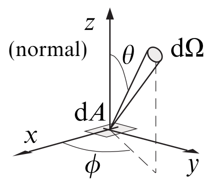
5.7 Radiation processes

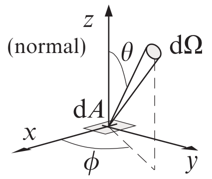
Radiometry[a]

Quantity	Equation	Units	Eq.	Symbols
Radiant energy[b]	$Q_e=\iiint L_e\cos\theta\,\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}t$	J	(5.145)	Q_e radiant energy L_e radiance (generally a function of position and direction) θ angle between dir. of $\mathrm{d}\Omega$ and normal to $\mathrm{d}A$ Ω solid angle
Radiant flux ("radiant power")	$\Phi_e=\dfrac{\partial Q_e}{\partial t}$	W	(5.146)	A area t time
	$=\iint L_e\cos\theta\,\mathrm{d}A\,\mathrm{d}\Omega$		(5.147)	Φ_e radiant flux
Radiant energy density[c]	$W_e=\dfrac{\partial Q_e}{\partial V}$	$\mathrm{J\,m^{-3}}$	(5.148)	W_e radiant energy density $\mathrm{d}V$ differential volume of propagation medium
Radiant exitance[d]	$M_e=\dfrac{\partial \Phi_e}{\partial A}$	$\mathrm{W\,m^{-2}}$	(5.149)	M_e radiant exitance
	$=\int L_e\cos\theta\,\mathrm{d}\Omega$		(5.150)	
Irradiance[e]	$E_e=\dfrac{\partial \Phi_e}{\partial A}$	$\mathrm{W\,m^{-2}}$	(5.151)	
	$=\int L_e\cos\theta\,\mathrm{d}\Omega$		(5.152)	E_e irradiance
Radiant intensity	$I_e=\dfrac{\partial \Phi_e}{\partial \Omega}$	$\mathrm{W\,sr^{-1}}$	(5.153)	I_e radiant intensity
	$=\int L_e\cos\theta\,\mathrm{d}A$		(5.154)	
Radiance	$L_e=\dfrac{1}{\cos\theta}\dfrac{\partial^2\Phi_e}{\mathrm{d}A\,\mathrm{d}\Omega}$	$\mathrm{W\,m^{-2}\,sr^{-1}}$	(5.155)	
	$=\dfrac{1}{\cos\theta}\dfrac{\partial I_e}{\partial A}$		(5.156)	



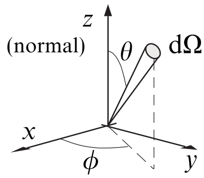
[a]Radiometry is concerned with the treatment of light as energy.
[b]Sometimes called "total energy." Note that we assume opaque radiant surfaces, so that $0\le\theta\le\pi/2$.
[c]The instantaneous amount of radiant energy contained in a unit volume of propagation medium.
[d]Power per unit area leaving a surface. For a perfectly diffusing surface, $M_e=\pi L_e$.
[e]Power per unit area incident on a surface.

Photometry[a]

Luminous energy ("total light")	$Q_\mathrm{v}=\iiint L_\mathrm{v}\cos\theta\,\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}t$ lm s (5.157)	Q_v luminous energy L_v luminance (generally a function of position and direction) θ angle between dir. of $\mathrm{d}\Omega$ and normal to $\mathrm{d}A$ Ω solid angle
Luminous flux	$\Phi_\mathrm{v}=\frac{\partial Q_\mathrm{v}}{\partial t}$ lumen (lm) (5.158) $=\iint L_\mathrm{v}\cos\theta\,\mathrm{d}A\,\mathrm{d}\Omega$ (5.159)	A area t time Φ_v luminous flux
Luminous density[b]	$W_\mathrm{v}=\frac{\partial Q_\mathrm{v}}{\partial V}$ lm s m^{-3} (5.160)	W_v luminous density V volume
Luminous exitance[c]	$M_\mathrm{v}=\frac{\partial \Phi_\mathrm{v}}{\partial A}$ lx (lm m^{-2}) (5.161) $=\int L_\mathrm{v}\cos\theta\,\mathrm{d}\Omega$ (5.162)	M_v luminous exitance
Illuminance ("illumination")[d]	$E_\mathrm{v}=\frac{\partial \Phi_\mathrm{v}}{\partial A}$ lm m^{-2} (5.163) $=\int L_\mathrm{v}\cos\theta\,\mathrm{d}\Omega$ (5.164)	
Luminous intensity[e]	$I_\mathrm{v}=\frac{\partial \Phi_\mathrm{v}}{\partial \Omega}$ cd (5.165) $=\int L_\mathrm{v}\cos\theta\,\mathrm{d}A$ (5.166)	E_v illuminance I_v luminous intensity
Luminance ("photometric brightness")	$L_\mathrm{v}=\frac{1}{\cos\theta}\frac{\partial^2 \Phi_\mathrm{v}}{\mathrm{d}A\,\mathrm{d}\Omega}$ cd m^{-2} (5.167) $=\frac{1}{\cos\theta}\frac{\partial I_\mathrm{v}}{\partial A}$ (5.168)	
Luminous efficacy	$K=\frac{\Phi_\mathrm{v}}{\Phi_\mathrm{e}}=\frac{L_\mathrm{v}}{L_\mathrm{e}}=\frac{I_\mathrm{v}}{I_\mathrm{e}}$ lm W^{-1} (5.169)	K luminous efficacy L_e radiance Φ_e radiant flux I_e radiant intensity
Luminous efficiency	$V(\lambda)=\frac{K(\lambda)}{K_\mathrm{max}}$ (5.170)	V luminous efficiency λ wavelength K_max spectral maximum of $K(\lambda)$

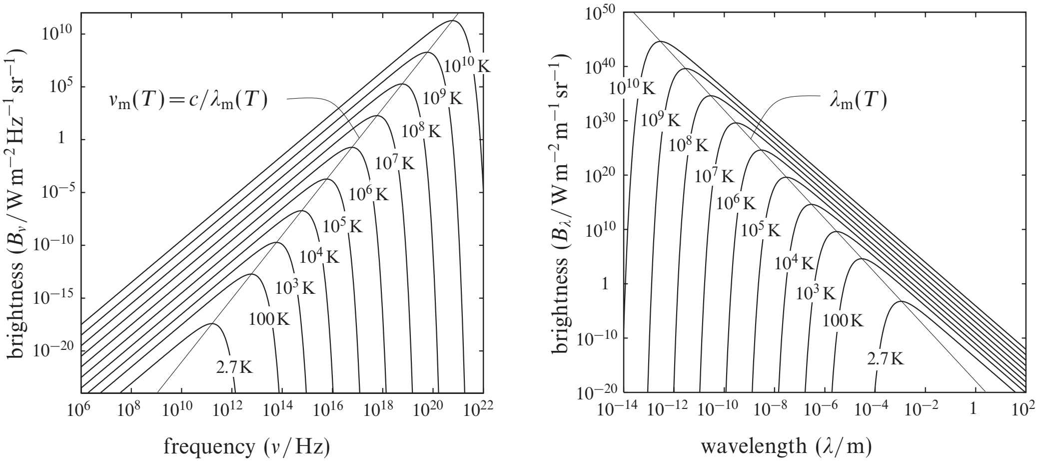
[a]Photometry is concerned with the treatment of light as seen by the human eye.
[b]The instantaneous amount of luminous energy contained in a unit volume of propagating medium.
[c]Luminous emitted flux per unit area.
[d]Luminous incident flux per unit area. The derived SI unit is the lux (lx). $1\,\mathrm{lx}=1\,\mathrm{lm\,m^{-2}}$.
[e]The SI unit of luminous intensity is the candela (cd). $1\,\mathrm{cd}=1\,\mathrm{lm\,sr^{-1}}$.

Radiative transfer[a]

Flux density (through a plane)	$F_\nu = \int I_\nu(\theta,\phi)\cos\theta \, \mathrm{d}\Omega \quad \mathrm{W\,m^{-2}\,Hz^{-1}}$ (5.171)	
Mean intensity[b]	$J_\nu = \frac{1}{4\pi}\int I_\nu(\theta,\phi)\,\mathrm{d}\Omega \quad \mathrm{W\,m^{-2}\,Hz^{-1}}$ (5.172)	F_ν flux density I_ν specific intensity ($\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$) J_ν mean intensity
Spectral energy density[c]	$u_\nu = \frac{1}{c}\int I_\nu(\theta,\phi)\,\mathrm{d}\Omega \quad \mathrm{J\,m^{-3}\,Hz^{-1}}$ (5.173)	u_ν spectral energy density Ω solid angle θ angle between normal and direction of Ω
Specific emission coefficient	$j_\nu = \frac{\epsilon_\nu}{\rho} \quad \mathrm{W\,kg^{-1}\,Hz^{-1}\,sr^{-1}}$ (5.174)	j_ν specific emission coefficient ϵ_ν emission coefficient ($\mathrm{W\,m^{-3}\,Hz^{-1}\,sr^{-1}}$) ρ density
Gas linear absorption coefficient ($\alpha_\nu \ll 1$)	$\alpha_\nu = n\sigma_\nu = \frac{1}{l_\nu} \quad \mathrm{m^{-1}}$ (5.175)	α_ν linear absorption coefficient n particle number density σ_ν particle cross section l_ν mean free path
Opacity[d]	$\kappa_\nu = \frac{\alpha_\nu}{\rho} \quad \mathrm{kg^{-1}\,m^2}$ (5.176)	κ_ν opacity
Optical depth	$\tau_\nu = \int \kappa_\nu \rho \, \mathrm{d}s$ (5.177)	τ_ν optical depth, or optical thickness $\mathrm{d}s$ line element
Transfer equation[e]	$\frac{1}{\rho}\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$ (5.178) or $\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\alpha_\nu I_\nu + \epsilon_\nu$ (5.179)	
Kirchhoff's law[f]	$S_\nu \equiv \frac{j_\nu}{\kappa_\nu} = \frac{\epsilon_\nu}{\alpha_\nu}$ (5.180)	S_ν source function
Emission from a homogeneous medium	$I_\nu = S_\nu(1 - \mathrm{e}^{-\tau_\nu})$ (5.181)	

[a] The definitions of these quantities vary in the literature. Those presented here are common in meteorology and astrophysics. Note particularly that the ambiguous term *specific* is taken to mean "per unit frequency interval" in the case of specific intensity and "per unit mass per unit frequency interval" in the case of specific emission coefficient.
[b] In radio astronomy, flux density is usually taken as $S = 4\pi J_\nu$.
[c] Assuming a refractive index of 1.
[d] Or "mass absorption coefficient."
[e] Or "Schwarzschild's equation."
[f] Under conditions of local thermal equilibrium (LTE), the source function, S_ν, equals the Planck function, $B_\nu(T)$ [see Equation (5.182)].

Blackbody radiation



Planck function[a]	$$B_\nu(T)=\frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1} \quad (5.182)$$ $$B_\lambda(T)=B_\nu(T)\frac{\mathrm{d}\nu}{\mathrm{d}\lambda} \quad (5.183)$$ $$=\frac{2hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda kT}\right)-1\right]^{-1} \quad (5.184)$$	B_ν surface brightness per unit frequency (W m^{-2} Hz^{-1} sr^{-1}) B_λ surface brightness per unit wavelength (W m^{-2} m^{-1} sr^{-1}) h Planck constant
Spectral energy density	$$u_\nu(T)=\frac{4\pi}{c}B_\nu(T) \quad \mathrm{J\,m^{-3}\,Hz^{-1}} \quad (5.185)$$ $$u_\lambda(T)=\frac{4\pi}{c}B_\lambda(T) \quad \mathrm{J\,m^{-3}\,m^{-1}} \quad (5.186)$$	c speed of light k Boltzmann constant T temperature $u_{\nu,\lambda}$ spectral energy density
Rayleigh–Jeans law ($h\nu \ll kT$)	$$B_\nu(T)=\frac{2kT}{c^2}\nu^2=\frac{2kT}{\lambda^2} \quad (5.187)$$	
Wien's law ($h\nu \gg kT$)	$$B_\nu(T)=\frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{kT}\right) \quad (5.188)$$	
Wien's displacement law	$$\lambda_m T=\begin{cases}5.1\times10^{-3}\,\mathrm{m\,K} & \text{for } B_\nu\\ 2.9\times10^{-3}\,\mathrm{m\,K} & \text{for } B_\lambda\end{cases} \quad (5.189)$$	λ_m wavelength of maximum brightness
Stefan–Boltzmann law[b]	$$M=\pi\int_0^\infty B_\nu(T)\,\mathrm{d}\nu \quad (5.190)$$ $$=\frac{2\pi^5k^4}{15c^2h^3}T^4=\sigma T^4 \quad \mathrm{W\,m^{-2}} \quad (5.191)$$	M exitance σ Stefan–Boltzmann constant ($\simeq 5.67\times10^{-8}$ W m^{-2} K^{-4})
Energy density	$$u(T)=\frac{4}{c}\sigma T^4 \quad \mathrm{J\,m^{-3}} \quad (5.192)$$	u energy density
Greybody	$$M=\epsilon\sigma T^4=(1-A)\sigma T^4 \quad (5.193)$$	ϵ mean emissivity A albedo

[a] With respect to the projected area of the surface. Surface brightness is also known simply as "brightness." "Specific intensity" is used for reception.

[b] Sometimes "Stefan's law." Exitance is the total radiated energy from unit area of the body per unit time.