Chapter 1. Electric Charges and Fields

Coulombs Law, Electrostatic Field and Electric Dipole

1 Mark Questions

1.Why do the electrostatic field lines not form closed loop?[All India 2014, Delhi 2012]

Ans. The electrostatic field lines do not form closed loop because no electric field lines exist inside the charged body

2.Why do the electric field lines never cross each other?[All India 2014]

Ans.At the intersection point, there would be two directions of electric field which is not possible so lines of forces never cross each other

3.Why must electrostatic field at the surface of a charge every point? Give reason. [Foreign 2014, Delhi 2012]

Ans.As, electric field inside a conductor is always zero. The electric lines of forces exert lateral pressure on each other leads to explain repulsion between like charges. Thus, in order to stable spacing, the lines are normal to the surface.

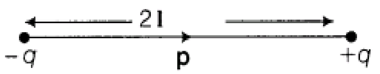
4.Two point charges g_x and q_2 are placed at a distance d apart as shown in the figure. The electric field intensity is zero at the point P on the line joining them as shown. Write two conclusions that you can draw from this. [Delhi2014c]

Ans.(i) The two point charges (q^ and q_2) should be of opposite nature.

(ii) Magnitude of charge q_1 must be greater than magnitude of charge q_2.

5.Define dipole moment of an electric dipole. Is it a scalar quantity or a vector quantity? [Foreign 2012; All India 2011]

Ans.Electric dipole moment of an electric dipole is equal to the product of its charges and the length of the electric dipole.It is denoted by p. Its unit is coulomb-metre.

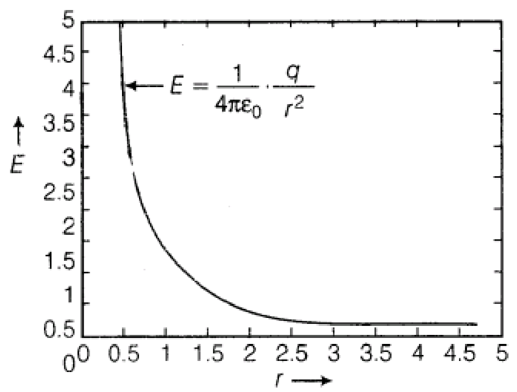


$$\mathbf{p} = q \times 2\mathbf{l}$$

It is a vector quantity and its direction is from negative charge towards positive charge

6.Draw a plot showing the variation of electric field (E) with distance r due to a point charge Q. [Delhi 2012]

Ans. The plot showing the variation of electric field and electric potential with distance r due to a point charge q is shown as below



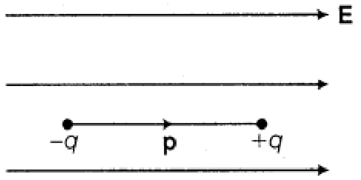
7.A proton is placed in a uniform electric field directed along the position X-axis. In which direction will it tend to move? [Delhi 2011 c]

Ans. Proton will tend to move along the X-axis in the direction of a uniform electric field.

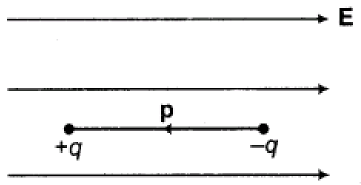
8. In which orientation, a dipole placed in a uniform electric field is in (i) stable (ii) unstable equilibrium?[Delhi 2011; All India 2008]

Ans.

(*i*) For stable equilibrium, the angle between **p** and **E** is 0°. **(1/2)**



(*ii*) For unstable equilibrium, the angle between **p** and **E** is 180°.



(1/2)

9.Two point charges having equal charges separated by lm distance experience a force of 8 N. What will be the force experienced by them if they are held in water at the same distance? (Given, K_{water} = 80).[Ail India 2010 C]

Ans.Two point charges system is taken from air to water keeping other variable (e.g., distance, magnitude of charge) unchanged. So, only factor which may affect the interacting force is dielectric constant of medium

Force acting between two point charges

$$F = \frac{1}{4\pi \varepsilon_0 K} \frac{q_1 q_2}{r^2} \quad \text{or} \quad F \propto \frac{1}{K}$$

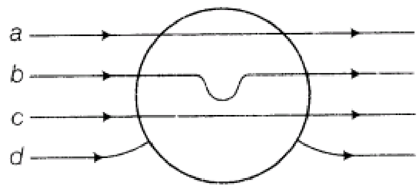
$$\Rightarrow \quad \frac{F_{air}}{F_{medium}} = K$$

$$\Rightarrow \quad \frac{8}{F_{water}} = 80$$

$$\Rightarrow \quad F_{water} = \frac{8}{80}$$

$$F_{water} = \frac{1}{10} \text{ N}$$

10.A metallic shpere is placed in a uniform electric field as shown in the figure. Which path is followed by electric field lines and why?[HOTS; Foreign 2010]



Ans. Path d is followed by electric field lines. Electric field intensity inside the metallic sphere will be zero, therefore, no electric lines of force exist inside the sphere, also lines fall normally on the surface. Electric field lines are always perpendicular to the surface of the conductor.

11.Point out right or wrong for the following statement.The mutual forces between two charges do not get affected by the presence of other charges.
Ans.Right, because mutual force acting between two point charges is proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them, i.e. independent of the other charges.

12.A dipole of dipole moment p is present in a uniform electric field E . Write the value of the angle between p and E for which the torque experienced by the dipole, is minimum. [Delhi 2009 c]
Ans. Since, torque (x) on the dipole in electric field E is

$$\tau = \mathbf{p} \times \mathbf{E}$$

$$\Rightarrow \quad |\tau| = pE\sin\theta$$

For minimum torque,

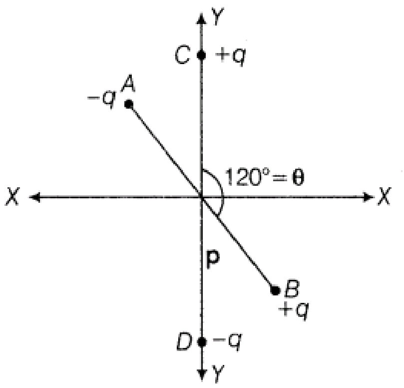
$$|\tau| = 0$$

$$\Rightarrow \quad pE\sin\theta = 0$$

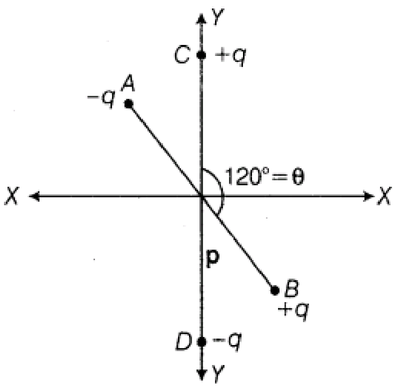
$$\sin\theta = 0$$

$$\Rightarrow \quad \theta = 0°, 180°$$

13.Two small identical dipoles AB and CD, each of dipole moment p are kept at an angle of 120° as

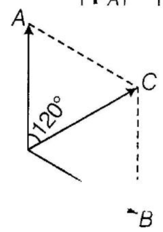


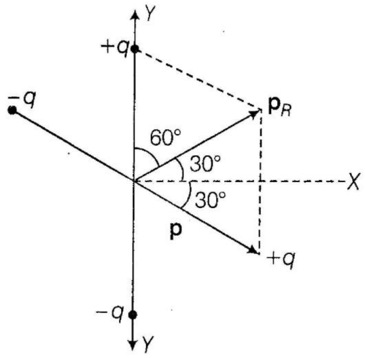
shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field (E) directed along + X-direction, what will be the magnitude and direction of the torque acting on this? [Delhi 2008]



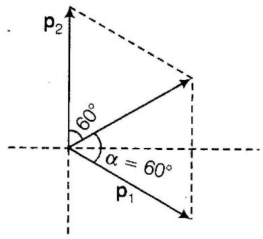
Ans.

$$|\mathbf{p}_A| = |\mathbf{p}_C| = p$$





The resultant p_R is a magnitude.



$$p_R = \sqrt{p_1^{\,2} + p_2^{\,2} + 2p_1 p_2 \cos\theta}$$

$$= \sqrt{p^2 + p^2 + 2p^2\cos\theta} = \sqrt{2p^2(1+\cos\theta)}$$

$$[\because p_1 = p_2 = p]$$

$$= \sqrt{2p^2 \times 2\cos^2\frac{\theta}{2}} = 2p\cos\frac{\theta}{2}$$

$$\tan\alpha = \frac{p_2 \sin\theta}{p_1 + p_2\cos\theta} = \frac{p\sin 120°}{p + p\cos 120°}$$

$$= \frac{p\sqrt{3}/2}{p - \frac{p}{2}} = \sqrt{3}$$

$$|\mathbf{p}_R| = 2p\cos\frac{\theta}{2} = 2p\cos\frac{120°}{2} = p$$

$\mathbf{p}_R$ will subtend an angle of 30° with X-axis.
Now, torque acting on the system

$$\tau = \mathbf{p}_R \times \mathbf{E} = p_R E \sin\theta = \frac{1}{2}pE$$

Torque will work to align the dipole in the direction of electric field **E**. **(1/2)**

2 Marks Questions

14.An electric dipole of length 4 cm when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm . Calculate the potential energy of the dipole if it has charge ± 8 nC.[Delhi 2014]

Ans.

Given, length $2a = 4\text{ cm} = 4 \times 10^{-2}$ m

Angle, $\theta = 60°$

Torque, $\tau = 4\sqrt{3}$ Nm

We know that, $\tau = Q(2a)E\sin\theta$

Electric field, $E = \dfrac{\tau}{Q(2a)\sin\theta}$

$$= \frac{4\sqrt{3}}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60°} \text{ N/C} \quad \textbf{(1)}$$

$$= 2.5 \times 10^{10} \text{ N/C}$$

$\therefore$ Potential energy, $C = -pE\cos\theta$

$= -Q(2a)E\cos\theta$

$$U = -8 \times 10^{-9} \times 4 \times 10^{-2} \times \frac{4\sqrt{3} \times \cos 60°}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60°}$$

$$= \frac{-4\sqrt{3}}{\sqrt{3}} \text{ J} = -4 \text{ J} \quad \textbf{(1)}$$

15.An electric dipole of length 2 cm when placed with its axis making an angle of 60° with a uniform of electric field,experiences a torque of $8\sqrt{3}$ Nm. Calculate the potential energy of the dipole if it has charge of ± 4 nC.[Delhi 2014]

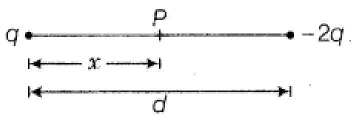
Ans.Refer to ans. 14 (Ans. 8J).

16.An electric dipole of length 1cm which placed with its axis making an angle of 60° with a uniform electric field,experiences a torque of $6\sqrt{3}$ Nm. Calculate the potential energy of the dipole if it has charge ± 2 nC. [Delhi 2014]

Ans.Refer to ans. 14 (Ans. -6J).

17.Two point charges q and – 2 q are kept d distance apart, find the location of the point relative to charge to q at which potential due to this system is zero.[All India 2014]

Ans.



(1)

Let P be the required point at distance x from charge q

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1(-2q)}{4\pi\varepsilon_0(d-x)} = 0$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{d-x}$$

$$x = \frac{d}{3} \quad \textbf{(1)}$$

So, required point is at a distance $d/3$ from charge q.

18.An electric dipole is placed in a uniform electric field E with its dipole moment p parallel to the field. Find

(i)the work done in turning the dipole till its dipole moment points in the direction opposite to E.

(ii)the orientation of the dipole for which the torque acting on it becomes maximum.[All India 2014 C]

Ans.

(*i*) We have $W = \int_{\theta_1}^{\theta_2} \tau d\theta$

$$\therefore \quad W = \int_0^{\pi} pE \sin\theta \, d\theta$$

$$= pE[-\cos\theta]_0^{\pi}$$

$$= 2pE$$

(*ii*) We know that,

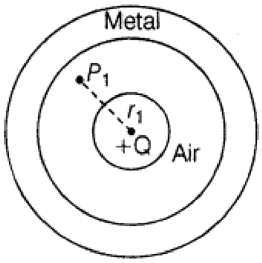
$$\tau = pE \sin\theta$$

If $\theta = \frac{\pi}{2}$, then τ is maximum

i.e. $\tau = pE \sin\frac{\pi}{2}$

$$\tau = pE \quad \text{(maximum)}$$

19.A small metal sphere carrying a charge +0 is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_x. [Delhi 2014 c]



Ans.

According to question, the charge on inner surface = – Q **(1)**

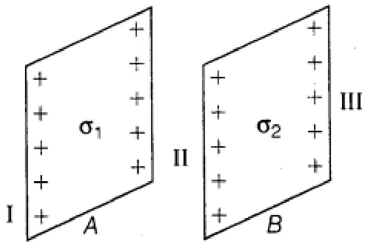
According to question, the charge on outer surface = + Q

Electric field at point P_1 is given by **(1)**

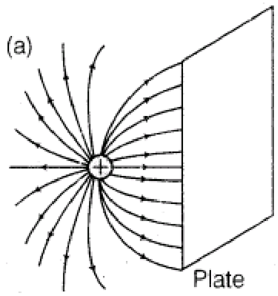
$$E = \frac{Q}{4\pi\varepsilon_0 r_1^2}$$

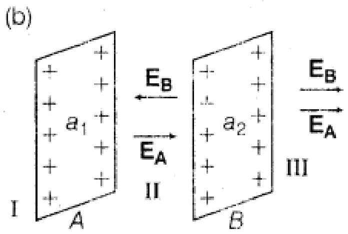
20.(i) Point charge (+Q) is kept in the vicinity of and uncharged conducting plate. Sketch electric field lines between the charge and the plate.

(ii)Two infinitely large plane thin parallel sheets having surface charge densities sigma1 and sigma2 (sigma1>sigma2) are shown in the figures. Write the magnitude and directions of net fields on the marked II and III. [Foreign 2014]



Ans.





In region II

The electric field due to the sheet of charge A will be from left of right (along the positive direction) and that due to the sheet of charge B will be from right to left (along the negative direction). Therefore, in region II we have

$$E = \frac{\sigma_1}{\varepsilon_0} + \left(-\frac{\sigma_2}{\varepsilon_0}\right)$$

$$\Rightarrow \quad E = \frac{1}{\varepsilon_0}(\sigma_1 - \sigma_2)$$

In region II

The electric fields due to both the charged sheets will be from left to right, i.e, along the positive direction. Therefore, in region III we have

$$E = \frac{\sigma_1}{\varepsilon_0} + \frac{\sigma_2}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\sigma_1 + \sigma_2)$$

21.Calculate the amount of work done in turning an electric dipole of dipole moment

3×10^{-8} C-m

from its position of unstable equilibrium to the position of stable equilibrium in a uniform electric field of intensity 10^3 N/C. [Foreign 2011]

Ans.

According to question,

For unstable equilibrium, the angle between **p** and **E** is $\theta_1 = 180°$

Finally, for stable equilibrium, $\theta_2 = 0°$ **(1/2)**

Required work done

$$W = pE(\cos\theta_1 - \cos\theta_2) \quad \textbf{(1/2)}$$

$$= 3 \times 10^{-8} \times 10^3(\cos 180° - \cos 0°)$$

$$[\because \cos 180° = -1, \cos 0° = +1]$$

$$W = -6 \times 10^{-5} \text{ J} \quad \textbf{(1)}$$

22. Plot a graph showing the variation of Coulomb force (F) *versus* $1/r^2$, where r is the distance between the two charges of each pair of charges ($1\ \mu C$, $2\ \mu C$) and ($1\ \mu C$, $-3\ \mu C$). Interpret the graphs obtained.

[All India 2011C]

Ans.

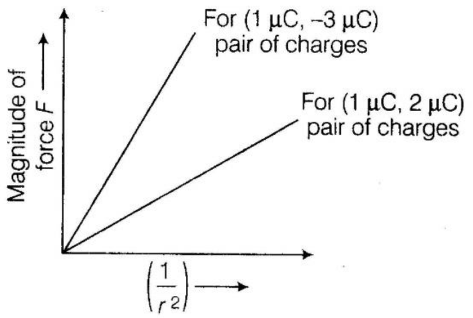
According to Coulomb's law, the magnitude of force acting between two stationary point charges is given by $F = \left(\frac{q_1 q_2}{4\pi\varepsilon_0}\right)\left(\frac{1}{r^2}\right)$

For given $q_1 q_2$, $\quad F \propto \left(\frac{1}{r^2}\right)$

The slope of F *versus* $\frac{1}{r^2}$, graph depends on $q_1 q_2$.

Magnitude of $q_1 q_2$ is higher for second pair.

$\therefore$ Slope of F *versus* $\frac{1}{r^2}$ graph. **(1)**



Corresponding to second pair ($1\ \mu C$, $-3\ \mu C$) is greater.

Higher the magnitude of product of charges q_1 and q_2, higher the slope. **(1)**

23.Two identical metallic spherical shells A and B having charges + 40 and -10Q are kept a certain distance apart. A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B, then spheres A and B are brought in contact and then separated. Find the charge on the spheres A and B.[All India 2011 c]

Ans.When two identical conducting charged spheres are brought in contact, then redistribution of charge takes place, i.e. the charge is equally divided on both the spheres. When C and A are placed in contact, charge of A equally divides in two spheres. Therefore, charge on each A and C = + 2Q

Now, C is placed in contact with B, then charge on each A and C becomes

$$\frac{2Q + (-10\ Q)}{2} = -4Q$$

When A and B are placed in contact, then charge on each A and B becomes

$$\frac{2Q + (-4Q)}{2} = -Q$$

(1)

24.A dipole with a dipole moment of magnitude p is in stable equilibrium in an electrostatic field of magnitude E. Find the work done in rotating this dipole to its position of unstable equilibrium. [All India 2010c]

Ans.

For stable equilibrium, the angle between **p** and **E** $\theta_1 = 0°$.

For unstable equilibrium, $\theta_2 = 180°$. **(1)**

Work done in rotating the dipole from angle θ_1 to θ_2

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$= pE(\cos 0° - \cos 180°)$$
$$W = 2pE$$ **(1)**

25.A dipole is present in an electrostatic field of magnitude 10 ^{6}N/C. If the work done in rotating it from its position of stable equilibrium to its position of unstable equilibrium is 2×10~23J, then find the magnitude of the dipole moment of this dipole.[All India 2010 C]

Ans.

Electric field intensity, $E = 10^6$ N/C

Work done, $W = 2 \times 10^{-23}$ J

Work done in rotating the dipole from stable equilibrium position to unstable equilibrium position.

$$W = pE(\cos 0° - \cos 180°) = 2pE$$ **(1)**

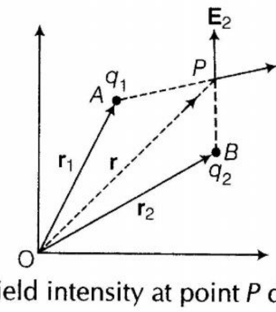
$$\therefore \quad p = \frac{W}{2E} = \frac{2 \times 10^{-23}}{2 \times 10^6} = 10^{-29} \text{ C-m}$$ **(1)**

26.Deduce the expression for the electric field E due to a system of two charges q_x and q_2 with position vectors i1 and r_2 at a point r with respect to common origin. [Delhi 2010c]

Ans.

Let two point charges q_1 and q_2 situated at points A and B have position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

$\because \quad \mathbf{AP} = \mathbf{r} - \mathbf{r}_1$ and $\mathbf{BP} = \mathbf{r} - \mathbf{r}_2$



Electric field intensity at point P due to q_1,

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{|\mathbf{BP}|^3} \mathbf{AP}$$

Similarly, $$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{|\mathbf{AP}|^3} \mathbf{BP}$$ **(1)**

$\therefore$ Net electric field intensity at point P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\mathbf{r} - \mathbf{r}_1|^3}(\mathbf{r} - \mathbf{r}_1) + \frac{q_2}{|\mathbf{r} - \mathbf{r}_2|^3}(\mathbf{r} - \mathbf{r}_2)\right]$$ **(1)**

27.The sum of two point charges is 7 microC. They repel each other with a force of 1 N when

kept 30 cm apart in free space. Calculate the value of each charge. [Foreign 2009]
Ans.

Let one of two charges is x μC. Therefore, other charge will be $(7-x)$ μC.

By Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ **(1)**

$$1 = 9 \times 10^9 \times \frac{(x \times 10^{-6})(7-x) \times 10^{-6}}{(0.3)^2}$$

$$9 \times 10^{-2} = 9 \times 10^{9-12} x(7-x)$$

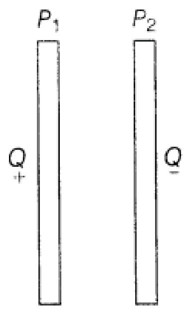
$$10 = x(7-x)$$

$$\therefore x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0$$

$$x = 2\ \mu C \text{ or } 5\ \mu C$$ **(1)**

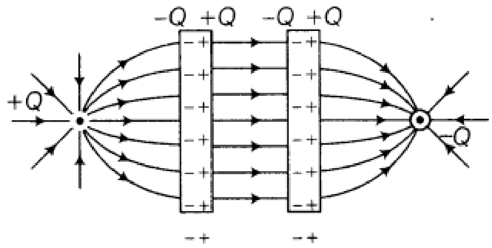
Therefore, charges are 2 μC and 5 μC.

28.Figure shows two large metal plates and P_2 tightly held against each other and placed between two equal and unlike point charges perpendicular to the line joining them.
(i)What will happen to the plates when they are released?
(ii)Draw the pattern of the electric field lines for the system.[HOTS; Foreign 2009]



Ans.(i) By electrostatic induction, charge induces on the plates and opposite nature of charge appears on the surface facing each other. Therefore, they start attracting towards each other.

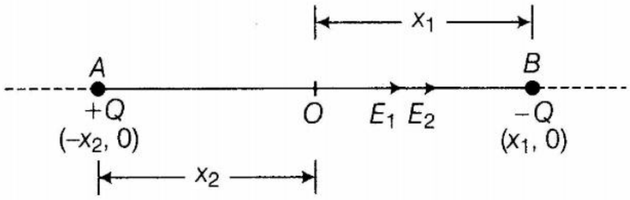
(*ii*) **(1)**



Field lines must be perpendicular to the plates. Also, equispaced field lines exist between two plates as electric field between, them is uniform.

29.Two charges + Q and -Q are kept at points ($-x_2$,0) and (x_t, 0) respectively, in the XY-plane. Find the magnitude and direction of the net electric field at the origin (0,0). [All India 2009 C]
Ans.

To find the electric field intensity at a point due to two charges, first of all find the individual electric fields due to both charges and then find the resultant field using vector addition.



Electric field intensity at O due to +Q charge,

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} \quad \text{(towards } B\text{)} \quad \ldots\text{(i)}$$

Electric field intensity at O due to −Q charge,

$$E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \quad \text{(towards } B\text{)} \quad \ldots\text{(ii)}$$

(1)

$\because$ E_1 and E_2 act along the same direction.

$\therefore$ Net electric field intensity at O,

$$E = E_1 + E_2 \quad \text{(towards } B\text{)}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2}$$

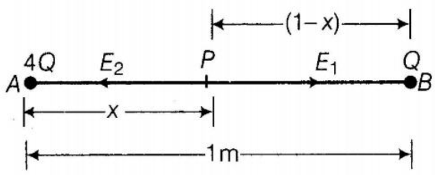
$$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]$$

(1)

30.Two point charges 4Q and Q are separated by 1 m in air. At what point on the line joining of charges, is the electric field intensity zero? [All India 2008]

Ans.

Let electric field intensity at any point P which lies at a distance x metre from 4Q be zero.



$\therefore$ Electric field intensity (E_1) due to 4Q at P

= Electric field intensity (E_2) due to +Q at P

As directions of $\mathbf{E}_1$ and $\mathbf{E}_2$ are in opposite directions.

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \times \frac{4Q}{x^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(1-x)^2}$$

(1)

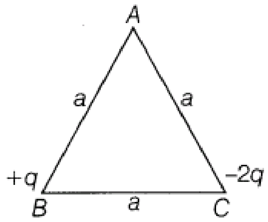
$$\text{or } \frac{4}{x^2} = \frac{1}{(1-x)^2} \Rightarrow \left(\frac{1-x}{x}\right)^2 = \frac{1}{4} \Rightarrow \frac{1-x}{x} = \frac{1}{2}$$

$$\frac{1}{x} - 1 = \frac{1}{2} \quad \Rightarrow \quad \frac{1}{x} = \frac{3}{2} \quad \therefore \quad x = \frac{2}{3}\text{ m}$$

Electric field intensity is zero at a point which lies at a distance $x = \frac{2}{3}$ m from +4Q charge on the line joining two charges. **(1)**

3 Marks Questions

31.Two point charges + q and -2q are placed at the vertices B and C of an equilateral triangle ABC of side a as given in the figure. Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.



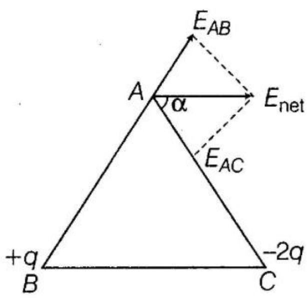
[All India 2014 C]

Ans.

(*i*) The magnitude,

$$|\mathbf{E}_{AB}| = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{a^2} = E \qquad \mathbf{(1/2)}$$

$$|\mathbf{E}_{AB}| = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{a^2} = 2E \qquad \mathbf{(1/2)}$$



$$E_{net} = \sqrt{(2E)^2 + E^2 + 2 \times 2E \times E \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{4E^2 + E^2 - 2E^2} = E\sqrt{3} \qquad \ldots(i) \quad \mathbf{(1/2)}$$

We know that, $E = \frac{q}{4\pi\varepsilon_0 a^2}$

So, $$E_{net} = \frac{q\sqrt{3}}{4\pi\varepsilon_0 a^2}$$

(*ii*) Direction of resultant electric field at vertex **(1/2)**

$$\tan\alpha = \frac{E_{AB}\sin 120°}{E_{AC} + E_{AB}\cos 120°} = \frac{E \times \frac{\sqrt{3}}{2}}{2E + E \times \left(-\frac{1}{2}\right)}$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

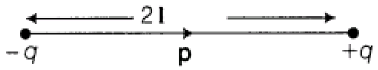
$$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad \mathbf{(1)}$$

$\alpha = 30°$ (with side *AC*)

32.Define the term electric dipole moment. Is it a scalar or vector? Deduce an expression for the electric field at a point on the equatorial plane of an electric dipole of length 2a.[All India 2013; Foreign 2009]

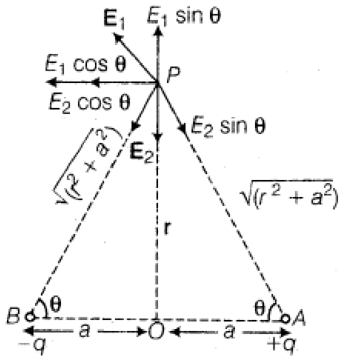
Ans.For electric dipole moment

Electric dipole moment of an electric dipole is equal to the product of its charges and the length of the electric dipole.It is denoted by p. Its unit is coulomb-metre.



$$\mathbf{p} = q \times 2\mathbf{l}$$

It is a vector quantity and its direction is from negative charge towards positive charge
Consider an electric dipole AB consists of two charges +q and -q separated by a distance 2a.
We have to find electric field at point P on equipotential line separated by a distance



Electric field at point P due to charge $+q$

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{[\sqrt{(r^2 + a^2)}]^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2 + a^2)} \text{ along } \mathbf{AP}$$

Electric field at point P due to charge $-q$

$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2 + a^2} \text{ along } \mathbf{PB}$$

$$E = E_1 \cos\theta + E_2 \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2 + a^2)} \cos\theta + \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2 + a^2)} \cos\theta$$

$$= 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2 + a^2)} \times \frac{a}{\sqrt{(r^2 + \ ^2)}}$$

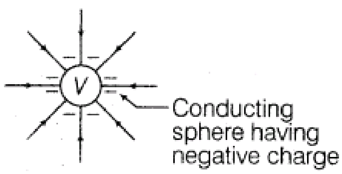
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q2a}{(r^2 + a^2)^{3/2}}$$

But $q \times 2a = P \quad \therefore \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{P}{(r^2 + a^2)^{3/2}}$

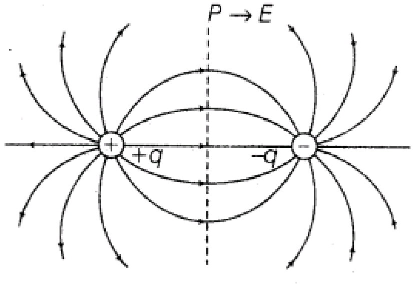
If $r >> a$, then $E = \frac{1}{4\pi\varepsilon_0} \times \frac{P}{r^3}$ **(1)**

33.Sketch the pattern of electric field lines due to
(i) a conducting sphere having negative charge on it.
(ii) an electric dipole.[All India 2011 C]

Ans.(i) Electric field lines due to a conducting sphere are shown in figure

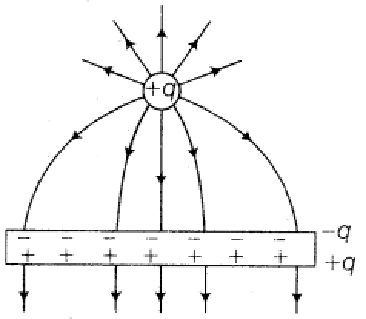


(ii) Electric field lines due to an electric dipole are shown in figure



34.A positive point charge (+ q) is kept in the vicinity of an uncharged conduction plate. Sketch electric field lines originated from the point on to the surface Of the plate. [All India 2009; HOTS]

Ans.Equal charge of opposite nature induces in the surface of conductor nearer to the source charge



Electric lines of forces should fall/normally 90° away on/from the conducting plate. **(1)**

4 Marks Questions

35.Deduce the expression for the torque acting on a dipole of dipole moment p in the presence of a uniform electric field E.[All India 2014; Delhi 2008]

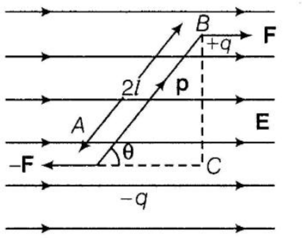
Ans. Torque on an Electric Dipole in a Uniform Electric Field Consider an electric dipole consisting of two charges – q and + q placed in a uniform external electric field of intensity The length of the electric dipole is 21.The dipole moment p makes an angle 0 with the direction of the electric field.

Two forces F and – F which are equal in magnitude and opposite in direction act on the dipole.

$$|\mathbf{F}| = |-\mathbf{F}| = qE$$

The net force is zero. Since, the two forces are equal in magnitude opposite in direction and act at different points, therefore, they constitute a couple.

A net torque x acts on the dipole about an axis passing through the mid-point of the dipole



Dipole in a uniform electric field

Now,

$\tau =$ either force $\times$ perpendicular distance BC between parallel forces

$$= qE(2l \sin\theta)$$

$$\tau = (q \times 2l)E \sin\theta \text{ or } \tau = pE \sin\theta$$

In vector notation,

$$\tau = \mathbf{p} \times \mathbf{E}$$

SI unit of torque is newton-metre (N-m) and its dimensional formula is $[ML^2T^{-2}]$.

Case 1 If $\theta = 0°$, then $\tau = 0$

The dipole is in stable equilibrium. **(1/2)**

Case 2 If $\theta = 90°$, then $\tau = pE$ (maximum value)

The torque acting on dipole will be maximum.

Case 3 If $\theta = 180°$, then $\tau = 0$ **(1)**

The dipole is in unstable equilibrium. **(3)**

36.While travelling back to his residence in the car, Dr. Pathak was caught up in a thunderstorm. It became very dark. He stopped driving the car and waited for thunderstorm to stop. Suddenly, he noticed a child walking alone on the road. He asked the boy to come inside the car till the thunderstorm stopped Dr.Pathak dropped the boy at his residence. The boy insisted that Dr. Pathak should meet his parents. The parents expressed their gratitude to Dr. Pathak for his concern for safety of the child.
Answer the following questions based on the above information
(i)Why is it safer to sit inside a car during a thunderstorm?
(ii)Which two values are displayed by Dr. Pathak in his action?
(iii)Which values are reflected in parents response to Dr. Pathak?
(iv)Give an example of similar action on your part in the past from everyday life.[Delhi 2013; VBQ]

Ans.(i)It is safer to be set inside a car during thunderstorm because the car acts like a Faraday cage. The metal in the car will shield you from any external electric fields and thus prevent the lightning from travelling within the car.
(ii) Awareness and humanity
(iii) Gratitude and obliged
(iv) I once came across to a situation where a puppy was stuck in the middle of a busy road during rain and was not able to go cross due to heavy flow, so I quickly rushed and helped him.

37.An electric dipole moment p is held in a uniform electric field E.
(i)Prove that no translation force acts on the dipole.
(ii)Hence, prove that the torque acting on the dipole is given by pE sin 0 indicating the direction along which it acts. [Foreign 2008]

Ans.(i)When dipole is placed in a unifrom electric field, then force on +q charge due to electric field E

$$\mathbf{F}_1 = q\mathbf{E} \text{ (along } \mathbf{E})$$

Force on $-q$ charge

$$\mathbf{F}_2 = -q\mathbf{E} \text{ (opposite to } \mathbf{E})$$

$\therefore$ Net force on dipole $F_{net} = \mathbf{F}_1 + \mathbf{F}_2$

$$= e\mathbf{E} + (-q\mathbf{E}) = 0$$

$$F_{net} = 0 \quad \textbf{(2)}$$

Net translating force on dipole is zero.

(ii)It is safer to be set inside a car during thunderstorm because the car acts like a Faraday cage. The metal in the car will shield you from any external electric fields and thus prevent the lightning from travelling within the car.

(a) Awareness and humanity

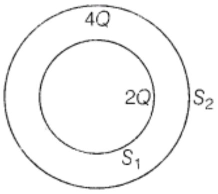
(b) Gratitude and obliged

(c) I once came across to a situation where a puppy was stuck in the middle of a busy road during rain and was not able to go cross due to heavy flow, so I quickly rushed and helped him.

Gauss's Law

1 Mark Questions

1.Consider two hollow concentric spheres S_1 and S_2 enclosing charges 20 and 40 respectively, as shown in the figure, (i) Find out the ratio of the electric flux through them, (ii) How will the electric flux through the spheres S_1 change if a medium of dielectric constant e_r is introduced in the space inside S_1 in place of air? Deduce the necessary expression. [All India 2014]



Ans.

(*i*) According to Gauss' theorem,

$$\phi = \frac{\Sigma q}{\varepsilon_0 \varepsilon} \propto \Sigma q$$

$$\frac{\phi_{S_1}}{\phi_{S_2}} = \frac{2Q}{2Q + 4Q} = \frac{1}{3}$$

(*ii*) If the medium is filled in S_1, then

$$\phi_{S_1} = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} = \frac{2Q}{\varepsilon_0 \varepsilon_r}$$

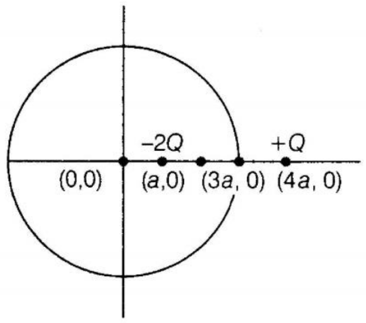
2.Two charges of magnitudes –20 and + O are located at points (a, 0) and (4a, 0), respectively. What is the electric flux due to these charges through a sphere of radius 3a with its centre at the origin? [All India 2013]

Ans.

Gauss' theorem states that the total electric flux linked with closed surface S is

$$\phi_E = \phi \mathbf{E} \cdot d\mathbf{S} = \frac{q}{E_0}$$

where, q is the total charge enclosed by the closed Gaussian (imaginary) surface.



The sphere enclose charge = $-2Q$

Therefore, $\phi = \frac{2Q}{\varepsilon_0}$ (inwards) **(1)**

3.A charge q is placed at the centre of a cube of side L. What is the electric flux passing through each face of the cube? [All India 2010; Foreign 2010]

Ans.

By Gauss' theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

where, q is the total charge enclosed by the closed surface.

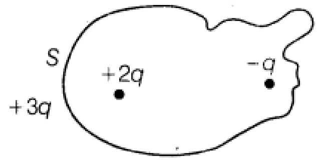
$\therefore$ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$

As charge is at centre, therefore, electric flux is symmetrically distributed through all 6 faces.

Flux linked with each face $= \frac{1}{6}\phi$

$$= \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\,\varepsilon_0}$$ **(1)**

4.Figure shows three point charges,+ 2q, – q and + 3q. Two charges + 2q and – q are enclosed within a surface S. What is the electric flux due to this configuration through the surface S? [Delhi 2010]



Ans.

Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0}$$

$$= \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$\Rightarrow$ $$\phi_S = \frac{q}{\varepsilon_0}$$ **(1)**

Charge $+3q$ is outside the closed surface S, therefore, it would not be taken into consideration in applying Gauss' theorem.

5.If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change?[All India 2009,2008]

Ans.Total charge enclosed by the Gaussian surface remains same even when radius is halved. Therefore, total electric flux remains constant as per Gauss' theorem. There will not be any change in electric flux through the Gaussian surface.

2 Marks Questions

6.Given a uniform electric held E =5 x 10^3 i N/C, find the flux of this held through a square of 10 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014]

Ans.

Given, electric field intensity

$\mathbf{E} = 5 \times 10^3\ \hat{\mathbf{i}}$ N/C

Magnitude of electric field intensity

$|\mathbf{E}| = 5 \times 10^3$ N/C

Side of square, $S = 10$ cm $= 0.1$ m

Area of square, $A = (0.1)^2 = 0.01\ m^2$

The plane of the square is parallel to the YZ-plane. **(1)**

Hence, the angle between the unit vector normal to the plane and electric field is zero.

i.e., $\theta = 0°$

∴ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \cos\theta$$

$$\phi = 5 \times 10^3 \times 0.01 \cos 0°$$

$$\phi = 50\ \text{N-m}^2/\text{C}$$ **(1)**

If the plane makes an angle of 30° with the x-axis, then $\theta = 60°$

∴ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \times \cos 60°$$

$$= 5 \times 10^3 \times 0.01 \times \cos 60°$$

$$= 25\ \text{N - m}^2/\text{C}$$

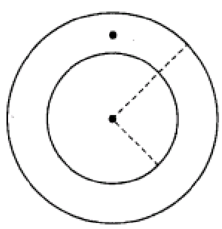
7.Given a uniform electric held E =2 x 10^3 i N/C, find the flux of this held through a square of side 20 cm, whose plane is parahel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014, HOTS]

Ans. Refer to ans. 6. (Ans 40 Nm^2/C)

8.Given a uniform electric held E = 4 x 10^3 i N/C. Find the flux of this field through a square of 5 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014, HOTS]

Ans.Refer to ans 6. (Ans. 5 Nm^2/C)

9.A sphere S_t of radius q enclosed a net charge Q. If there is another concentric sphere S_2 of radius r_2(r_2 > q) enclosing charge 20, hnd the ratio of the electric hux through S_1 and S_2. How will the electric flux through sphere Sj change if a medium of dielectric constant K is introduced in the space inside S_2 in place Of air? [All India 2014]



Ans.

According to Gauss' law,

Flux through S_1,

$$\phi_1 = \frac{Q}{\varepsilon_0} \quad \text{...(i)}$$

(1/2)

Flux through S_2,

$$\phi_2 = \frac{Q + 2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0} \quad \text{...(ii)}$$

Ratio of flues $= \frac{\phi_1}{\phi_2}$

From Eqs. (i) and (ii), we get

$$= \frac{Q}{\varepsilon_0} \times \frac{\varepsilon_0}{3Q} = \frac{1}{3}$$

(1)

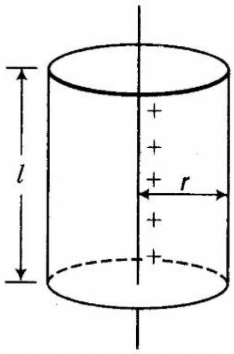
There is no change in the flux through S_1 with dielectric medium inside the sphere S_2. **(1/2)**

10. A thin straight infinitely long conduction wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **[All India 2011]**

Ans. A thin straight conducting wire will be a uniform linear charge distribution.

Let q charge be enclosed by the cylindrical surface.

$\therefore$ Linear charge density, $\lambda = \frac{q}{l}$



Charge enclosed by the cylindrical surface

$$\therefore \quad q = \lambda l \quad \text{...(i)}$$ **(1/2)**

By Gauss' theorem,

$\therefore$ Total electric flux through the surface of cylinder

$$\phi = \frac{q}{\varepsilon_0} \quad \text{[Gauss' theorem]}$$

(1)

$$\therefore \quad \phi = \frac{\lambda l}{\varepsilon_0} \quad \text{[From Eq. (i)]}$$

(1/2)

11.Two charged conducting spheres of radii r_t and r_2 connected to each other by a wire. Find the ratio of electric fields at the surfaces of the two spheres. [Delhi 2011 c]

Ans.

When two charged conducting spheres are connected, then charge flows between the two till their potentials become same.

Electric potential on the surface of connected charged conducting spheres would be equal.

i.e. $$V_1 = V_2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

[Assuming q_1 and q_2 are charges on the spheres connected to each other and r_1, r_2 are their radii.]

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2} \quad \text{...(i)}$$

(1)

Now, ratio of electric field intensities

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1^2}}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2^2}} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2}$$

$$\frac{E_1}{E_2} = \left(\frac{q_1}{q_2}\right) \times \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} \quad \text{[From Eq. (i)]}$$

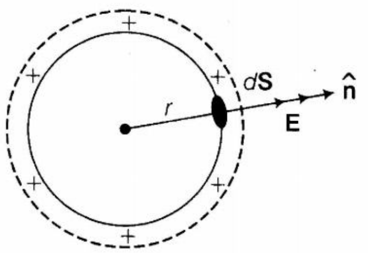
$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

(1)

12. Show that the electric field at the surface of a charged conductor is given by $E = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$, where σ is the surface charge density and $\hat{\mathbf{n}}$ is a unit vector normal to the surface in the outward direction. **[All India 2010]**

Ans.

Let q charge be uniformly distributed over the spherical shell of radius r.



∴ Surface charge density on spherical shell

$$\sigma = \frac{q}{4\pi r^2} \quad \text{...(i)}$$

(1/2)

∵ Electric field intensity on the surface of spherical shell

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{\mathbf{n}}$$

(1/2)

[∵ **E** acts along radially outward and along $\hat{\mathbf{n}}$]

$$\mathbf{E} = \frac{\left(\frac{q}{4\pi r^2}\right)}{\varepsilon_0}\hat{\mathbf{n}}, \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}} \quad \text{...(ii)}$$

(1)

13.A spherical conducting shell of inner radius R_1 and outer radius R_2 has a charge Q. A charge q is placed at the centre of the shell. [All India 2010c]

(i)What is the surface charge density on the (a) inner surface, (b) outer surface of the shell?

(ii)Write the expression for the electric field at a point to $x>R_2$ from the centre of the shell.

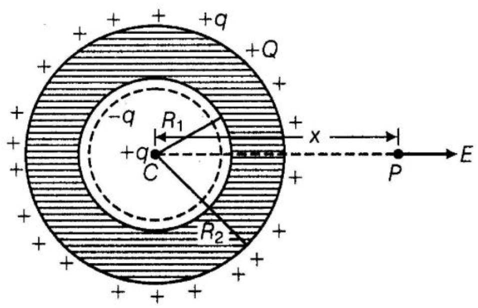
Ans.Here, two points are important

(i) Charge resides on the outer surface of spherical conductor (skin effect).

(ii) Equal charge of opposite nature induces in the surface of conductor nearer to source charge.

(*i*) (*a*) Charge produced on inner surface due to induction = $-q$

∴ Surface charge density of inner surface = $\frac{-q}{4\pi R_1^2}$



When charge $-q$ is induced on inner walls, then equal charge $+q$ is produced at outer surface.

(*b*) Charge on outer surface = $q + Q$

∴ Surface charge density of outer surface

$$= \frac{q+Q}{4\pi R_2^2}$$ **(1)**

(*ii*) Electric field intensity at point *P* separated by a distance $x (x > R_2)$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{(q+Q)}{x^2}$$

[along *CP* and away from spherical shell] **(1)**

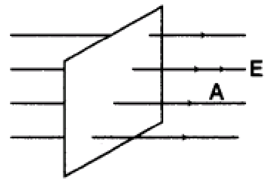
Whole charge is assumed to be concentrated at the centre.

14.Define electric flux. Write its SI unit. A charge gis enclosed by a spherical surface of radius [All India 2009]

Ans. The total electric flux linked with a surface is equal to the total number of electric lines of force passing through the surface when surface is held normal to the direction of electric field.

$\phi = EA$

If surface is placed in non-uniform, electric fild then electric field.



Total electric flux linked with the closed surface

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S}$$

The SI unit of electric flux is N-m^2/C. **(2)**

15.Draw the shapes of the suitable Gaussian surfaces while applying Gauss' law to calculate

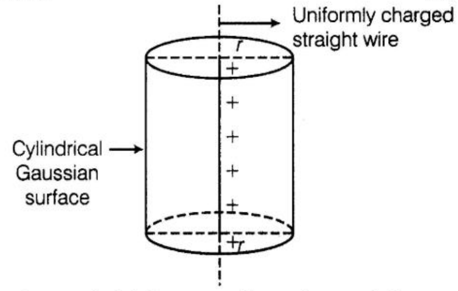
the electric field due to

(i)a uniformly charged long straight wire.

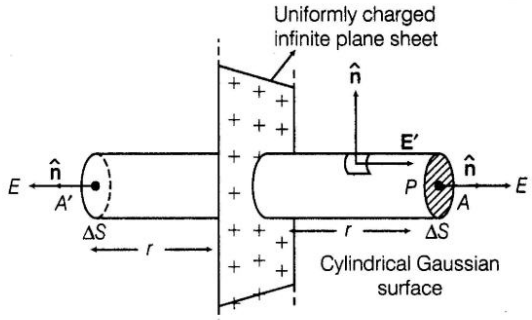
(ii)a uniformly charged infinite plane sheet. [Delhi 2009 C]

Ans. The surface that we choose for application of Gauss' theorem is called Gaussian surface. We usually choose a spherical Gaussian surface.

(*i*) Electric field due to a long straight wire of sheet **(1)**



(*ii*) Electric field due to a plane sheet of charge



16. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0\,\mu C/m^2$

(*i*) Find the charge on the sphere.

(*ii*) What is the total electric flux leaving the surface of the sphere? **[Delhi 2009C]**

Ans.

(*i*) Radius of sphere, $R = \frac{d}{2} = \frac{2.4}{2} = 1.2$ m

Surface charge density,

$\sigma = 80 \times 10^{-6}$ C/m^2

$\therefore \quad \sigma = \frac{q}{4\pi R^2}$

or $\quad q = 4\pi R^2 \sigma$

Charge on the sphere

$q = 4 \times 3.14 \times (1.2)^2 \times 80 \times 10^{-6}$ C

$q = 1.447 \times 10^{-3}$ C

(*ii*) According to Gauss' theorem,

Electric flux, $\phi = \frac{q}{\varepsilon_0}$

$\phi = \frac{1.447 \times 10^{-3}}{8.85 \times 10^{-12}}$ Nm^2/C

$\phi = 1.63 \times 10^8$ Nm^2/C

17. Two large parallel thin metallic plates are placed close to each other. The plates have surface charge densities of opposite signs and of magnitude $2.0 \times 10^{-12} C/m^2$. Calculatethe electric field intensity

(*i*) in the outer region of the plates

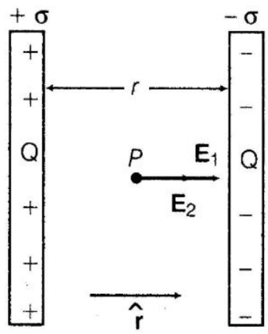
(*ii*) in the interior region between the plates. **[Foreign 2008]**

Ans.

Let $\hat{\mathbf{r}}$ be a unit vector directed from left to right.

Let *P* and *Q* are two points in the outer and inner region of two plates, respectively.

Charge densities on plates are $+\sigma$ and $-\sigma$.



$$\therefore \quad \sigma = 2 \times 10^{-12} \text{ C/m}^2$$

(*i*) Electric field at point *P* in the outer region of the plates

$$\mathbf{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} \text{ or } \mathbf{E}_2 = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$\therefore$ Net field in the outer region of the plates (i.e., at *P*)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} = 0 \quad \textbf{(1)}$$

(*ii*) Electric field at point Q in the interior of two plates

$$\mathbf{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

$\mathbf{E} = \dfrac{\sigma}{\varepsilon_0}$ [From positive plate to negative plate]

But, $\sigma = 2 \times 10^{-2} \text{ C/m}^2$

$$\therefore \quad E = \frac{2 \times 10^{-12}}{8.85 \times 10^{-12}}$$

$$E = 2.25 \times 10^{-1} \text{ N/C} \quad \textbf{(1)}$$

18. A point charge causes and electric flux -3×10^{-14} Nm^2/C to pass through a spherical Gaussian surface.

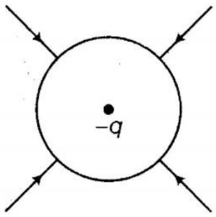
(*i*) Calculate the value of the point charge.

(*ii*) If the radius of the Gaussian surface is double, how much flux would pass through the surface?

[Foreign 2008]

Ans.

(*i*) By Gauss' theorem, total electric flux through closed Gaussian surface is given by



$$\phi = \frac{q}{\varepsilon_0}$$

$\therefore$ $$q = \phi\varepsilon_0$$

But, electric flux passing through the surface

$$\phi = -3\times10^{-14}Nm^2/C$$

$\therefore$ $$q = (-3\times10^{-14})\times 8.85\times10^{-12}$$

$$= -26.55\times10^{-26}C$$

$$q = -2.655\times10^{-25}C$$ **(1)**

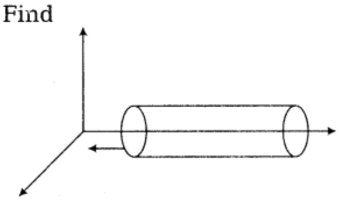
(*ii*) Electric flux passing through the surface remains unchanged because it depends only on charge enclosed by the surface and is independent of its size. **(1)**

3 Marks Questions

19.A hollow cylindrical box of lenght 1 m and area of cross-section 25 cm^2 is placed in a three-dimensional coordinate system as shown in the figure.

The electric field in the region is given by $E = 50\,x\hat{i}$, where E is in NC^{-1} and x is in metre.

Find



(*i*) net flux through the cylinder.

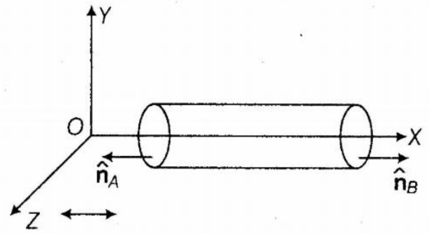
(*ii*) charge enclosed by the cylinder.

[Delhi 2013]

Ans.

(*i*) Given, $E = 50 \times i$

and $\Delta S = 25\ cm^2 = 25 \times 10^{-4}\ m^2$



As the electric field is only along the *X*-axis, so flux will pass only through the cross-section of the cylinder.

Magnitude of electric field at cross-section *A*,

$$E_A = 50 \times 1 = 50\ NC^{-1}$$

Magnitude of electric field at cross-section *B*,

$$E_B = 50 \times 2 = 50\ NC^{-1}$$ **(1/2)**

The corresponding electric fluxes are

$\phi_A = \mathbf{E}_A \cdot \Delta\mathbf{S} = 50 \times 25 \times 10^{-4} \times \cos 180°$

$= -0.125\ Nm^2C^{-1}$

$\phi_B = \mathbf{E}_B \cdot \Delta\mathbf{S} = 100 \times 25 \times 10^{-4} \times \cos 0°$

$= 0.25\ Nm^2C^{-1}$

So, the net flux through the cylinder, **(1)**

$\phi = \phi_A + \phi_B$

$= 0.125 + 0.25 = 0.125\ Nm^2C^{-1}$

(*ii*) Using Gauss' law,

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{q}{\varepsilon_0}$$

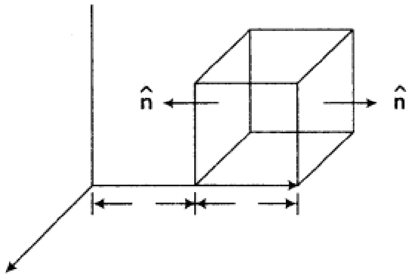
$$\Rightarrow \quad 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q = 8.85 \times 0.125 \times 10^{-12}$$

$$q = 1.1 \times 10^{-12}\ C$$ **(1/2)**

So, the charge enclosed by the cylinder is 1.1×10^{-12} C. **(1)**

20.State Gauss' law in electrostatics. A cube which each side a is kept in an electric field given by E = as shown in the figure, where C is a positive dimensional constant. Find out



(i)the electric flux through the cube

(ii)the net charge inside the cube.[Foreign 2012]

Ans.

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

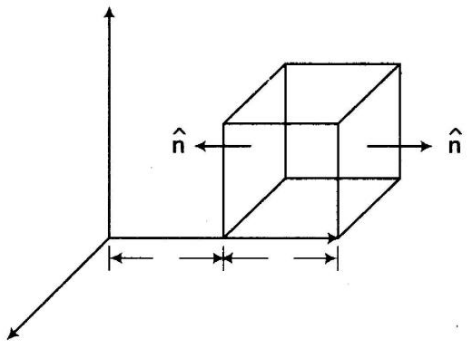
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of E at left face,

$E_L = Cx = Ca$ [$x = a$ at left face]

Magnitude of E at right face

$E_R = Cx$
$= C2a = 2aC$ [$x = 2a$ at right face]

Thus, corresponding fluxes are

$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$
$= -aC \times a^2$ [As, $\theta = 180°$]

$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta$ [$\because \theta = 0°$]
$= 2aCa^2$
$= 2a^3C$ **(1)**

(*i*) Now, net flux through the cube is
$= \phi_L + \phi_R$
$= -a^3C + 2a^3C$
$= a^3C \text{ Nm}^2\text{C}^{-1}$ **(1)**

(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge q inside the cube.

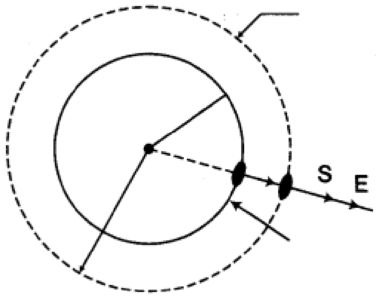
We have $\phi = \frac{q}{\varepsilon_0}$

or $q = \phi\,\varepsilon_0$

$q = a^3C\varepsilon_0$ coulomb **(1)**

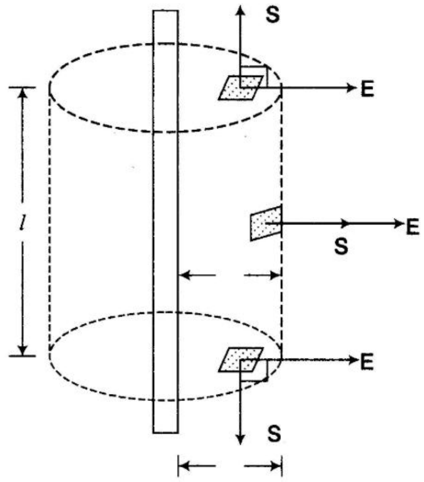
21.Using Gauss' law, obtain the expression for the electric field due to uniformly charged spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for r > R and r < R.[All India 2011]

Ans.Let us consider charge +q is uniformly distributed over a spherical shell of radius R. Let £ is to be obtained at P lying outside of spherical shell.



Eat any point is radially outward (if charge q is positive) and has same magnitude at all points which lie at the same distance r from centre of spherical shell such that r> R.

Therefore, Gaussian surface is concentric sphere of radius r such that $r > R$. **(1/2)**



Cylindrical Gaussian surface for line charge

Since, Gaussian surface enclosed charge q inside it.

By Gauss' theorem,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \Rightarrow \oint E\, dS \cos 0° = \frac{q}{\varepsilon_0}$$

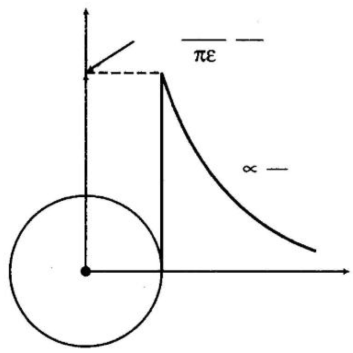
[$\because$ **E** and d**S** are along the same direction]

$$E \oint dS = \frac{q}{\varepsilon_0}$$

[$\because$ Magnitude of E is same at every point on Gaussian surface]

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad \textbf{(1)}$$

Now, graph



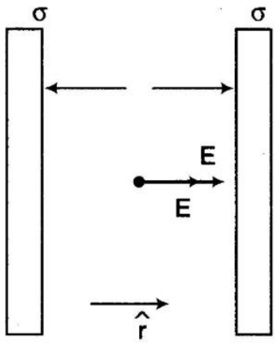
Variation of E with r for a spherical shell of charge **(1)**

22. Use Gauss' law to derive the expression for the electric field between two uniformly charge parallel sheets with surface charge densities σ and $-\sigma$, respectively.

[All India 2009]

Ans.

Let us consider two uniformly charge, large parallel sheets carrying charge densities $+\sigma$ and $-\sigma$ respectively, are separated by a small distance from each other.



By Gauss' law, it can be proved that electric field intensity due to a uniformly charged infinite plane sheet at any nearby is given

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{...(i)}$$

(1)

The electric field is directed normally outward from the plane sheet if nature of charge on sheet is positive and normally inward if charge is of negative nature.

Let $\hat{\mathbf{r}}$ represents unit vector directed from positive plate to negative plate.

Now, Electric Field Intensity (EFI) at any point P between the two plates is given by

(*i*) $\mathbf{E}_1 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [Due to positive plate]

(*ii*) $\mathbf{E}_2 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [Due to negative plate]

(1)

$\therefore$ Electric field intensity at P point is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

Thus, a uniform electric field is produced between the two infinite parallel plane sheet of charge which is directed from positive plate to negative plate. **(1)**

23.State Gauss' law in electrostatics. Using this law, derive an expression for the electric field due to a uniformly charged infinite plane Sheet. [Delhi 2009]

Ans.

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

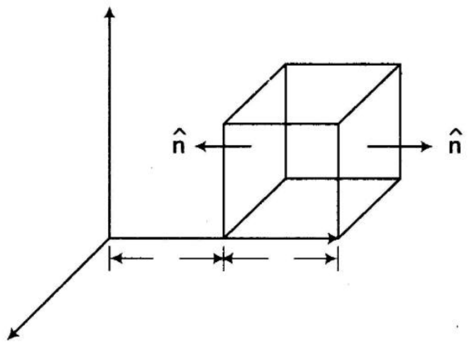
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, $\mathbf{E}$ is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.

Magnitude of E at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of E at right face

$$E_R = Cx$$
$$= C2a = 2aC \quad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$

$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$ **(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1)**

(*ii*) Net charge inside the cube

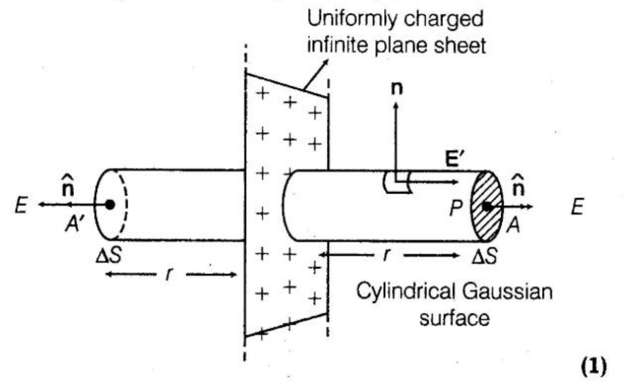
Again, we can use Gauss' law to find total charge q inside the cube.

We have $\phi = \frac{q}{\varepsilon_0}$

or $q = \phi\,\varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$ **(1)**

Let us consider a large plane sheet of charge having surface charge density sigma



(1)

Let electric field is to be obtained at a point P at a distance r from it. It is obvious that Gaussian surface will be a cylinder of cross-sectional area A and length $2r$ with its axis perpendicular to plane sheet of charge.

Now, applying Gauss' law over the closed Gaussian surface.

$$\oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$ **(1)**

$$\int_{CSA} \mathbf{E}\cdot d\mathbf{S} + \int_{CSA} \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$\int_{CSA} E\,dS\,\cos 0^\circ + \int_{CSA} E\,dS\,\cos 90^\circ = \frac{q}{\varepsilon_0}$$

[As **E** and $d\mathbf{S}$ are along the same direction by at CSA **E** perpendicular to $d\mathbf{S}$]

$$\int_{CSA} dS = \frac{q}{\varepsilon_0} \Rightarrow E \times 2A = \frac{q}{\varepsilon_0} \quad \text{...(i)}$$

$$E = \frac{q}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \quad \text{[From Eq. (i)]}$$

$\therefore$ Electric field intensity $= \dfrac{\sigma}{2\varepsilon_0}$

The direction of E is normal to the plane sheet and directed away from sheet when charge on plate is positive and *vice-versa*. **(1)**

Closed cylinder comprises of two caps and Curved Surface Area (CSA).

24. State Gauss' law in electrostatics. Using this law derive an expression for the electric field due to a long straight wire of linear charge density λ C/m. **[Delhi 2009]**

Ans.(i)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

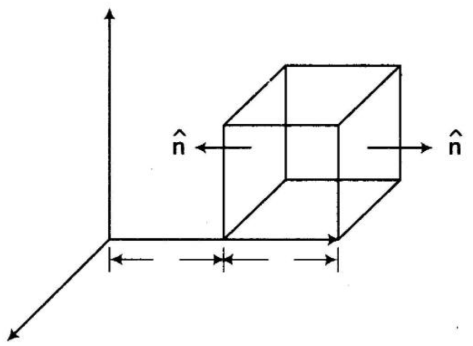
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of E at left face,

$E_L = Cx = Ca$ [$x = a$ at left face]

Magnitude of E at right face

$E_R = Cx$
$= C2a = 2aC$ [$x = 2a$ at right face]

Thus, corresponding fluxes are

$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$
$= -aC \times a^2$ [As, $\theta = 180°$]

$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta$ [$\because \theta = 0°$]
$= 2aCa^2$
$= 2a^3C$ **(1)**

(*i*) Now, net flux through the cube is
$= \phi_L + \phi_R$
$= -a^3C + 2a^3C$
$= a^3C \text{ Nm}^2\text{C}^{-1}$ **(1)**

(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge q inside the cube.

We have $\phi = \frac{q}{\varepsilon_0}$

or $q = \phi\, \varepsilon_0$
$q = a^3C\varepsilon_0$ coulomb **(1)**

(*ii*) Let us consider a long straight wire carrying $+q$ charge on its length l and linear charge density λ C/m.

$$\because \qquad \lambda = \frac{q}{l}$$

$$\Rightarrow \qquad q = \lambda l \qquad \text{...(i)}$$

Let electric field intensity is to be obtained at a distance r from it. Since, magnitude of **E** due to long charged wire is same at every point which lie at the same distance from the wire. So, Gaussian surface will be a cylinder of radius r and length l such that wire lies along the axis as shown in figure.

(1)

$\because$ Angle between **E** and $d\mathbf{S}$ is 90° at caps, whereas 0° at any point on curved surface of a cylinder.

Now, applying Gauss' theorem

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad \textbf{(1)}$$

$$\int_{CSA} \mathbf{E} \cdot d\mathbf{S} + \int_{CSA} \mathbf{E} \cdot d\mathbf{S} = \frac{\lambda}{\varepsilon_0} \qquad \text{[From Eq. (i)]}$$

[CSA = Close Surface Area]

$$\int_{CSA} E\, dS \cos 90° + \int_{CSA} E\, dS \cos 0° = \frac{\lambda}{\varepsilon_0}$$

(S_1 and S_3 are caps and S_2 represents CSA)

$$0 + \int_{CSA} E\, dS = \frac{\lambda l}{\varepsilon_0} \qquad [\because \cos 90° = 0]$$

$$E \int_{CSA} dS = \frac{\lambda l}{\varepsilon_0}$$

[$\because E$ is a constant at every point at CSA]

$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \qquad \left[\because \int_{CSA} dS = 2\pi r l\right]$$

$$E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \textbf{(1)}$$

4 Marks Questions

25.Using Gauss' law, deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point

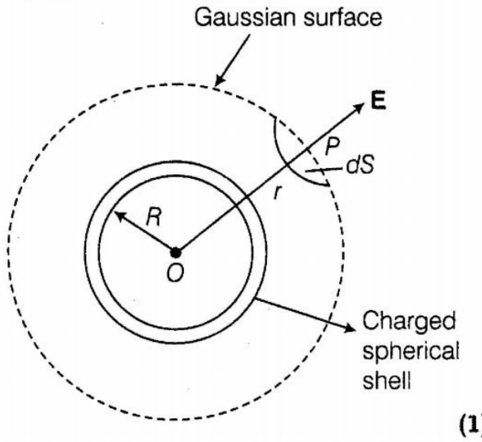
(i)outside the shell

(ii)inside the shell

Plot a graph showing variation of electric field as a function of r > R and r < R.{r being the distance from the centre of the shell) [All India 2013]

Ans.

Electric field due to a uniformly charged thin spherical shell.



(1)

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point P at a distance $r\ (r > R)$ from its centre. Draw the Gaussian surface through point P, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O.

Let $\mathbf{E}$ be the electric field at point P. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \quad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\, dS$$

$\therefore$ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\, dS = E \oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \quad \phi = E \times 4\pi r^2 \quad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is q. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$

(1)

(*ii*) **When point *P* lies inside the spherical shell**

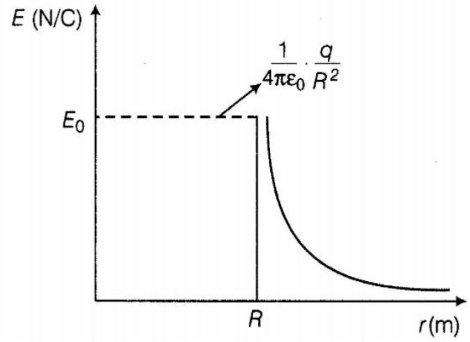
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

$$E \times 4\pi r^2 = 0$$

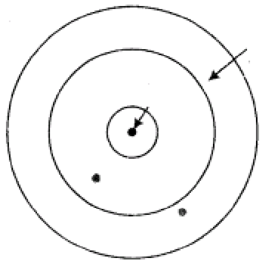
i.e., $E = 0$ [for $r < R$]

Graph showing the variation of electric field as a function of *r* **(2)**



26.(i) Define electric flux. Write its SI unit.

(ii) A small metal sphere carrying charge + Q is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure.Use Gauss' law to find the expressions for the electric field at points P_l and P_2.



(iii)Draw the pattern of electric field lines in this arrangement.[Delhi 2012 C]

Ans.

(*i*) **Electric flux** Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m^2/C **(1)**

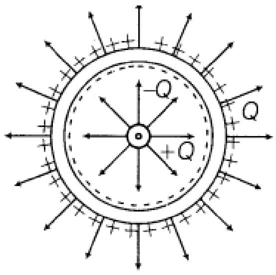
(*ii*) Using Gauss' theorem,

$$E \times 4\pi r_1^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_1^2}$$

Field at point $P_2 = 0$, because the electric field inside the conductor is zero. **(1)**

(iii) The electric field lines due to arrangement is shown as below:



Charges will be uniformly distributed on all the surfaces hence, all field lines will be uniformly separated.

27.Define electric flux. Write its SI unit, (ii) Using Gauss' law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of distance from it. How is the field directed if
(a)the sheet is positively charged
(b)negatively charged? [Delhi 2012]

Ans.(i) Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m^2/C

(ii)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

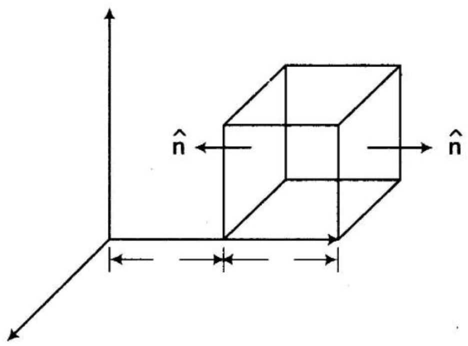
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS\,\cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of E at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of E at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As}, \theta = 180°]$$
$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$ **(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1)**

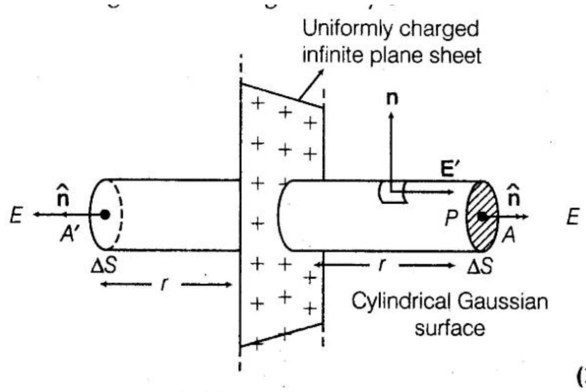
(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge q inside the cube.

We have $\phi = \dfrac{q}{\varepsilon_0}$

or $q = \phi\, \varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$ **(1)**

Let us consider a large plane sheet of charge having surface charge density sigma



(1)

Let electric field is to be obtained at a point P at a distance r from it. It is obvious that Gaussian surface will be a cylinder of cross-sectional area A and length $2r$ with its axis perpendicular to plane sheet of charge.

Now, applying Gauss' law over the closed Gaussian surface.

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \quad \textbf{(1)}$$

$$\int_{CSA} \mathbf{E} \cdot d\mathbf{S} + \int_{CSA} \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$\int_{CSA} E\,dS \cos 0° + \int_{CSA} E\,dS \cos 90° = \frac{q}{\varepsilon_0}$$

[As **E** and $d\mathbf{S}$ are along the same direction by at CSA **E** perpendicular to $d\mathbf{S}$]

$$\int_{CSA} dS = \frac{q}{\varepsilon_0} \Rightarrow E \times 2A = \frac{q}{\varepsilon_0} \quad ...(i)$$

$$E = \frac{q}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \quad \text{[From Eq. (i)]}$$

$\therefore$ Electric field intensity $= \dfrac{\sigma}{2\varepsilon_0}$

The direction of E is normal to the plane sheet and directed away from sheet when charge on plate is positive and *vice-versa*. **(1)**

Closed cylinder comprises of two caps and Curved Surface Area (CSA).

The field directed

- Normally away from the sheet when sheet is positively charged.
- Normally inward towards the sheet when plane sheet is negatively charged.

28.(i) State Gauss' law. Use it to deduce the expression for the electric field due to a uniformly charged thin spherical shell at points

- inside the shell and
- outside the shell.

(ii) Two identical metallic spheres A and B having charges +40 and – 10O are kept a certain distance apart. A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B. Then, spheres A and Bare brought in contact and then separated. Find the charges on the spheres A and B.[All India 2011C]

Ans.(i)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

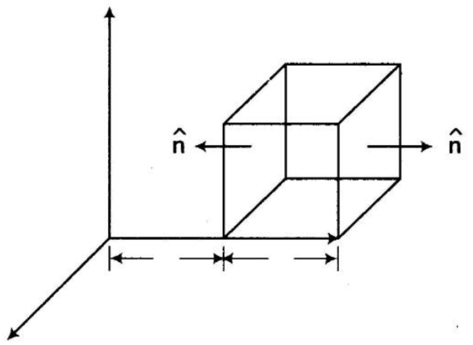
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

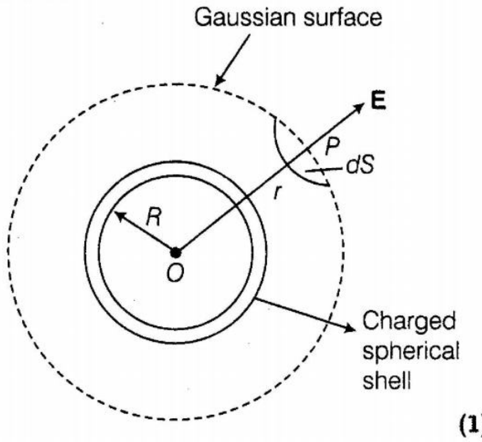
Also, $$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\, dS \cos 90° = 0$ through it.



Electric field due to a uniformly charged thin spherical shell.



(1)

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point P at a distance $r\ (r > R)$ from its centre. Draw the Gaussian surface through point P, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O.

Let **E** be the electric field at point P. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \qquad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\, dS$$

∴ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\, dS = E \oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \qquad \phi = E \times 4\pi r^2 \qquad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is q. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$ **(1)**

(*ii*) **When point *P* lies inside the spherical shell**

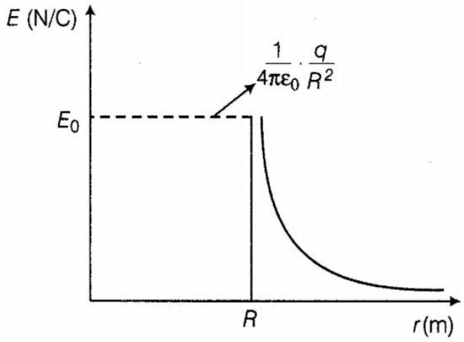
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

$$E \times 4\pi r^2 = 0$$

i.e., $E = 0$ $[\text{for } r < R]$

Graph showing the variation of electric field as a function of *r* **(2)**



(ii)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

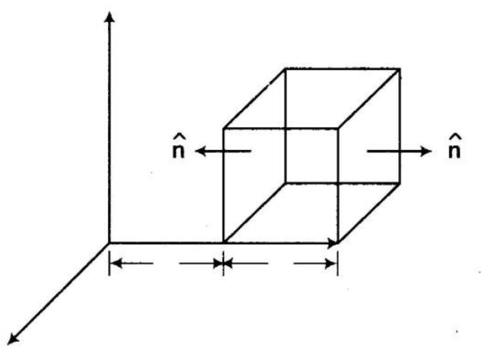
$$\phi = \frac{q}{\varepsilon_0}$$

Here, ε_0 is the absolute permittivity of the free space and q is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in *X*-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of E at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of E at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L \, dS \, \cos\theta$$
$$= -\, aC \times a^2 \qquad [\text{As}, \theta = 180°]$$
$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC \, dS \, \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$ **(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -\, a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1)**

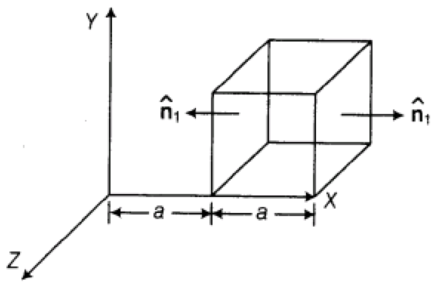
(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge q inside the cube.

We have $\phi = \frac{q}{\varepsilon_0}$

or $q = \phi \, \varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$ **(1)**

29.(i) Define electric flux. Write its SI unit, (ii) The electric field components due to a charge inside the cube of side 0.1 m are shown below.



$$E_x = \alpha x,$$

where, $\alpha = 500$ N/C - m,

$$E_y = 0, E_z = 0$$

Calculate

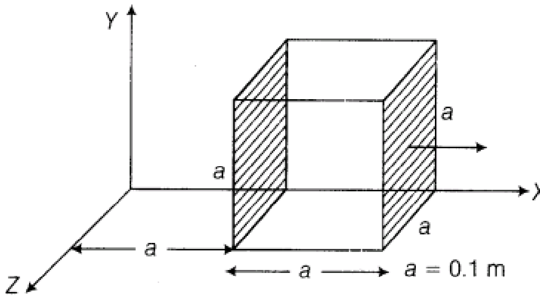
(*a*) the flux through the cube and

(*b*) the charge inside the cube.

[All India 2008]

Ans.(i)Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m^2/C

(ii) The electric field is directed along +X-axis. Therefore, angle between E and A for left face is 180°, whereas for right face is 0°. The angle between E and A on four non-shaded faces is 90°. Therefore, flux linked with these four faces is zero.



(1)

(a) Total electric flux through the cube (ϕ)

$$\phi = \phi_L + \phi_R + 0$$

$[\because \phi_{\text{for non-shaded face}} = 0]$

where, flux linked with left face,

$$\phi_L = E_1 \times a^2 \cos 180°$$

where, flux linked with right face,

$$\phi_R = E_2 \times a^2 \cos 0°$$

Total flux passing through cube

$$\therefore \quad \phi = E_1 a^2 \cos 180° + E_2 a^2 \cos 0°$$

$$= -E_1 a^2 + E_2 a^2$$ **(1/2)**

$[\because E_1 = \alpha x = 500 \times 0.1, E_2 = \alpha y = 500 \times 0.2]$

$$= -(500 \times 0.1) \times (0.1)^2 + (500 \times 0.2) \times (0.1)^2$$

$$\phi = -0.5 + 1 = 0.5 \text{ N-m}^2/\text{C}$$

$$\phi = 0.5 \text{ N-m}^2/\text{C}$$ **(1)**

(b) By Gauss' theorem,

$$\phi = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad q = \phi \varepsilon_0$$ **(1)**

$$\therefore \quad q = (0.5) \times 8.85 \times 10^{-12}$$

$$= 4.425 \times 10^{-12} \text{C}$$

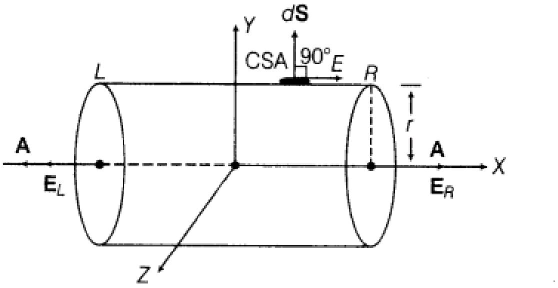
Charge inside the cube = 4.425×10^{-12}C

(1/2)

30. (i) Define electric flux. Write its SI unit.

(ii) A uniform electric field $\mathbf{E} = E_x \hat{\mathbf{i}}$ N/C for $x > 0$ and $\mathbf{E} = -E_x \hat{\mathbf{i}}$ N/C for $x < 0$ are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the X-axis. Find out the net outward flux. Using the Gauss' law, write the expression for the net charge within the cylinder. **[Delhi 2008C]**

Ans.(i)Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m^2/C (*ii*) Electric flux linked with curved surface



$$\mathbf{E}\cdot\mathbf{dS} = E\,dS\,\cos 90^\circ = 0 \qquad \textbf{(1/2)}$$

According to the problem,

$$\mathbf{E}_R = E_x\hat{\mathbf{i}} \qquad [\text{For } x > 0]$$

$$\mathbf{E}_L = -E_x\hat{\mathbf{i}} \qquad [\text{For } x < 0]$$

∴ Total flux linked with the right circular cylinder

$$\phi = \phi_R + \phi_L \qquad \textbf{(1/2)}$$

where, $\phi_R = \mathbf{E}_R\cdot\mathbf{A} = E_R A\cos 0^\circ$

$$= E_x\left(\frac{\pi r^2}{10^4}\right) \qquad [\because A = \pi r^2]$$

$$\phi_L = \mathbf{E}_L\cdot\mathbf{A} = E_L A\cos 0^\circ = E_x\left(\frac{\pi r^2}{10^4}\right)$$

$$[1\text{ cm} = 10^{-2}\text{ m}]$$

$$\therefore\ \phi = \phi_R + \phi_L$$

$$= [E_x(\pi r^2) + E_x(\pi r^2)]\times 10^{-4}$$

$$= 2\pi r^2 E_x \times 10^{-4}\text{ N-m}^2/\text{C} \qquad \textbf{(1)}$$

By Gauss' theorem, $\phi = \dfrac{q}{\varepsilon_0}$

∴ Net charge $q = \phi\varepsilon_0$

$$= 2\pi r^2 E_x\,\varepsilon_0 \times 10^{-4}\text{ C} \qquad \textbf{(1)}$$

31. (*i*) Using Gauss' law, derive an expression for electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and the density σ C/m^2. Draw the field lines when the charge density of the sphere is

(*a*) positive and

(*b*) negative

(*ii*) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m^2.

Calculate the

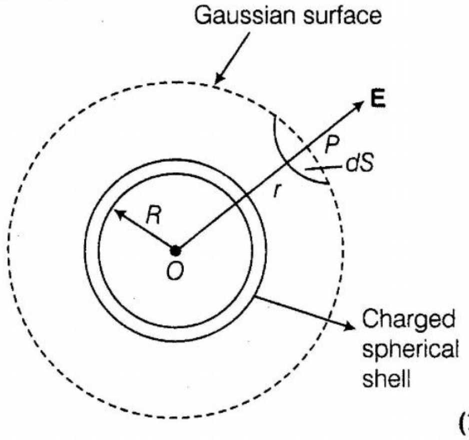
(*a*) charge on the sphere and

(*b*) total electric flux through the sphere. **[Delhi 2008]**

Ans. The electric lines of force emerge from the positive charge and comes into the negative charge.

(i)

Electric field due to a uniformly charged thin spherical shell.



(1)

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point P at a distance r $(r > R)$ from its centre. Draw the Gaussian surface through point P, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O.

Let **E** be the electric field at point P. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \qquad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\, dS$$

$\therefore$ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\, dS = E \oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \qquad \phi = E \times 4\pi r^2 \qquad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is q. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$ **(1)**

(*ii*) **When point *P* lies inside the spherical shell**

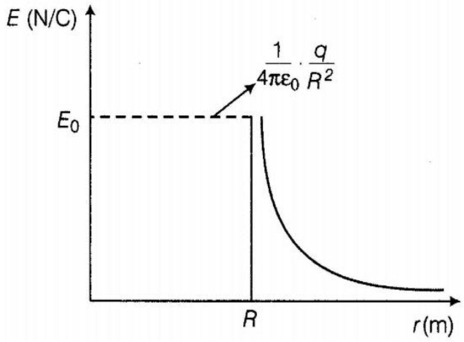
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

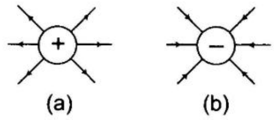
$$E \times 4\pi r^2 = 0$$

i.e., $E = 0$ [for $r < R$]

Graph showing the variation of electric field as a function of *r* **(2)**



Electric field lines due to positive and negative charged spherical shell are as given below in figures (a) and (b) I respectively



(*ii*) (*a*) Charge density, **(1)**

$$\sigma = \frac{q}{4\pi r^2}$$

$$\therefore\ q = 4\pi r^2 \sigma$$

$$= 4 \times 3.14 \times \left(\frac{2.5}{2}\right)^2 \times 100 \times 10^{-6}$$

$$= 1.9625 \times 10^{-3} \text{C}$$ **(1)**

(*b*) By Gauss' law,

Total electric flux through the sphere

$$\phi = \frac{q}{\varepsilon_0}$$

$$= \frac{1.9625 \times 10^{-3}}{8.85 \times 10^{-12}}$$

$$= 2.2 \times 10^8 \text{ N-m}^2/\text{C}$$ **(1)**