CBSE Test Paper 04
Chapter 8 Gravitation

1. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth? **1**

   a. Larger by a factor of 1.23
   b. Smaller by a factor of 0.5
   c. Larger by a factor of 1.11
   d. Smaller by a factor of 0.63

2. The escape speed of a projectile on the earth's surface is 11.2 km sec^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets. **1**

   a. 29.7 km/s
   b. 31.6 km/s
   c. 28.7 km/s
   d. 32.7 km/s

3. Deimos, a moon of Mars, is about 12 km in diameter with mass 2.0 $\times 10^{15}$ kg. Suppose you are stranded alone on Deimos and want to play a one-person game of baseball. You would be the pitcher, and you would be the batter! With what speed would you have to throw a baseball so that it would go into a circular orbit just above the surface and return to you so you could hit it? **1**

   a. 4.7 m/s
   b. 5.1 m/s
   c. 4.3 m/s
   d. 4.9 m/s

4. Determine the kinetic energy a 5 000-kg spacecraft must have in order to escape the Earth's gravitational field. Mass of the earth = 6.0 $\times 10^{24}$ kg; radius of the earth = 6.4 $\times 10^{6}$ m; G = 6.67 $\times 10^{-11}$ N m^2 kg^{-2} **1**

a. 3.14×10^{11} J
b. 3.04×10^{11} J
c. 3.34×10^{11} J
d. 3.53×10^{11} J

5. According to Kepler's Law of periods, The __________ of the time period of revolution of a planet is proportional to the cube of the __________ of the ellipse traced out by the planet **1**

   a. square, semi-minor axis
   b. square, semi-major axis
   c. cube, semi-major axis
   d. cube, semi-major axis

6. Why a tennis ball bounces higher on hills than on plains? **1**

7. The distance of Pluto from the Sun is 40 times the distance of earth if the masses of earth and Pluto be equal, what will be the ratio of gravitational forces of sun on these planets? **1**

8. Can we determine the mass of a satellite by measuring its time period? **1**

9. If T be the period of a satellite revolving just above the surface of a planet, whose average density is ρ, show that ρT^2 is a universal constant. **2**

10. The asteroid Pallas has an orbital period of 4.62 yr. Find the semi-major axis of its orbit. Given, G =6.67 $\times 10^{-11}$ Nm kg^{-2}, mass of the sun = 1.99 $\times 10^{30}$ kg and 1 yr = 3.156 $\times$ 10^7 s. **2**

11. Show that for a two-particle system $\overrightarrow{F_{12}} = \overrightarrow{F_{21}}$. **2**

12. Two stationary particles of masses M_1 and M_2 are a distance d part. A third particle lying on the line joining the particles experiences no resultant gravitational force. What is the distance of this particle from M_1? **3**

13. Three equal masses of M kg each are fixed at the vertices of an equilateral triangle

ABC. **3**

[Take AO = BO = CO = 1 m]

i. What is the force acting on a mass 2M placed at the centroid of the triangle?
ii. What is the force if the mass at the centroid is doubled?

14. Assuming the earth to be a sphere of uniform mass density, how much would body weigh halfway down to the centre of the earth if it weighted 250 N on the surface? **3**

15. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force on a sphere of mass m placed at midpoint of the line joining spheres? What is the potential at the mid-point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable? **5**

Answer

1. d. Smaller by a factor of 0.63

   Explanation: Time period of revolution of earth around sun T_e= 1 Year

   Time period of revolution of planet around sun,T_p=0.5 Year

   Orbital size of earth, r_e= 1 A.U

   Orbital size of the planet,r_p=?

   Applying Kepler's third law we get:

   $\frac{r_p{}^3}{r_e{}^3} = \frac{T_p{}^2}{T_e{}^2} \Rightarrow \frac{r_p{}^3}{r_e{}^3} = \left(\frac{\frac{1}{2}}{1}\right)^2 = \frac{1}{4}$
   $\Rightarrow \frac{r_p}{r_e} = \left(\frac{1}{4}\right)^{1/3}$
   $\Rightarrow r_p = \left(\frac{1}{4}\right)^{1/3} r_e$
   As r_e= 1 A.U (Given)
   $\Rightarrow r_p = \left(\frac{1}{4}\right)^{1/3}$
   $\Rightarrow r_p = 0.63\text{ A.U}$

2. b. 31.6 km/sec

   Explanation: Escape velocity of a projectile from the Earth, V_{esc}= 11.2 km/s

   Projection velocity of the projectile, V_p=3V_{esc}

   Mass of the projectile = m
   Velocity of the projectile far away from the Earth=V_f

   Total energy of the projectile on the earth $= \frac{1}{2}mv_p{}^2 - \frac{1}{2}mv_{esc}{}^2$
   Gravitational potential energy of the projectile far away from the Earth is zero.
   Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f{}^2$
   From the law of conservation of energy, we have
   $= \frac{1}{2}mv_p{}^2 - \frac{1}{2}mv_{esc}{}^2 = \frac{1}{2}mv_f{}^2$
   $v_f = \sqrt{v_p{}^2 - v_{esc}{}^2}$
   $= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$

$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$
$$= \sqrt{8}v_{esc}$$
$$= \sqrt{8} \times 11.2 = 31.68 \text{ km/sec}$$

3. a. 4.7 m/s

Explanation: $v = \sqrt{\frac{GM}{r}}$

$Here, G = 6.67 \times 10^{-11} N\ m^2\ kg^{-2}$

$M = 2 \times 10^{15} kg$

$r = \frac{12000}{2} = 6000m$

$\Rightarrow v = \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{15}}{6000}}$

$\Rightarrow v = 4.7 \text{ m/sec}$

4. a. 3.14×10^{11} J

Explanation: Kinetic energy required for spacecraft to escapr the earth's gravitational field

$= \frac{1}{2} m_s {v_{esc}}^2$

$we\ know,\ V_{esc} = \sqrt{\frac{2GM}{R}}$

$Here\ G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

$M = 6 \times 10^{24}\ kg$

$R = 6.4 \times 10^6\ m$

$\Rightarrow V_{esc} = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}}$

$= \sqrt{\frac{12 \times 6.67 \times 10^7}{6.4}}$

$= 1.12 \times 10^4\ m/\text{sec}$

Kinetic energy required for spacecraft to escapr the earth's gravitational field

$= \frac{1}{2} m_s {v_{esc}}^2$

Here m_s = mass of spacecraft = 5000 kg

$\therefore K.E = \frac{1}{2} \times 5000 \times 1.12 \times 10^4\ J$

$\Rightarrow K.E = 2500 \times 1.12 \times 10^4\ J$

$\Rightarrow K.E\ = 3.14 \times 10^{11}\ J$

5. b. square, semi-major axis

Explanation: Kepler's 3rd Law: $T^2 = a^3$. Kepler's 3rd law is a mathematical formula. It means that if you know the period of a planet's orbit (T = how long it takes the planet to go around the Sun), then you can determine that planet's distance from the Sun (a is the length of the semimajor axis of the planet's orbit)

6. A tennis ball bounces higher on hills than on plains because the acceleration due to gravity on hills is less than that on the surface of the earth (effect of height).

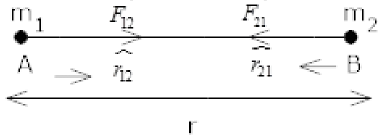
7. $\frac{F_{es}}{F_{ps}} = \left(\frac{r_p}{r_e}\right) = \left(\frac{40r_e}{r_e}\right)^2$
   = 1600 : 1

8. No, By measuring its time period, we cannot determine the mass of a satellite.

9. If a satellite revolves in a circular orbit of radius r, then its time period is given by
   $T = 2\pi\sqrt{\frac{r^3}{GM}}$
   If a satellite is revolving very near the planet's surface, then r = R = radius of planet and
   M = $\frac{4}{3}\pi R^3 \rho$
   Hence $\mathrm{T} = 2\pi\sqrt{\frac{R^3}{G.\frac{4}{3}\pi R^3 \rho}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$
   $\mathrm{T}^2 = \frac{3\pi}{\mathrm{G}\rho} \text{or } \rho\mathrm{T}^2 = \frac{3\pi}{\mathrm{G}}$
   But $\frac{3\pi}{Gp}$ is a universal constant. Hence, it is clear that the product $\rho\mathrm{T}^2$ is a universal constant.

10. $T^2 = \frac{4\pi^2 a^3}{GM_s}$
    $\Rightarrow a = \left[\frac{GM_s T^2}{4\pi^2}\right]^{1/3}$
    Substituting values, we get $a = 4.15 \times 10^{11} m.$

11. $\overrightarrow{F}_{12} = \frac{Gm_1m_2}{r^2}\widehat{r_{21}}$ ......(1)



$\overrightarrow{F}_{21} = \frac{Gm_1m_2}{r^2}\widehat{r_{12}}$ ....(2)

& (1) and (2) cam be written as $\left(\hat{a}=\frac{\hat{a}}{1a1}\right)$

$\overrightarrow{F}_{12}=\frac{Gm_1m_2}{r^3}\overrightarrow{r}_{21}$

$\overrightarrow{F}_{21}=\frac{Gm_1m_2}{r^3}\overrightarrow{r}_{12}$

Since $\overrightarrow{r}_{12}=\overrightarrow{r}_{21}$

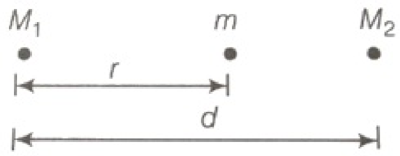
$\rightarrow\overrightarrow{F}_{21}=\frac{-Gm_1m_2}{r^3}\overrightarrow{r}_{21}$

$\rightarrow\overrightarrow{F}_{21}=-\overrightarrow{F}_{12}$

Hence proved.

12. The force on m towards M_1 is $F=G\frac{M_1m}{r^2}$

The force on m towards M_2 is $F=G\frac{M_2m}{(d-r)^2}$



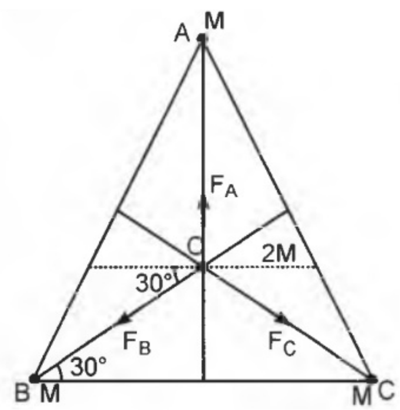
Equating two forces,

$\Rightarrow G\ \frac{M_1m}{r^2}=G\frac{M_2m}{(d-r)^2}$

$\left(\frac{\text{d - r}}{\text{r}}\right)^2=\frac{\text{M}_2}{\text{M}_1}$

$\Rightarrow\ \frac{d}{r}-1=\frac{\sqrt{\text{M}_2}}{\sqrt{\text{M}_1}}$

$\therefore$ distance of an particle from m,$r=d\left(\frac{\sqrt{\text{M}_1}}{\sqrt{\text{M}_1}+\ \sqrt{\text{M}_2}}\right)$

13. 

i. As AO = BO = CO = 1 m, hence we have

$\left|\overrightarrow{F_A}\right|=\left|\overrightarrow{F_B}\right|=\left|\overrightarrow{F_C}\right|=\frac{GM\cdot 2M}{(1)^2}=2GM^2$

If we consider direction parallel to BC as x-axis and perpendicular direction as y-axis, then as shown in figure, we have

$\vec{F}_A = 2GM^2\hat{j}$

$\vec{F}_B = \left(-2GM^2\cos 30^\circ\hat{i} - 2GM^2\sin 30^\circ\hat{j}\right)$

and $\overrightarrow{F_C} = \left(2GM^2\cos 30^\circ\hat{i} - 2GM^2\sin 30^\circ\hat{j}\right)$

Therefore, the net force on mass 2M placed at the centroid O is given by,

$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$

$= 2GM^2\left[j + \left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{i}\right) + \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)\right]$

$= 0$

ii. If mass at the centroid of the triangle gets doubled, even then the net force on it will be zero.

14. Weight of the body at the earth's surface

$w = mg = 250N$...............(i)

Acceleration due to gravity at depth d from the earth's surface

$g' = g\left(1 - \frac{d}{R}\right)$

here, $d = \frac{R}{2}$

$\therefore g' = g\left(1 - \frac{R/2}{R}\right)$ = g $\left(1 - \frac{1}{2}\right)$

$\Rightarrow g' = \frac{g}{2}$

$\therefore$ The weight of the body at depth d

$\Rightarrow w' = mg' = \frac{mg}{2}$

Using Eq. (i) we get

$w' = \frac{250}{2} = 125N$

$\therefore$ Weight of the body will be $125N$.

15. The situation is represented in the given figure:

Mass of each sphere, M = 100 kg. Separation between spheres r =1.0 m. X is the midpoint between the spheres. Gravitational force on mass m, $F_{net} = F_A + F_B$ where $F_A = \frac{GMm}{\frac{r}{2}}$ and $F_B = \frac{GMm}{\frac{r}{2}}$

Both the forces are attractive. They are equal in magnitude and opposite in direction. Hence net force on 'm' is zero. gravitational potential at X:

$V = \frac{-GM}{x}\ here\ x = \frac{r}{2}$ for each sphere

$V_{net} = \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4\frac{GM}{r}$

$V_{net} = \frac{4 \times 6.67 \times 10^{-11} \times 100}{r}$

$V_{net} = -2.67 \times 10^{-3} J/kg$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.