Long Answer Type Questions

[4 marks]

Que 1. The sum of the 4th and 8th term of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.

Sol. We have, $a_4 + a_8 = 24$

$\Rightarrow$ $a + (4 – 1) d + a + (8 – 1) d = 24$ $\Rightarrow$ $2a + 3d + 7d = 24$

$\Rightarrow$ $2a + 10d = 24$ $\Rightarrow$ $2 (a + 5d) = 24$

$\therefore$ $a + 5d = 12$ …(i)

and, $a_6 + a_{10} = 44$

$\Rightarrow$ $a + (6 – 1) d + a + (10 – 1) d = 44$ $\Rightarrow$ $2a + 5d + 9d = 44$

$\Rightarrow$ $2a + 14d = 44$ $\Rightarrow$ $a + 7d = 22$ …(ii)

Subtracting (i) from (ii), we have

$2d = 10$

$\therefore$ $d = \frac{10}{2} = 5$

Putting the value of d in equation (i), we have

$a + 5 \times 5 = 12$ $\Rightarrow$ $a = 12 – 25 = -13$

Here, $a = -13, d = 5$

Hence, first three terms are

$-13, -13 + 5, -13 + 2 \times 5$ $i.e.,$ $-13, -8, -3$

Que 2. The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and 12th term.

Sol. We have, $S_n = 3n^2 - 4n$ (i)

Replacing n by $n - 1$, we get

$$S_{n-1} = 3(n-1)^2 - 4(n-1) \quad (ii)$$

We know,

$$a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$$

$$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$$

$$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$$

So, nth term $a_n = 6n - 7$ $\qquad (iii)$

To get the AP, substituting $n = 1,2,3, \ldots$ respectively in(iii), we get

$a_1 = 6 \times 1 - 7 = -1, a_2 = 6 \times 2 - 7 = 5$

$a_3 = 6 \times 3 - 7 = 11, \ldots$

Hence, AP is $-1, 5, 11, \ldots$

Also, to get 12^{th} term, substituting $n = 12$in(iii), we get

$a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$

Que 3. Divide 56 into four parts which are in AP such that the ratio of product of extremes to the product of means is $5:6$.

Sol. Let the four parts be $a - 3d, a - d, a + d, a + 3d$.

Given, $(a - 3d) + (a - d) + (a + d) + (a + 3d) = 56$

$\Rightarrow \quad 4a = 56$ or $a = 14$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$

$\Rightarrow \quad \frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6} \quad \Rightarrow \quad 6(196 - 9d^2) = 5(196 - d^2) \quad [\because a = 14]$

$\Rightarrow \quad 6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

$\Rightarrow \quad 49d^2 = 6 \times 196 - 5 \times 196 = 196$

$\Rightarrow \quad d^2 = 4$ or $d = \pm 2$

$\therefore$ Required parts are $14 - 3 \times 2, 14 - 2, 14 + 2, 14 + 3 \times 2$

Or $\quad 14 - 3(-2), 14 + 2, 14 - 2, 14 + 3(-2)$

$i.e.,$ $\quad 8, 12, 16, 20$

Que 4. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

Sol. Let $'a'$ be the first term and $'d'$ be the common difference.

nth term of AP is $\quad a_n = a + (n - 1)d$

And sum of AP is $\quad S_n = \frac{n}{2}[2a + 9d]$

Sum of first 10 terms = $210 = \frac{10}{2}[2a + 9d]$

$\Rightarrow \quad 42 = 2a + 9d \Rightarrow \quad 2a + 9d = 42 \qquad \ldots(i)$

15^{th} term from the last $= (50 - 15 + 1th) = 36th$ term

$\Rightarrow \quad a_{36} = a + 35d$

Sum of last 15 terms $= 2565 = \frac{15}{2}[2a_{36} + (15 - 1)d]$

$\Rightarrow \quad 2565 = \frac{15}{2}[2(a + 35d) + 14d$

$\Rightarrow \quad 2565 = 15[a + 35d + 7d]$

$\Rightarrow \quad a + 42d = 171$

Subtracting (ii) from (i) we get

$9d - 84d = 42 - 342 \qquad \Rightarrow \qquad 75d = 300$

$\Rightarrow \quad d = \frac{300}{75} = 4$

Putting the value of d in (ii)

$42 \times 4 + a = 171 \qquad \Rightarrow \qquad a = 171 - 168$

$\Rightarrow \quad a = 3$

So, the AP formed is $3, 7, 11, 15, \ldots \ldots$ and 199.

Que 5. If S_n denotes the sum of the first n terms of an AP, prove that $S_{30} = 3(S_{30} = 3(S_{20} - S_{10})$.

Sol. $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \qquad \Rightarrow \qquad S_{30} = 30a + 435d \qquad \ldots(i)$

$\Rightarrow \; S_{20} = \frac{20}{2}[2a + 19d] \qquad \Rightarrow \qquad S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \qquad \Rightarrow \qquad S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30} \qquad [\text{From })(i)]$

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

Que 6. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?

Sol. Let total time be n minutes
Total distance covered by thief $= 100n$ metres

Total distance covered by policeman $= 100 + 110 + 120 + \cdots + (n-1)$ terms

$\therefore \qquad 100n = \frac{n-1}{2}[100(2) + (n-2)10]$

$\Rightarrow \qquad 200n = (n-1)(180 + 10n) \qquad \Rightarrow \qquad 10n^2 - 30n - 180 = 0$

$\Rightarrow \qquad n = 6$

Policeman took $(n-1) = (6-1) = 5$ minutes to catch the thief.

Que 7. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceeding the house numbered X is equal to sum of the numbers of houses following X. Find value of X.

Sol. The numbers of houses are 1, 2, 3, 4 ……….49.

The numbers of the houses are in AP, where $a = 1$ and $d = 1$

Sum of n terms of an $AP = \frac{n}{2}[2a + (n-1)d]$

Let Xth number house be the required house.

Sum of number of houses preceding Xth house is equal to $S_{x-1} i.e.,$

$S_{X-1} = \frac{X-1}{2}[2a + (X-1-1)d] \qquad \Rightarrow \qquad S_{X-1} = \frac{X-1}{2}[2 + (X-2)]$

$\Rightarrow \qquad S_{X-1} = \frac{X-1}{2}[2 + X - 2 \qquad \Rightarrow \qquad S_{X-1} = \frac{X(X-1)}{2}$

Sum of numbers of houses following X^{th} house is equal to $S_{49} - S_X$

$= \frac{49}{2}[2a + (49-1)d] - \frac{X}{2}(2a + (X-1)d$

$= \frac{49}{2}[2 + 48] - \frac{X}{2}(2 + X - 1) = \frac{49}{2}(50) - \frac{X}{2}(X+1)$

$= 25(49) - \frac{X}{2}(X+1)$

Now, we are given that

Sum of number of houses before X is equal to sum of number of houses after X.

$i.e., S_{X-1} = S_{49} - S_X$

$\Rightarrow \qquad \frac{X(X-1)}{2} = 25(49) - X\frac{(X+1)}{2} \qquad \Rightarrow \qquad \frac{X^2}{2} - \frac{X}{2} = 1225 - \frac{X^2}{2} - \frac{X}{2}$

$\Rightarrow \qquad X^2 = 1225 \qquad \Rightarrow \qquad X = \sqrt{1225}$

$\Rightarrow \qquad X = \pm 35$

Since number of houses is positive integer, $\therefore X = 35$