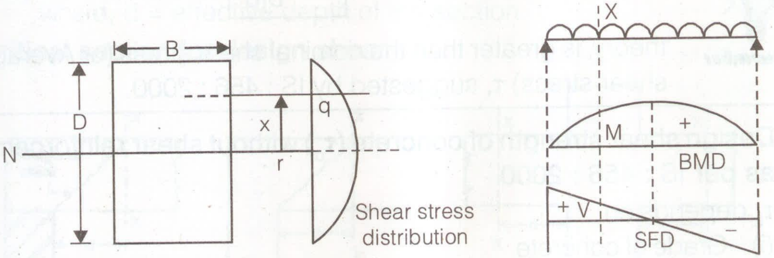
4. SHEAR

SHEAR STRESS

(a) For Homogeneous beam

$$q = \frac{V}{IB} \cdot A\overline{Y}$$



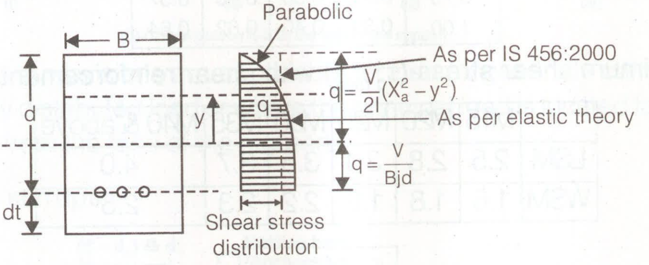
where, q = shear stress at any section

V = shear force at any section

$A\overline{Y}$ = Moment of area of section above the point of consideration

I = Moment of inertia of section = $\frac{BD^3}{12}$

(b) For Reinforced concrete beam



(i) Shear stress above N.A

$$q = \frac{V}{2I} \cdot (X_a^2 - y^2) \qquad q_{max} = \frac{V}{2I} \cdot X_a^2 \text{ at } y = 0$$

(ii) Shear stress below N.A

$$q = \frac{V}{Bjd}$$

As per IS 456 : 2000

Nominal shear stress, $\tau_v = \frac{V}{Bd}$

> **Remember**
>
> The maximum shear stress $q = \frac{V}{bjd}$ obtained from elastic theory, is greater than the nominal shear stress (or Average shear stress) τ, suggested by IS : 456 : 2000.

- **Design shear strength of concrete (τ_c) without shear reinforcement as per IS : 456 : 2000**

τ_c depends on
(i) Grade of concrete
(ii) Percentage of steel,

$$p = \frac{A_{st}}{Bd} \times 100$$

where, A_{st} = Area of steel
B = Width of the Beam
d = Effective depth of the beam

	WSM		LSM	
P	M 20	M 25	M 20	M 25
$0 \le 0.15$	0.18	0.19	0.28	0.29
0.25	0.22	0.23	0.36	0.36
0.50	0.30	0.31	0.48	0.49
0.75	0.35	0.36	0.56	0.57
1.00	0.39	0.40	0.62	0.64

- **Maximum shear stress ($\tau_{c\,max}$) with shear reinforcement is**

	M15	M20	M25	M30	M35	M40 & above
LSM	2.5	2.8	3.1	3.5	3.7	4.0
WSM	1.6	1.8	1.9	2.2	2.3	2.5

$$\tau_v \not> \tau_{c\,max}$$

- **Minimum shear reinforcement (As per IS 456 : 2000)**

$$\frac{A_{sv}}{BS_v} \ge \frac{0.4}{0.87 f_y}$$ → This is valid for both W.S.M. and L.S.M.

or $$S_v \le \frac{2.175 f_y A_{sv}}{B}$$ where, A_{sv} = Area of shear reinforcement
S_v = Spacing of shear reinforcement

- **Spacing of shear reinforcement**

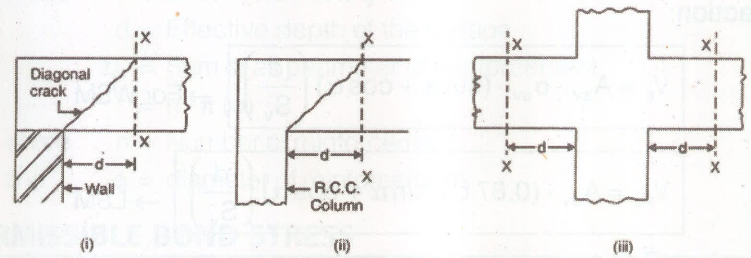
Maximum spacing is minimum of (i), (ii) and (iii)

(i) $$S_v = \frac{2.175 f_y A_{sv}}{B}$$

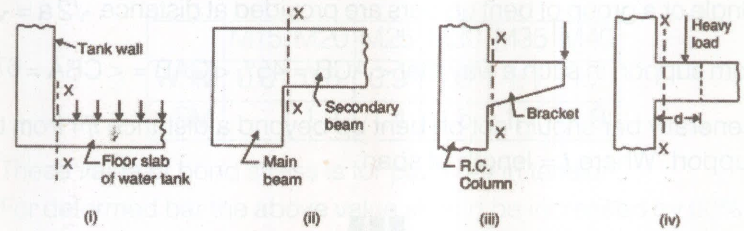
(ii) 300 mm

(iii) 0.75d → For vertical stirrups
d → For inclined stirrups
where, d = effective depth of the section

- **Critical section for design shear**



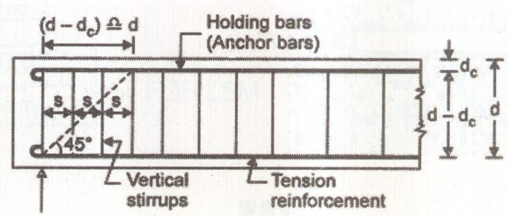
(a) Critical section X-X at d from the face of the support



(b) Critical section X-X at the face of the support

The above provisions are applicable for beams generally carrying uniformly distributed load or where the principal load is located faster than 2d from the face of the support.

- **Vertical stirrups:**



Shear force 'V_s' will be
Resisted by shear
Reinforcement provided in 'd' length of the beam,

$$\boxed{V_s = \left(\frac{d}{S_v}\right) A_{sv} \cdot \sigma_{sv}} \rightarrow \text{For WSM}$$

where, A_{sv} = Cross-sectional area of stirrups
S_v = Centre to centre spacing of stirrups

$$\boxed{V_{su} = \left(\frac{d}{S_v}\right) A_{sv} (0.87\, f_y)} \rightarrow \text{For LSM}$$

- **Inclined stirrups : or a series of bars bent-up at different cross-section:**

$$\boxed{V_s = A_{sv} \cdot \sigma_{sv} \cdot (\sin\alpha + \cos\alpha)\left(\frac{d}{S_v}\right)} \rightarrow \text{For WSM}$$

$$\boxed{V_{su} = A_{sv} \cdot (0.87\, f_y)(\sin\alpha + \cos\alpha)\left(\frac{d}{S_v}\right)} \rightarrow \text{LSM}$$

- **Bent up Bars:**

  Single or a group of bent up bars are provided at distance $\sqrt{2}a = \sqrt{2}jd$ from support in such a way that $< ACB = 45°$, $< CAB = < CBA = 67\frac{1}{2}°$.

  Generally bar should not be bent up beyond a distance $l/4$ from the support. Where l = length of span.

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