11. Structural Dynamics.

11.1 Introduction:-

It is performed on a structure if load varies with time (earthquake load, wind load etc). This analysis becomes important for flexible and important structures.

11.2 Damping:-

The process by which free vibration (without any permanent loading) is seazed after some time is called damping. In actual structure, many mechanism contributes to damping.

1) Friction at connections.
2) Closing and opening of micro-cracks in concrete
3) Friction between structural element etc.

It is very difficult to establish any mathematical equation for this mechanism. As a result, damping in actual structure is usually represented in very idealised manner.

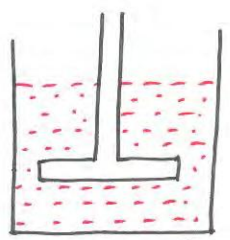
$$f_{damping} \propto \dot{x} \quad \left(\because \dot{x} = \frac{dx}{dt}\right)$$

$$f_{damping} = C\dot{x}$$

Velocity

damping coefficient

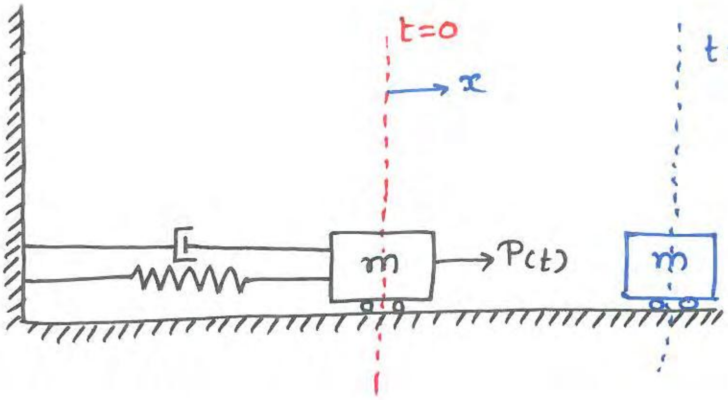
Damping force



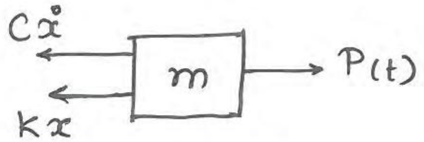
Dash pot

Representation.

11.3 Generalised Equation of Motion of Single Degree of freedom (SDOF) system :-



FBD at $t=t$



from Newton's Second Law :-

$$\Sigma F = ma$$

$$\Rightarrow P(t) \underbrace{-kx - c\dot{x}}_{\text{-ve becoz opp 'x'}} = m\ddot{x}$$

$$\Rightarrow \boxed{m\ddot{x} + c\dot{x} + kx = P(t)}$$

11.4 Types of Vibration :-

A) Free Vibration :-

1) Undamped

2) Damped.

B) Forced Vibration :-

1) Undamped

2) Damped.

11.4.1 Undamped Free Vibration:-

Equation of motion:

$$m\ddot{x} + kx = 0$$

$$\Rightarrow m\frac{d^2x}{dt^2} + kx = 0$$

Solution of this equation:-

$$x(t) = A\sin(\omega_n t + \delta)$$

where, $\omega_n = \sqrt{\frac{k}{m}}$ = Natural angular frequency. (rad/sec)

A & δ are calculated from initial condition:-

Case I:- At $t=0$

$$x(t) = x_0$$

$$\dot{x}(t) = 0$$

Case II:- At $t=0$

$$x(t) = 0$$

$$\dot{x}(t) = \dot{x}_0$$

Case III:- At $t=0$

$$x(t) = x_0$$

$$\dot{x}(t) = \dot{x}_0$$

Considering case III:-

$$x(t) = A\sin(\omega_n t + \delta)$$

At $t=0$

$$\Rightarrow x_0 = A\sin(\omega_n \times 0 + \delta)$$

$$\Rightarrow x_0 = A\sin\delta \quad \text{------} \quad (i)$$

Now,

$$\dot{x}(t) = A\omega_n\cos(\omega_n t + \delta)$$

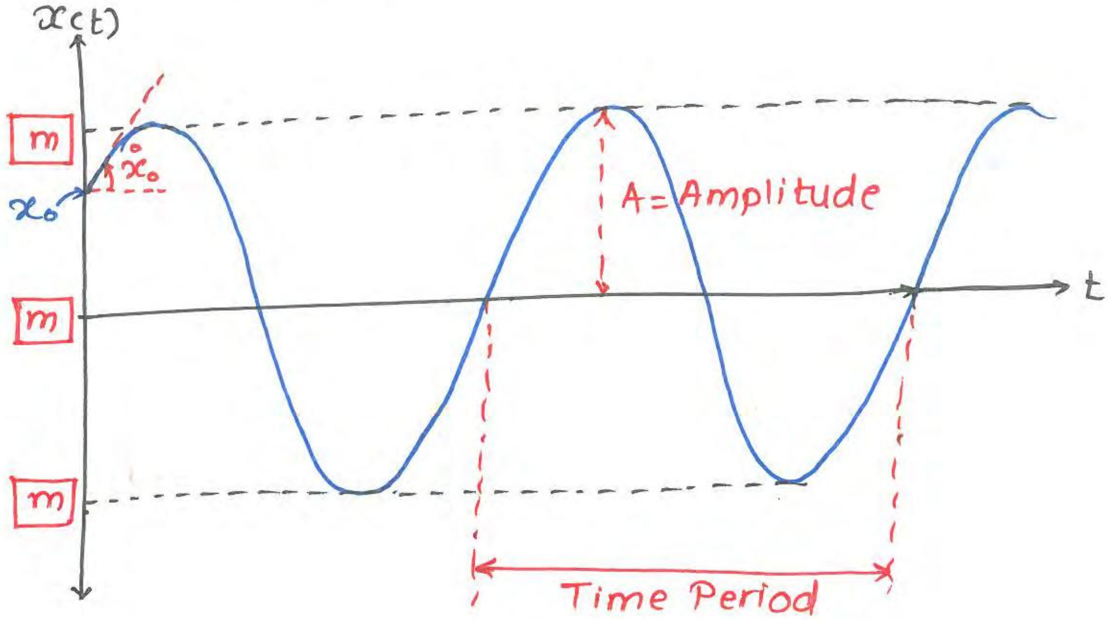
At $t=0$

$$\Rightarrow \dot{x}_0 = A\omega_n\cos(\omega_n \times 0 + \delta)$$

$$\dot{x}_0 = A\omega_n\cos\delta \quad \text{----} \quad (ii)$$

From equation (i) and (ii) A and δ are calculated.

$$x(t) = A \sin(\omega_n t + \delta)$$



- Time period:-

$$\omega_n t = 2\pi$$

$$\Rightarrow \quad t = \frac{2\pi}{\omega_n}$$

- Frequency:-

$$f = \frac{1}{t}$$

$$\Rightarrow \quad f = \frac{\omega_n}{2\pi} \quad \text{(cycle/sec)}$$

- Maximum Velocity:-

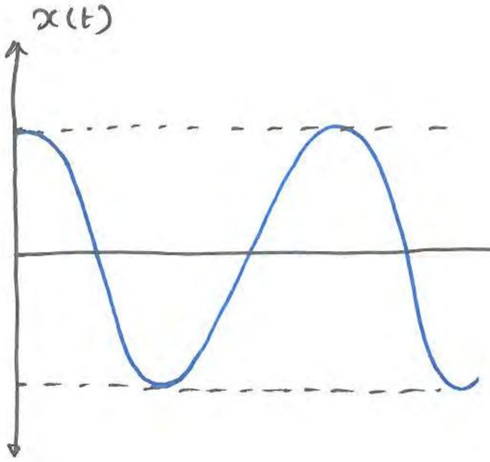
$$\dot{x}(t) = A\omega_n \cos(\omega_n t + \delta)$$

$$\dot{x}_{max} = A\omega_n$$

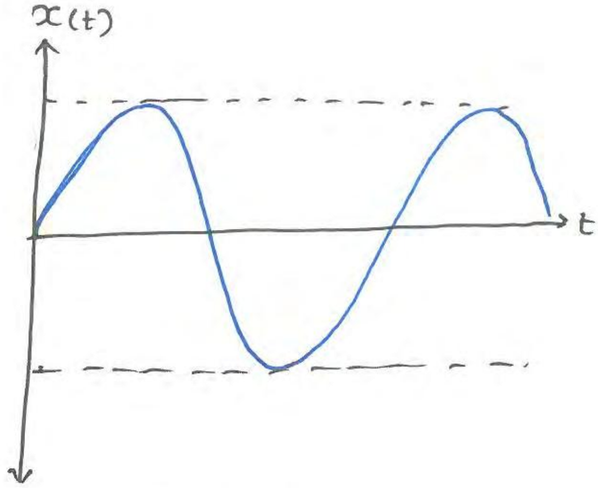
- Maximum Acceleration:-

$$\ddot{x}(t) = -A\omega_n^2 \sin(\omega_n t + \delta)$$

$$\ddot{x}_{max} = A\omega_n^2$$



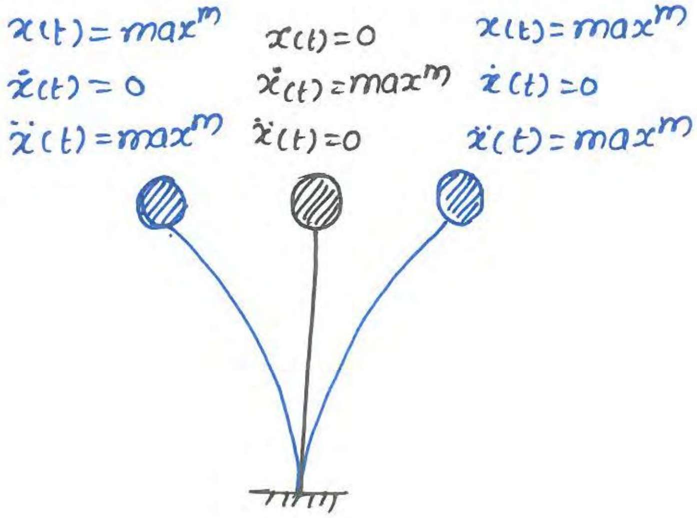
Case I



Case II

- **Phase Difference (δ) :-**

  It is the time lag between two sinusoidal quantities of same frequency.



11.4.2 Damped Free Vibration:-

Equation of Motion :-

$$m\ddot{x} + C\dot{x} + kx = 0$$

Critical Damping (C_{cr}) :-

Minimum damping of any system corresponding to which there is just no vibration.

$C < C_{cr}$ - Vibration (Under damped)

$C = C_{cr}$ - Just No vibration (Critically damped)

$C > C_{cr}$ - Never vibrates (Over damped)

where,

$$\boxed{C_{cr} = 2m\omega_n} \quad **$$

Now, damping of any system is represented in terms of critical damping of that system.

$$\xi = \frac{C}{C_{cr}} = \text{damping factor}$$

$$\Rightarrow \quad \boxed{C = 2\xi m\omega_n} \quad **$$

where,

$$\omega_n = \sqrt{\frac{k}{m}}$$

Now,

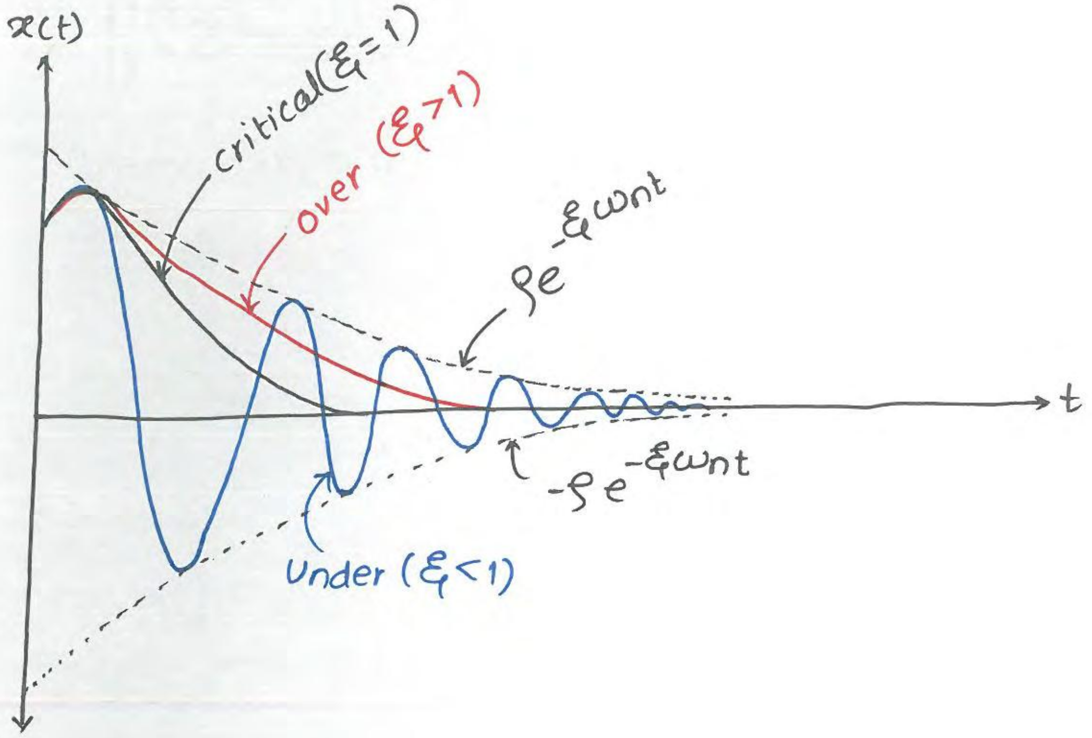
$$m\ddot{x} + C\dot{x} + kx = 0$$

$$\Rightarrow m\frac{d^2x}{dt^2} + C\frac{dx}{dt} + kx = 0$$

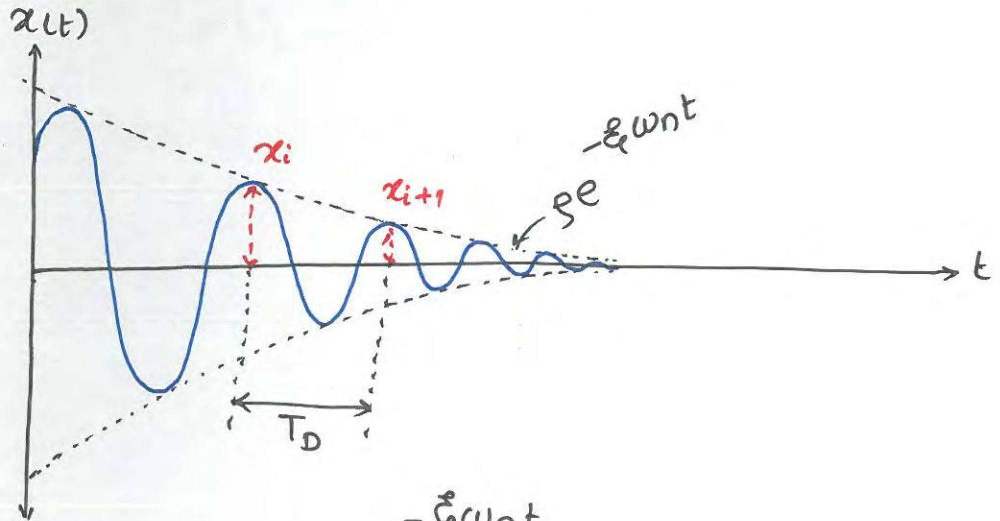
Solution of this equation:-

$$x_{(t)} = e^{-\xi\omega_n t}\left[x_0\cos\omega_D t + \frac{\dot{x}_0 + \xi\omega_n x_0}{\omega_D}\sin\omega_D t\right]$$

where, $\omega_D = \omega_n\sqrt{1-\xi^2}$ = Damped natural frequency.



- Determination of Damping Factor (ξ):-



$$\frac{x_i}{x_{i+1}} = \frac{\rho e^{-\xi\omega_n t}}{\rho e^{-\xi\omega_n (t+T_D)}}$$

$$\Rightarrow \frac{x_i}{x_{i+1}} = e^{\xi\omega_n T_D}$$

$$\Rightarrow \frac{x_i}{x_{i+1}} = e^{\xi\omega_n \frac{2\pi}{\omega_D}}$$

$$\Rightarrow \frac{x_i}{x_{i+1}} = e^{\xi\omega_n \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$\Rightarrow \frac{x_i}{x_{i+1}} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \log \frac{x_i}{x_{i+1}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$ ... Logarithmic decrement.

Considering 1^{st} cycle and $1^{st}+j^{th}$ cycle.

$$\frac{x_1}{x_{1+j}} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \cdots\cdots \times \frac{x_j}{x_{1+j}}$$

$$\Rightarrow \frac{x_1}{x_{1+j}} = e^{\frac{j2\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \log \frac{x_1}{x_{1+j}} = \frac{2\pi j\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \boxed{\xi = \frac{1}{2\pi j} \log\left(\frac{x_1}{x_{1+j}}\right)}$$ (for small ξ, $\sqrt{1-\xi^2} \approx 1$)

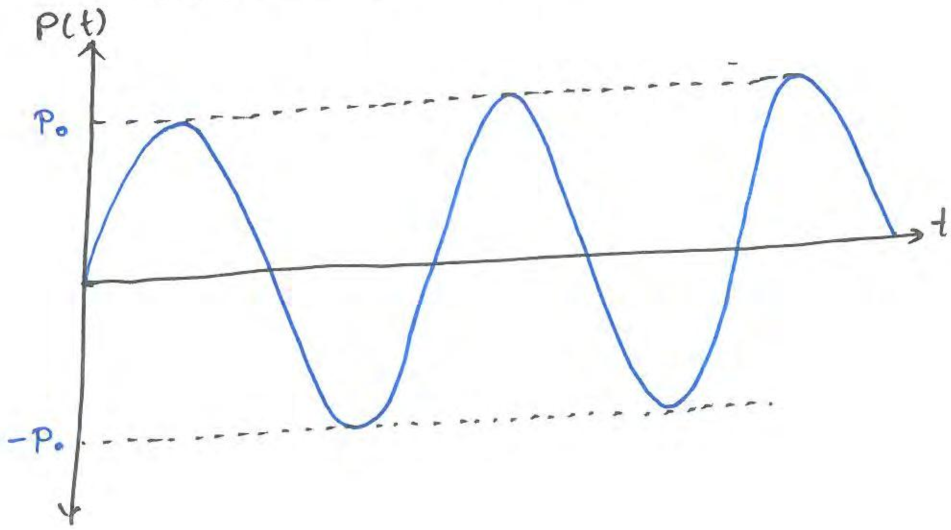
11.4.3 Undamped Forced Vibration.

Equation of motion:-

$$m\ddot{x} + kx = P(t)$$

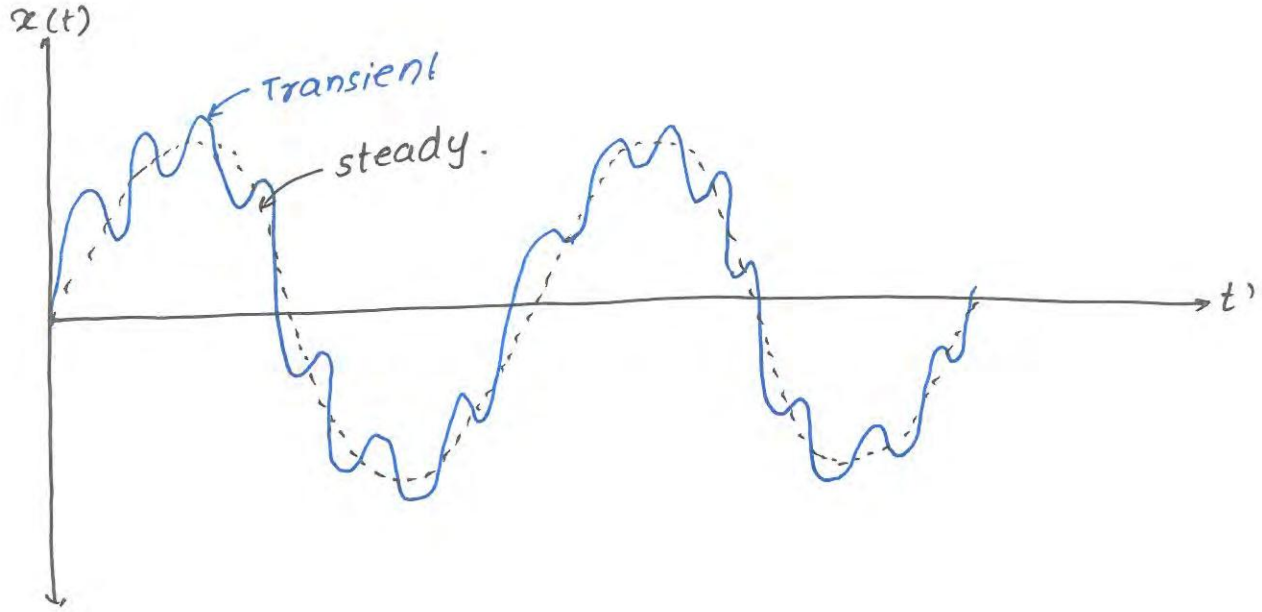
considering harmonic exitation

$$m\ddot{x} + kx = P_0 \sin \omega t$$



Solution of above equation is :-

$x(t) = \text{-----------}$



• Response factor:-

It is the ratio of maximum dynamic response to maximum static response.

$$\text{Response factor } (R) = \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$$

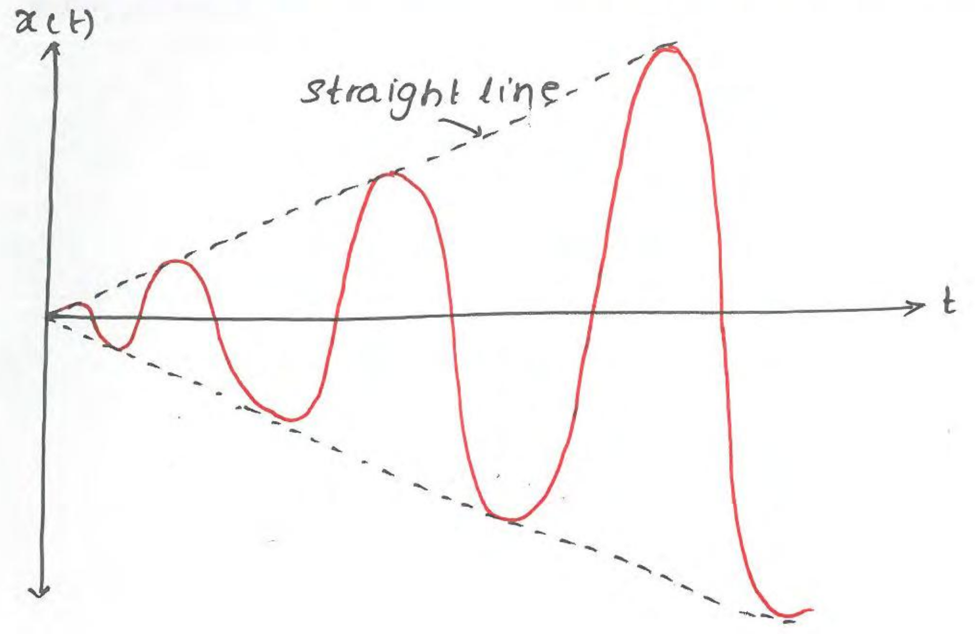
$$\Rightarrow \quad \frac{\{x(t)\}_{max}}{\{x_{st}\}_{max}} = \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$$

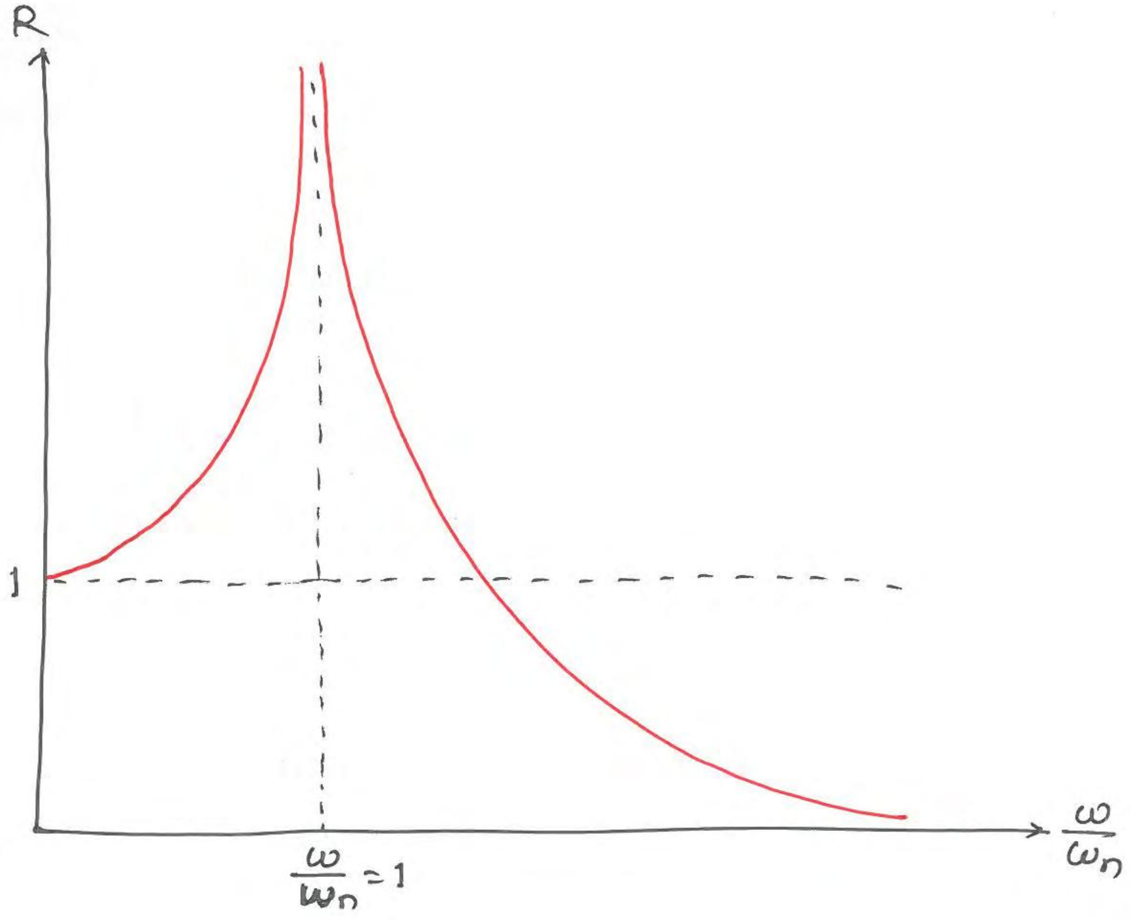
$$\Rightarrow \{x(t)\}_{max} = \frac{P_o/k}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|} \qquad \left(\text{Bcoz } (x_{st})_{max} = \frac{P_o}{K}\right)$$

ω = forcing angular frequency.

ω_n = Natural angular frequency.

For $\omega = \omega_n$ Response factor $= \infty$ (Resonance)





11.4.4 Damped Forced Vibration:-

equation of motion:-

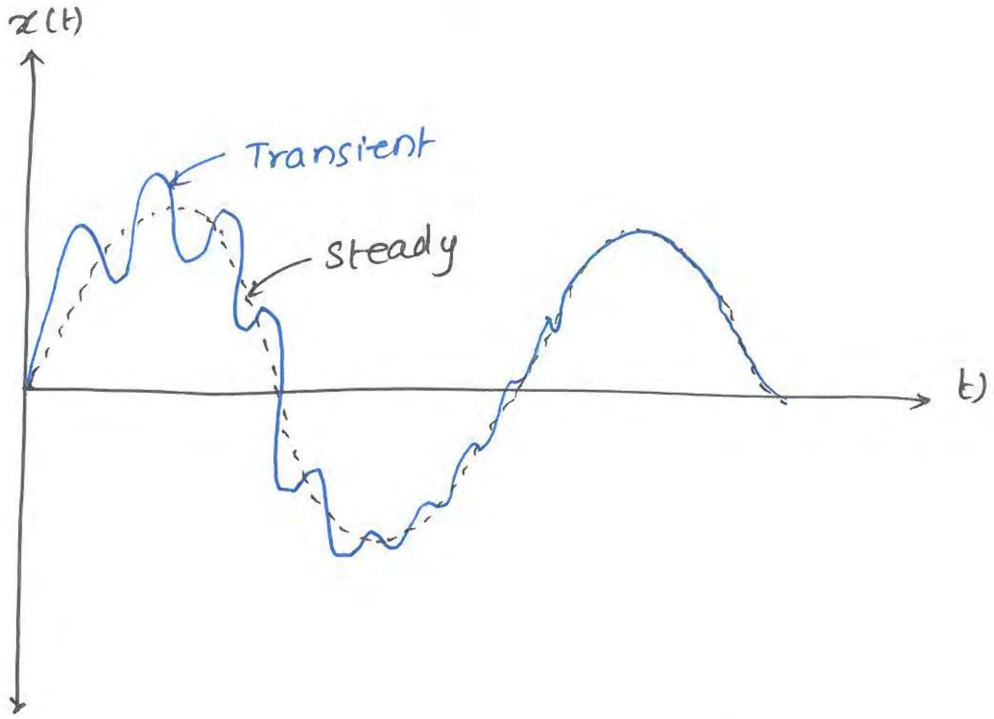
$$m\ddot{x} + c\dot{x} + kx = P(t)$$

Considering harmonic excitation

$$m\ddot{x} + c\dot{x} + kx = P_0 \sin \omega t$$

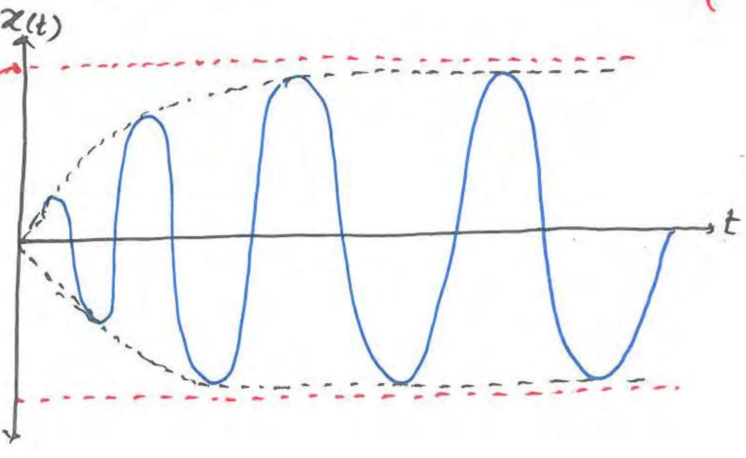
Solution of this equation is:-

$x(t) =$ - - - - - - - - - - - - -

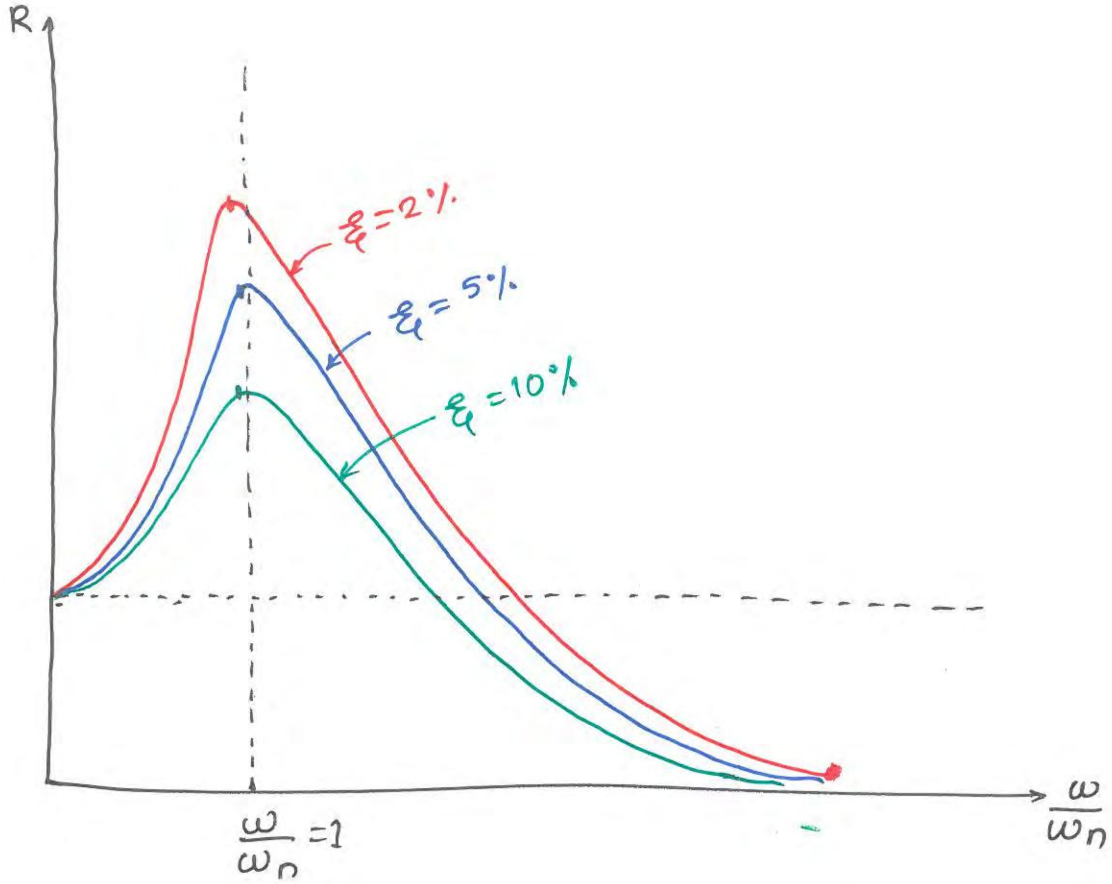


$$\text{Response Factor } (R) = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

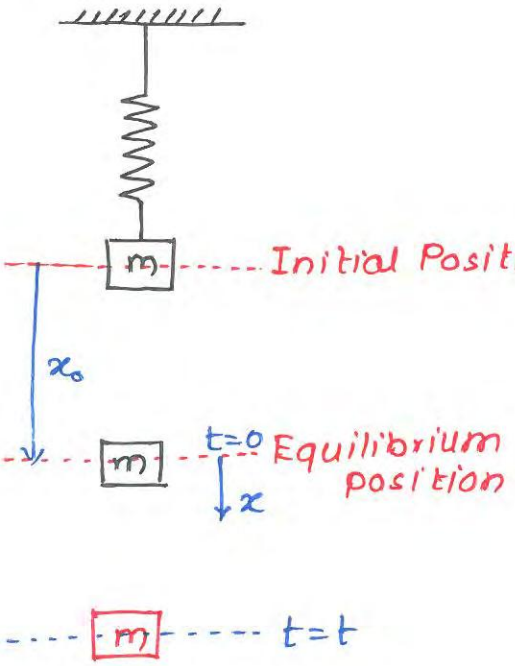
For $\omega = \omega_n$, Response factor $= \frac{1}{2\xi}$



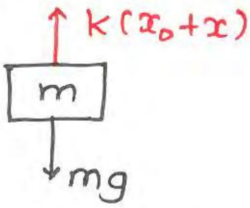
$\frac{\{x_{st}\}_{max}}{2\xi}$



11.5 Other equations of Motion :-



FBD at $t = t$



$$\Sigma F = ma$$

$$\Rightarrow \quad mg \underbrace{- \, k(x_0 + x)}_{\text{-ve bcoz opp 'x'}} = m\ddot{x}$$

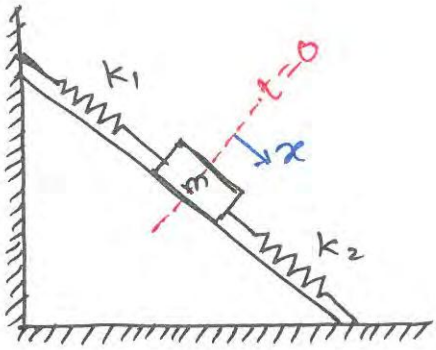
$$\Rightarrow \quad \cancel{mg} - \cancel{kx_0} - kx = m\ddot{x}$$

$$(\because kx_0 = mg)$$

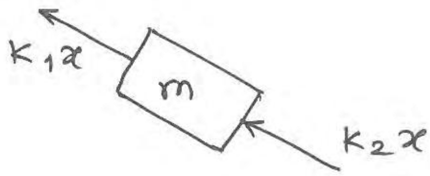
$$\Rightarrow \quad \boxed{m\ddot{x} + kx = 0}$$

• Note:

If weight of system is balanced by stiffness of system then vibration starts about equilibrium position. It means, weight of system can be removed from FBD while writing equation of motion.



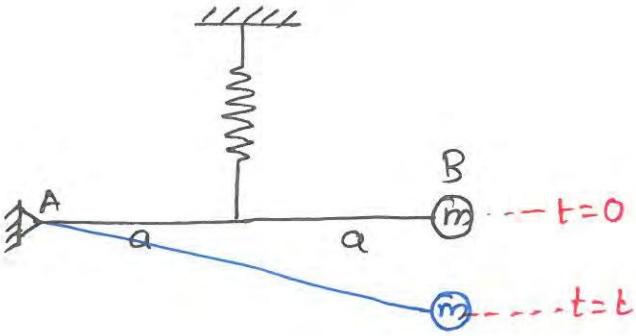
FBD at t = t



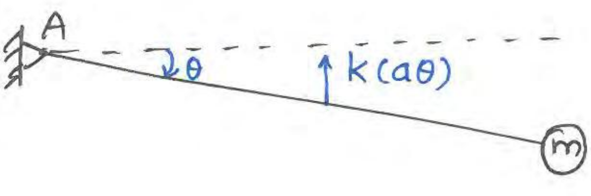
$$\Sigma F = ma$$

$$\underbrace{-k_1 x - k_2 x}_{\text{-ve bcoz opp 'x'}} = m\ddot{x}$$

$$\Rightarrow \boxed{m\ddot{x} + (k_1 + k_2)x = 0}$$



FBD at t=t

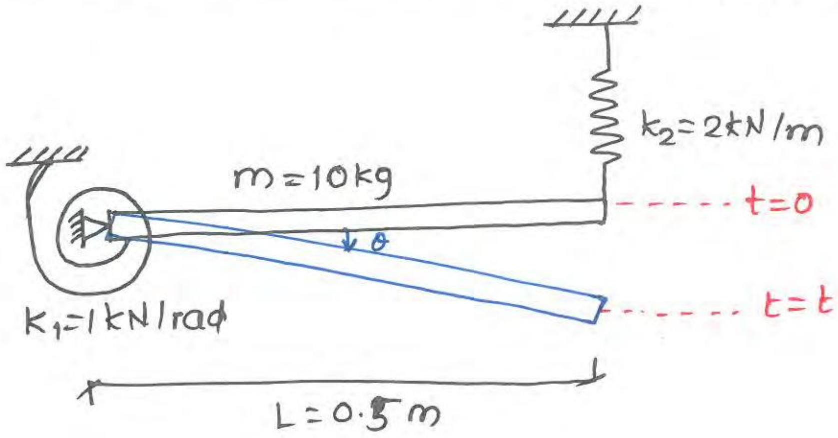


$$\Sigma T_A = I\alpha$$

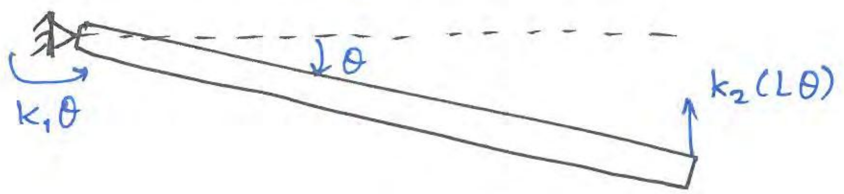
$$\Rightarrow \underbrace{-k(a\theta)a}_{\text{-ve bcoz opp'}\theta\text{'}} = m(2a)^2\ddot{\theta}$$

$$\Rightarrow \boxed{4m\ddot{\theta} + k\theta = 0}$$

Ex. Calculate undamped natural frequency:-

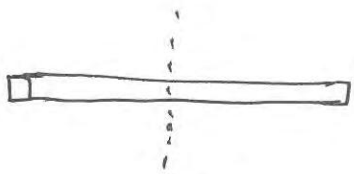


FBD at $t=t$



$$\Sigma T_A = I\alpha$$

$$\underbrace{-k_1\theta - k_2(L\theta)L}_{\text{-ve bcoz opp '}\theta\text{'}} = \frac{mL^2}{3}\ddot{\theta}$$

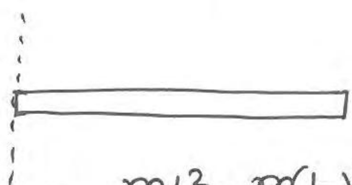


$I = \frac{mL^2}{12}$

$$\Rightarrow \quad \frac{mL^2}{3}\ddot{\theta} + (k_1 + k_2L^2)\theta = 0$$

Now,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_1 + k_2L^2}{mL^2/3}}$$

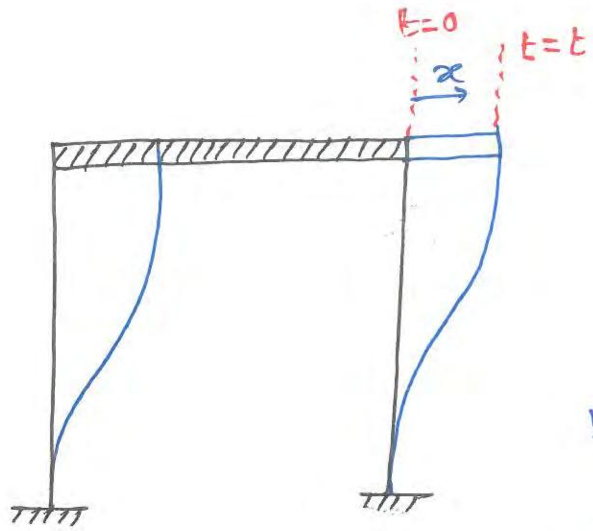


$I = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2$

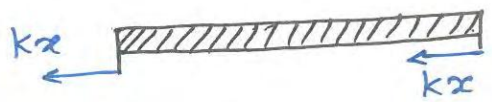
$I = \frac{mL^2}{3}$

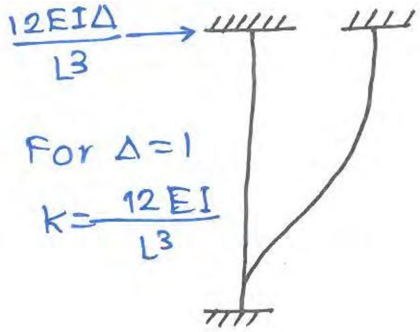
$$= \sqrt{\frac{1000 + 2000 \times 0.5^2}{10 \times 0.5^2/3}}$$

$$= 42.43 \text{ rad/sec}$$



FBD at t=t

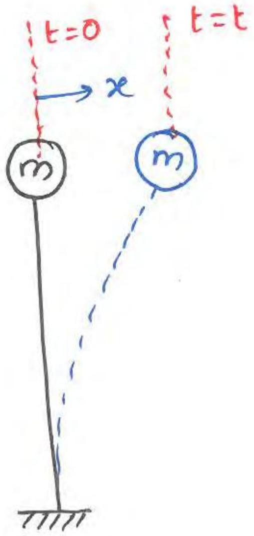




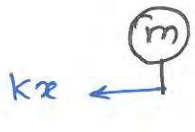
$$\Sigma F = ma$$

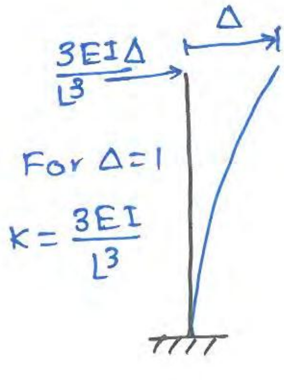
$$\Rightarrow \underbrace{-kx - kx}_{\text{-ve bcoz opp 'x'}} = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + 2kx = 0$$



FBD at t=t

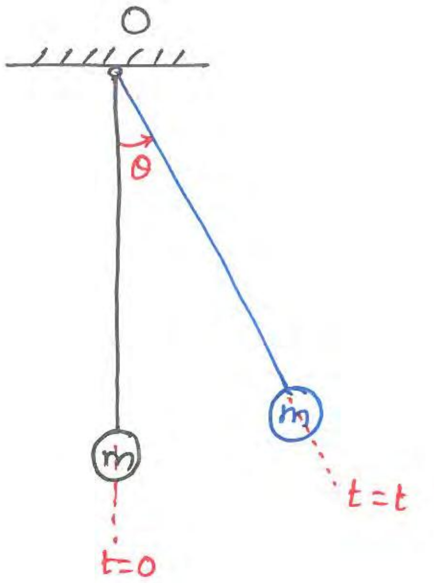




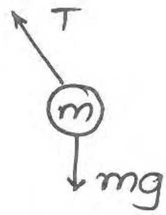
$$\Sigma F = ma$$

$$\Rightarrow -kx = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + kx = 0$$



FBD at $t=t$



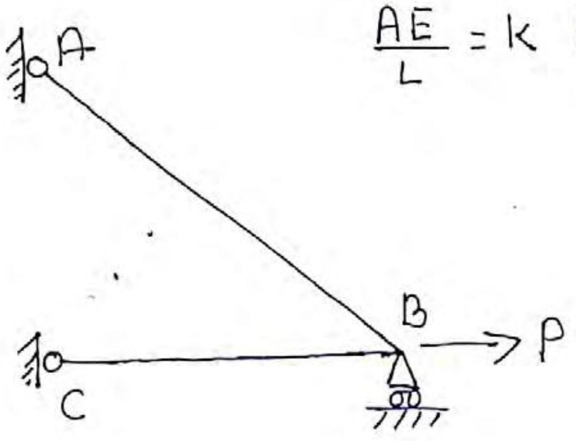
mg is considered bcoz this is not balanced by stiffness of system

$$\Sigma \tau_0 = I\alpha$$

$$\Rightarrow \underbrace{-mg(L\sin\theta)}_{\text{-ve bcoz opp '}\theta\text{'}} = mL^2\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \frac{g}{L}\theta = 0 \quad \left(\because \text{for small } \theta \; \sin\theta \simeq \theta\right)$$

Ex. What is the horizontal displacement of B due to load P?

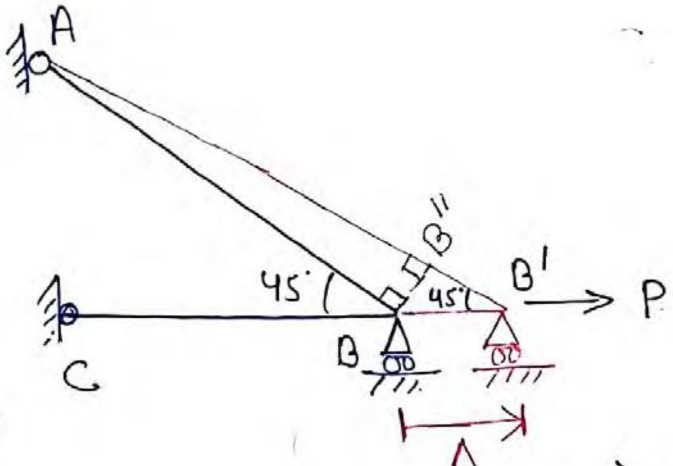


$\frac{AE}{L} = k$ for all members.

Solution

DSI = 1 bcoz both members are axial.
KI = 1

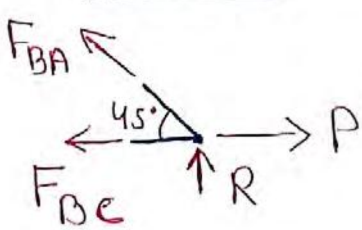
Using displacment method to solve. Assuming horizontal deflection at B is Δ.



$$F_{BC} = \frac{AE}{L} BB'$$
$$= k\Delta$$

$$F_{AB} = \frac{AE}{L} B'B''$$
$$= k\,(BB' \cos 45)$$
$$= \frac{k\Delta}{\sqrt{2}}$$

Joint B'

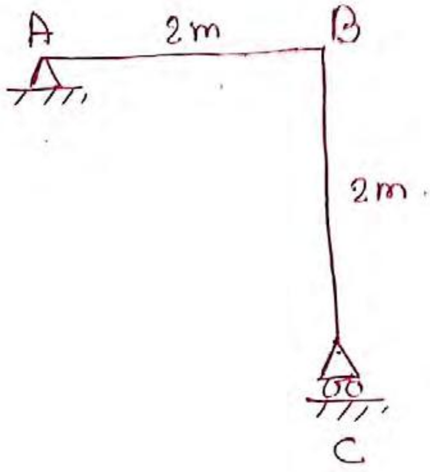


$\Sigma F_x = 0$

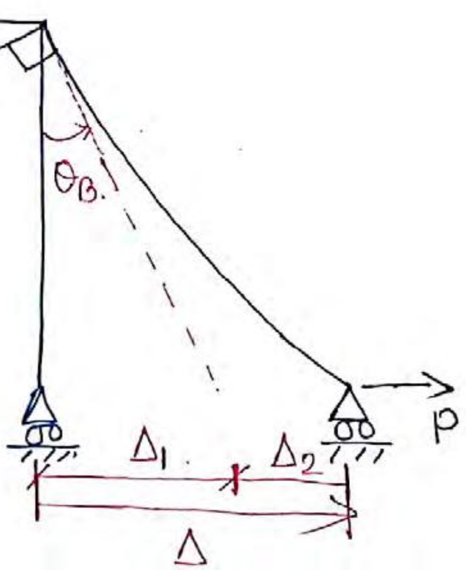
$\Rightarrow -F_{BC} - F_{BA}\cos 45^\circ + P = 0$

$\Rightarrow \boxed{\Delta = \frac{P}{1.5k}}$

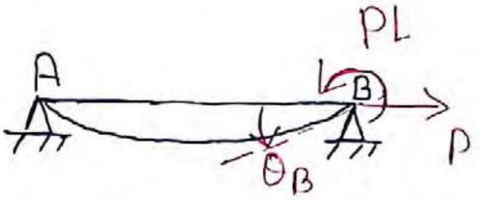
Ex What is the force required for displacing support C horizontally through a distance Δ?

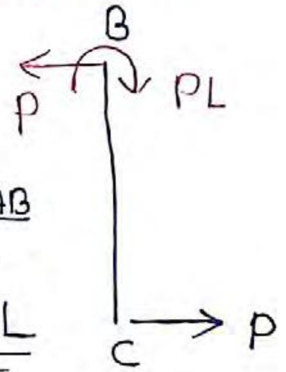


Solution



FBD





$$\theta_B = \frac{ML_{AB}}{3EI}$$

$$= \frac{PL \times L}{3EI}$$

$$= \frac{PL^2}{3EI}$$

Now,

$$\Delta = \Delta_1 + \Delta_2$$

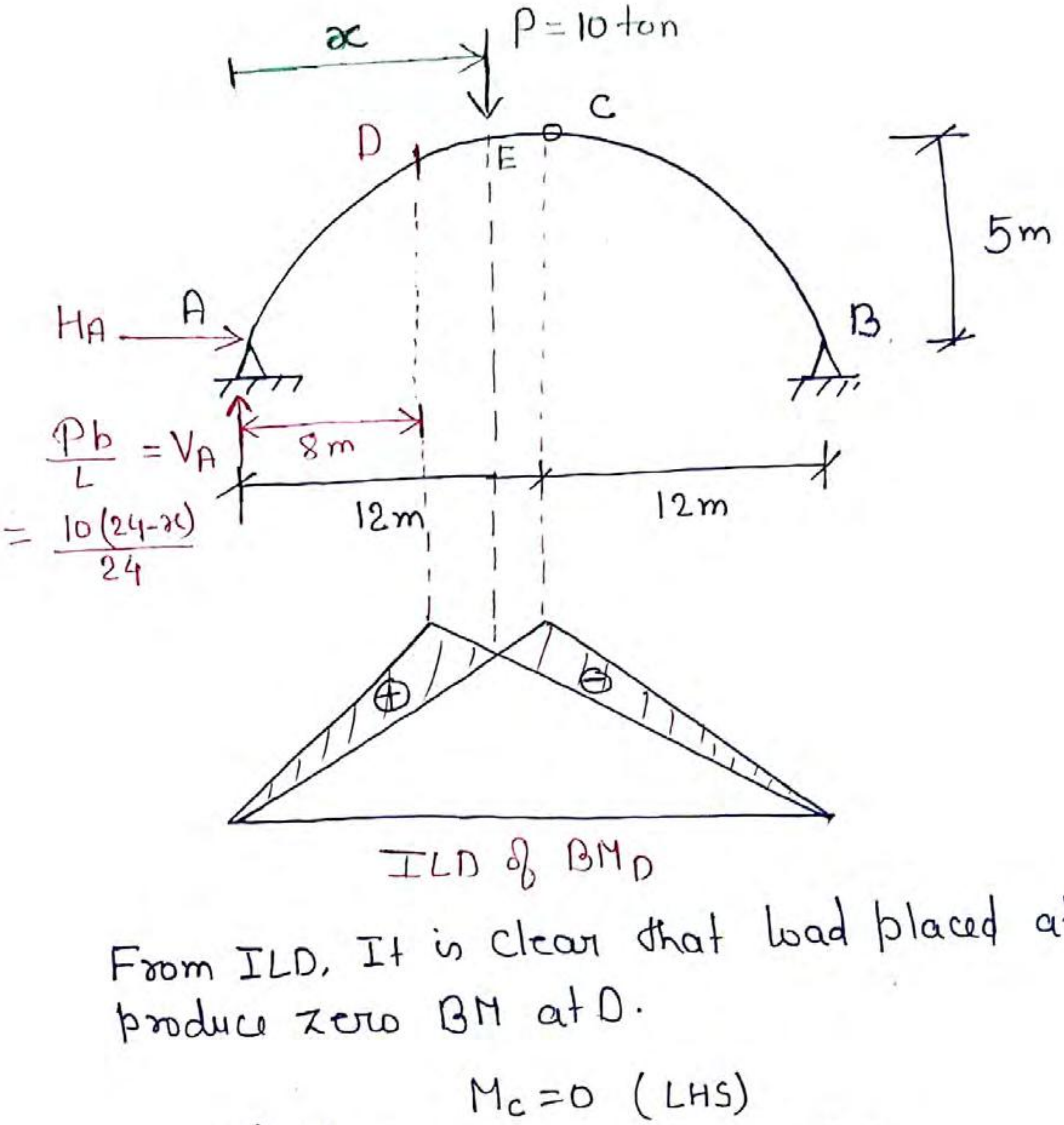
$$\Rightarrow \quad \Delta = \theta_B L_{BC} + \frac{PL_{BC}^3}{3EI}$$

$$\Rightarrow \quad \Delta = \frac{PL^2}{3EI} \times L + \frac{PL^3}{3EI}$$

$$\Rightarrow \quad P = \frac{3EI\Delta}{2L^3}$$

$$\Rightarrow \quad \boxed{P = \frac{3EI\Delta}{16.}}$$

Ex A symmetrical 3-hinged arch (Parabolic) of span 24m & central rise 5m carries a single vertical point load of 10 ton. Locate the position of load on arch in order that the bending moment is zero at section 8m from left hinge. For this position of load, calculate the bending moment under the load.



ILD of BM_D

From ILD, It is clear that load placed at E will produce zero BM at D.

$$M_C = 0 \ (\text{LHS})$$

$$\Rightarrow V_A \times \frac{L}{2} - H_A \times 5 - P\left(\frac{L}{2} - x\right) = 0 .$$

$$\Rightarrow \quad H_A = ??$$

Now,

$$BM_D = 0$$

$$\Rightarrow V_A \times 8 - H_A \times y_D = 0$$

$$\Rightarrow \frac{10(24-x)}{24} \times 8 - H_A \left\{ \frac{4 \times 5}{24^2} \; 8(24-8) \right\} = 0.$$

$$\Rightarrow \quad x = \boxed{10.29\,m.}$$

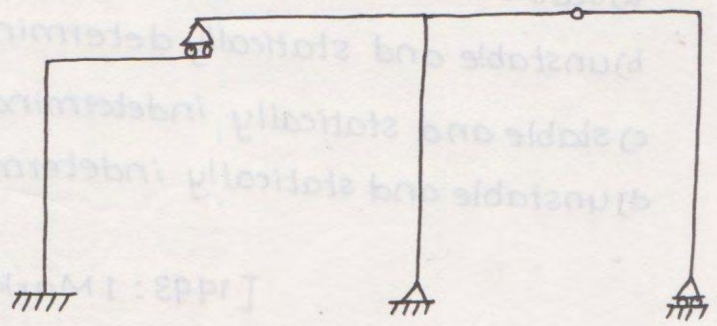
Now,

$$BM_E = V_A \times x - H_A \, y_x$$

$$= \frac{10(24-10.29)}{24} \times 10.29 - H_A \left\{ \frac{4 \times 5}{24} \; 10.29(24-10.29) \right\}$$

$$= \boxed{45.6 \text{ kNm.}}$$

Q.1. A plane structure shown in figure is



a) Stable and determinate

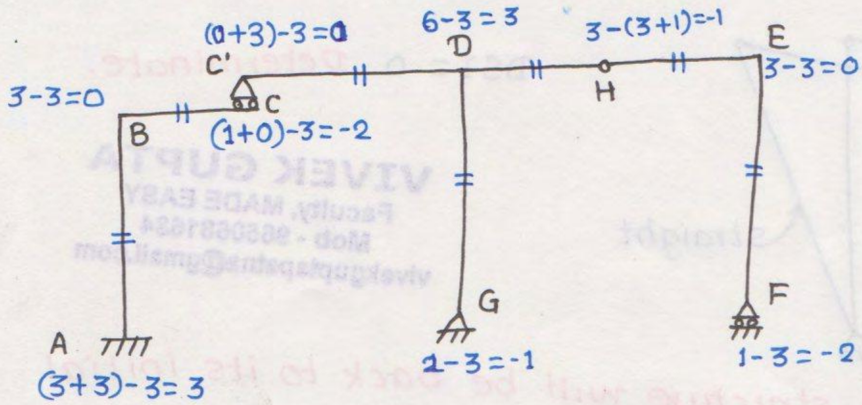
b) stable and indeterminate

c) unstable and determinate

d) unstable and indeterminate.

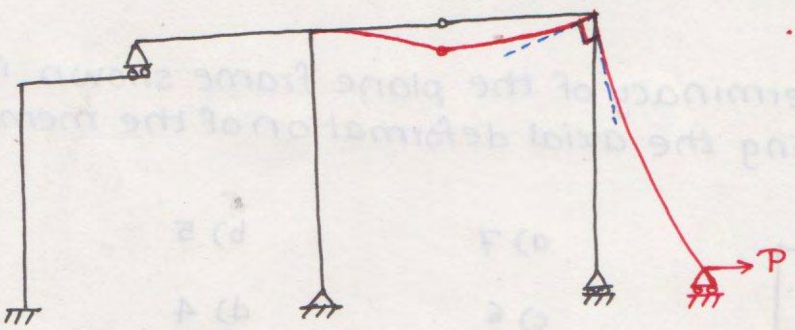
[1992: 1 Mark]

Solution:



Seq. ABCC'DHEFG

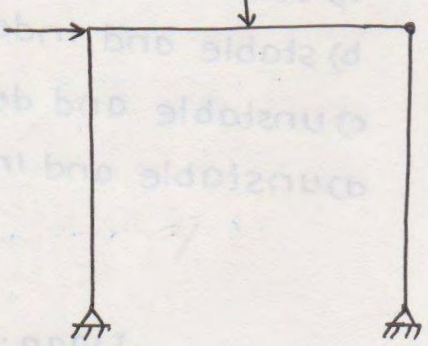
DSI = 0 ≤ 0 Determinate



After removal of load, structure will be back to its initial position so stable.

Ans: a) Stable and determinate.

Q.2. The plane frame shown in figure is.



a) stable and statically determinate

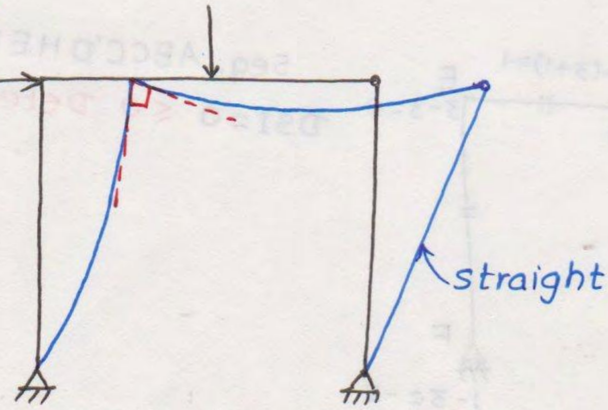
b) unstable and statically determinate

c) stable and statically indeterminate

d) unstable and statically indeterminate.

[1993 : 1 Mark]

Solution:

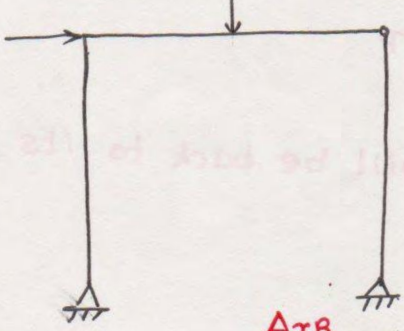


DSI = 0 Determinate.

After removal of load structure will be back to its initial position so stable.

Ans: a) stable and statically determinate.

Q.3. The kinematic indeterminacy of the plane frame shown in figure is (disregarding the axial deformation of the members).



a) 7

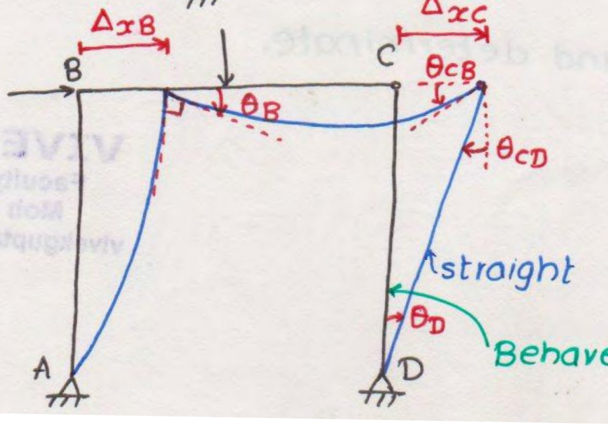
b) 5

c) 6

d) 4

[1993 : 1 Mark]

Solution:

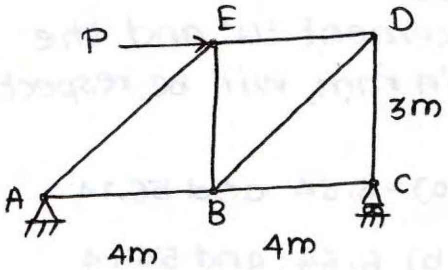


KI = 4 (θ_A, θ_B, Δx_B, θ_{CB},

$\Delta x_C = \Delta x_B$

$\theta_{CD} = \theta_D = \dfrac{\Delta x_C}{L} = \dfrac{\Delta x_B}{L}$)

Q.5. What is the ratio of the forces in the members AB, BE and AE of the pin-jointed truss shown in the figure given below?

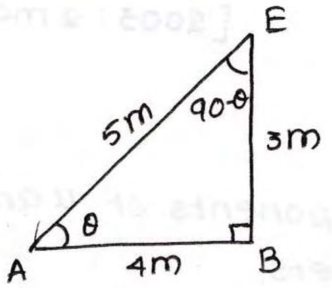


a) 5 : 4 : 3   b) 4 : 3 : 5

c) $\left(\frac{1}{4}\right) : \left(\frac{1}{3}\right) : \left(\frac{1}{5}\right)$ d) None of these

[ESE : 2005]

considering ΔABE,



$\sin\theta = 3/5$

$\cos\theta = 4/5$

from Lami's theorem,

$$\frac{F_{AB}}{\sin(90-\theta)} = \frac{F_{BE}}{\sin\theta^\circ} = \frac{F_{AE}}{\sin 90^\circ} = k$$

$F_{AB} = k\cos\theta = \frac{4}{5}\cdot k$

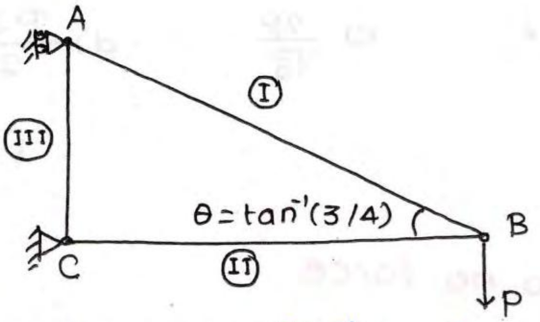
$F_{BE} = \frac{3}{5}k$

$F_{AE} = k$

$F_{AB} : F_{BE} : F_{AE}$

4 : 3 : 5

Q.6. A cantilever truss carries a concentrated load P as shown in the figure below:



What are the magnitudes of axial forces in the members I, II and III, respectively?

a) 1.00P, 1.33P and 1.67P

b) 1.67P, 1.33P and 1.00P

c) 1.33P, 0.75P and 0.60P

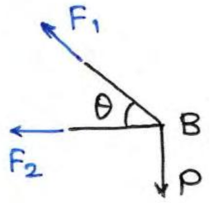
d) 0.60P, 0.75P and 1.00P

[ESE: 2006]

Solution: $\theta = \tan^{-1}(3/4) \Rightarrow \tan\theta = \frac{3}{4} \Longrightarrow \sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$

.... (3, 4, 5 ⇒ pythagorean triple)

Joint B:



$\Sigma F_y = 0$

$F_1 \sin\theta - P = 0$

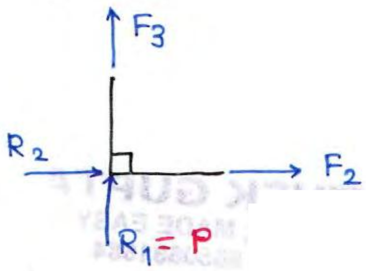
$F_1 = \frac{P}{\sin\theta} = \frac{P}{3/5} \Rightarrow F_1 = 1.67P$ ......(i)

$\Sigma F_x = 0$

$-F_2 - F_1\cos\theta = 0$

$F_2 = -F_1\cos\theta = -\frac{P}{\tan\theta} = -\frac{P}{3/4} \Rightarrow F_2 = 1.33P$ ...(ii)

Joint C:



$\Sigma F_y = 0$

$F_3 + R_1 = 0$

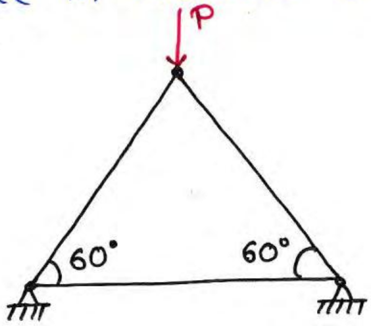
$F_3 = -R_1 = -P$ .....(iii)

From (i), (ii) and (iii)

Ans: b) 1.67P, 1.33P, 1.00P

(F_1 , F_2 , F_3)

Q.7 The force in member BC is?



a) $\frac{P}{2\sqrt{2}}$ b) Zero

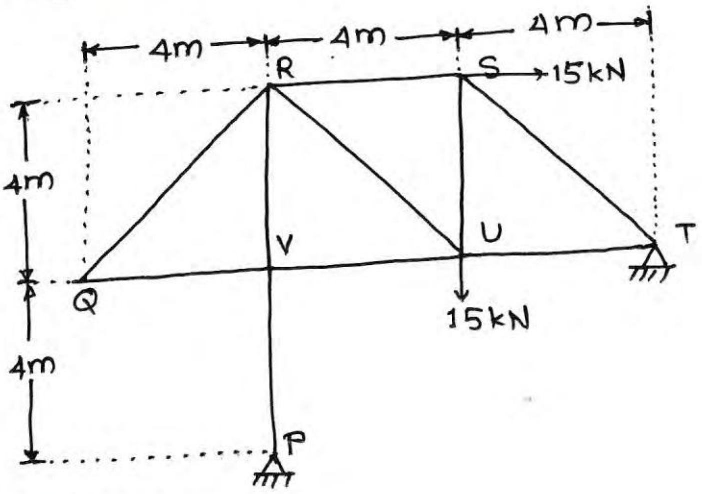
c) $\frac{2P}{\sqrt{3}}$ d) $\frac{P\sqrt{3}}{2}$

[ESE: 2013]

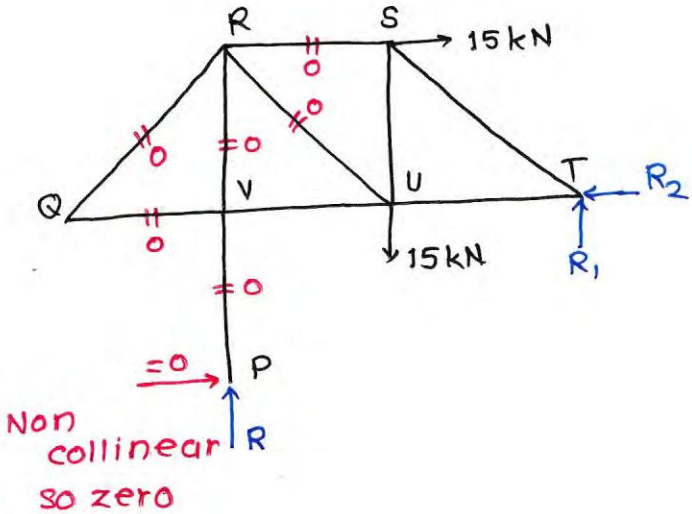
Solution:

No elongation in BC so no force.

Q8. The pin-jointed 2-D truss is loaded with a horizontal force of 15 kN at joint S and another 15 kN vertical force at joint U as shown in figure. Find the force in member RS (in kN) and report your answer taking tension as +ve and compression as -ve.



Solution:

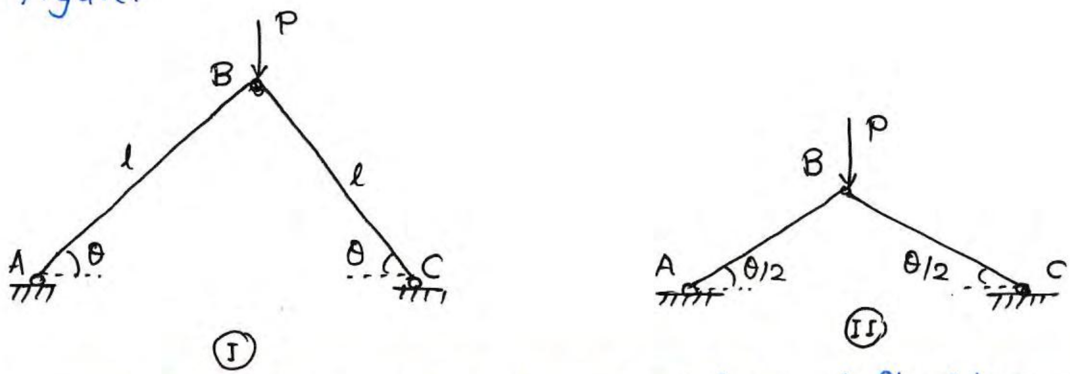


$\Sigma M_T = 0$

$\Rightarrow 15\times4 - 15\times4 + R\times8 = 0$

$\Rightarrow R = 0$

Q9. Which one of the following is the correct statements regarding the force and deflection at point B in trusses I and II shown in the figure?



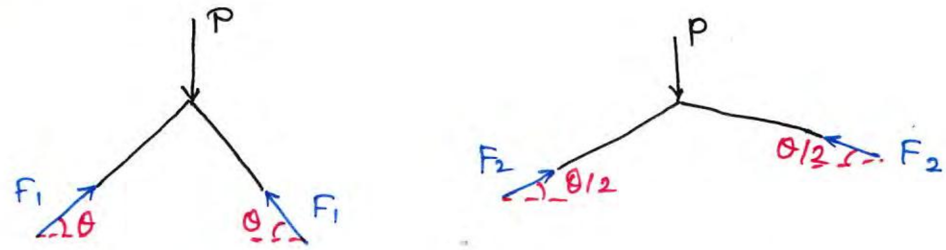
a) (I) will have less member force and less deflection at B compared to (II)

b) (I) will have less member force and more deflection at B compared to (II)

c) (I) will have more member force and deflection at B compared to (II)

d) (I) will have more member force and less deflection at B compared to (II)

[IAS : 2005]

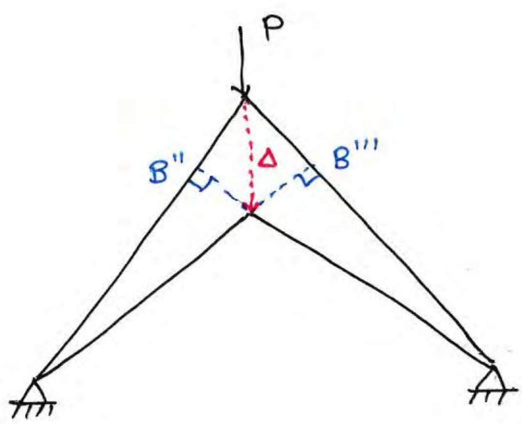


$$2F_1 \sin\theta = P$$

$$\Rightarrow \quad F_1 = \frac{P}{2\sin\theta}$$

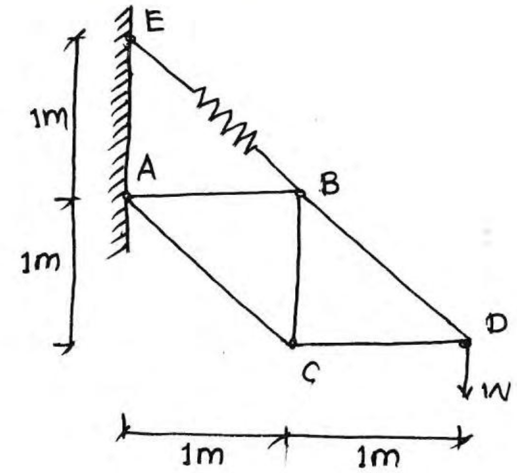
$$F_2 = \frac{P}{2\sin\theta/2}$$

$$F_2 > F_1$$

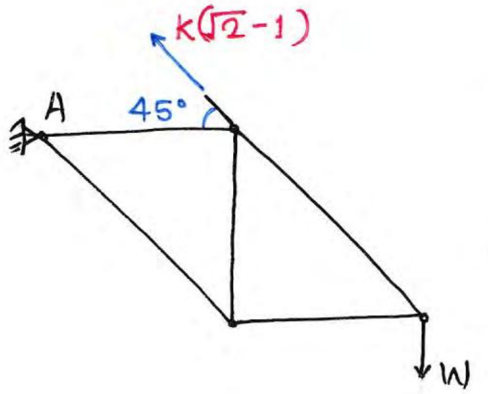


If $F_2 > F_1$, then $\delta_2 > \delta_1$ so $\Delta_2 > \Delta_1$

Q.10. AB, AC, BC, CD and BD are pin-connected rods. Point B is attached to point E by a spring whose unstretched length is 1m and whose spring constant is 4 kN/m. Assume member are inextensible and neglect the weight of all the bars and the spring. The magnitude of the load W applied at D that makes CD horizontal is ____ N.



Solution:-



$\Sigma M_A = 0$

$\Rightarrow \quad W \times 2 - k(\sqrt{2}-1)\sin 45^\circ \times 1 = 0$

$\Rightarrow W = 0.585$ kN

$\Rightarrow W = 585$ N.

Q11 A beam PQRS is 18 m long and is simply supported at points Q and R 10m apart. Overhangs PQ and RS are 3m and 5m respectively. A train of two point loads of 150 kN and 100 kN, 5m, apart, crosses this beam from left to right with 100kN load leading.

1) The maximum sagging moment under the 150kN load anywhere is

a) 500kNm b) 450 kNm c) 400 kNm d) 375 kNm

[2003 : 2Marks]

2) During the passage of the loads, the maximum and the minimum reactions at support R, in kN are respectively.

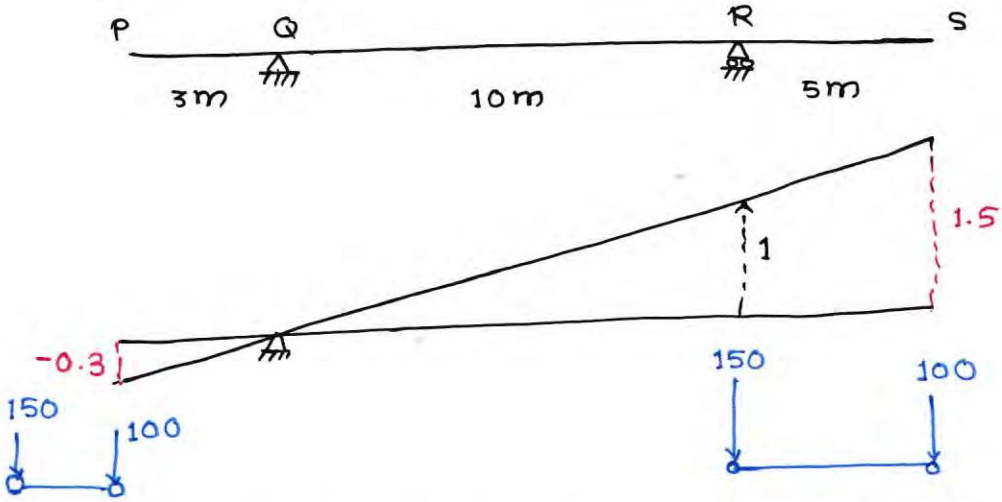
a) 300 and -30 b) 300 and -25

c) 225 and -30 d) 225 and -25 [2003 : 2Marks]

3) The maximum hogging moment in the beam anywhere is

a) 300kN·m b) 450kN·m c) 500kN·m d) 750 kNm

[2003 : 2 Marks]



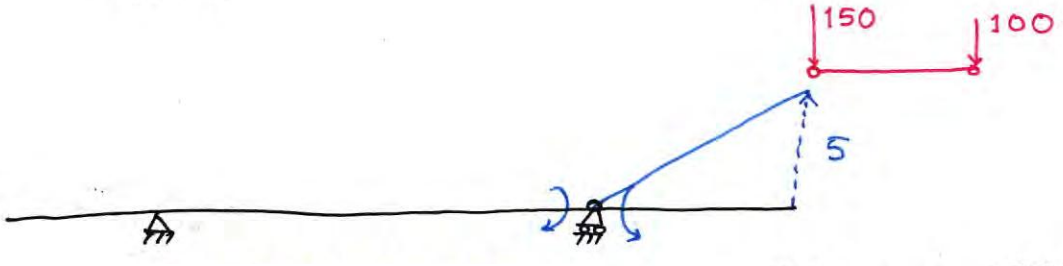
ILD of reaction at R

$V_{R,min} = 150 \times 0 + 100(-0.3)$

$= -30\,kN$

$V_{R\,max} = 150 \times 1 + 100 \times 1.5$

$= 300\,kN$

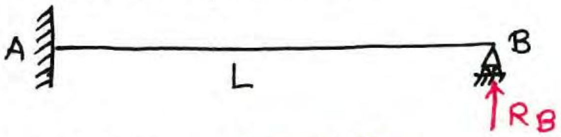


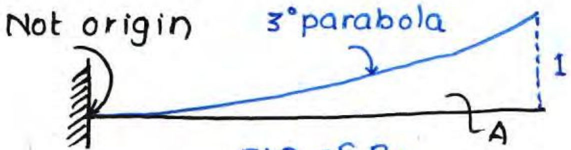
$BM_{max} = 150 \times 5$

$= 750\,kNm.$

Q.12. What is the area of influence line diagram for the reaction at the hinged end of uniform propped cantilever beam of span L?

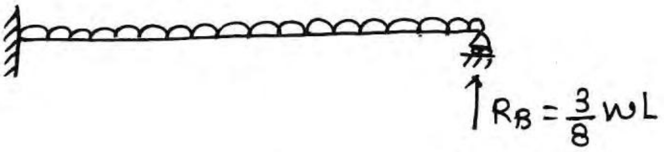
a) $\frac{L}{8}$ b) $\frac{L}{2}$ c) $\frac{L}{4}$ d) $\frac{3L}{8}$ [ESE: 2009]

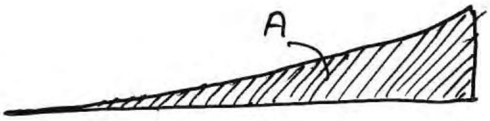




ILD of R_B

Assuming beam is subjected to udl





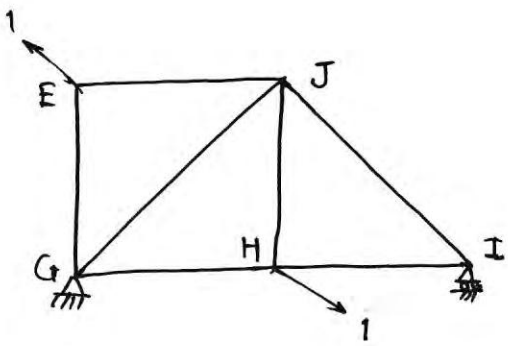
ILD of R_B

$$w \times \text{Area of ILD of } R_B = R_B$$

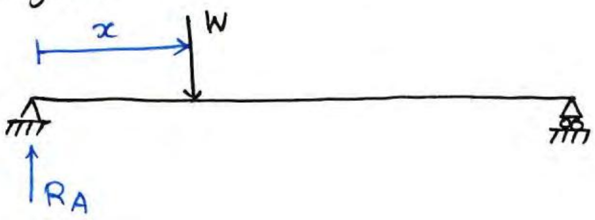
$$w \times A = \frac{3}{8} wL$$

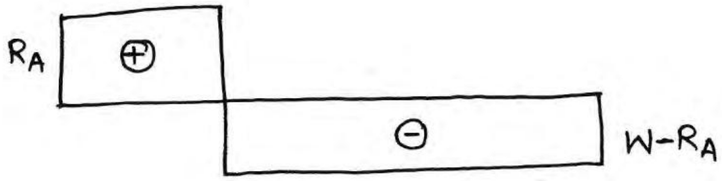
$$\boxed{A = \frac{3}{8} L}$$

Q.13.



Q.14. When a single point load W travels over a simply supported beam, what is shape of the graph for maximum positive or negative shear force? [IAS : 2006]





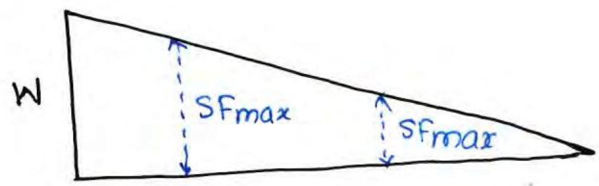
SFD

Maximum +ve SF. $= R_A$

$$= \frac{W(L-x)}{L}$$

At $x=0$, $SF_{max} = W$

$x = L$, $SF_{max} = 0$



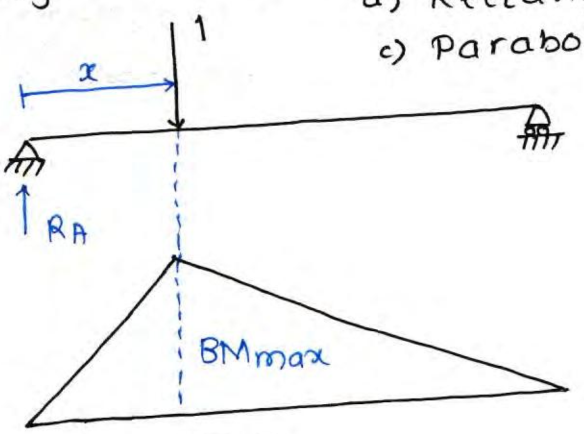
⇒ Triangle with maximum ordinate W at a support.

ILD of SF_{max}

Q.15. What is the shape of influence line diagram for the maximum bending moment in respect of a simply supported beam?

a) Rectangular
b) Triangular
c) Parabolic
d) Circular

[ESE:2005]



BMD

Maximum $BM = R_A \cdot x$

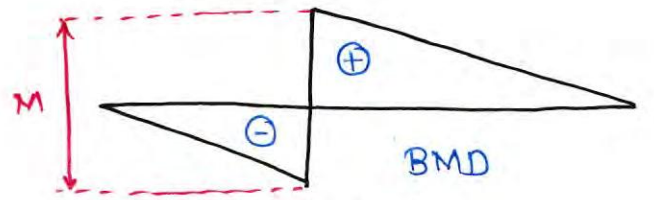
$$= \left(\frac{L-x}{L}\right) \cdot x$$

= Parabolic

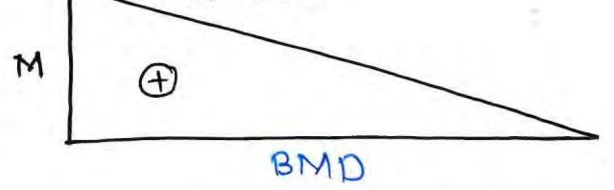
Q16 An applied couple 'M' is moving on a simply supported beam of span L as shown in given figure. The absolute maximum bending moment developed in the beam is ?

a) $M/2$ b) M c) $3M/2$ d) $2M$

[ESE: 2001]

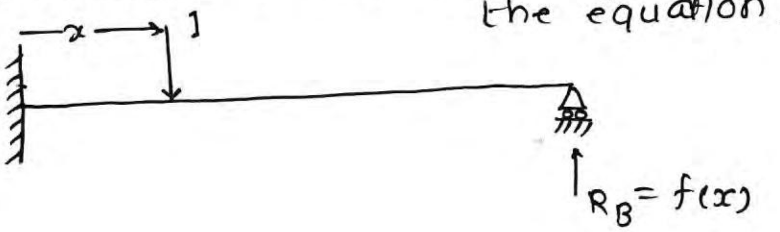


For $x = 0$



so absolute maximum is M

Q17 For the propped cantilever shown in the figure, the influence line for reaction at the propped end is given by $y_1 = f(x)$. The influence line ordinate (y_2) for moment at A is given by the equation __ [ESE: 2002]



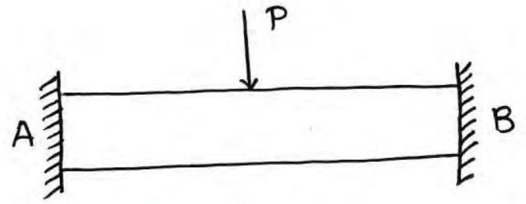
$$M_A = 1 \times x - R_B \times L$$

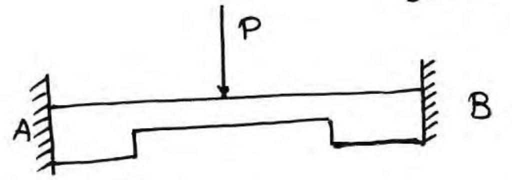
$$= x - f(x) \cdot L$$

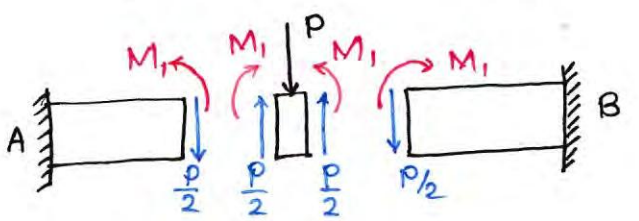
Q.18 A fixed beam of uniform section is carrying a point load at its mid span. If the moment of inertia of the middle half length is now reduced to half its previous value, then the fixed end moments will

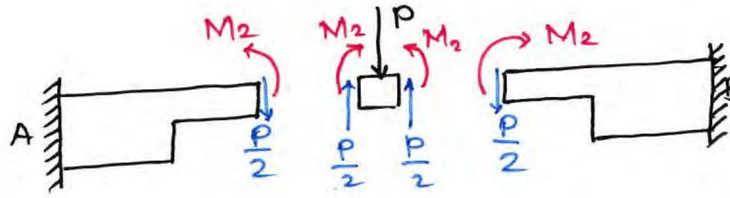
a) increase b) decrease c) remain const. d) Change their directions

[ESE: 1997]









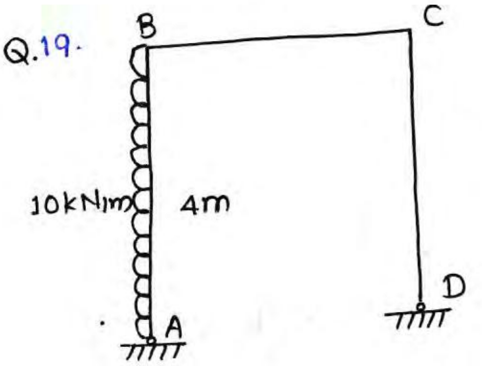
$M_A = \frac{P}{2} \times \frac{L}{2} - M_1$

$M_A = \frac{P}{2} \times \frac{L}{2} - M_2$

$M_1 > M_2$

Since $M_1 > M_2$ so M_A of 2nd structure is higher.

Q.19.



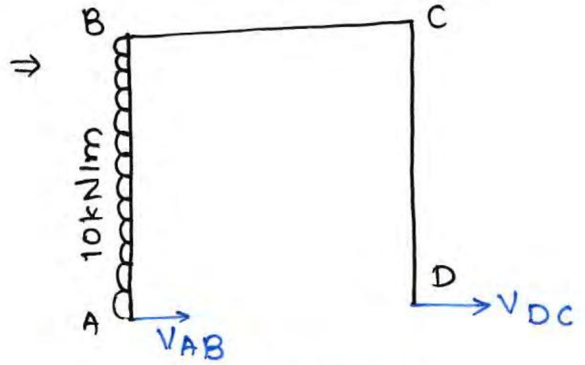
Consider the portal frame shown in the figure, with both lower ends hinged. Which one of the following represents the equilibrium equation among horizontal forces?

a) $M_{BC} + M_{CB} = 40$ kNm b) $M_{BC} + M_{CB} = 80$ kNm

c) $M_{BA} + M_{CD} = 20$ kNm d) $M_{BA} + M_{CD} = 40$ kNm

[IAS: 2010]

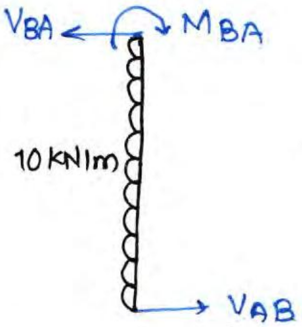
⇒



$\Sigma F_x = 0$

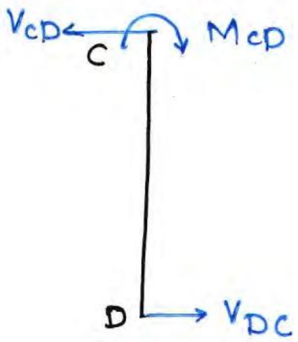
$V_{AB} + V_{DC} + 10 \times 4 = 0$ ---(i)

For V_{AB}:-



$\Sigma M_B = 0$

$\Rightarrow V_{AB} = \frac{M_{BA} - 10 \times 4 \times 2}{4}$



$\Sigma M_c = 0$

$\Rightarrow \quad V_{DC} = \dfrac{M_{CD}}{4}$

from eq^n (i) :-

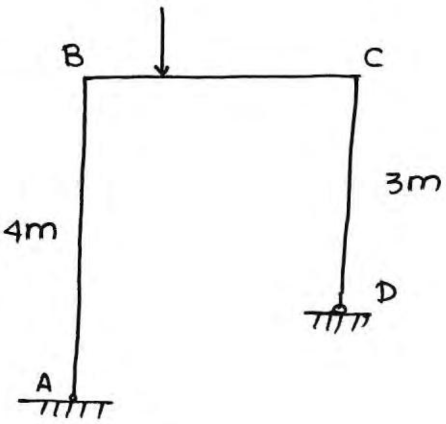
$M_{BA} - 80 + M_{CD} + 160 = 0$

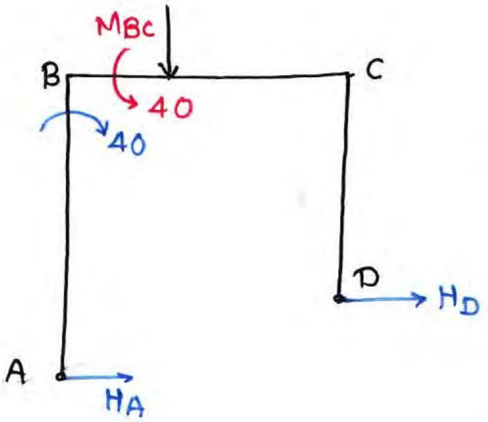
$M_{BA} + M_{CD} = -80$

$-M_{BC} - M_{CB} = -80$

$\boxed{M_{BC} + M_{CB} = 80}$

Q. 20.

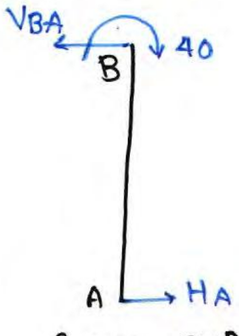




$\Sigma F_x = 0$

$\Rightarrow \quad H_A + H_D = 0 \quad \ldots\ldots (i)$

For H_A :-

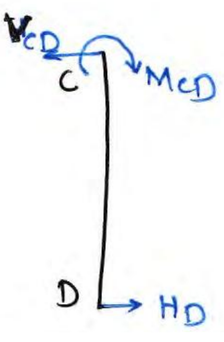


$\Sigma M_B = 0$

$\Rightarrow H_A = \frac{40}{4} = 10\,kN.$

from eq^n(i) $\Rightarrow H_D = -10\,kN.$

For $M_{CD} \Rightarrow$

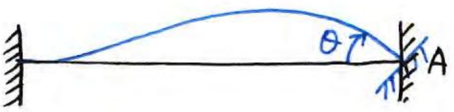


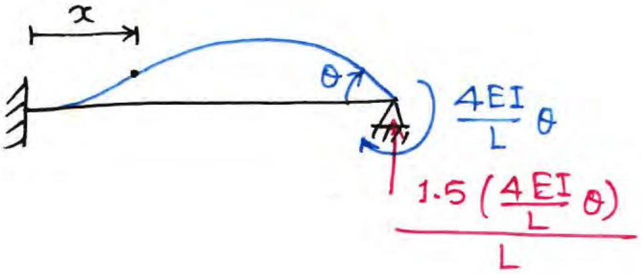
$\Sigma M_c = 0 \quad \Rightarrow M_{CD} - H_D \times 3 = 0$

$M_{CD} - (-10) \times 3 = 0$

$\Rightarrow M_{CD} = -30\,kNm.$

Q.21.



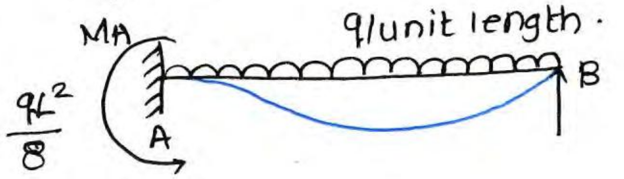


$BM_x = 0$

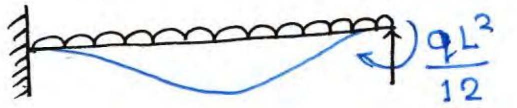
$$\Rightarrow -\frac{4EI}{L}\theta + \frac{1.5\left(\frac{4EI}{L}\theta\right)}{L}(L-x) = 0$$

$$\Rightarrow \quad x = L/3$$

Q.22. The propped cantilever AB carries a uniformly distributed load of q/unit length. In this condition the moment reaction $M_A = \frac{qL^2}{8}$. What is the clockwise moment required at B to make the slope of deflection curve zero?

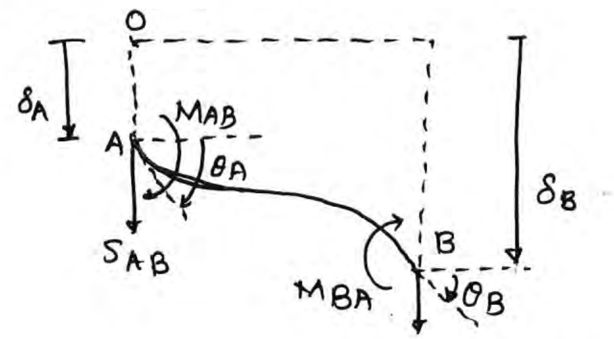


$\Rightarrow$



Q23. A member co-ordinate system is shown in the given figure. The symmetric stiffness square matrix obtained for the member AB of length 'l' with flexural rigidity 'EI' by using the slope deflection equation and rules of matrix multiplication is as follows:

$$\begin{bmatrix} M_{AB} \\ S_{AB} \\ M_{BA} \\ S_{BA} \end{bmatrix} = \frac{2EI}{l} \begin{bmatrix} \text{Symmetric} \\ \text{Stiffness} \\ \text{square} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} \theta_A \\ \delta_{A/l} \\ \theta_B \\ \delta_{B/l} \end{bmatrix}$$



The correct sequence of elements of the first row of the symmetric stiffness square matrix is

a) 2, 3, 1 and 3 b) 2, 3, -1 and -3

c) 2, 3, 1 and -3 d) 1, 3, 2 and -3

[ESE: 2000]

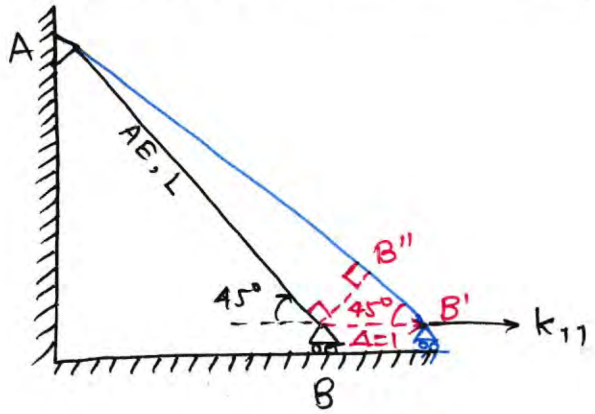
⇒

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3(\delta_B - \delta_A)}{L}\right)$$

$$= 0 + \frac{2EI}{L}\left(2\theta_A + 3\frac{\delta_A}{L} + \theta_B - 3\frac{\delta_B}{L}\right)$$

Ans:- 2, 3, 1 and -3

Q24. Horizontal stiffness coefficient, K_{11} of bar AB is given by



a) $\frac{AE}{2\sqrt{2}l}$ b) $\frac{AE}{2l}$

c) $\frac{AE}{l}$ d) $\frac{2AE}{l}$

[MPSC: 2013 (III)]

Elongation of AB $= B'B''$

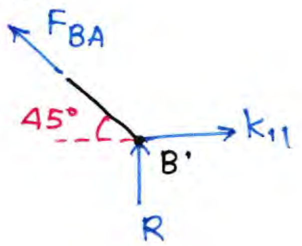
$= \Delta \cos 45°$

$= 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$F_{AB} = \frac{AE}{L} \cdot B'B''$

$= \frac{AE}{\sqrt{2}L}$

Joint B'



$\Sigma F_x = 0$

$\Rightarrow -F_{BA} \cos 45° + k_{11} = 0$

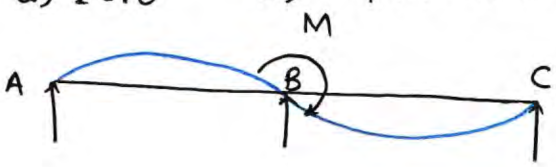
$\Rightarrow k_{11} = \frac{AE}{\sqrt{2}L} \times \frac{1}{\sqrt{2}}$

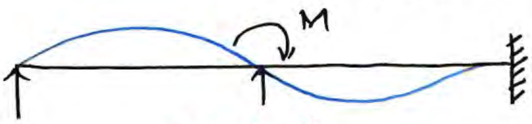
$\Rightarrow k_{11} = \frac{AE}{2L}$

Q25. A continuous beam ABC is loaded with a concentrated moment at B. End reaction at C is calculated as R_o. Keeping all conditions same, if the end C is fixed, what is the Reaction (R_c) at C?

a) Zero b) Equal to R_o c) Greater than R_o d) Less than R_o

[IAS: 2008]





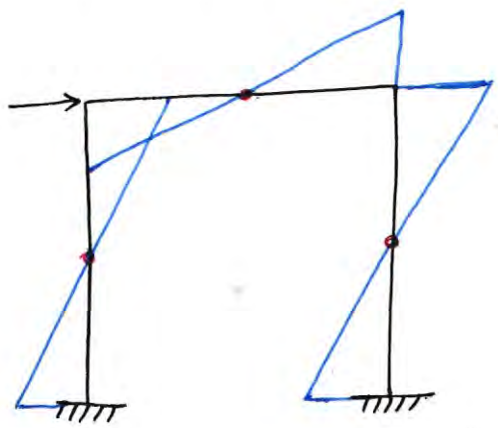
$DF = \frac{3}{7}, \frac{4}{7}$

$R_o = \frac{(M/2)}{L}$

$R_c = \frac{1.5\left(\frac{4}{7}M\right)}{L}$

$R_c > R_o$

Q26. An statically indeterminate building frame may be converted to a statically determinate one by assuming



a) hinges at mid-height of column

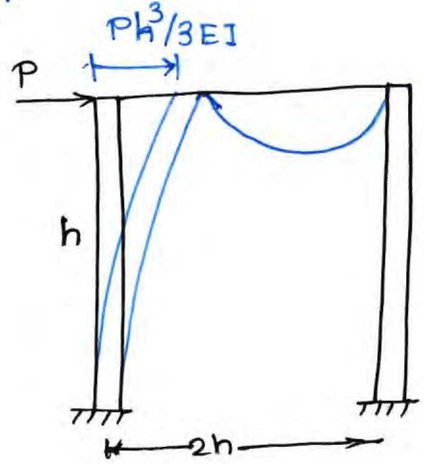
b) hinges at the mid-span of the beams

c) hinges at both mid-height of columns and mid-span of beams

d) one support as fixed at base and other support on rollers

[ESE: 2004].

Q.27.



If the flexural rigidity of the beam BC of the portal frame shown in the given figure is assumed to be zero, then the horizontal displacement of the beam would be:

a) $\frac{Ph^3}{3EI}$ b) $\frac{Ph^3}{24EI}$

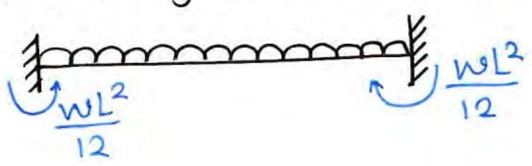
c) $\frac{Ph^3}{12EI}$ d) $\frac{Ph^3}{6EI}$

[IAS: 2000]

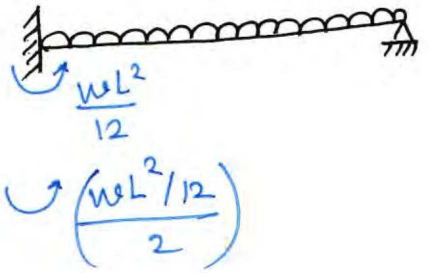
Q.28 A beam AB is fixed at both ends and carries a uniformly distributed load of intensity p per unit length run over its entire length. Due to some constructional defects, the end B is now reduced to a simple support. The percentage increase in bending moment at A is:-

a) 25 b) 50 c) 75 d) 100

[IAS : 1999]



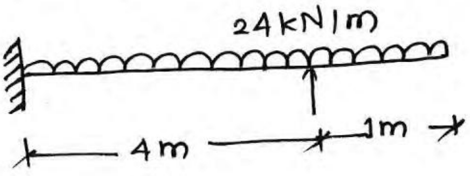
⇒

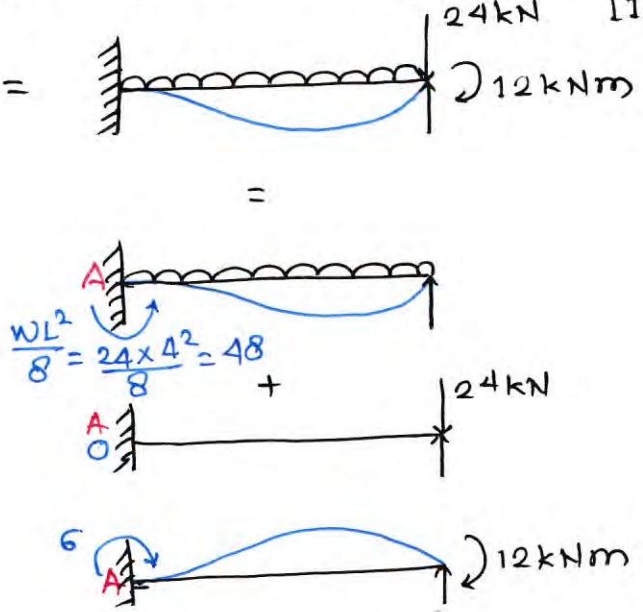


Q.29 What is the moment at joint A for the beam shown?

a) −32 kNm b) −22 kNm c) −42 kNm d) −52 kNm

[IAS: 2009]



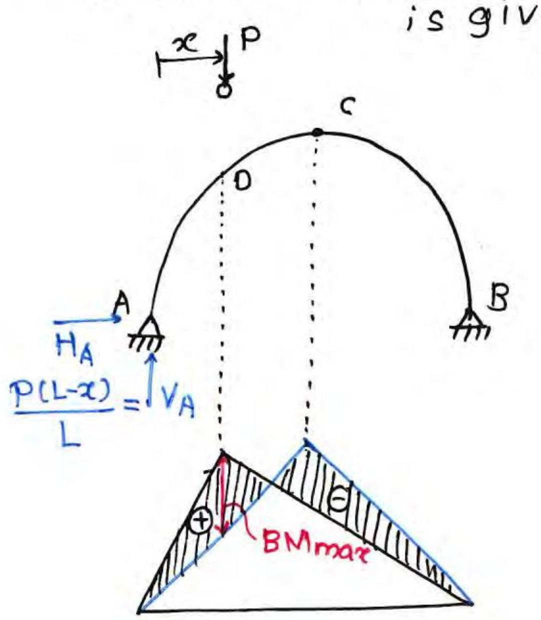


$M_A = -48 + 6$

$= -42$ kNm.

Q30. The absolute maximum bending moment due to a single rolling load in a three-hinged parabolic arch, span 'L' and central rise 'h', having one of its hinges at the crown, occurs at a distance 'x' from either support. The value of 'x' is given by:

a) $\frac{L}{2\sqrt{3}}$ b) $\frac{L}{4}$ c) $\frac{L}{3\sqrt{2}}$ d) $\frac{L}{3+\sqrt{3}}$ [IAS: 2001]



$M_C = 0$ (L.H.S.)

$\Rightarrow V_A \times \frac{L}{2} - P\left(\frac{L}{2} - x\right) - H_A \cdot h = 0$

$\Rightarrow \frac{P(L-x)}{L} \times \frac{L}{2} - P\left(\frac{L}{2} - x\right) - H_A \cdot h = 0$

$\Rightarrow H_A = \frac{Px}{2h}$

$BM_{max} = V_A \cdot x - H_A \cdot y_D$

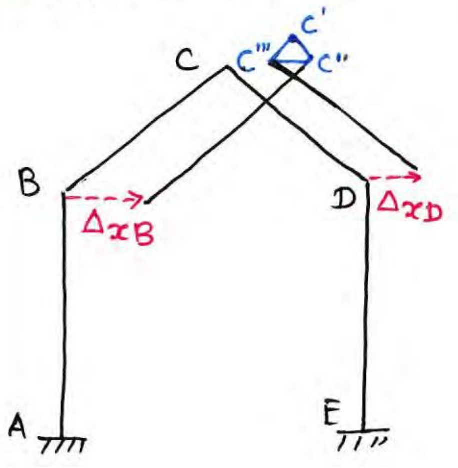
$= \frac{P(L-x)}{L} \cdot x - \frac{Px}{2h}\left\{\frac{4h}{L^2} \cdot x(L-x)\right\}$

For absolute maximum BM.

$\frac{d(BM_{max})}{dx} = 0$

$\Rightarrow \boxed{x = 0.211L}$

Q31 What is the KI of a given structure?

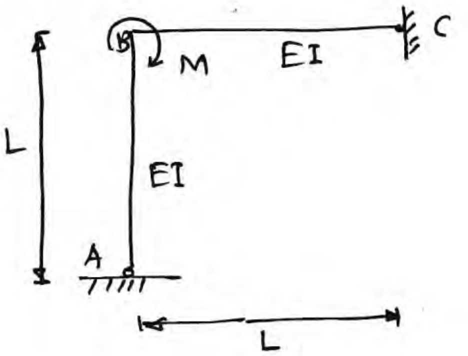


$C''C''' = \Delta_{xB} - \Delta_{xD}$

Δ_{xC} & Δ_{yC} = $f(\Delta_{xB}$ & $\Delta_{xD})$

$KI = 5$ $(\Delta_{xB}, \theta_B, \Delta_{xD}, \theta_D, \theta_C)$

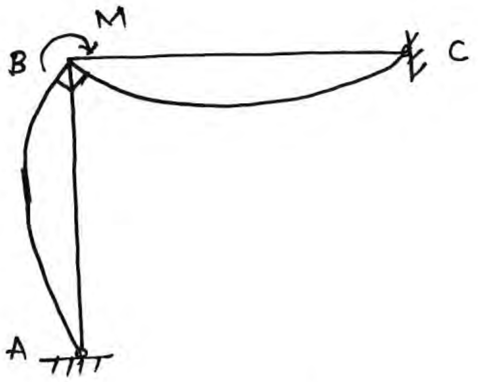
Q32. What is the rotation of the member at C for a frame as shown in figure below?

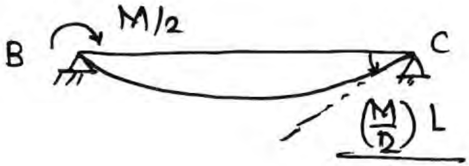


a) $\frac{ML}{3EI}$ b) $\frac{ML}{4EI}$

c) $\frac{ML}{6EI}$ d) $\frac{ML}{12EI}$

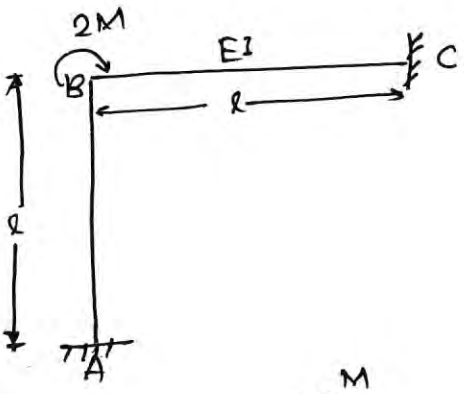
[ESE : 2009]





$$\frac{\left(\frac{M}{2}\right)L}{6EI} = \frac{ML}{12EI}$$

Q33. Members AB and BC in the figure shown are identical. Due to a moment 2M applied at B, what is the value of axial force in the member AB?



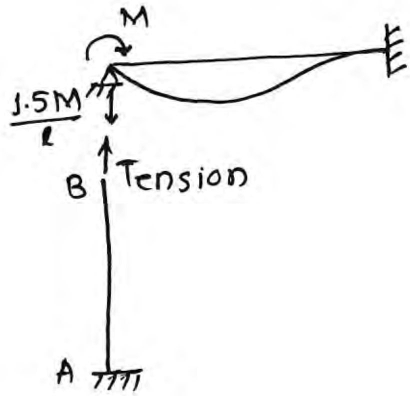
a) M/l (compression)

b) M/l (tension)

c) $1.5M/l$ (compression)

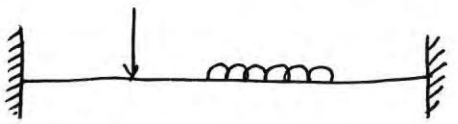
d) $1.5M/l$ (tension)

[IAS : 2005]



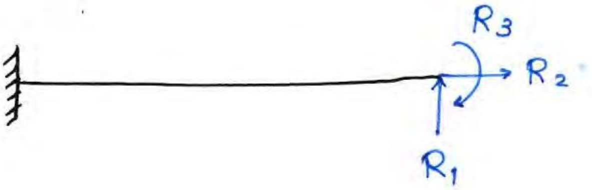
Q34. A beam fixed at the ends and subjected to lateral loads only is statically indeterminate and the degree of indeterminacy is

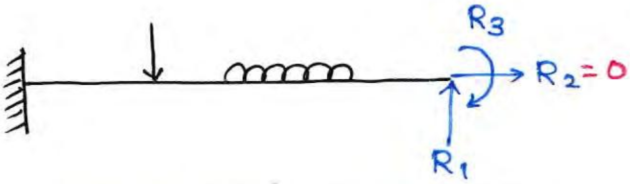
a) One b) Two c) Three d) Four. [1994 : 2 Marks]



DSI = 3, for generalised loading

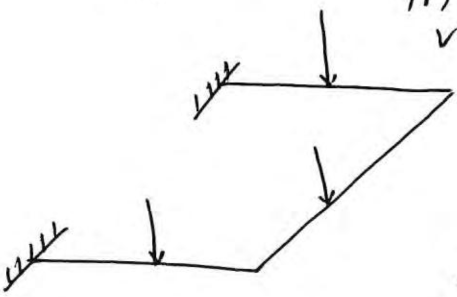
DSI = 2 for vertical loading

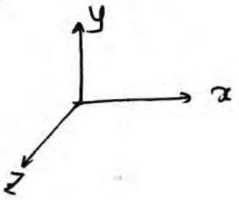




since $R_2 = 0$ so DSI = 2

Q.35 The degree of static indeterminacy of a rigidly jointed frame in a horizontal plane and subjected to vertical loading, as shown in figure below, is :-





⇒ At support B

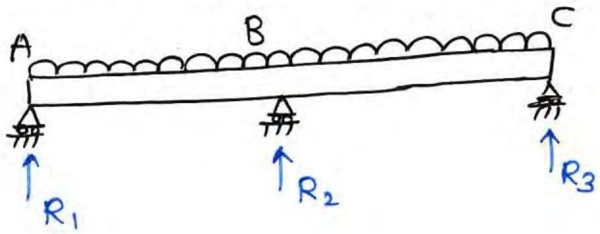
$F_x = 0$ $M_x \neq 0$

$F_y \neq 0$ $M_y = 0$

$F_z = 0$ $M_z \neq 0$

so, DSI = 3

• CONCEPT :-



$\Sigma F_x = 0$

$\Rightarrow \quad 0 = 0 \quad \ldots (i)$

$\Sigma F_y = 0$

$\Rightarrow \quad R_1 + R_2 + R_3 - wL = 0 \quad \ldots (ii)$

$\Sigma M_z = 0$

$\Rightarrow \quad \Sigma M_A = 0$

$\Rightarrow \quad -R_2 \times \frac{L}{2} - R_3 \times L + wL \times \frac{L}{2} = 0 \quad \ldots (iii)$

Since $\Sigma F_x = 0$ is useless equation so DSI = 1.

Q.36. The beam supported by 3 links and loaded as shown in the figure is

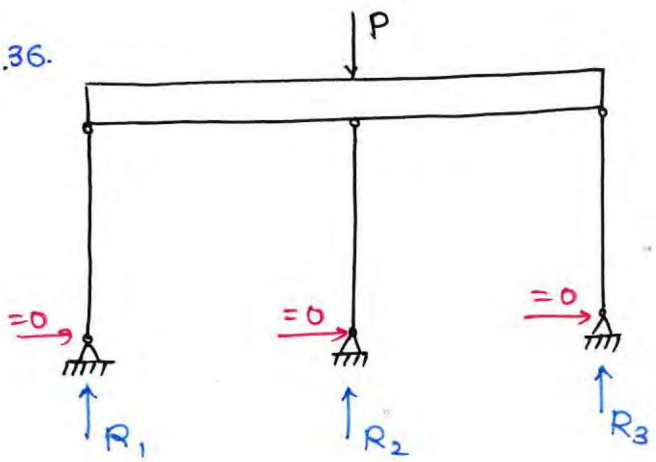
a) Stable and determinate

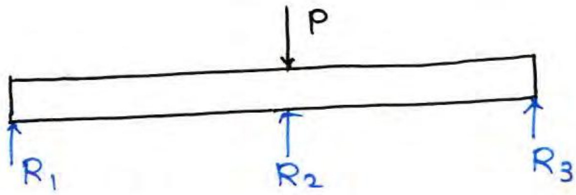
b) Unstable

c) Stable and indeterminate.

d) unstable but determinate.

[1991 : 1 mark]





DSI = 1

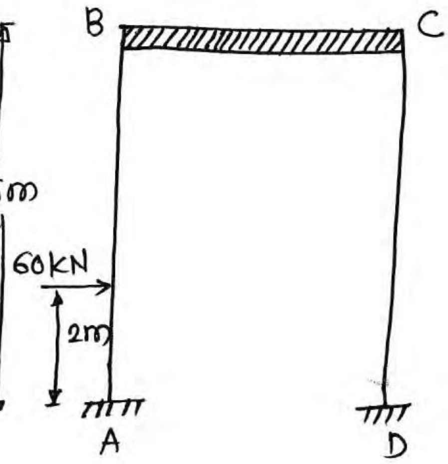
and stable for vertical loading.

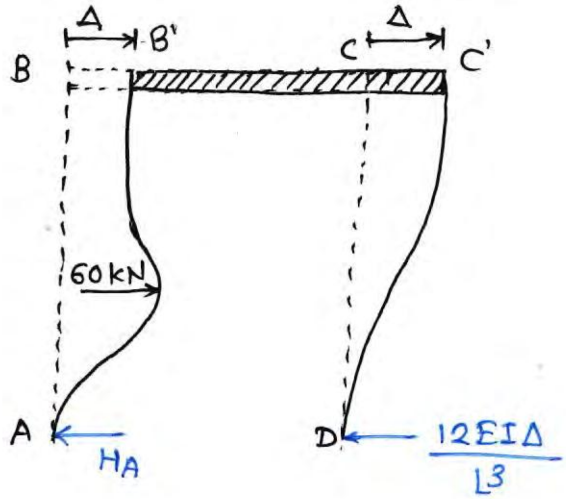
Q37. A symmetrical portal frame with the horizontal beam element very stiff compared to column elements is loaded as shown. What is the horizontal reaction at support A

a) 10 kN b) 30 kN

c) 40 kN d) 50 kN

[IAS : 2008]

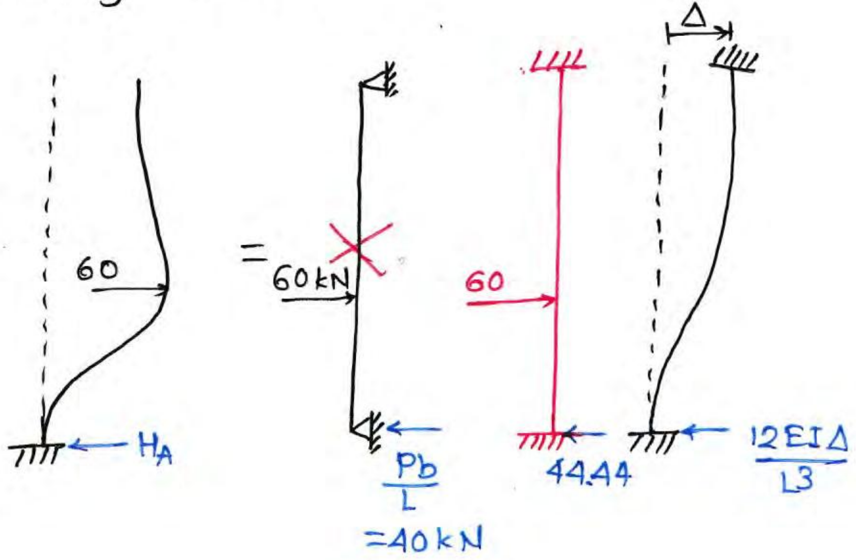




$$\Sigma F_x = 0$$

$$\Rightarrow 60 - H_A - \frac{12EI\Delta}{L^3} = 0 \quad \cdots (i)$$

Taking member AB:-



$$H_A = \cancel{40}\ 44.44 + \frac{12EI\Delta}{L^3} \quad \cdots (ii)$$

from eq^n (i) and (ii)

$$\Rightarrow 60 - \cancel{40}\ 44.44 - \frac{12EI\Delta}{L^3} - \frac{12EI}{L^3} = 0$$

$$\Rightarrow \frac{12EI\Delta}{L^3} = \cancel{10}\ 7.78$$

from eq^n (ii)

$$H_A = \cancel{40} + \cancel{10} = 50$$

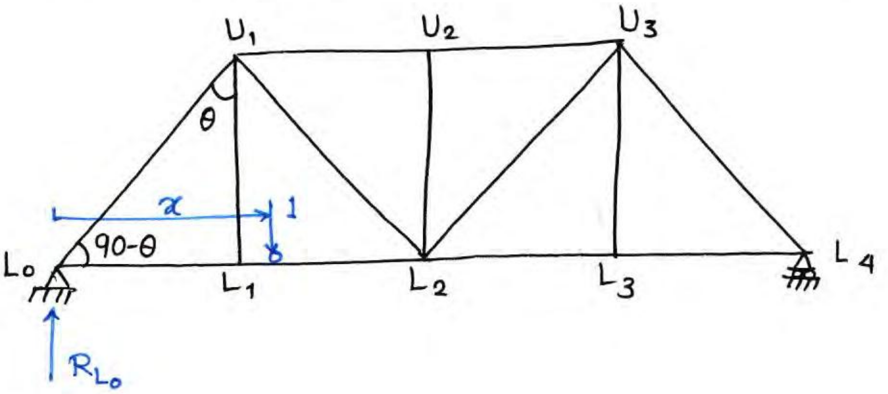
$$44.44 + 7.78 = 52.22 \text{ kN.}$$

Q.38 Consider the following statements:

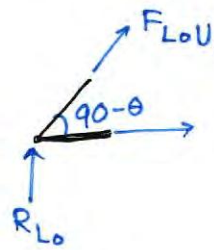
For the N-girder shown in the given figure, ILD for force in the member L_0U_1 by is obtained by

1) Multiplying the ordinate of ILD for shear in the panel L_0L_1 by $\sec\theta$
2) dividing the ordinate of ILD for moment at L_1, by $\cos\theta \times L_0L_1$
3) dividing the ordinate of ILD for moment at L_1, by L_1U_1.

[IAS: 1995]



ILD of $F_{L_0U_1}$:-



$\Sigma F_y = 0$

$\Rightarrow R_{L_0} + F_{L_0U_1}\cos\theta = 0$

$\Rightarrow F_{L_0U_1} = \dfrac{-R_{L_0}}{\cos\theta}$

① ILD of shear in panel $L_0L_1 = R_{L_0}$

② ILD of moment at $L_1 = R_{L_0} \times L_0L_1$

Ans:- 1 and 2 are correct.