and O′ be the centres of two circles with radii 10 cm and 8 cm respectively.

So $\qquad OP = 10 \text{ cm}, O'P = 8 \text{ cm}$

and $\qquad PQ = 12 \text{ cm}$

then $\qquad PL = \frac{1}{2}PQ = 6 \text{ cm}$

In $\triangle OLP$, $OP^2 = OL^2 + LP^2$

$\Rightarrow \qquad OL^2 = OP^2 - LP^2$

$\qquad\quad OL = \sqrt{(10)^2 - 6^2} = \sqrt{64} = 8$ cm

In $\triangle O'LP$,

$\qquad\quad O'L = \sqrt{O'P^2 - LP^2} = \sqrt{8^2 - 6^2}$

$\qquad\qquad\quad = \sqrt{64 - 36}$

$\qquad\quad O'L = \sqrt{28}$ cm

$\qquad\qquad\quad = 5{\cdot}29$ cm

Distance between centres

$\qquad\quad OO' = OL + LO'$

$\qquad\qquad\quad = (8 + 5{\cdot}29)$ cm

$\qquad\qquad\quad = 13{\cdot}29$ cm.

Question 22. AB and CD are two chords of a circle such that AB = 6 cm, CD = 12 cm and AB || CD. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution : Let AB and CD be two parallel chords

of a circle with centre O such that AB = 6 cm and CD = 12 cm. Let the radius of the circle be r cm. Draw OP ⊥ AB and OQ ⊥ CD. Since, AB || CD and OP ⊥ AB, OQ ⊥ CD. Therefore, points O, Q and P are collinear. Clearly, PQ = 3 cm.

Let OQ = x cm. Then, OP = (x + 3) cm.

In right triangles OAP and OCQ, we have

$\qquad OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$\Rightarrow \qquad r^2 = (x + 3)^2 + 3^2$ and $r^2 = x^2 + 6^2$
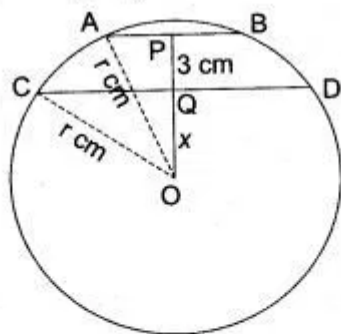
$$\left[\because AP = \frac{1}{2} AB = 3 \text{ cm and } CQ = \frac{1}{2} CD = 6 \text{ cm}\right]$$

$\Rightarrow \qquad (x + 3)^2 + 3^2 = x^2 + 6^2$

(on equating the value of r^2)

$\Rightarrow \quad x^2 + 6x + 9 + 9 = x^2 + 36$

$\Rightarrow \qquad\qquad 6x = 18 \Rightarrow x = 3$ cm

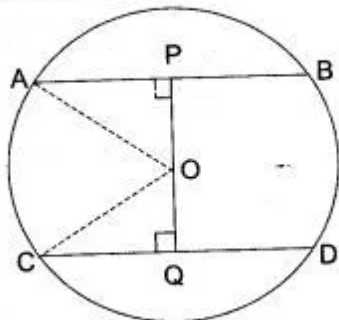Putting the values of x in $r^2 = x^2 + 6^2$, we get

$\qquad\qquad r^2 = 3^2 + 6^2 = 45$

$\Rightarrow \qquad r = \sqrt{45}$ cm $= 6{\cdot}7$ cm

Hence, the radius of the circle is 6·7 cm. Ans.

Question 23. In Fig. O is the centre of the circle with radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB | | CD, AB = 8 cm and CD = 6 cm. Determine PQ.

Solution : Join OA and OC.

Since, the perpendicular from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently, $AP = PB = \frac{1}{2} AB = 3$ cm

and $CQ = QD = \frac{1}{2} CD = 4$ cm

In right triangles OAP and OCQ, we have

$$OA^2 = OP^2 + AP^2 \text{ and } OC^2 = OQ^2 + CQ^2$$
$$\Rightarrow \quad 5^2 = OP^2 + 3^2 \text{ and } 5^2 = OQ^2 + 4^2$$
$$\Rightarrow \quad OP^2 = 5^2 - 3^2 \text{ and } OQ^2 = 5^2 - 4^2$$
$$\Rightarrow \quad OP^2 = 16 \text{ and } OQ^2 = 9$$
$$\Rightarrow \quad OP = 4 \text{ and } OQ = 3$$
$$\therefore \quad PQ = OP + OQ = (4+3) \text{ cm} = 7 \text{ cm.}$$

Question 24. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, find the radius of the circle.

Solution : Let O be the centre of the given circle and let it's radius be r cm. Draw OP ⊥ AB and OQ ⊥ CD. Since, AB | | CD. Therefore, points P, O and Q are collinear. So, PQ = 17 cm.
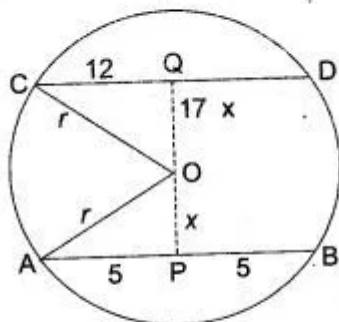
Let OP = x cm. Then, OQ = (17 − x) cm.

Join OA and OC. Then, OA = OC = r.

Since, the perpendicular from the centre to a chord of the circle bisects the chord.

$\therefore$ AP = PB = 5 cm and CQ = QD = 12 cm

In right triangles OAP and OCQ, we have

$$OA^2 = OP^2 + AP^2 \text{ and } OC^2 = OQ^2 + CQ^2$$

$\Rightarrow \qquad r^2 = x^2 + 5^2 \qquad\qquad ...(i)$

and $\qquad\qquad r^2 = (17-x)^2 + 12^2 \qquad ...(ii)$

$\Rightarrow \qquad x^2 + 5^2 = (17-x)^2 + 12^2$

[On equating the values of r^2]

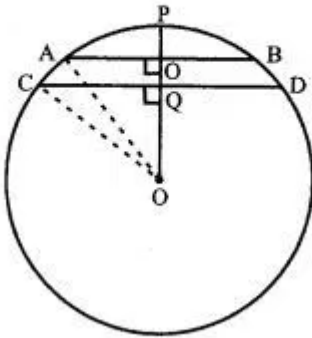$\Rightarrow \qquad x^2 + 25 = 289 - 34x + x^2 + 144$

$\Rightarrow \qquad\quad 34x = 408 \Rightarrow x = 12 \text{ cm}.$

Putting $x = 12$ cm in (i), we get

$$r^2 = 12^2 + 5^2 = 169 \Rightarrow r = 13 \text{ cm}$$

Hence, the radius of the circle is 13 cm.   **Ans.**

Question 25. In Fig. O is the centre of the circle of radius 5 cm. OP $\perp$ AB, OQ $\perp$ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.

Solution : Join OA and OC.

Since, the perpendicular from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently,

$$AP = PB = \frac{1}{2} AB = 3 \text{ cm}$$

and $\quad CQ = QD = \frac{1}{2} CD = 4 \text{ cm}$

In the right angled triangle OAP, we have

$$OA^2 = OP^2 + AP^2$$

$\Rightarrow \qquad 5^2 = OP^2 + 3^2$

$\Rightarrow \quad OP^2 = 5^2 - 3^2 = 16 \text{ cm}^2 \Rightarrow OP = 4 \text{ cm}^2$

In the right angled triangle OCQ we have

$$OC^2 = OQ^2 + CQ^2$$

$\Rightarrow \qquad 5^2 = OQ^2 + 4^2$

$\Rightarrow \quad OQ^2 = 5^2 - 4^2 = 9 \text{ cm}^2 \Rightarrow OQ = 3 \text{ cm}$

$\therefore \qquad PQ = PO - QO$

$\qquad\qquad = OP - OQ = (4-3) \text{ cm} = 1 \text{ cm}.$

Question 26. In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD.
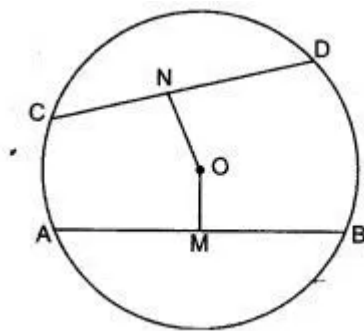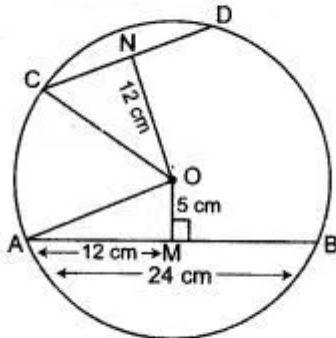
AB = 24 cm. OM = 5 cm, ON = 12 cm. Find the

(i) radius of the circle.

(ii) length of chord CD.

Solution :

AB = 24 cm, ON = 12 cm, OM = 5 cm.

(i) In $\triangle$ AOM, $OA^2 = OM^2 + AM^2$

$$= (5)^2 + (12)^2$$

$$= 25 + 144 = 169$$

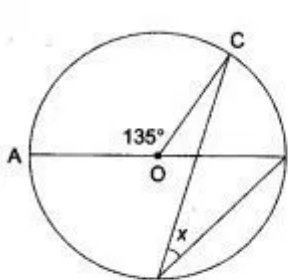$$OA = 13 \text{ cm}$$

Thus, radius of the circle is 13 cm.

(ii) In $\triangle$ CON, $OC^2 = ON^2 + CN^2$

$$(13)^2 = (12)^2 + CN^2$$

$$[\because OC = OA = 13 \text{ (Radius)}]$$

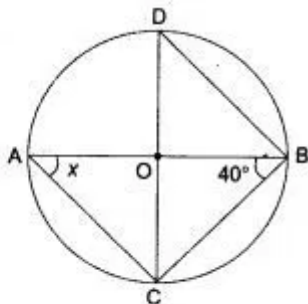$$169 - 144 = CN^2$$

$$CN^2 = 25$$

$$CN = 5$$

Thus length of chord CD $= 2 \, CN$

$$= 2 \times 5$$

$$= 10 \text{ cm.} \quad \text{Ans.}$$

Question 27. If O is the centre of the circle, find the value of x in each of the following figures

(i)   (ii)
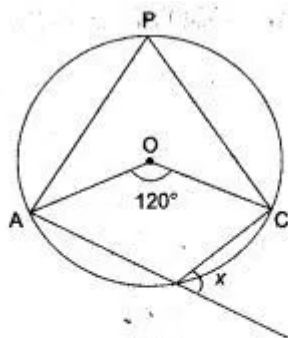
(iii)

Solution : (i) We have $AOC = 135°$

then $COB = 180° - 135° = 45°$

but $x = \dfrac{COB}{2} = \dfrac{45}{2}^° = 22\dfrac{1}{2}^°$ Ans.

(ii) In BOC,

$OB = OC$ (radii)

So $OCB = OBC = 40°$

In BOC,

$BOC = 180° - (40 + 40)°$

$= 180° - 80°$

So $x = \dfrac{BOC}{2}$

$= \dfrac{100°}{2} = 50°$ Ans.

$= \dfrac{1}{2} \; 120° = 60°$

(i

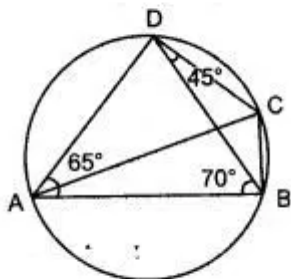Since, $ABCD$ is a cyclic quadrilateral

$CBD = x$ is the exterior angle

so $x = APC = 60°$.

Question 28. In the given figure, $BAD = 65°$

$ABD = 70°$, $BDC = 45°$

(i) Prove that AC is a diameter of the circle.

(ii) Find ACB

Solution : Given

$BAD = 65°$

$ABD = 70°$

$BDC = 45°$

(i) In ∠ABD,
∠BAD + ∠ABD + ∠ADB
$= 180°$
$65° + 70° +$ ∠ADB $= 180°$
{Sum of three angles of a △ }
∠ADB $= 180° - (65° + 70°)$
$= 45°$
∠ADC $=$ ∠ADB $+$ ∠BDC
$45° + 45° = 90°$
AC is the diameter of the circle.
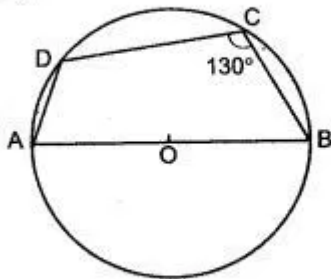[Angle in a semi circle is 90°]   Proved
(ii)   ∠ACB $=$ ∠ADB $= 45°$
{Angles in the same segment of a circle}

Question 29. In the given figure, AB is the diameter of a circle with centre O.
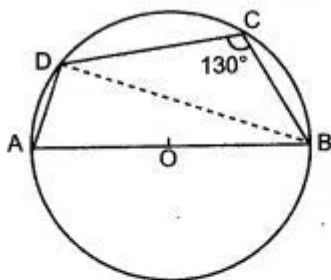∠BCD $= 130°$. Find :
(i)   ∠DAB
(ii)   ∠DBA

Solution : (i)   ∠DAB $+$ ∠BCD $= 180°$
[opp. angles of a cyclic quadrilateral]
∠DAB $+ 130° = 180°$

∠DAB $= 180° - 130°$
∠DAB $= 50°$ Ans.
(ii)   ∠ADB $= 90°$ (angle in semi-circle)
In △ADB
∠DAB $+$ ∠ADB $+$ ∠DBA $= 180°$
                (angle sum property)
$50° + 90° +$ ∠DBA $= 180°$
∠DBA $= 180° - 140°$
∠DBA $= 40°$ Ans.

Question 30. In ABCD is a cyclic quadrilateral; O is the centre of the circle. If ∠BOD = 160°, find the measure of ∠BPD.

Solution : Consider the arc BCD of the circle. This arc makes angle $\angle BOD = 160°$ at the centre of the circle and $\angle BAD$ at a point A on the circumference.
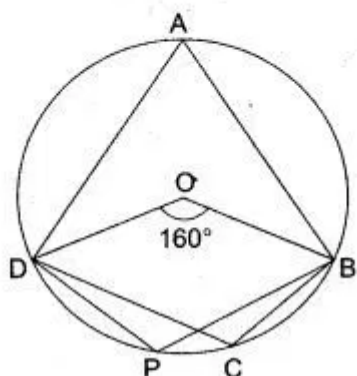
$$\therefore \qquad \angle BAD = \frac{1}{2} \angle BOD = 80°$$

Now, ABPD is a cyclic quadrilateral

$\therefore \ \angle BAD + \angle BPD = 180°$
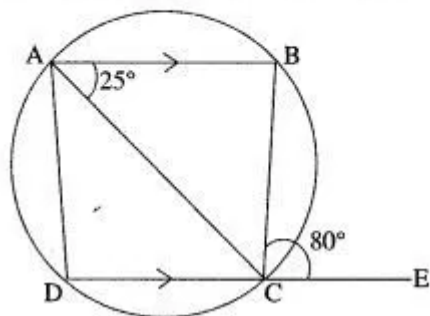
$\Rightarrow \qquad 80° + \angle BPD = 180°$

$\Rightarrow \qquad \qquad \angle BPD = 100°$

$\Rightarrow \qquad \qquad \angle BCD = 100°$

$\qquad [\because \angle BPD$ and $\angle BCD$ are angles in the same segment $\therefore \angle BCD = \angle BPD] = 100°$ Ans.

Question 31. In the given below figure, AB is parallel to DC, $\angle BCE = 80°$ and $\angle BAC = 25°$, find

(i) $\angle CAD$, (ii) $\angle CBD$, (iii) $\angle ADC$

Solution : (i) $\angle CAD = \angle BCE - \angle CAB$

$$= 80° - 25°$$

$$= 55°$$

$\because$ Ext. of cyclic is equal to opp. int. Ans.

(ii) $\qquad \angle CBD = \angle CAD$

$\qquad \qquad$ [Angles in the same segment]

$$= 55° \qquad \text{Ans.}$$

(iii) $\qquad \angle ADC = 180° - \angle DAB$

$$= 180° - 80°$$

$$= 100° \qquad \text{Ans.}$$

**Question 32.** In the given figure O is the centre of the circle, ∠ BAD = 75° and chord BC = chord CD. Find:

(i) ∠ BOC (ii) ∠ OBD (iii) ∠ BCD.
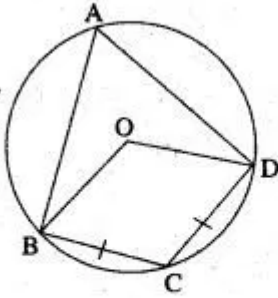
Solution :

(i)   ∠ BOD = 2 · ∠ BAD

           = 2 × 75° = 150°

     ∠ BOC = ∠ COD

∵   BC = CD

∴   ∠ BOD = 2 ∠ BOC

∴ $\angle BOC = \dfrac{1}{2} \angle BOD = 75°$ Ans.

(ii) $\angle OBD = \dfrac{1}{2}(180° - \angle BOD)$

            $= \dfrac{1}{2}(180° - 150°) = 15°$ Ans.

(iii)   ∠ BCD = 180° − ∠ BAD

            (opp. ∠s of a cyclic quardilateral

                         is supplementary)

          = 180° − 75°

          = 105°   Ans.

Question 33. In Fig. ΔPQR is an isosceles triangle with PQ = PR and m∠PQR = 35°. Find m∠ QSR and m∠QTR.
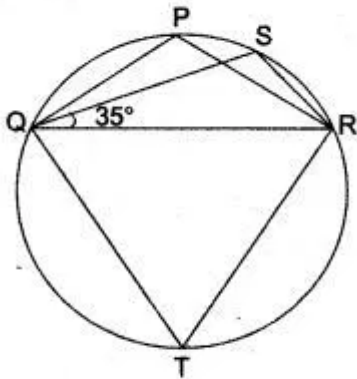
Solution : In ΔPQR, we have

         PQ = PR

⇒   ∠ PQR = ∠ PRQ

⇒   ∠ PRQ = 35°

∴   ∠ QPR = 180° − (∠ PQR + ∠ PRQ)

            = 180° − (35° + 35°) = 110°.

Since, PQTR is a cyclic quadrilateral.

∴   ∠ P + ∠ T = 180°

⇒   ∠ T = 180° − 110° = 70°   Ans.

In cyclic quadrilateral QSRT, we have

       ∠ S + ∠ T = 180°

⇒   ∠ S = 180° − 70° = 110°.   Ans.

Question 34. In Fig., chord ED is parallel to the diameter AC of the circle. Given ∠ CBE = 65°, calculate ∠ DEC.
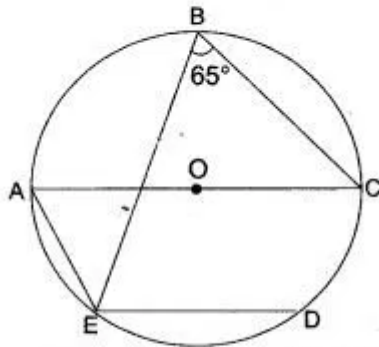
Solution : Consider the arc CDE. We find that ∠CBE and ∠CAE are the angles in the same segment of arc CDE.

∴   ∠ CAE = ∠ CBE

⇒   ∠ CAE = 65°   [∵ ∠ CBE = 65°]

Since, AC is the diameter of the circle and the angle in a semi-circle is a right angle. Therefore, ∠ AEC = 90°.

Now, in Δ ACE, we have

∠ ACE + ∠ AEC + ∠ CAE = 180°

⇒   ∠ ACE + 90° + 65° = 180°

⇒   ∠ ACE = 25°

But ∠DEC and ∠ ACE are alternate angles, because AC I I DE.

∴   ∠ DEC = ∠ ACE = 25°.   Ans.

Question 35. In the figure, ∠ DBC = 58°, BD is diameter of the circle. Calculate :

(i)   ∠ BDC   (ii)   ∠ BEC

(iii)   ∠ BAC
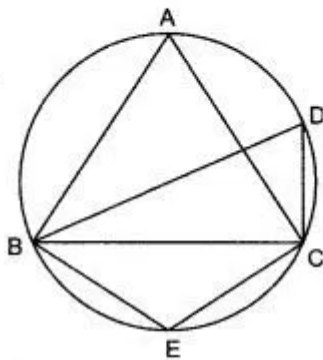
Solution :

∠ DBC = 58°   (given)

Now, BD is the diameter

∠ BCD = 90°

                   (angle in a semicircle)

In Δ BDC

∠ BDC + 90° + 58° = 180°

                (Sum of the angles of a triangle)

∴   ∠ BDC = 180 – (90 + 58) = 32°

(ii) BECD is a cyclic quadrilateral
∴ ∠BEC + ∠BDC = 180°

(Opp. angles of a cyclic quadrilateral)

∴ ∠BEC = 180 – ∠BDC

= 180° – 32° = 148°   Ans.

(iii) ∠BAC = ∠BDC = 32°

(angles in the same segment of a circle)

Ans.

Question 36. In the given circle with centre O, ∠ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C. Find ∠ADC and ∠DCT.
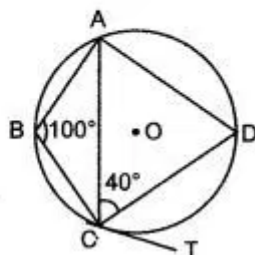
Solution : Given

$$\left.\begin{array}{l} ∠ABC = 100° \\ ∠ACD = 40° \end{array}\right\} \text{Given}$$

∠ABC + ∠ADC = 180°

{opposite angles of a cyclic quadrilateral}

100 + ∠ADC = 180°

∴ ∠ADC = 180 – 100 = 80°

Also

∠ACD + ∠ADC + ∠CAD = 180°

{sum of angles of a Δ}

40° + 80° + ∠CAD = 180°

∠CAD = 180 – 120 = 60°

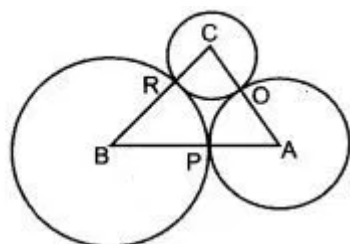Now, ∠DCT = ∠CAD

= 60°

{Alternate segment theorem}

Ans.

Question 37. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.
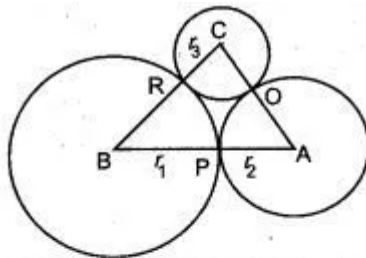

Solution :

AB = 10 cm

BC = 8 cm

AC = 6 cm

Let the radii of three circle be r_1, r_2 & r_3 (shown in fig.)

$$r_1 + r_2 = 10 = AB \quad ...(1)$$
$$r_2 + r_3 = 6 = AC \quad ...(2)$$
$$r_3 + r_1 = 8 = BC \quad ...(3)$$

Adding (1), (2) and (3), we get

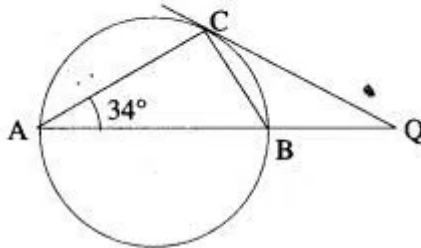$$2(r_1 + r_2 + r_3) = 10 + 6 + 8 = 24$$
$$r_1 + r_2 + r_3 = 12 \quad ...(4)$$

Subtract (4) and (1) $\Rightarrow r_3 = 12 - 10 = 2$ cm

Subtract (4) and (2) $\Rightarrow r_1 = 12 - 6 = 6$ cm

Subtract (4) and (3) $\Rightarrow r_2 = 12 - 8 = 4$ cm

Question 38. In the given figure, AB is a diameter. The tangent at C meets AB produced at Q. If $\angle CAB = 34°$, find :

(i) $\angle CBA$, (ii) $\angle CQA$.

(i) AB is diameter.

$\therefore$ $\qquad \angle ACB = 90°$

Angle in semicircle is rt. angle

$\therefore$ In $\triangle ACB$,

$$\angle A + \angle C + \angle B = 180$$
$$34 + 90 + \angle B = 180$$
$$\angle B = 180 - (90 + 34)$$
$$= 180 - 124$$
$\therefore$ $\qquad \angle CBA = 56°$

(ii) Now CQ is tangent

$\therefore$ $\quad \angle QCB = \angle CAB$

$\qquad$ (Alternate segment angle)

$$= 34°$$

and $\angle CBQ = 180 - \angle CBA$

$$= 180 - 56 = 124°$$

$\therefore$ $\quad \angle CQA = 180 - (\angle QCB + \angle CBQ)$
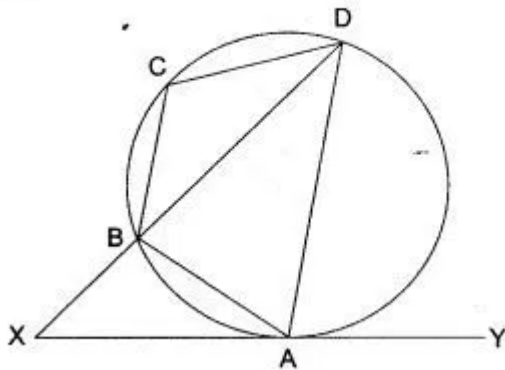
$$= 180 - (34 + 124)$$
$$= 180 - 158 = 22° \qquad \text{Ans.}$$

Question 39. In the joining figure shown XAY is a tangent. If $\angle BDA = 44°$, $\angle BXA = 36°$, Calculate :

(i) $\angle BAX$ (ii) $\angle DAY$ (iii) $\angle DAB$
(iv) $\angle BCD$.

Solution : (i) $\angle BAX = \angle BDA = 44°$
[Angles in the alternate segment]   Ans.

(ii) $\angle ABD = \angle BXA + \angle BAX$
            $= 36° + 44° = 80°$

[Ext. angle of a Δ = sum of int. opp. angles]

$\therefore$ $\angle DAY = \angle ABD = 80°$

[Angles in alternate segment] Ans.

(iii) $\angle DAB = 180 - [\angle BAX + \angle DAY]$
 $= 180 - (44° + 80°) = 56°$

                     Ans.

(iv) $\angle BCD = 180° - \angle DAB$
            $= 180° - 56° = 124°$ Ans.

[Opp. $\angle s$ of a cyclic quad. are supplementary]

Question 40. P and Q are the centre of circles of radius 9 cm and 2 cm respectively; PQ = 17 cm. R is the centre of circle of radius x cm, which touches the above circles externally, given that $\angle PRQ = 90°$. Write an equation in x and solve it.

Solution : Let the circle with centre R touch the given two circles at A and B. Then, P, A, R are collinear and Q, B, R are collinear.
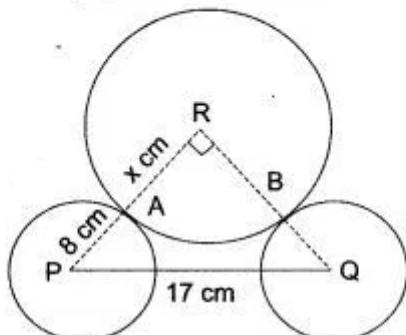
Since, $\angle PRQ = 90°$, by Pythagoras theorem,
          $PQ^2 = PR^2 + QR^2$
$\Rightarrow$ $17^2 = (9 + x)^2 + (2 + x)^2$
$\Rightarrow$ $x^2 + 11x - 102 = 0$
$\Rightarrow$ $(x + 17)(x - 6) = 0$
$\Rightarrow$ $x = 6$ cm

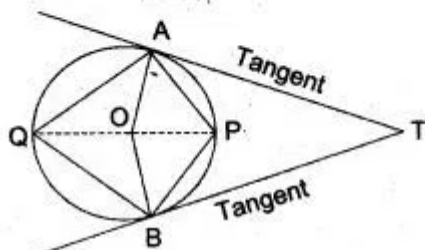 $(x = -17$ is not possible).

                      Ans.

Question 41. In the adjoining diagram TA and TB are tangents, O is the centre. If $\angle PAT = 35°$ and $\angle PBT = 40°$. Calculate :

(i) $\angle AQP$ (ii) $\angle BQP$

(iii) $\angle AQB$ (iv) $\angle APB$

(v) $\angle AOB$ (vi) $\angle ATB$.

Solution : (i) $\angle AQP = \angle PAT = 35°$

           [Angles are in alternate segment] Ans.

(ii) $\angle BQP = \angle PBT = 40°$

           [Angles are in alternate segment] Ans.

(iii) $\angle AQB = \angle AQP + \angle BQP$

                $= 35° + 40° = 75°$.   Ans.

(iv) $\angle APB + \angle AQB = 180°$

                [Opp. $\angle$s of a cyclic

          quadrilateral are supplementary]

$\therefore$ $\angle APB + 75° = 180°$

$\therefore$ $\angle APB = 105°$.   Ans.

(v) $\cdot$ $\angle AOB = 2\angle AQB$

            $= 2(75°) = 150°$.   Ans.

      [Angle at the centre = 2 Angle at the
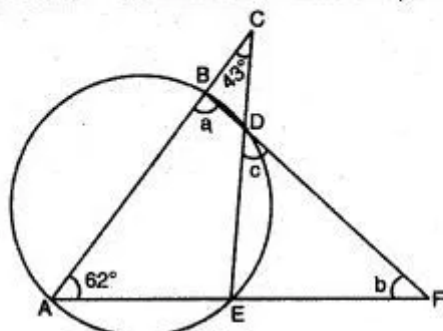
                        circumference]

(vi) In quadrilateral AOBT :

$\angle ATB = 360° - (\angle OAT + \angle OBT$

                       $+ \angle AOB)$

    $= 360° - (90° + 150° + 90°) = 30°$

[$\angle OAT = \angle OBT = 90°$ rad, $\perp$ tangent] Ans.

Question 42. In the given figure, if $\angle ACE = 43°$ and $\angle CAF = 62°$ find the value of a, b and c.

Solution : ABDE is cyclic quadrilateral.

$\therefore$ $\angle ABD + \angle AED = 180°$

and $\angle EAB + \angle BDE = 180°$

Now in $\triangle ACE$

$\angle A + \angle C + \angle E = 180°$

$62° + 43° + \angle E = 180°$

              $\angle E = 180° - 105° = 75°$

so $\angle ABD + \angle AED = 180°$

$\therefore \quad a + 75° = 180°$

$\therefore \quad a = 105°$

$\angle EDF = \angle BAE$

(exterior angle of cyclic quadrilateral)

$62° = c$

$\therefore \quad c = 62°$

In $\triangle ABF$,

$\angle ABF + \angle BAF + \angle BFA$

$= 180°$

$105° + 62° + b = 180°$

$167° + b = 180°$

$b = 180° - 167°$

$b = 13°$

$\therefore \quad a = 105°, b = 13°$

and $\quad c = 62°.$ Ans.

Question 43. In the given figure O is the centre of the circle. Tangents at A and B meet at C.

If $\angle ACO = 30°$, find

(i) $\angle BCO$ (ii) $\angle AOB$

(iii) $\angle APB$
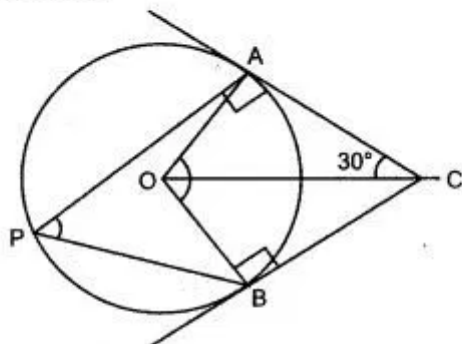
Solution :

$\angle BCO = \angle ACO = 30°$ Ans.

($\therefore$ C is the intensecting point of tangents AC and BC)

(ii) $\angle OAC = \angle OBC = 90°$

$\angle ACO = 30°$ Given

$\therefore \quad \angle AOC = \angle BOC$

$= 180° - (90° + 30°)$

($\therefore$ sum of the 3 angles of a $\triangle$ is 180°)

$\angle AOC = 180° - 120°$

$\angle AOC = 60°$

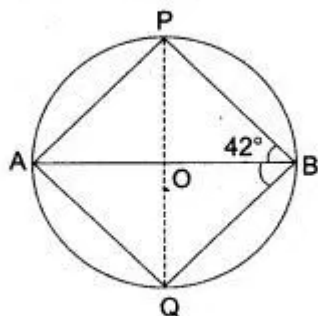$\therefore \quad \angle AOB = \angle AOC + \angle BOC$

$= 60° + 60°$

$\angle AOB = 120°$ Ans.

(iii) $\angle APB = \dfrac{1}{2} \angle AOB = \dfrac{120°}{2} = 60°$

Ans.

($\therefore$ Angle substended at the remaining part of the circle is half the $\angle$ substended at the centre}

Question 44. In the following figure, O is the centre of the circle, $\angle PBA = 42°$. Calculate :

(i) $\angle APB$ (ii) $\angle PQB$ (iii) $\angle AQB$.

Solution : (i) In circle $C(O, r)$

AB is the diameter

So $\angle APB = 90°$ (Angle in semi-circle)

(ii) Now in $\triangle APB$

$$\angle PAB = 180 - (\angle APB + \angle ABP)$$
$$= 180 - (90° + 42°)$$
$$= 180° - 132° = 48°$$
$$\angle PQB = \angle PAB = 48°$$

(Angles of the same segment)

Hence

$$\angle PQB = 48°.$$ Ans.

(iii) AQBP is a cyclic quadrilateral.

Therefore
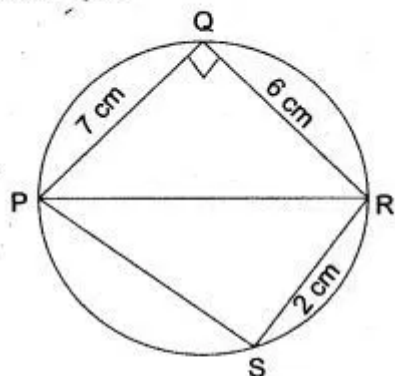
$$\angle APB + \angle AQB = 180°$$
$$\Rightarrow \quad 90° + \angle AQB = 180°$$
$$\Rightarrow \quad \angle AQB = 180° - 90° = 90°.$$ Ans.

Question 45. In the figure alongside PR is a diameter of the circle, PQ = 7 cm; QR = 6 cm and RS = 2 cm. Calculate the perimeter of the cyclic quadrilateral PQRS.

Solution : PR is the diameter of the circle

then $\angle PQR = 90°$,

(Angle in semi-circle)

In $\triangle PQR$, $PR = \sqrt{(7)^2 + (6)^2}$

$$= \sqrt{49 + 36} = \sqrt{85}$$

Similarly

$$\angle PSR = 90°$$

In $\triangle$PSR,   PS $= \sqrt{PR^2 - SR^2}$

$= \sqrt{85 - 4} = \sqrt{81}$

.   PS $= 9$ cm

Perimeter of cyclic quadrilateral

PQRS $=$ PQ + QR + RS + PS

$= 7$ cm $+ 6$ cm ± 2 cm $+ 9$ cm

$= 24$ cm.   **Ans.**

Question 46. In the adjoining figure, PQ is the diameter, chord SR is parallel to PQ. Given $\angle$PQR = 58°. Calculate :

(i) $\angle$RPQ

(ii) $\angle$STP (T is a point on the minor arc).
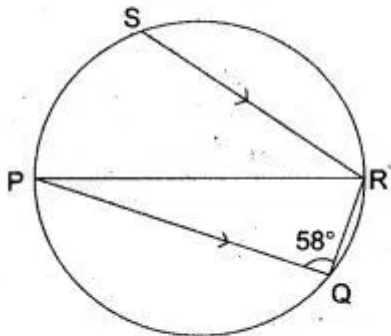
Solution : $\angle$PRQ $= 90°$

($\angle$ in a semi-circle)

In $\triangle$ PQR,

$\angle$RPQ + $\angle$PQR + $\angle$PRQ $= 180°$

(∵ The sum of the three $\angle$s of a $\triangle$ is 180°)

$\Rightarrow$ $\angle$RPQ + 58° + 90° $= 180°$

$\Rightarrow$ $\angle$RPQ + 148° $= 180°$

$\Rightarrow$ $\angle$RPQ $= 180° - 148°$

$\Rightarrow$ $\angle$RPQ $= 32°$ **Ans.**

(ii) ∵   PQ ‖ SR

and RP intersects them

$\angle$PRS $= \angle$RPQ

(Alternate angles)

∴ $\angle$PRS $= 32°$

∵ PTSR is a cyclic quadrilateral.

∴ $\angle$PTS + $\angle$PRS $= 180°$

(∵ Opposite $\angle$s of a cyclic quadrilateral are supplementary)

$\Rightarrow$ $\angle$PTS + 32° $= 180°$

$\Rightarrow$ $\angle$PTS $= 180° - 32° = 148°$

$\Rightarrow$ $\angle$STP $= 148°$.   **Ans.**

Question 47. C is a point on the minor arc AB of the circle, with centre O. Given $\angle$ ACB = p°, $\angle$ AOB = q°.

(i) Express q in terms of p.

(ii) Calculate p if ACBO is a parallelogram.

(iii) If ACBO is a parallelogram, then find the value of q + p.
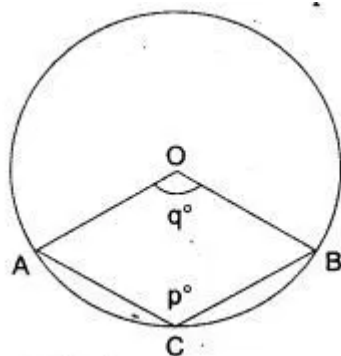
Solution : (i) Reflex

$$AOB = 360° - q°$$

$$ACB = \frac{1}{2} \text{reflex} \quad AOB$$

(angle at the centre property)

$$p° = \frac{1}{2}(360° - q°)$$
$$2p° = 360° - q°$$
$$q° = 360° - 2p°$$
$$q = 360° - 2p. \qquad \text{Ans.}$$

(ii) If ACBO is a parallelogram, then

$$p = q$$
$$q = 360° - 2p$$
$$p = 360° - 2p$$
$$p + 2p = 360°$$
$$3p = 360°$$
$$p = \frac{360°}{3} = 120°. \qquad \text{Ans.}$$
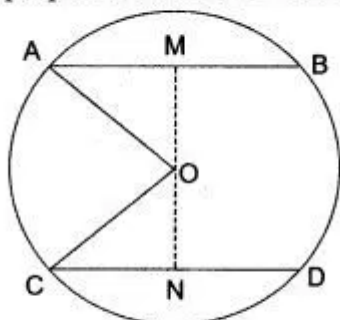
(iii) If ACBO is a parallelogram, then

$$p = q$$

Also

$$p = 120° \qquad \text{[From (ii)]}$$
$$p + q = p + p = 2p$$
$$= 2 \quad 120° = 240°. \qquad \text{Ans.}$$

Question 48. AB, CD are parallel chords of a circle 7 cm apart. If AB = 6 cm, CD = 8 cm, find the radius of the circle.

Solution : Let O be the centre of the circle OM and ON are perpendiculars on AB and CD.

MON is one straight line.

Here $AM = \frac{1}{2} AB = 3$ cm, $CN = \frac{1}{2} CD = 4$ cm

Let ON = x cm and radius OA = OC = r cm

From right angled triangle OCN,

$$ON^2 = OC^2 - CN^2$$
[By Pythagoras Theorem]
$$x^2 = r^2 - 16 \qquad \text{...(1)}$$
From right angled triangle OAM,
$$OM^2 = OA^2 - AM^2$$
[By Pythagoras Theorem]
$$(7 - x)^2 = r^2 - 9 \qquad \text{...(2)}$$
From (1) and (2),
$$(7 - x)^2 - x^2 = 7$$
$$49 + x^2 - 14x - x^2 = 7$$
$$14x = 42$$
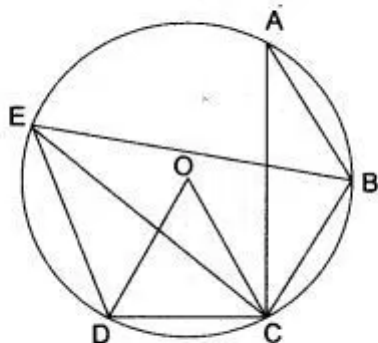$$x = 3$$
From (1), $\quad r^2 = x^2 + 16$
$$= 9 + 16 = 25$$
$$r = 5 \text{ cm}$$
Hence, the radius of the circle is 5 cm.   Ans.

Question 49. In the adjoining diagram, chords AB, BC and CD are equal. O is the centre of the circle. If ∠ABC = 120°, calculate :

(i)   ∠BAC   (ii)   ∠BEC
(iii) ∠BED   (iv)   ∠COD.

Solution : (i) In △ABC
$$\angle ABC + \angle BAC + \angle BCA = 180°$$
(∵ The sum of three ∠s of a △ is 180°)
$$120° + \angle BAC + \angle BCA = 180°$$
[∵ ∠ABC = 120° (given)]
$$\angle BAC + \angle BCA = 60°$$
But, $\qquad BA = BC$
$$\angle BAC + \angle BAC = 60°$$
$$2\angle BAC = 60°$$
$$\angle BAC = 30°. \qquad \text{Ans}$$
(ii) $\qquad \angle BEC = \angle BAC = 30°. \quad$ Ans.
(iii) $\qquad AB = BC = CD.$
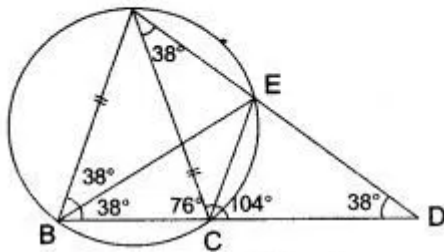$$\text{Arc } AB = \text{Arc } BC = \text{Arc } CD$$

Now, $\quad$ COB $= 2$ CAB

$\qquad = 2 \quad 30° = 60°$

$\qquad$ DOC $= \quad$ COB $= 60°$

$\qquad$ DEC $= \dfrac{1}{2} \quad$ DOC $= \dfrac{1}{2} \quad 60°$

$\qquad = 30°$

$\therefore \qquad \angle BED = \angle BEC + \angle DEC$

$\qquad \doteq \angle BAC + \angle DEC$

$\qquad = 30° + 30° = 60°. \qquad$ Ans.

(iv) $\qquad \angle COD = 60°. \qquad$ Ans.

Question 50. In the figure, AB = AC = CD, angle, ADC = 38°. Calculate :

(i) Angle ABC $\qquad$ (ii) Angle BEC.

Solution : $\because \quad$ AC $=$ CD

$\therefore \qquad \angle$ CAD $= \angle$ ADC $= 38°$

Now in $\triangle$ ACD,

$\angle$ ACD $+ \angle$ CAD $+ \angle$ ADC $= 180°$

$\Rightarrow \qquad \angle$ ACD $+ 38° + 38° = 180°$

$\Rightarrow \qquad \angle$ ACD $= 104°$

Now $\qquad \angle$ ACB $+ \angle$ ACD $= 180°$

$\Rightarrow \qquad \angle$ ACB $+ 104° = 180°$

$\Rightarrow \qquad \angle$ ACB $= 76°$

Again, $\because \qquad$ AB $=$ AC

$\therefore \qquad \angle$ ABC $= \angle$ ACB $= 76°. \qquad$ Ans.

(ii) In $\triangle$ABC

$\angle$ BAC $+ \angle$ ABC $+ \angle$ ACB $= 180°$

$\Rightarrow \qquad \angle$ BAC $+ 76° + 76° = 180°$

$\Rightarrow \qquad \angle$ BAC $= 28°$
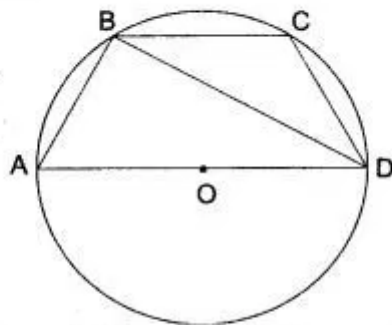
Now $\qquad \angle$ BEC $= \angle$ BAC $= 28°.$

[Angles subtended by the same chord] Ans.

Question 51. In the figure given alongside, AD is the diameter of the circle. If $\angle BCD = 130°$, calculate :

(i) $\angle$ DAB   (ii) $\angle$ ADB.

Solution : (i) Since ABCD is a cyclic quadrilateral.

∴ Its opposite angles are supplementary.

∴ $\angle DAB + \angle BCD = 180°$

$\Rightarrow$ $\angle DAB = 180° - \angle BCD$

            $= 180° - 130°$

            $= 50°.$ Ans.

(ii) Since, angle in the semi-circle is a right angle.

∴ In $\triangle$ ABD, $\angle ABD = 90°$

Since, the sum of the angle of a $\triangle$ is $180°$

∴ From $\triangle$ ABD,

    $\angle ABD + \angle ADB + \angle DAB = 180°$

    $90° + \angle ADB + 50° = 180°$

             $\angle ADB = 180° - (90° - 50°)$

             $\angle ADB = 180° - 140°$

             $\angle ADB = 40°$ Ans.

Question 52. In the figure AC is the diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles p, q, r in term of x.
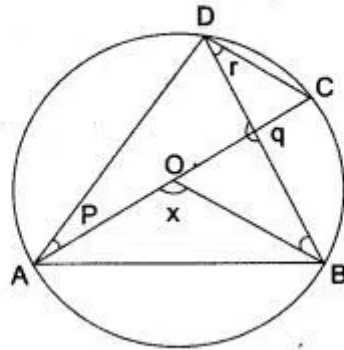
Solution. $\angle ADB = \dfrac{1}{2}\angle AOB = \dfrac{x}{2}$

$\angle ADB = 90° - r$

$\angle ADB = \angle ACB = q$
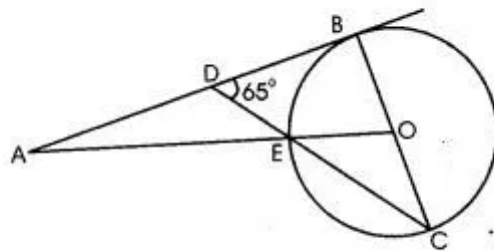
Combining these, we get

$$\dfrac{x}{2} = 90° - r = q$$

$\Rightarrow \qquad 2r = 180° - x$

and $\qquad x = 2q$

$\angle DAC = \angle CAB$

$\qquad\qquad = \angle BDC$

$\Rightarrow \qquad p = r = \dfrac{1}{2}(180° - x)$ Ans.

Question 53. In the following figure O is the centre of the circle and AB is a tangent to it at point B. $\angle BDC = 65°$. Find $\angle BAO$.

Solution. As AB is a tangent to the circle at B and OB is radius, $OB + AB \Rightarrow \angle CBD = 90°$

In $\triangle BCD$,

$\angle BCD + \angle CBD + \angle BDC = 180°$

$$\angle BCD + 90° + 65° = 180°$$
$$\angle BCD + 155° = 180°$$
$$\angle BCD = 180° - 155°$$
$$\angle BCD = 25°$$
$$\angle BOE = 2 \angle BCE$$

[angle at centre = double the

angle at the remaining part of circle]

$$\Rightarrow \qquad \angle BOE = 2 \times 25° = 50°$$
$$\angle BOA = 50°$$

In $\triangle$ BOA,
$$\angle BAO + \angle ABO + \angle BOA = 180°$$
$$\Rightarrow \qquad \angle BAO + 90° + 50° = 180°$$
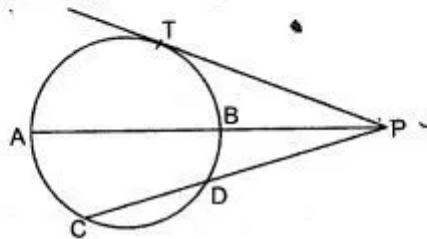$$\Rightarrow \qquad \angle BAO + 140° = 180°$$
$$\Rightarrow \qquad \angle BAO = 180° - 140°$$
$$\Rightarrow \qquad \angle BAO = 40° \qquad \text{Ans.}$$

Question 54. In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7·8 cm, PD = 5 cm, PB = 4 cm. Find :
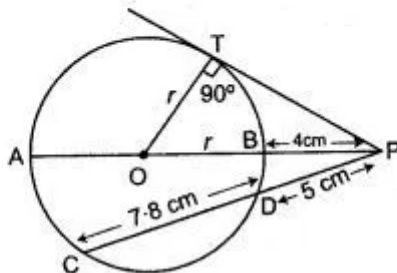
(i) AB.

(ii) The length of tangent PT.

Solution : Given that
$$CD = 7·8 \text{ cm, } PD = 5 \text{ cm,}$$
$$PB = 4 \text{ cm}$$

As we know,
$$PT^2 = PD \times PC$$
$$PT^2 = PD \times (PD + CD)$$
$$PT^2 = 5 \times 12·8$$
$$PT^2 = 64$$
$$\Rightarrow \qquad PT = 8 \text{ cm}$$

Now in $\triangle$ POT
$$PO^2 = OT^2 + PT^2$$
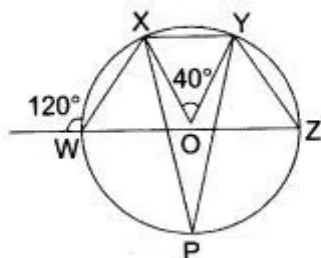$$(r + 4)^2 = r^2 + 64$$

$$r^2 + 16 + 8r = r^2 + 64$$
$$8r = 48$$
$$r = 6$$

(i)   Thus $AB = 2r = 12$ cm

(ii)   Length of tangent $PT = 8$ cm.

Question 55. In the figure alongside O is the centre of circle $\angle XOY = 40°$, $\angle TWX = 120°$ and XY is parallel to TZ.

Find :

(i) $\angle XZY$,   (ii) $\angle YXZ$,   (iii) $\angle TZY$.

Solution : Construction : Take a point P on the circumference of the circle. Join XP and YP.

Determination of Angles : (i) $\angle XOY = 2\angle XPY$

(Angle subtended by an arc of a circle at the centre is twice the angle subtended by that arc at any point on the circumference of the circle)

$$\Rightarrow \qquad 40° = 2\angle XPY$$
$$[\because \angle XOY = 40° \text{ (given)}]$$
$$\Rightarrow \qquad \angle XPY = \frac{40°}{2} = 20°$$
$$\angle XZY = 20° \quad [\because \angle XPY = \angle XZY]$$

Angles in a same segment of a circle are equal.

(ii) $\angle XWT + \angle XWZ = 180°$

(Linear Pair Axiom)

$$120° + \angle XWZ = 180°$$
$$\Rightarrow \qquad \angle XWZ = 180° - 120° = 60°$$
$$\Rightarrow \quad \angle XWZ + \angle XYZ = 180°$$

(Opposite angles of a cyclic quadrilateral are supplementary)

$$\Rightarrow \qquad 60° + \angle XYZ = 180°$$
$$\Rightarrow \qquad \angle XYZ = 180° - 60° = 120°$$
$$\therefore \Delta XYZ, \angle YXZ + \angle XYZ + \angle XZY = 180°$$

(The sum of the three angles of triangle is 180°)

$$\Rightarrow \angle YXZ + 120° + 20° = 180°$$
$$\Rightarrow \qquad \angle YXZ + 140° = 180°$$
$$\Rightarrow \qquad \angle YXZ = 180° - 140° = 40°. \text{ Ans.}$$
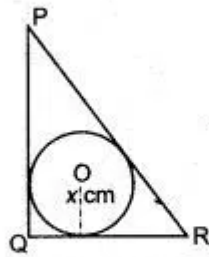
(iii) $\because$ XY || TZ and transversal YZ intersects then

$$\angle XYZ + \angle TZY = 180°$$

(Sum of the consecutive interior angles is 180°)

$$\Rightarrow \qquad 120° + \angle TZY = 180°$$
$$\Rightarrow \qquad \angle TZY = 180° - 120° = 60°.$$

Question 56. In $\triangle$ PQR, PQ = 24 cm, QR = 7 cm and $\angle$ PQR = 90°. Find the radius of the inscribed circle.
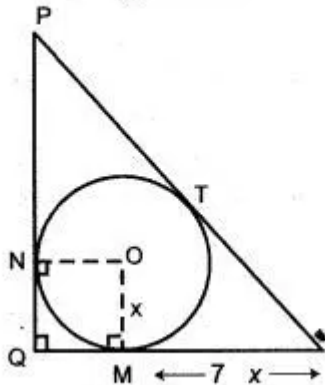

Solution : OM $\perp$ QR
ON $\perp$ PQ

$$\begin{bmatrix} \text{Tangents and radius} \\ \text{perpendicular to each other} \end{bmatrix}$$

OM = ON (r)
QM = QN

(Tangents from an external point)

$\Rightarrow$ QMON is a square.

$\Rightarrow$ QM = OM = ON = QN = x cm.

So, mR = $(7 - x)$ cm
PN = $(24 - x)$ cm.
PT = PN = 24 $- x$

and, mR = RT = 7 $- x$

[Tangents from an external point]

$\Rightarrow$ PR = PT + RT
= 24 $- x + 7 - x = 31 - 2x$

Now, In $\triangle$ PQR

PR² = PQ² + QR²
= 24² + 7²
= 576 + 49 = 625
PR = 25 cm

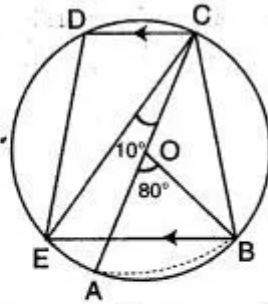$\Rightarrow$ 31 $- 2x$ = 25
$\Rightarrow$ $2x$ = 31 $-$ 25
$2x$ = 6
$\Rightarrow$ x = 3 cm. **Ans.**

Question 57. In the diagram given alongside, AC is the diameter of the circle, with centre O. CD and BE are parallel. Angle AOB = 80° and angle ACE = 10°. Calculate :

(i) ∠ BEC   (ii) ∠ BCD   (iii) ∠ CED.

Solution : From the figure, we have

$$\angle AOB = 80°$$

$$\angle ACE = 10°$$

(i) $$\angle BOC = 180° - \angle AOB$$

$$= 180° - 80°$$

$$= 100°$$

$$\angle BEC = \frac{1}{2} \angle BOC$$

[∵ ∠ subtended at the centre and ∠ subtend by E by arc BC]

$$= \frac{1}{2} \times 100°$$

∴ $$\angle BEC = 50°.$$ Ans.

(ii) $$\angle ACB = \frac{1}{2} \angle AOB$$

[∵ ∠s subtended by arc AB at the centre and at C]

$$= \frac{1}{2} \times 80°$$

$$= 40°$$

$$\angle ECD = \angle BEC$$

[∵ Alt. ∠s as CD || BE]

$$= 50°$$

$$\angle BCD = \angle ACB + \angle ECA + \angle ECD$$

$$= 40° + 10° + 50°$$

$$= 100°.$$ Ans.

(iii) BCDE is a cyclic quadrilateral,

[∵ Its opposite ∠s are supplementary]

⇒ $$\angle BED = \angle 180° - \angle BCD$$

$$= 180° - 100°$$ [From (ii)]

$$= 80°$$

$$\angle BEC + \angle CED = 80°$$

⇒ $$\angle CED = 80° - \angle BED$$

$$= 80° - 50°$$ [From (i)]

∴ $$\angle CED = 30°.$$ Ans.