

## Exercise 7.5

### Answer 1e.

In order to condense the given expression, use the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_3 2x + \log_3 y = \log_3 2xy$$

Therefore, to condense the expression  $\log_3 2x + \log_3 y$ , you need to use the product property of logarithms.

### Answer 1gp.

Apply the quotient property of logarithms.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$

and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_6 \frac{5}{8} = \log_6 5 - \log_6 8$$

Substitute 0.898 for  $\log_6 5$  and 1.161 for  $\log_6 8$  and simplify.

$$\begin{aligned}\log_6 5 - \log_6 8 &\approx 0.898 - 1.161 \\ &= -0.263\end{aligned}$$

Thus, the logarithm evaluates to  $-0.263$ .

### Answer 1q.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 16 when 4 is raised to that number.

The number 4 to a power of 2 gives 16.

$$4^2 = 16$$

We know that  $\log_b y$  is defined as  $\log_b y = x$  if and only if  $b^x = y$ , where  $b$  and  $y$  are positive numbers and  $b \neq 0$ .

In the given equation,  $b$  is 4,  $y$  is 16, and  $x$  is 2. Thus,  $\log_4 16 = 2$ .

### Answer 2e.

If  $a, b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using common Describing two ways to evaluate  $\log_7 12$  using a calculator

Using common logarithms in first way

$$\log_7 12$$

$$\begin{aligned} &= \frac{\log 12}{\log 7} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 1.277 && [\text{Simplify}] \end{aligned}$$

Using natural logarithms in Second way

$$\log_7 12$$

$$\begin{aligned} &= \frac{\ln 12}{\ln 7} && \left[ \text{Use the formula } \log_c a = \frac{\ln a}{\ln c} \right] \\ &\approx 1.277 && [\text{Simplify}] \end{aligned}$$

Therefore the answer is  $\boxed{1.277}$ .

### Answer 2gp.

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n$$

Using  $\log_6 5 \approx 0.898$  and  $\log_6 8 \approx 1.161$  to evaluate the logarithm

$$\log_6 40$$

$$\begin{aligned} &= \log_6 (5 \cdot 8) && [\text{Write 40 as } 5 \cdot 8] \\ &= \log_6 5 + \log_6 8 && [\text{Product property}] \\ &\approx 0.898 + 1.161 && [\text{Use the given values of } \log_6 5 \text{ and } \log_6 8] \\ &= 2.059 && [\text{Add}] \end{aligned}$$

Therefore the answer is  $\boxed{2.059}$ .

### Answer 2q.

To help you find the value of, ask yourself what power of  $b$  gives you  $y$ .

Evaluating the logarithm without using a calculator

$$\log_5 1$$

5 to what power gives 1?

$$\log_5 1 = 0$$

Therefore the answer is  $\boxed{0}$ .

**Answer 3e.**

Let us use the quotient property of logarithms.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln 6 - \ln 2 = \ln \frac{6}{2}$$

Simplify.

$$\ln \frac{6}{2} = \ln 3$$

Thus, the correct choice is **B**.

**Answer 3gp.**

Rewrite 64 as  $8^2$ .

$$\log_6 64 = \log_6 8^2$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$ , and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_6 8^2 = 2 \log_6 8$$

Substitute 1.161 for  $\log_6 8$  and simplify.

$$\begin{aligned} 2 \log_6 8 &\approx 2(1.161) \\ &= 2.322 \end{aligned}$$

Thus, the logarithm evaluates to 2.322.

**Answer 3q.**

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 8 when 8 is raised to that number.

The number 8 to a power of 1 gives 8.

$$8^1 = 8$$

We know that  $\log_b y$  is defined as  $\log_b y = x$  if and only if  $b^x = y$ , where  $b$  and  $y$  are positive numbers and  $b \neq 0$ .

In the given equation,  $b$  is 8,  $y$  is 8, and  $x$  is 1. Thus,  $\log_8 8 = 1$ .

**Answer 4e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Matching the expression with the logarithm that has the same value

$$2 \ln 6$$

$$= \ln 6^2 \quad [\text{Power property}]$$

$$= \ln 36 \quad [\text{Simplify}]$$

Therefore the answer is option D.  $\ln 36$ .

**Answer 4gp.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n$$

Using  $\log_6 5 \approx 0.898$  and  $\log_6 8 \approx 1.161$  to evaluate the logarithm

$$\log_6 125$$

$$= \log_6 (5 \cdot 5 \cdot 5) \quad [\text{Write 40 as } 5 \cdot 8]$$

$$= \log_6 5 + \log_6 5 + \log_6 5 \quad [\text{Product property}]$$

$$\approx 0.898 + 0.898 + 0.898 \quad [\text{Use the given values of } \log_6 5]$$

$$= 2.694 \quad [\text{Add}]$$

Therefore the answer is 2.694.

**Answer 4q.**

To help you find the value of, ask yourself what power of  $b$  gives you  $y$ .

Evaluating the logarithm without using a calculator

$$\log_{1/2} 32$$

$1/2$  to what power gives 32?

$$\log_{1/2} 32 = -5$$

Therefore the answer is -5.

**Answer 5e.**

Let us use the power property of logarithm.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$6 \ln 2 = \ln 2^6$$

Simplify.

$$\ln 2^6 = \ln 64$$

Thus, the correct choice is **A**.

### Answer 5gp.

First, apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$ , and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 3x^4 = \log 3 + \log x^4$$

Next, let us apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 3 + \log x^4 = \log 3 + 4 \log x$$

Thus, the expression can be expanded as  $\log 3 + 4 \log x$ .

### Answer 5q.

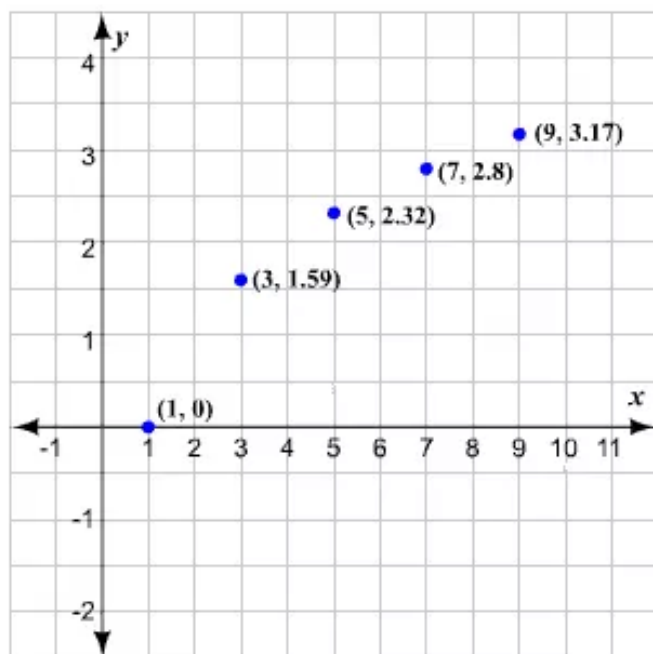
In order to graph the function, choose some values for  $x$  and find the corresponding  $y$ -values.

Organize the results in a table.

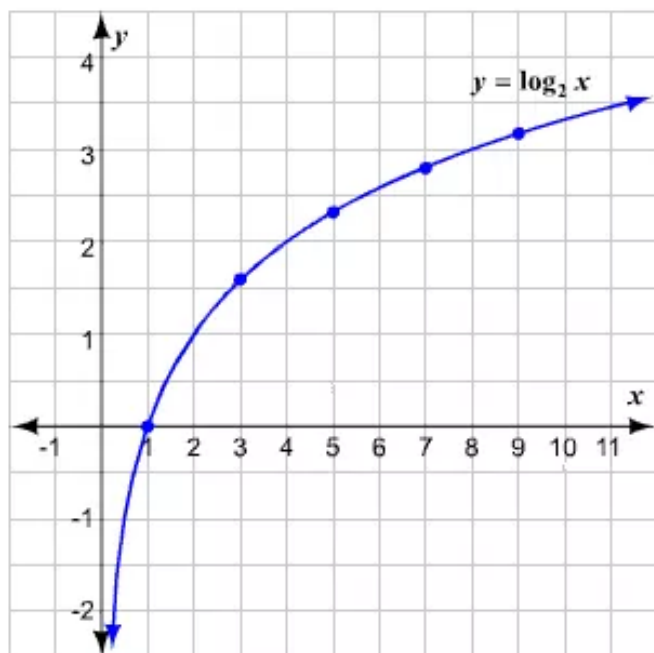
$x$	1	3	5	7	9
$y$	0	1.59	2.32	2.80	3.17

The points are  $(1, 0)$ ,  $(3, 1.59)$ ,  $(5, 2.32)$ ,  $(7, 2.80)$ , and  $(9, 3.17)$ .

Plot the points on a coordinate plane.



Join the points using a smooth curve.



The set of input values is the domain and the set of output values is the range. The domain consists of all nonnegative values of  $x$  and the range consists of all the values of  $y$ .

Thus, the domain of the given function is  $x > 0$  and the range of the given function is all real numbers.

#### Answer 6e.

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

Matching the expression with the logarithm that has the same value

$$\ln 6 + \ln 2$$

$$= \ln(6 \cdot 2) \quad [\text{Product property}]$$

$$= \ln 12 \quad [\text{Simplify}]$$

Therefore the answer is option C.  $\ln 12$ .

#### Answer 6gp.

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

**Answer 7e-1.**

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

Apply the quotient property.

$$\log \frac{12}{4} = \log 12 - \log 4$$

Substitute 0.602 for  $\log 4$  and 1.079 for  $\log 12$  and simplify.

$$\begin{aligned}\log 12 - \log 4 &\approx 1.079 - 0.602 \\ &= 0.477\end{aligned}$$

Thus, the logarithm evaluates to 0.477.

**Answer 7gp.**

The change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a, b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula for common logarithms.

$$\log_5 8 = \frac{\log 8}{\log 5}$$

Use a calculator and evaluate.

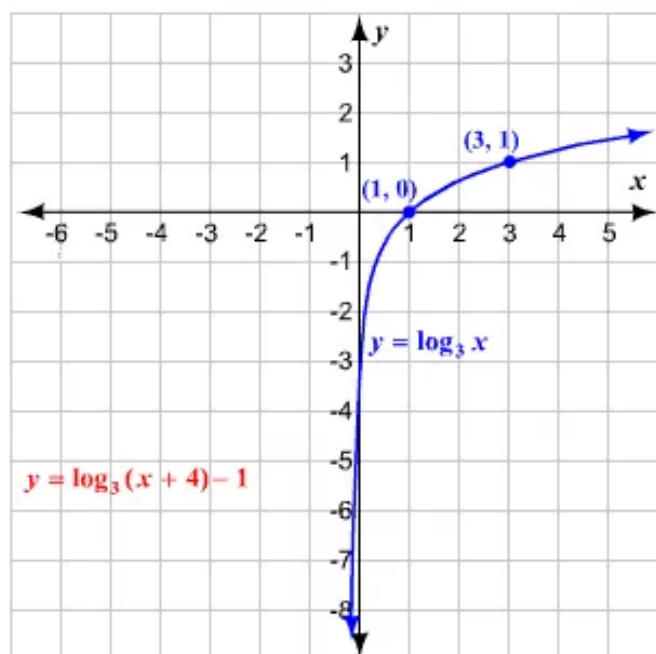
$$\frac{\log 8}{\log 5} \approx 1.292$$

Thus,  $\log_5 8$  evaluates to 1.292.

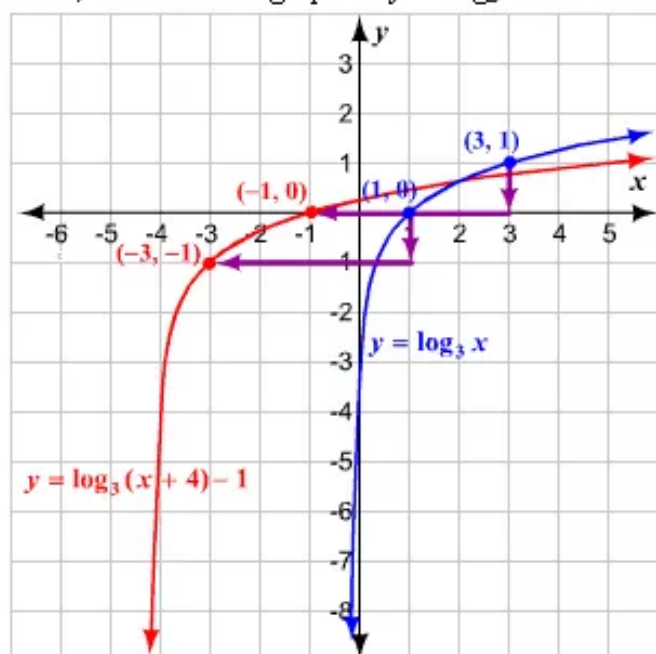
### Answer 7q.

In order to graph the given function, first, graph the parent function  $y = \log_3 x$  on a coordinate plane.

The graph passes through the points  $(1, 0)$  and  $(3, 1)$ .



Now, translate the graph of  $y = \log_3 x$  left 4 units and down 1 unit.



The resulting graph passes through the points  $(-3, -1)$  and  $(-1, 0)$ .

The set of input values is the domain and the set of output values is the range.

Thus, the domain of the given function is  $x > -4$  and the range of the given function is all real numbers.



**Answer 8e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n$$

Using  $\log 4 \approx 0.602$  and  $\log 12 \approx 1.079$  to evaluate the logarithm

$$\log 48$$

$$= \log(4 \cdot 12) \quad [\text{Write 48 as } 4 \cdot 12]$$

$$= \log 4 + \log 12 \quad [\text{Product property}]$$

$$\approx 0.602 + 1.079 \quad [\text{Use the given values of } \log 4 \text{ and } \log 12]$$

$$= 1.681 \quad [\text{Add}]$$

Therefore the answer is  $\boxed{1.681}$ .

**Answer 8gp.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_8 14$$

$$= \frac{\log 14}{\log 8} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

$$\approx 1.269 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{1.269}$ .

**Answer 8q.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b (mn) = \log_b m + \log_b n \quad [\text{Product property}]$$

Expanding the expression

$$\log_2 5x$$

$$= \log_2 (5 \cdot x) \quad [\text{Write } 5x \text{ as } 5 \cdot x]$$

$$= \log_2 5 + \log_2 x \quad [\text{Product property}]$$

$$= \frac{\log 5 + \log x}{\log 2}$$

Therefore the answer is  $\boxed{\frac{\log 5 + \log x}{\log 2}}$ .

**Answer 9e.**

Rewrite 16 as  $4^2$ .

$$\log 16 = \log 4^2$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 4^2 = 2 \log 4$$

Substitute 0.602 for  $\log 4$  and simplify.

$$\begin{aligned} 2 \log 4 &\approx 2(0.602) \\ &= 1.204 \end{aligned}$$

Thus, the logarithm evaluates to 1.204.

**Answer 9gp.**

The change-of-base formula is  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$ , and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula for common logarithms.

$$\log_{26} 9 = \frac{\log 9}{\log 26}$$

Use a calculator and evaluate.

$$\frac{\log 9}{\log 26} \approx 0.674$$

Thus,  $\log_{26} 9$  evaluates to 0.674.

**Answer 9q.**

Apply the power property.

The power property of logarithm is given as  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_5 x^7 = 7 \log_5 x$$

Thus, the given expression can be expanded as  $7 \log_5 x$ .

**Answer 10e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n$$

Using  $\log 4 \approx 0.602$  to evaluate the logarithm

$$\log 64$$

$$= \log 4^3 \quad \left[ \text{Write } 64 \text{ as } 4^3 \right]$$

$$= 3 \log 4 \quad \left[ \text{Power property} \right]$$

$$= 3(0.602) \quad \left[ \text{Use the given value of } \log 4 \right]$$

$$= 1.806 \quad \left[ \text{Multiply} \right]$$

Therefore the answer is  $\boxed{1.806}$ .

### Answer 10gp.

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_{12} 30$$

$$= \frac{\log 30}{\log 12} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

$$\approx 1.369 \quad \left[ \text{Simplify} \right]$$

Therefore the answer is  $\boxed{1.369}$ .

### Answer 10q.

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b (mn) = \log_b m + \log_b n \quad \left[ \text{Product property} \right]$$

$$\log_b (m^n) = n \log_b m \quad \left[ \text{Power property} \right]$$

Expanding the expression

$$\ln 5xy^3$$

$$= \ln(5 \cdot x \cdot y^3) \quad \left[ \text{Write } 5xy^3 \text{ as } 5 \cdot x \cdot y^3 \right]$$

$$= \ln 5 + \ln x + \ln y^3 \quad \left[ \text{Product property} \right]$$

$$= \ln 5 + \ln x + 3 \ln y \quad \left[ \text{Power property} \right]$$

Therefore the answer is  $\boxed{\ln 5 + \ln x + 3 \ln y}$ .

### Answer 11e.

Rewrite 144 as  $12^2$ .

$$\log 144 = \log 12^2$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 12^2 = 2 \log 12$$

Substitute 1.079 for  $\log 12$  and simplify.

$$\begin{aligned} 2 \log 12 &\approx 2(1.079) \\ &= 2.158 \end{aligned}$$

Thus, the logarithm evaluates to 2.158.

### Answer 11gp.

Let  $I$  be the original intensity. Then, the tripled intensity of the sound will be  $3I$ .

The increase in loudness will be the difference in loudness of the two intensities  $L(3I)$  and  $L(I)$  which is equal to  $L(3I) - L(I)$ .

We know that the loudness  $L(I)$  of the sound in decibels is given by  $L(I) = 10 \log \frac{I}{I_0}$ , where  $I$  is the intensity, and  $I_0$  is the intensity of a barely audible sound which is about  $10^{-12}$  watts per square meter. Thus,  $L(3I) = 10 \log \frac{3I}{I_0}$ .

Substitute  $10 \log \frac{I}{I_0}$  for  $I$ , and  $10 \log \frac{3I}{I_0}$  for  $10I$  in  $L(3I) - L(I)$ .

$$L(3I) - L(I) = 10 \log \frac{3I}{I_0} - 10 \log \frac{I}{I_0}$$

Factor out the common term 10.

$$10 \log \frac{3I}{I_0} - 10 \log \frac{I}{I_0} = 10 \left( \log \frac{3I}{I_0} - \log \frac{I}{I_0} \right)$$

Apply the product property of logarithms on the term  $\log \frac{3I}{I_0}$  and expand the logarithmic

expression. The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$10 \left( \log \frac{3I}{I_0} - \log \frac{I}{I_0} \right) = 10 \left( \log 3 + \log \frac{I}{I_0} - \log \frac{I}{I_0} \right)$$

Simplify.

$$\begin{aligned}10 \left( \log 3 + \log \frac{I}{I_0} - \log \frac{I}{I_0} \right) &= 10 \log 3 \\ &\approx 10(0.48) \\ &\approx 4.8\end{aligned}$$

The loudness increases by about 4.8 decibels.

### Answer 11q.

First, apply the quotient property.

The quotient property of logarithm is given as  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_3 \frac{6y^4}{x^8} = \log_3 6y^4 - \log_3 x^8$$

Next, let us apply the product property.

The product property of logarithm is given as  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_3 6y^4 - \log_3 x^8 = \log_3 6 + \log_3 y^4 - \log_3 x^8$$

Now, apply the power property.

The power property of logarithm is given as  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_3 6 + \log_3 y^4 - \log_3 x^8 = \log_3 6 + 4 \log_3 y - 8 \log_3 x$$

Thus, the expression can be expanded as  $\log_3 6 + 4 \log_3 y - 8 \log_3 x$ .

### Answer 12e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Using  $\log 4 \approx 0.602$  and  $\log 12 \approx 1.079$  to evaluate the logarithm

$$\begin{aligned}\log \frac{1}{3} \\&= \log \frac{4}{12} && \left[ \text{Write } \frac{1}{3} \text{ as } \frac{4}{12} \right] \\&= \log 4 - \log 12 && [\text{Quotient property}] \\&\approx 0.602 - 1.079 && [\text{Use the given values of } \log 4 \text{ and } \log 12] \\&= 0.477 && [\text{Add}]\end{aligned}$$

Therefore the answer is  $\boxed{-0.477}$ .

### Answer 12q.

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Condensing the expression

$$\begin{aligned}\log_3 5 - \log_3 20 \\&= \log_3 \left( \frac{5}{20} \right) && [\text{Quotient property}] \\&= \log_3 \left( \frac{1}{4} \right) && [\text{Divide}]\end{aligned}$$

Therefore the answer is  $\boxed{\log_3 \left( \frac{1}{4} \right)}$ .

### Answer 13e.

Rewrite  $\frac{1}{4}$  as  $4^{-1}$ .

$$\log \frac{1}{4} = \log 4^{-1}$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 4^{-1} = (-1) \log 4$$

Substitute 0.602 for  $\log 4$  and simplify.

$$\begin{aligned}(-1)\log 4 &\approx (-1)(0.602) \\ &= -0.602\end{aligned}$$

Thus, the logarithm evaluates to  $-0.602$ .

### Answer 13q.

Apply the product property.

The product property of logarithm is given as  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln 6 + \ln 4x = \ln(6 \cdot 4x)$$

Simplify.

$$\ln(6 \cdot 4x) = \ln 24x$$

Thus, we can condense the expression as  $\ln 24x$ .

### Answer 14e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Using  $\log 4 \approx 0.602$  and  $\log 12 \approx 1.079$  to evaluate the logarithm

$$\begin{aligned}\log \frac{1}{12} \\ &= \log \frac{4}{48} && \left[ \text{Write } \frac{1}{12} \text{ as } \frac{4}{48} \right] \\ &= \log 4 - \log 48 && [\text{Quotient property}] \\ &= \log 4 - \log(4 \cdot 12) && [\text{Write } 48 \text{ as } 4 \cdot 12] \\ &= \log 4 - \log 4 - \log 12 && [\text{Product property}] \\ &\approx 0.602 - 0.602 - 1.079 && [\text{Use the given values of } \log 4 \text{ and } \log 12] \\ &= -1.079 && [\text{Add}]\end{aligned}$$

Therefore the answer is  $\boxed{-1.079}$ .

### Answer 14q.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Condensing the expression

$$\log_6 5 + 3 \log_6 2$$

$$= \log_6 5 + \log_6 2^3 \quad [\text{Power property}]$$

$$= \log_6 (5 \cdot 2^3) \quad [\text{Product property}]$$

$$= \log_6 40 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{\log_6 40}$ .

### Answer 15e.

Apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_3 4x = \log_3 4 + \log_3 x$$

Thus, the expression can be expanded as  $\log_3 4 + \log_3 x$ .

### Answer 15q.

First, apply the power property.

The power property of logarithm is given as  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$4 \ln x - 5 \ln x = \ln x^4 - \ln x^5$$

Next, let us apply the quotient property.

The quotient property of logarithm is given as  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln x^4 - \ln x^5 = \ln \frac{x^4}{x^5}$$

Simplify.

$$\ln \frac{x^4}{x^5} = \ln \frac{1}{x}$$

Thus, we can condense the expression as  $\ln \frac{1}{x}$ .

### Answer 16e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad [\text{Product property}]$$



Expanding the expression

$$\ln 15x$$

$$= \ln(15 \cdot x) \quad [\text{Write } 15x \text{ as } 15 \cdot x]$$

$$= \ln 15 + \ln x \quad [\text{Product property}]$$

Therefore the answer is  $\boxed{\ln 15 + \ln x}$ .

**Answer 16q.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_3 10$$

$$= \frac{\log 10}{\log 3} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

$$\approx 2.096 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{2.096}$ .

**Answer 17e.**

First, apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 3x^4 = \log 3 + \log x^4$$

Next, let us apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log 3 + \log x^4 = \log 3 + 4 \log x$$

Thus, the expression can be expanded as  $\log 3 + 4 \log x$ .

**Answer 17q.**

Change-of-base formula is given as  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_7 14 = \frac{\log 14}{\log 7}$$

Use a calculator and evaluate.

$$\frac{\log 14}{\log 7} \approx 1.36$$

Thus,  $\log_7 14$  evaluates to 1.36.

**Answer 18e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b m^n = n \log_b m \quad [\text{Power property}]$$

Expanding the expression

$$\begin{aligned} \log_5 x^5 \\ &= 5 \log_5 x \quad [\text{Power property}] \\ &= \frac{5 \log x}{\log 5} \end{aligned}$$

Therefore the answer is  $\boxed{\frac{5 \log x}{\log 5}}$ .

**Answer 18q.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\begin{aligned} \log_5 24 \\ &= \frac{\log 24}{\log 5} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 1.253 \quad [\text{Simplify}] \end{aligned}$$

Therefore the answer is  $\boxed{1.253}$ .

**Answer 19e.**

Apply the quotient property of logarithms.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_2 \frac{2}{5} = \log_2 2 - \log_2 5$$

Thus, the expression can be expanded as  $\log_2 2 - \log_2 5$ .

**Answer 19q.**

Change-of-base formula is given as  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_8 40 = \frac{\log 40}{\log 8}$$

Use a calculator and evaluate.

$$\frac{\log 40}{\log 8} \approx 1.77$$

Thus,  $\log_8 40$  evaluates to 1.77.

**Answer 20e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Expanding the expression

$$\begin{aligned} \ln \frac{12}{5} \\ &= \ln \frac{3 \cdot 4}{5} \\ &= \ln 12 - \ln 5 \quad [\text{Quotient property}] \end{aligned}$$

Therefore the answer is  $\boxed{\ln 12 - \ln 5}$ .

**Answer 20q.**

The sound of alarm clock has an intensity of  $I = 10^{-4}$  watts per square meter. And given model  $L(I) = 10 \log \frac{I}{I_0}$ , Where  $I_0 = 10^{-12}$  watts per square meter

$$\begin{aligned}
 L(I) &= 10 \log \frac{I}{I_0} \\
 L(10^{-4}) &= 10 \log \frac{10^{-4}}{10^{-12}} && \left[ \text{Let } I = 10^{-4} \text{ and } I_0 = 10^{-12} \right] \\
 &= 10 \log (10^{-4} \cdot 10^{12}) && \left[ \text{Because } \frac{1}{n^{-a}} = n^a \right] \\
 &= 10 \log (10^{-4+12}) && \left[ \text{Because } m^a m^b = m^{a+b} \right] \\
 &= 10 \log 10^8 && \left[ \text{Simplify} \right] \\
 &= 10 \cdot 8 && \left[ \log 10^8 = 8 \right] \\
 &= 80 && \left[ \text{Multiply} \right]
 \end{aligned}$$

Thus the alarm clock's loudness  $L(10^{-4}) = 80\text{DB}$

**Answer 21e.**

Apply the quotient property.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_4 \frac{x}{3y} = \log_4 x - \log_4 3y$$

Thus, the expression can be expanded as  $\log_4 x - \log_4 3y$ .

**Answer 22e.**

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Expanding the expression

$$\begin{aligned}
 &\ln 4x^2y \\
 &= \ln 4 \cdot x^2 \cdot y && \left[ \text{Write } 4x^2y \text{ as } 4 \cdot x^2 \cdot y \right] \\
 &= \ln 4 + \ln x^2 + \ln y && \left[ \text{Product property} \right] \\
 &= \ln 4 + 2 \ln x + \ln y && \left[ \text{Power property} \right]
 \end{aligned}$$

Therefore the answer is  $\ln 4 + 2 \ln x + \ln y$ .

### Answer 23e.

First, apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_7 5x^3yz^2 = \log_7 5 + \log_7 x^3 + \log_7 y + \log_7 z^2$$

Next, let us apply the power property.

The power property of logarithm is given by  $\log_b m^n = n\log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_7 5 + \log_7 x^3 + \log_7 y + \log_7 z^2 = \log_7 5 + 3\log_7 x + \log_7 y + 2\log_7 z$$

Thus,  $\log_7 5x^3yz^2$  can be expanded as  $\log_7 5 + 3\log_7 x + \log_7 y + 2\log_7 z$ .

### Answer 24e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n\log_b m \quad \text{Power property}$$

Expanding the expression

$$\log_6 36x^2$$

$$= \log_6 6^2 x^2 \quad \left[ \text{Write } 36x^2 \text{ as } 6^2 x^2 \right]$$

$$= \log_6 6^2 + \log_6 x^2 \quad \left[ \text{Product property} \right]$$

$$= 2\log_6 6 + 2\log_6 x \quad \left[ \text{Power property} \right]$$

$$= \frac{2\log 6 + 2\log x}{\log 6}$$

Therefore the answer is  $\boxed{\frac{2\log 6 + 2\log x}{\log 6}}$ .

### Answer 25e.

First, apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln x^2 y^{\frac{1}{3}} = \ln x^2 + \ln y^{\frac{1}{3}}$$

Next, let us apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln x^2 + \ln y^{\frac{1}{3}} = 2 \ln x + \frac{1}{3} \ln y$$

Thus, the given expression can be expanded as  $2 \ln x + \frac{1}{3} \ln y$ .

### Answer 26e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Expanding the expression

$$\log 10x^3$$

$$= \log(10 \cdot x^3) \quad \left[ \text{Write } 10x^3 \text{ as } 10 \cdot x^3 \right]$$

$$= \log 10 + \log x^3 \quad \left[ \text{Product property} \right]$$

$$= \log 10 + 3 \log x \quad \left[ \text{Power property} \right]$$

Therefore the answer is  $\boxed{\log 10 + 3 \log x}$ .

### Answer 27e.

Rewrite  $\sqrt{x}$  as  $x^{\frac{1}{2}}$ .

$$\log_2 \sqrt{x} = \log x^{\frac{1}{2}}$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log x^{\frac{1}{2}} = \frac{1}{2} \log x$$

Thus, the given expression can be expanded as  $\frac{1}{2} \log x$ .

**Answer 28e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Expanding the expression

$$\begin{aligned} \ln \frac{6x^2}{y^4} &= \ln 6x^2 - \ln y^4 && [\text{Quotient property}] \\ &= \ln(6 \cdot x^2) - \ln y^4 && [\text{Write } 6x^2 \text{ as } 6 \cdot x^2] \\ &= \ln 6 + \ln x^2 - \ln y^4 && [\text{Product property}] \\ &= \ln 6 + 2 \ln x - 4 \ln y && [\text{Power property}] \end{aligned}$$

Therefore the answer is  $\boxed{\ln 6 + 2 \ln x - 4 \ln y}$ .

**Answer 29e.**

Rewrite  $\sqrt[4]{x^3}$  as  $x^{\frac{3}{4}}$ .

$$\ln \sqrt[4]{x^3} = \ln x^{\frac{3}{4}}$$

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln x^{\frac{3}{4}} = \frac{3}{4} \ln x$$

Thus, the given expression can be expanded as  $\frac{3}{4} \ln x$ .

**Answer 30e.**

Let  $b$ ,  $m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Expanding the expression

$$\begin{aligned}\log_3 \sqrt{9x} \\&= \log_3 (9x)^{1/2} \\&= \log_3 (9^{1/2} \cdot x^{1/2}) \quad \left[ \text{Write } 9x^{1/2} \text{ as } 9^{1/2} \cdot x^{1/2} \right] \\&= \log_3 9^{1/2} + \log_3 x^{1/2} \quad \left[ \text{Product property} \right] \\&= \frac{\frac{1}{2} \log 9 + \frac{1}{2} \log x}{\log 3}\end{aligned}$$

Therefore the answer is  $\boxed{\frac{\frac{1}{2} \log 9 + \frac{1}{2} \log x}{\log 3}}$ .

### Answer 31e.

The error in the expansion is that the two parts should be added and not multiplied.

Apply the product property and expand the logarithmic expression.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log_2 5x = \log_2 5 + \log_2 x$$

Thus, the expression can be expanded as  $\log_2 5 + \log_2 x$ .

### Answer 32e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Describing and correct the error in expanding the logarithmic expression

$$\ln 8x^3 = 3 \ln 8 + \ln x$$

First solve expression  $\ln 8x^3$

$$\begin{aligned}\ln 8x^3 \\&= \ln (2^3 \cdot x^3) \quad \left[ \text{Write } 8x^3 \text{ as } 2^3 \cdot x^3 \right] \\&= \ln 2^3 + \ln x^3 \quad \left[ \text{Product property} \right] \\&= 3 \ln 2 + 3 \ln x \quad \left[ \text{Power property} \right]\end{aligned}$$

Thus, error in the logarithmic expression is  $\ln 8x^3 = 3 \ln 8 + \boxed{\ln x}$ . And the correct answer is  $\boxed{3 \ln 2 + 3 \ln x}$ .



### Answer 33e.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

Apply the quotient property of logarithms.

$$\log_4 7 - \log_4 10 = \log_4 \frac{7}{10}$$

Thus, the we can condense the expression as  $\log_4 \frac{7}{10}$ .

### Answer 34e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

Condensing the expression

$$\ln 12 - \ln 4$$

$$= \ln \frac{12}{4} \quad [\text{Quotient property}]$$

$$= \ln 3 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{\ln 3}$ .

### Answer 35e.

First, apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$2 \log x + \log 11 = \log x^2 + \log 11$$

Next, let us apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log x^2 + \log 11 = \log 11x^2$$

Thus, we can condense the expression as  $\log 11x^2$ .

### Answer 36e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Condensing the expression

$$\begin{aligned} & 6 \ln x + 4 \ln y \\ &= \ln x^6 + \ln y^4 && \text{[Power property]} \\ &= \ln(x^6 \cdot y^4) && \text{[Product property]} \\ &= \ln x^6 y^4 && \text{[Simplify]} \end{aligned}$$

Therefore the answer is  $\boxed{\ln x^6 y^4}$ .

### Answer 37e.

First, apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$5 \log x - 4 \log y = \log x^5 - \log y^4$$

Next, let us apply the quotient property.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\log x^5 - \log y^4 = \log \frac{x^5}{y^4}$$

Thus, we can condense the expression as  $\log \frac{x^5}{y^4}$ .

### Answer 38e.

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Condensing the expression

$$5 \log_4 2 + 7 \log_4 x + 4 \log_4 y$$

$$\text{[Power property]} \quad \Rightarrow \log_4 2^5 + \log_4 x^7 + \log_4 y^4 \quad [1]$$

$$\text{[Product property]} \quad \Rightarrow \log_4 (2^5 \cdot x^7 \cdot y^4) \quad [2]$$

$$\text{[Simplify]} \quad \Rightarrow \log_4 32x^7 y^4 \quad [3]$$

$$\boxed{y^4}. \quad \text{Therefore the answer is } \boxed{\log_4 32x^7 y^4}.$$

**Answer 39e.**

First, apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln 40 + 2 \ln \frac{1}{2} + \ln x = \ln 40 + \ln \left( \frac{1}{2} \right)^2 + \ln x$$

Next, let us apply the product property.

The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\ln 40 + \ln \left( \frac{1}{2} \right)^2 + \ln x = \ln \left[ 40 \cdot \left( \frac{1}{2} \right)^2 \cdot x \right]$$

Simplify.

$$\begin{aligned} \ln \left[ 40 \cdot \left( \frac{1}{2} \right)^2 \cdot x \right] &= \ln \left[ 40 \cdot \left( \frac{1}{4} \right) \cdot x \right] \\ &= \ln 10x \end{aligned}$$

Thus, we can condense the expression as  $\ln 10x$ .

**Answer 40e.**

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Condensing the expression

$$\begin{aligned} &\log_5 4 + \frac{1}{3} \log_5 x \\ &= \log_5 4 + \log_5 x^{1/3} && \text{[Power property]} \\ &= \log_5 4 + \log_5 \sqrt[3]{x} && \text{[Write } x^{1/3} \text{ as } \sqrt[3]{x}] \\ &= \log_5 (4 \cdot \sqrt[3]{x}) && \text{[Product property]} \\ &= \log_5 4\sqrt{x} && \text{[Simplify]} \end{aligned}$$

Therefore the answer is  $\boxed{\log_5 4\sqrt{x}}$ .

**Answer 41e.**

First, apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$6 \ln 2 - 4 \ln y = \ln 2^6 - \ln y^4$$

Next, let us apply the quotient property.

The quotient property of logarithm is given by  $\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$\begin{aligned} \ln 2^6 - \ln y^4 &= \ln \frac{2^6}{y^4} \\ &= \ln \frac{64}{y^4} \end{aligned}$$

Thus, we can condense the expression as  $\ln \frac{64}{y^4}$ .

**Answer 42e.**

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b \frac{m}{n} = \log_b m - \log_b n \quad \text{Quotient property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

Condensing the expression

$$2(\log_3 20 - \log_3 4) + 0.5 \log_3 4$$

$$= 2 \left( \log_3 \frac{20}{4} \right) + 0.5 \log_3 4 \quad [\text{Quotient property}]$$

$$= 2 \log_3 5 + \frac{1}{2} \log_3 4 \quad \left[ \text{Simplify; write } 0.5 \text{ as } \frac{1}{2} \right]$$

$$= \log_3 5^2 + \log_3 4^{1/2} \quad [\text{Power property}]$$

$$= \log_3 5^2 + \log_3 \sqrt{4} \quad [\text{Write } 4^{1/2} \text{ as } \sqrt{4}]$$

$$= \log_3 25 + \log_3 2 \quad [\text{Simplify}]$$

$$= \log_3 50 \quad [\text{Product property}]$$

Therefore the answer is  $\boxed{\log_3 50}$ .

**Answer 43e.**

Apply the power property.

The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$3 \log_4 6 = \log_4 6^3$$

Simplify.

$$\log_4 6^3 = \log_4 216$$

Thus, the correct answer is choice C.

**Answer 44e.**

Let  $b, m$  and  $n$  be positive numbers such that  $b \neq 1$ .

$$\log_b mn = \log_b m + \log_b n \quad \text{Product property}$$

$$\log_b m^n = n \log_b m \quad \text{Power property}$$

The option D is not correct. Because

$$\log_3 48 = \log_3 8 + 2 \log_3 3 \quad [\text{Given}]$$

Firstly solve right-hand side and compare each statements of left-hand side

$$= \log_3 8 + \log_3 3^2 \quad [\text{Power property}]$$

$$= \log_3 (8 \cdot 3^2) \quad [\text{Write 48 as } 16 \cdot 3]$$

$$= \log_3 72 \quad [\text{Product property}]$$

Thus, the statement **D** is not correct.

**Answer 45e.**

The change-of-base formula is  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a, b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula.

$$\log_4 7 = \frac{\log 7}{\log 4}$$

Use a calculator and evaluate.

$$\frac{\log 7}{\log 4} \approx 1.404$$

Thus,  $\log_4 7$  evaluates to 1.404.

**Answer 46e.**

If  $a, b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm  $\log_5 13$

$$\begin{aligned} &= \frac{\log 13}{\log 5} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 1.594 && [\text{Simplify}] \end{aligned}$$

Therefore the answer is  $\boxed{1.594}$ .

#### Answer 47e.

The change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_3 15 = \frac{\log 15}{\log 3}$$

Use a calculator and evaluate.

$$\frac{\log 15}{\log 3} \approx 2.465$$

Thus,  $\log_3 15$  evaluates to 2.465.

#### Answer 48e.

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm  $\log_8 22$

$$\begin{aligned} &= \frac{\log 22}{\log 8} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 1.487 && [\text{Simplify}] \end{aligned}$$

Therefore the answer is  $\boxed{1.487}$ .

#### Answer 49e.

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_3 6 = \frac{\log 6}{\log 3}$$

Use a calculator and evaluate.

$$\frac{\log 6}{\log 3} \approx 1.631$$

Thus,  $\log_3 6$  evaluates to 1.631.

**Answer 50e.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_5 14$$

$$\begin{aligned} &= \frac{\log 14}{\log 5} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 1.639 && [\text{Simplify}] \end{aligned}$$

Therefore the answer is 1.639.

**Answer 51e.**

The change-of-base formula is  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_6 17 = \frac{\log 17}{\log 6}$$

Use a calculator and evaluate.

$$\frac{\log 17}{\log 6} \approx 1.581$$

Thus,  $\log_6 17$  evaluates to 1.581.

**Answer 52e.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_2 28$$

$$= \frac{\log 28}{\log 2} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

$$\approx 4.808 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{4.808}$ .

### Answer 53e.

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a, b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_7 19 = \frac{\log 19}{\log 7}$$

Use a calculator and evaluate.

$$\frac{\log 19}{\log 7} \approx 1.513$$

Thus,  $\log_7 19$  evaluates to 1.513.

### Answer 54e.

If  $a, b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_4 48$$

$$= \frac{\log 48}{\log 4} \quad \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

$$\approx 2.793 \quad [\text{Simplify}]$$

Therefore the answer is  $\boxed{2.793}$ .



**Answer 55e.**

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_9 27 = \frac{\log 27}{\log 9}$$

Use a calculator and evaluate.

$$\frac{\log 27}{\log 9} \approx 1.5$$

Thus,  $\log_9 27$  evaluates to 1.5.

**Answer 56e.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\log_8 32$$

$$= \frac{\log 32}{\log 8}$$

$$\approx 1.667$$

$$\left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right]$$

[Simplify]

Therefore the answer is 1.667.

**Answer 57e.**

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_6 \frac{24}{5} = \frac{\log \frac{24}{5}}{\log 6}$$

Use a calculator and evaluate.

$$\frac{\log \frac{24}{5}}{\log 6} \approx 0.875$$

Thus,  $\log_6 \frac{24}{5}$  evaluates to 0.875.

### Answer 58e.

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\begin{aligned} \log_2 \frac{15}{7} &= \log_2 15 - \log_2 7 && [\text{Quotient property}] \\ &= \frac{\log 15}{\log 2} - \frac{\log 7}{\log 2} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 3.907 - 2.808 && [\text{Simplify}] \\ &= 1.099 && [\text{Subtract}] \end{aligned}$$

Therefore the answer is 1.099.

### Answer 59e.

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

Apply the change-of-base formula using common logarithms.

$$\log_3 \frac{9}{40} = \frac{\log \frac{9}{40}}{\log 3}$$

Use a calculator and evaluate.

$$\frac{\log \frac{9}{40}}{\log 3} \approx -1.358$$

Thus,  $\log_3 \frac{9}{40}$  evaluates to -1.358.

**Answer 60e.**

If  $a$ ,  $b$  and  $c$  be positive numbers such that  $c \neq 1$ .

$$\text{In particular, } \log_c a = \frac{\log a}{\log c}$$

Using the change-of-base formula to evaluate the logarithm

$$\begin{aligned} \log_7 \frac{3}{16} &= \log_7 3 - \log_7 16 && [\text{Quotient property}] \\ &= \frac{\log 3}{\log 7} - \frac{\log 16}{\log 7} && \left[ \text{Use the formula } \log_c a = \frac{\log a}{\log c} \right] \\ &\approx 0.565 - 1.425 && [\text{Simplify}] \\ &= -0.86 && [\text{Subtract}] \end{aligned}$$

Therefore the answer is  $\boxed{-0.86}$ .

**Answer 61e.**

When we use the change-of-base formula, the base should go in the denominator. The error is that the base is written in the numerator.

Change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

When we apply this formula, we get

$$\log_3 7 = \frac{\log 7}{\log 3}.$$

$$\text{Thus, } \log_3 7 = \frac{\log 7}{\log 3}.$$

**Answer 62e.**

Sound intensity for a sound with intensity  $I$  (in watts per square meter), the loudness  $L(I)$  of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

Where  $I_0$  is the intensity of a barely audible sound (about  $10^{-12}$  watts per square meter).

a) Finding the decibel level of the sound made by barking dog, where  $I = 10^{-4} \text{ W / m}^2$ .

Substitute  $I = 10^{-4}$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$$\begin{aligned} L(10^{-4}) &= 10 \log \frac{10^{-4}}{10^{-12}} && \text{Substitute} \\ &= 10 \log 10^{-4-(-12)} && \text{Quotient property} \\ &= 10 \log 10^8 && \text{Subtract exponent} \\ &= 10 \cdot 8 && \log 10^8 = 8 \\ &= 80 && \text{Multiply} \end{aligned}$$

Thus loudness of the sound is 80 decibels.

b) Finding the decibel level of the sound made by ambulance siren, where  $I = 10^0 \text{ W / m}^2$ .

Substitute  $I = 10^0$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$$\begin{aligned} L(10^0) &= 10 \log \frac{10^0}{10^{-12}} && \text{Substitute} \\ &= 10 \log 10^{0-(-12)} && \text{Quotient property} \\ &= 10 \log 10^{12} && \text{Subtract exponent} \\ &= 10 \cdot 12 && \log 10^{12} = 12 \\ &= 120 && \text{Multiply} \end{aligned}$$

Thus loudness of the sound is 120 decibels.

c) Finding the decibel level of the sound made by bee, where  $I = 10^{-6.5} \text{ W / m}^2$ .

Substitute  $I = 10^{-6.5}$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$$\begin{aligned} L(10^{-6.5}) &= 10 \log \frac{10^{-6.5}}{10^{-12}} && \text{Substitute} \\ &= 10 \log 10^{-6.5-(-12)} && \text{Quotient property} \\ &= 10 \log 10^{5.5} && \text{Subtract exponent} \\ &= 10 \cdot 5.5 && \log 10^{5.5} = 5.5 \\ &= 55 && \text{Multiply} \end{aligned}$$

Thus loudness of the sound is 55 decibels.

**Answer 63e.**

We know that the loudness  $L(I)$  of the sound in decibels is given by  $L(I) = 10 \log \frac{I}{I_0}$ , where  $I$  is the intensity, and  $I_0$  is the intensity of a barely audible sound which is about  $10^{-12}$  watts per square meter.

Substitute  $10^3$  for  $I$  and  $10^{-12}$  in the formula.

$$L(I) = 10 \log \frac{10^3}{10^{-12}}$$

Apply the quotient property of logarithms. The quotient property of logarithm is given by

$\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m$ ,  $n$ , and  $b$  are positive numbers such that  $b \neq 1$ .

$$10 \log \frac{10^3}{10^{-12}} = 10(\log 10^3 - \log 10^{-12})$$

Simplify.

$$\begin{aligned} 10(\log 10^3 - \log 10^{-12}) &= 10[3 - (-12)] \\ &= 10(15) \\ &= 150 \end{aligned}$$

The decibel level of the trumpet with the given intensity is 150 decibels.

**Answer 64e.**

a) For the given statement  $\log_b (M+N) = \log_b M + \log_b N$ .

Let  $b=10$ ,  $M=2$  and  $N=3$ ; to show the statement is false in general.

Substitute all values in the given statement.

$$\log_{10} (2+3) = \log_{10} 2 + \log_{10} 3 \quad \text{Substitute}$$

$$\log_{10} 5 = \log_{10} 2 + \log_{10} 3 \quad \text{Add}$$

$$0.6990 = 0.3010 + 0.4771 \quad \text{Use a calculator}$$

$$0.6990 \neq 0.7781 \quad \text{Not equal}$$

Using  $b=10$ ,  $M=2$  and  $N=3$ ; we get  $0.6990 \neq 0.7781$ , so the original statement is false.

b) For the given statement  $\log_b (M-N) = \log_b M - \log_b N$ .

Let  $b=10$ ,  $M=3$  and  $N=2$ ; to show the statement is false in general.

Substitute all values in the given statement.

$$\log_{10} (3-2) = \log_{10} 3 - \log_{10} 2 \quad \text{Substitute}$$

$$\log_{10} 1 = \log_{10} 3 - \log_{10} 2 \quad \text{Subtract}$$

$$0 = 0.4771 - 0.3010 \quad \text{Use a calculator}$$

$$0 \neq 0.1761 \quad \text{Not equal}$$

Using  $b=10$ ,  $M=3$  and  $N=2$ ; we get  $0 \neq 0.1761$ , so the original statement is false.

### Answer 65e.

We have to prove the product property of logarithm,  $\log_b mn = \log_b m + \log_b n$ .

Let  $x = \log_b m$ , and  $y = \log_b n$ . By the definition of logarithm, we get  
 $m = b^x$  and  $n = b^y$ .

First, evaluate the left side of the equation. For this, substitute  $b^x$  for  $m$ , and  $b^y$  for  $n$ .  
 $\log_b mn = \log_b (b^x b^y)$

Apply the product of powers property of exponents.  
 $\log_b (b^x b^y) = \log_b (b^{x+y})$

Rewrite the equation using the property of logarithms,  $\log_b b^x = x$ .  
 $\log_b (b^{x+y}) = x + y$

Replace  $x$  with  $\log_b m$  and  $y$  with  $\log_b n$ .  
 $x + y = \log_b m + \log_b n$

Thus, the product property of logarithms is proved.

### Answer 66e.

Using the given hint and properties of exponents to prove the property of logarithms:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

We need to somehow manipulate the left sides of the equation so that it equals to the right side.

Let  $x = \log_b m$  and  $y = \log_b n$  then  $m = b^x$  and  $n = b^y$ .

Now we substitute these two expressions for  $m$  and  $n$  in the left sides of the equation.

$$\begin{aligned}\log_b \frac{m}{n} &= \log_b \frac{b^x}{b^y} && \text{Substitute} \\ &= \log_b b^{x-y} && \text{Quotient rule} \\ &= x - y && \log_b b^x = x \\ &= \log_b m - \log_b n && \text{Substitute } x = \log_b m \text{ and } y = \log_b n\end{aligned}$$

So the left side is now equal to right side so we have proven the rule.

**Answer 67e.**

We have to prove the power property of logarithm,  $\log_b m^n = n \log_b m$ .

Let  $x = \log_b m$ . By the definition of logarithm, we get  $m = b^x$ .

First, evaluate the left side of the equation. For this, substitute  $b^x$  for  $m$ .

$$\log_b m^n = \log_b (b^x)^n$$

Apply the power of a power property of exponents.

$$\log_b (b^x)^n = \log_b b^{nx}$$

Rewrite the equation using the property of logarithms,  $\log_b b^x = x$ .

$$\log_b b^{nx} = nx$$

Replace  $x$  with  $\log_b m$ .

$$nx = n \log_b m$$

Thus, the power property of logarithms is proved.

**Answer 68e.**

Using the given hint and properties of exponents to prove the property of logarithms:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

Let  $x = \log_b a$ ,  $y = \log_b c$  and  $z = \log_c a$  then  $a = b^x$ ,  $c = b^y$  and  $a = c^z$ .

$$a = c^z$$

$$\log_b a = \log_b c^z$$

Take the base  $b$  log of both sides

$$\log_b a = z \log_b c$$

Move the exponent  $z$  using the property of logarithms

$$\frac{\log_b a}{\log_b c} = z$$

Divide both sides by  $\log_b c$

$$\frac{\log_b a}{\log_b c} = \log_c a$$

Substitute the value of  $z = \log_c a$

Now, it's proved the change of base formula.



**Answer 69e.**

We know that the loudness  $L(I)$  of the sound in decibels is given by  $L(I) = 10 \log \frac{I}{I_0}$ , where  $I$  is the intensity, and  $I_0$  is the intensity of a barely audible sound which is about  $10^{-12}$  watts per square meter.

The intensity of the sound of each conversation is given as  $1.4 \times 10^{-5} \text{ W/m}^2$ . Thus, the intensity of sound for the three conversations will be  $3(1.4 \times 10^{-5}) \text{ W/m}^2$  or  $4.2 \times 10^{-5} \text{ W/m}^2$ .

Substitute  $4.2 \times 10^{-5}$  for  $I$  and  $10^{-12}$  in the formula.

$$L(I) = 10 \log \frac{4.2 \times 10^{-5}}{10^{-12}}$$

Apply the quotient property of logarithms. The quotient property of logarithm is given by

$\log_b \frac{m}{n} = \log_b m - \log_b n$ , where  $m$ ,  $n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$10 \log \frac{4.2 \times 10^{-5}}{10^{-12}} = 10 [\log (4.2 \times 10^{-5}) - \log 10^{-12}]$$

Use a calculator and simplify.

$$\begin{aligned} 10 [\log (4.2 \times 10^{-5}) - \log 10^{-12}] &= 10 [-4.38 - (-12)] \\ &\approx 10(7.6) \\ &\approx 76 \end{aligned}$$

The decibel level of the combined conversations in the room will be about 76 decibels.

**Answer 70e.**

Sound intensity for a sound with intensity  $I$  (in watts per square meter), the loudness  $L(I)$  of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

Where  $I_0$  is the intensity of a barely audible sound (about  $10^{-12}$  watts per square meter).



Finding the decibel level of the sound made by all five cars in the parking garage, where  $I = 5(3.2 \times 10^{-4}) W/m^2$ .

Substitute  $I = 5(3.2 \times 10^{-4})$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$L(10^{-4}) = 10 \log \frac{5(3.2 \times 10^{-4})}{10^{-12}}$	Substitute
$= 10 \log 16 \times 10^{-4-(-12)}$	Quotient property
$= 10 \log (16 \times 10^8)$	Subtract exponent
$= 10 \cdot 9.204$	$\log(16 \times 10^8) = 9.204$
$= 92.04$	Multiply

Thus loudness of the sound is 92.04 decibels.

### Answer 71e.

Let  $I$  be the intensity of the sound of an average TV show. Then, the intensity of the sound of TV ads will be  $10I$ .

The increase in loudness will be the difference in loudness of the two intensities  $L(10I)$  and  $L(I)$  which is equal to  $L(10I) - L(I)$ .

We know that the loudness  $L(I)$  of the sound in decibels is given by  $L(I) = 10 \log \frac{I}{I_0}$ , where  $I$  is the intensity, and  $I_0$  is the intensity of a barely audible sound which is about  $10^{-12}$  watts per square meter. Thus,  $L(10I) = 10 \log \frac{10I}{I_0}$ .

Substitute  $10 \log \frac{I}{I_0}$  for  $I$ , and  $10 \log \frac{10I}{I_0}$  for  $10I$  in  $L(10I) - L(I)$ .

$$L(10I) - L(I) = 10 \log \frac{10I}{I_0} - 10 \log \frac{I}{I_0}$$

Factor out the common term 10.

$$10 \log \frac{10I}{I_0} - 10 \log \frac{I}{I_0} = 10 \left( \log \frac{10I}{I_0} - \log \frac{I}{I_0} \right)$$

Apply the product property of logarithms on the term  $\log \frac{10I}{I_0}$  and expand the logarithmic expression. The product property of logarithm is given by  $\log_b mn = \log_b m + \log_b n$ , where  $m, n$  and  $b$  are positive numbers such that  $b \neq 1$ .

$$10 \left( \log \frac{10I}{I_0} - \log \frac{I}{I_0} \right) = 10 \left( \log 10 + \log \frac{I}{I_0} - \log \frac{I}{I_0} \right)$$

Simplify.

$$\begin{aligned} 10 \left( \log 10 + \log \frac{I}{I_0} - \log \frac{I}{I_0} \right) &= 10 \log 10 \\ &= 10(1) \\ &= 10 \end{aligned}$$

The TV ad is 10 decibels louder than that of an average TV show.

### Answer 72e.

Sound intensity for a sound with intensity  $I$  (in watts per square meter), the loudness  $L(I)$  of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

Where  $I_0$  is the intensity of a barely audible sound (about  $10^{-12}$  watts per square meter).

Finding the decibel level of the sound made by blue whale, where  $I = 10^{6.8} W / m^2$ .

Substitute  $I = 10^{6.8} W / m^2$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$L(10^4) = 10 \log \frac{10^{6.8}}{10^{-12}}$	Substitute
$= 10 \log 10^{6.8 - (-12)}$	Quotient property
$= 10 \log 10^{18.8}$	Subtract exponent
$= 10 \cdot 18.8$	$\log 10^{18.8} = 18.8$
$= 188$	Multiply

Thus loudness of the sound made by blue whale is 188 decibels .

Now, finding the decibel level of the sound made by a human, where  $I = 10^{0.8} W / m^2$ .

Substitute  $I = 10^{0.8} W / m^2$  and  $I_0 = 10^{-12}$  in the formula  $L(I) = 10 \log \frac{I}{I_0}$

$$\begin{aligned} L(10^{-4}) &= 10 \log \frac{10^{0.8}}{10^{-12}} && \text{Substitute} \\ &= 10 \log 10^{0.8 - (-12)} && \text{Quotient property} \\ &= 10 \log 10^{12.8} && \text{Subtract exponent} \\ &= 10 \cdot 12.8 && \log 10^{12.8} = 12.8 \\ &= 128 && \text{Multiply} \end{aligned}$$

Thus loudness of the sound made by a human is 128 decibels.

The blue whale's intensity is greater than the human's by a factor of  $10^6$ . The blue whale's decibel level is greater by +60. For every increase in the decibel level by +10.

### Answer 73e.

- a. Apply the power property of logarithm to expand the expression. The power property of logarithm is given by  $\log_b m^n = n \log_b m$ , where  $m$ ,  $n$ , and  $b$  are positive numbers such that  $b \neq 1$ .
- $$s = 2 \log_2 f$$
- b. Substitute each value of  $f$  in the given expression and evaluate the corresponding values of  $s$ .

Replace  $f$  with 1.414 in  $s = 2 \log_2 f$ .

$$s = 2 \log_2 1.414$$

Apply the change-of-base formula using common logarithms to evaluate. The change-of-base formula is given by  $\log_c a = \frac{\log_b a}{\log_b c}$ , where  $a$ ,  $b$  and  $c$  are positive numbers such that  $b \neq 1$  and  $c \neq 1$ .

$$2 \log_2 1.414 = 2 \left( \frac{\log 1.414}{\log 2} \right)$$

Use a calculator and evaluate.

$$\begin{aligned} 2 \left( \frac{\log 1.414}{\log 2} \right) &\approx 2(0.4997) \\ &\approx 1 \end{aligned}$$

Similarly, find the value of  $s$  for the other values of  $f$  and complete the table.

$f$	1.414	2.000	2.828	4.000	5.657	8.000	11.314	16.000
$s$	1	2	3	4	5	6	7	8

We note that the amount of light that strikes the film in the camera increases by about 1 each time.

- c. Replace  $s$  with 9 in  $s = 2 \log_2 f$  and evaluate the value of  $f$ .
- $$9 = 2 \log_2 f$$

Divide each side by 2.

$$\frac{9}{2} = \frac{2 \log_2 f}{2}$$

$$\frac{9}{2} = \log_2 f$$

Apply the definition of logarithm with base  $b$ . The logarithm of  $y$  with base  $b$  is defined as  $\log_b y = x$  if and only if  $b^x = y$ .

$$f = 2^{\frac{9}{2}}$$

$$\approx 22.627$$

Thus, the ninth  $f$ -stop will be about 22.627.

### Answer 74e.

Consider the function is:

$$s(h) = 2 \ln(100h) \quad \dots\dots (1)$$

where  $s(h)$  is the speed of wind (in knots) at an altitude of  $h$  meters.

(a)

We need to determine the amount increase in wind when the altitude doubles.

Consider  $s'$  be the speed of wind when the altitude is doubles.

Therefore

$$\begin{aligned} s'(h) &= 2 \ln(100 \cdot 2h) \\ &= 2 \ln(2 \cdot 100h) \\ &= 2 [\ln 2 + \ln(100h)] \quad [\text{Since } \ln(ab) = \ln a + \ln b] \\ &= 2 \ln 2 + 2 \ln(100h) \\ &= 1.386 + s(h) \quad [\text{From equation (1)}] \end{aligned}$$

$$s'(h) - s(h) = 1.386$$

is the amount increases in wind when the altitude doubles is 1.386 knots. Thu

(b)

From the function (1), we have

$$\begin{aligned}s(h) &= 2 \ln(100h) \\&= 2 \log_e 10 (\log_{10} 100h) && \left[ \text{Since } \log_a b = \log_a c \log_c a \right] \\&= 2 \log_e 10 (\log 10^2 + \log h) \\&= 2 \log_e 10 (2 + \log h) \\s(h) &= \frac{2}{\log e} (2 + \log h) && \left[ \text{Since } \log_a b = \frac{1}{\log_b a} \right]\end{aligned}$$

Therefore

$$\boxed{s(h) = \frac{2}{\log e} (2 + \log h)}$$

### Answer 75e.

In order to add two matrices, add the elements in corresponding positions. The resultant matrix will have the same dimensions as the matrices that are added.

$$\begin{bmatrix} 5 & -8 \\ 12 & 20 \end{bmatrix} + \begin{bmatrix} 6 & 7 \\ -2 & -19 \end{bmatrix} = \begin{bmatrix} 5+6 & -8+7 \\ 12-2 & 20-19 \end{bmatrix}$$

Add.

$$\begin{bmatrix} 5+6 & -8+7 \\ 12-2 & 20-19 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ 10 & 1 \end{bmatrix}$$

$$\text{Therefore, } \begin{bmatrix} 5 & -8 \\ 12 & 20 \end{bmatrix} + \begin{bmatrix} 6 & 7 \\ -2 & -19 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ 10 & 1 \end{bmatrix}.$$

### Answer 76e.

Subtract the matrices.

If the matrices are the same size ( $m \times n$ ), we just subtract the entries.

$$\begin{aligned}& \begin{bmatrix} -7 & 11 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} -9 & 17 \\ -13 & 1 \end{bmatrix} \\&= \begin{bmatrix} 2 & -6 \\ 19 & 2 \end{bmatrix} && \left[ \begin{array}{l} \text{Write } -7 - (-9) = 2, 11 - 17 = -6, \\ 6 - (-13) = 19, 3 - 1 = 2 \end{array} \right]\end{aligned}$$

$$\text{Thus the answer is } \boxed{\begin{bmatrix} 2 & -6 \\ 19 & 2 \end{bmatrix}}.$$

**Answer 77e.**

We multiply each element in the matrix by the scalar to multiply a matrix by a scalar.  
 Multiply each element in the matrix by 3.

$$3 \begin{bmatrix} 1.7 & 2.4 & 6.8 \\ 9.2 & 5.3 & 7.2 \end{bmatrix} = \begin{bmatrix} 3(1.7) & 3(2.4) & 3(6.8) \\ 3(9.2) & 3(5.3) & 3(7.2) \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} 3(1.7) & 3(2.4) & 3(6.8) \\ 3(9.2) & 3(5.3) & 3(7.2) \end{bmatrix} = \begin{bmatrix} 5.1 & 7.2 & 20.4 \\ 27.6 & 15.9 & 21.6 \end{bmatrix}$$

Therefore,

$$3 \begin{bmatrix} 1.7 & 2.4 & 6.8 \\ 9.2 & 5.3 & 7.2 \end{bmatrix} = \begin{bmatrix} 5.1 & 7.2 & 20.4 \\ 27.6 & 15.9 & 21.6 \end{bmatrix}.$$

**Answer 78e.**

Solving the equation

$$\sqrt{x+12} + 4 = 11$$

$$\sqrt{x+12} + 4 - 4 = 11 - 4 \quad [\text{Subtract 4 from both sides}]$$

$$\sqrt{x+12} = 7 \quad [\text{Simplify}]$$

$$(x+12)^{1/2} = 7 \quad [\text{Write } \sqrt{x+12} \text{ as } (x+12)^{1/2}]$$

$$(x+12)^{\frac{1}{2} \cdot 2} = (7)^2 \quad [\text{Squaring both sides}]$$

$$x+12 = 49 \quad [\text{Simplify}]$$

$$x+12-12 = 49-12 \quad [\text{Subtract 12 from both sides}]$$

$$x = 37 \quad [\text{Subtract}]$$

Now, to check this proposed solution replace  $x$  with 37 in original equation

Check

$$\sqrt{x+12} + 4 = 11 \quad [\text{This is the original equation}]$$

$$\sqrt{37+12} + 4 \stackrel{?}{=} 11 \quad [\text{Let } x = 37]$$

$$\sqrt{49} + 4 \stackrel{?}{=} 11 \quad [\text{Add}]$$

$$7 + 4 \stackrel{?}{=} 11 \quad [\text{Find square roots}]$$

$$11 = 11 \quad [\text{Add}]$$

Therefore the solution is  $\boxed{37}$ .

**Answer 79e.**

Subtract 6 from both sides of the equation to isolate the radical.

$$\begin{aligned}\sqrt[3]{x+10} + 6 - 6 &= 4 - 6 \\ \sqrt[3]{x+10} &= -2\end{aligned}$$

Take the cube of both sides of the equation to eliminate the radical.

$$\begin{aligned}(\sqrt[3]{x+10})^3 &= (-2)^3 \\ x + 10 &= -8\end{aligned}$$

Solve for  $x$ .

Subtract 10 from both the sides.

$$\begin{aligned}x + 10 - 10 &= -8 - 10 \\ x &= -18\end{aligned}$$

The solution to the equation appears to be  $-18$ .

Substitute  $-18$  for  $x$  in the original equation and check.

$$\begin{aligned}\sqrt[3]{x+10} + 6 &= 4 \\ \sqrt[3]{-18+10} + 6 &\stackrel{?}{=} 4 \\ \sqrt[3]{-8} + 6 &\stackrel{?}{=} 4 \\ -2 + 6 &\stackrel{?}{=} 4 \\ 4 &= 4 \quad \checkmark\end{aligned}$$

The solution is  $-18$ .

**Answer 80e.**

**Solving the equation**

$$\sqrt{x+6} = \sqrt{3x-14}$$

$$(\sqrt{x+6})^2 = (\sqrt{3x-14})^2$$

$$x+6 = 3x-14$$

$$x+6-x+14 = 3x-14-x+14$$

$$20 = 2x$$

$$\frac{20}{2} = \frac{2x}{2}$$

$$10 = x$$

[Squaring both sides]

[Simplify]

[Subtract  $x$  and Add 14 to both sides]

[Simplify]

[Divide both sides by 2]

[Simplify]

Now, to check this proposed solution replace  $x$  with 10 in original equation

Check

$$\sqrt{x+6} = \sqrt{3x-14} \quad [\text{This is the original equation}]$$

$$\sqrt{10+6} \stackrel{?}{=} \sqrt{3(10)-14} \quad [\text{Let } x = 10]$$

$$\sqrt{10+6} \stackrel{?}{=} \sqrt{30-14} \quad [\text{Multiply}]$$

$$\sqrt{16} \stackrel{?}{=} \sqrt{16} \quad [\text{Find square roots}]$$

$$4 = 4 \quad [\text{Add}]$$

Therefore the solution is  $\boxed{10}$ .

### Answer 81e.

Cube each side of the given equation.

$$\left(\sqrt[3]{2x-7}\right)^3 = \left(\sqrt[3]{8-x}\right)^3$$

$$2x-7 = 8-x$$

Add  $x+7$  to each side, and simplify.

$$2x-7+x+7 = 8-x+x+7$$

$$3x = 15$$

Divide each side by 3.

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

The solution appears to be 5.

Check  $x = 5$  in the original equation.

$$\sqrt[3]{2x-7} = \sqrt[3]{8-x}$$

$$\sqrt[3]{2(5)-7} \stackrel{?}{=} \sqrt[3]{8-5}$$

$$\sqrt[3]{10-7} \stackrel{?}{=} \sqrt[3]{3}$$

$$\sqrt[3]{3} \stackrel{?}{=} \sqrt[3]{3}$$

$$1 = 1 \checkmark$$

The solution is 5.



**Answer 82e.**

Solving the equation

$$\sqrt{x-1} = x-3$$

$$(\sqrt{x-1})^2 = (x-3)^2 \quad [\text{Squaring both sides}]$$

$$x-1 = x^2 - 6x + 9 \quad [\text{Simplify; } (A-B)^2 = A^2 - 2AB + B^2]$$

$$x^2 - 7x + 10 = 0 \quad [\text{Subtract } x \text{ and Add } 1 \text{ to both sides}]$$

$$(x-2)(x-5) = 0 \quad [\text{Factor}]$$

$$x-2 = 0 \quad \text{or} \quad x-5 = 0$$

$$x = 2 \quad \text{or} \quad x = 5$$

Now, to check this proposed solution replace  $x$  with 2 in original equation

Check

$$\sqrt{2-1} = x-3 \quad [\text{This is the original equation}]$$

$$\sqrt{2-1} \stackrel{?}{=} 2-3 \quad [\text{Let } x = 2]$$

$$\sqrt{1} \stackrel{?}{=} -1 \quad [\text{Simplify}]$$

$$\pm 1 \stackrel{?}{=} -1 \quad [\text{Find square roots}]$$

$$-1 = -1$$

Now, Let  $x = 5$  in the original equation

$$\sqrt{2-1} = x-3 \quad [\text{This is the original equation}]$$

$$\sqrt{5-1} \stackrel{?}{=} 5-3 \quad [\text{Let } x = 5]$$

$$\sqrt{4} \stackrel{?}{=} 2 \quad [\text{Simplify}]$$

$$2 = 2 \quad [\text{Find square roots}]$$

Therefore the solution is  $\boxed{2, 5}$ .**Answer 83e.**

Take the square of both sides of the equation.

$$(x+2)^2 = (\sqrt{9x+28})^2$$

Simplify the left side, and expand the right side of the equation.

$$x^2 + 4x + 4 = 9x + 28$$

Write the equation in standard form.

$$x^2 + 4x + 4 - 9x - 28 = 0$$

$$x^2 - 5x - 24 = 0$$

Factor the equation.

$$(x + 3)(x - 8) = 0$$

Apply the zero-product property and solve for  $x$ .

$$x + 3 = 0 \quad \text{or} \quad x - 8 = 0$$

$$x = -3 \quad \text{or} \quad x = 8$$

The solution to the equation appears to be  $-3$  or  $8$ .

Check the solutions by substituting them in the original equation.

Check  $x = -3$

$$x + 2 = \sqrt{9x + 28}$$

$$-3 + 2 \stackrel{?}{=} \sqrt{9(-3) + 28}$$

$$-1 \stackrel{?}{=} \sqrt{-27 + 28}$$

$$-1 = 1 \quad \times$$

Check  $x = 8$

$$x + 2 = \sqrt{9x + 28}$$

$$8 + 2 \stackrel{?}{=} \sqrt{9(8) + 28}$$

$$10 \stackrel{?}{=} \sqrt{72 + 28}$$

$$10 = 10 \quad \checkmark$$

The solution is  $8$ .

#### Answer 84e.

Using a calculator to evaluate the expression

$$e^8$$

$$= 2,980.957987$$

$$\left[ \text{Use the calculator to find } e^8 = 2,980.957987 \right]$$

Therefore the answer is  $\boxed{2,980.957987}$ .

#### Answer 85e.

First, you have to press the  $\boxed{2\text{nd}}$  key on a calculator. Then, press the  $\boxed{e^x}$  key and enter  $\boxed{-}$  key, followed by the number  $6$  in the calculator. Now, press the  $\boxed{)}$  key.

Finally, press the  $\boxed{\text{ENTER}}$  key to get the result.

The display is  $0.002478752$ . This result might vary slightly depending on the calculator you use.

Round the result to the nearest thousandth.

$$0.002478752 \approx 0.002$$

Thus, the value of  $e^{-6}$  is about  $0.002$ .

**Answer 86e.**

Using a calculator to evaluate the expression

$$e^{3.5}$$

$$= 33.1155 \quad \left[ \text{Use the calculator to need } e^{3.5} = 33.1155 \right]$$

Therefore the answer is  $\boxed{33.1155}$ .

**Answer 87e.**

First, you have to press the  $\boxed{2\text{nd}}$  key on a calculator. Then, press the  $\boxed{e^x}$  key and enter  $\boxed{-}$  key, followed by the number 0.4 in the calculator. Now, press the  $\boxed{)}$  key.

Finally, press the  $\boxed{\text{ENTER}}$  key to get the result.

The display is 0.670320046. This result might vary slightly depending on the calculator you use.

Round the result to the nearest thousandth.

$$0.670320046 \approx 0.670$$

Thus, the value of  $e^{-0.4}$  is about 0.670.

**Answer 88e.**

Using a calculator to evaluate the expression

$$\log 12$$

$$= 1.079181246 \quad \left[ \text{Use the calculator to need } \log 12 = 1.079181246 \right]$$

Therefore the answer is  $\boxed{1.079181246}$ .

**Answer 89e.**

First, you have to press the  $\boxed{\log}$  key on a calculator. Then, press the number 1.8 in the calculator. Now, press the  $\boxed{)}$  key.

Finally, press the  $\boxed{\text{ENTER}}$  key to get the result.

The display is 0.255272505. This result might vary slightly depending on the calculator you use.

Round the result to the nearest thousandth.

$$0.255272505 \approx 0.255$$

Thus, the value of  $\log 1.8$  is about 0.255.

**Answer 90e.**

Using a calculator to evaluate the expression  
 $\ln 24$

$$= 1.380211242 \quad \left[ \text{Use the calculator to need } \ln 24 = 1.380211242 \right]$$

Therefore the answer is  $\boxed{1.380211242}$ .

**Answer 91e.**

First, you have to press the  $\boxed{\ln}$  key on a calculator. Then, press the number 8.49 in the calculator. Now, press the  $\boxed{)}$  key.

Finally, press the  $\boxed{\text{ENTER}}$  key to get the result.

The display is 2.138889. This result might vary slightly depending on the calculator you use.

Round the result to the nearest thousandth.  
 $2.138889 \approx 2.139$

Thus, the value of  $\ln 8.49$  is about 2.139.