Long Answer Questions

Q. 1. Answer the following questions

(i) What is induced emf? Write Faraday's law of electromagnetic induction. Express it mathematically.

(ii) A conducting rod of length 'l', with one end pivoted, is rotated with a uniform angular speed 'ω' in a vertical plane, normal to a uniform magnetic field 'B'. Deduce an expression for the emf induced in this rod. [CBSE Delhi 2013, 2012]

If resistance of rod is R, what is the current induced in it?

Ans. (i) Induced emf: The emf developed in a coil due to change in magnetic flux linked with the coil is called the induced emf.

Faraday's Law of Electromagnetic Induction: On the basis of experiments, Faraday gave two laws of electromagnetic induction:

(i) When the magnetic flux linked with a coil or circuit changes, an emf is induced in the coil. If coil is closed, the current is also induced. The emf and current last so long as the change in magnetic flux lasts. The magnitude of induced emf is proportional to the rate of change of magnetic flux linked with the circuit. Thus if $\Delta\varphi$ is the change in magnetic flux linked in time Δt then rate of change of flux is

$\frac{\Delta\varphi}{\Delta t}$,

So emf induced $\varepsilon \propto \frac{\Delta\varphi}{\Delta t}$

2. The emf induced in the coil (or circuit) opposes the cause producing it.

$\varepsilon \propto -\frac{\Delta\varphi}{\Delta t}$

Here the negative sign shows that the induced emf ε′opposes the change in magnetic flux.

$\varepsilon = -K\frac{\Delta\varphi}{\Delta t}$ where K is a constant of proportionality which depends on units chosen for φ, t and ε. In SI system the unit of flux φ is weber, unit of time t is second and unit of emf ε′ is volt and $K=1$

$\therefore \quad \varepsilon = -\frac{\Delta\varphi}{\Delta t} \quad ...(i)$

If the coil contains N– turns of insulated wire, then the flux linked with each turn will be same and the emf induced in each turn will be in the same direction, hence the emfs of all turns will be added. Therefore the emf induced in the whole coil,

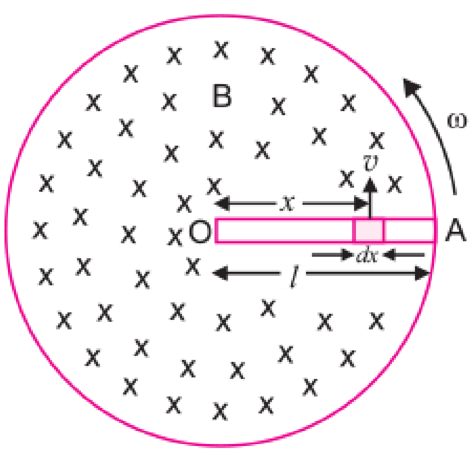
$\varepsilon = -N\frac{\Delta\varphi}{\Delta t} = -\frac{\Delta(N\varphi)}{\Delta t} ...(ii)$

Nφ is called the **effective magnetic flux or the number of flux linkages in the coil and may be denoted by φ**

(ii) Expression for Induced emf in a Rotating Rod

Consider a metallic rod OA of length l which is rotating with angular velocity ω in a uniform magnetic field B, the plane of rotation being perpendicular to the magnetic field. A rod may be supposed to be formed of a large number of small elements. Consider a small element of length dx at a distance x from centre. If v is the linear velocity of this element, then area swept by the element per second = v dx

The emf induced across the ends of element



$$d\varepsilon = B\frac{\mathrm{dA}}{\mathrm{dt}} = \mathrm{Bv\ \ dx}$$

But $v = x\omega$

$\therefore \quad d\varepsilon = \mathrm{B}\ x\ \omega\ dx$

$\therefore \quad$ The emf induced across the rod

$$\varepsilon = \int_0^l B\ x\omega\ \mathrm{dx} = B\omega \int_0^l x\ \ \mathrm{dx}$$

$$= B\omega\left[\frac{x^2}{2}\right] = B\omega\left[\frac{l^2}{2} - 0\right] = \frac{1}{2}B\omega I^2$$

Current induced in rod $I = \frac{\varepsilon}{R} = \frac{1}{2}\frac{B\omega l^2}{R}$.

If circuit is closed, power dissipated $= \frac{\varepsilon^2}{R} = \frac{B^2\omega^2 l^2}{4R}$

Q. 2. Answer the following questions

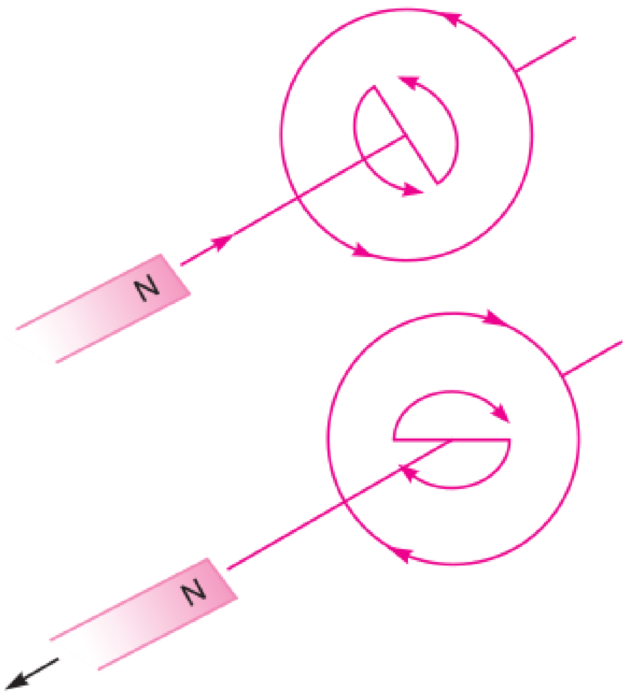
(1) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce an induced current which opposes the change of magnetic flux that produces it.

(2) The current flowing through an inductor of self-inductance L is continuously increasing. Plot a graph showing the variation of

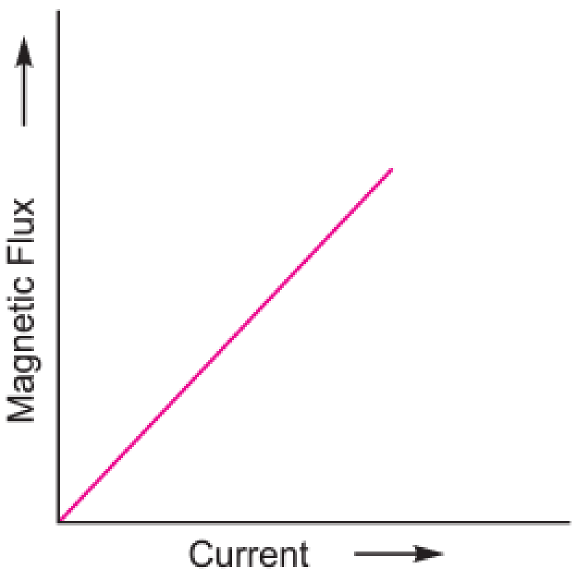
(i) Magnetic flux versus the current
(ii) Induced emf versus dI/dt
(iii) Magnetic potential energy stored versus the current. [CBSE Delhi 2014]

Ans. (i) When the North pole of a bar magnet moves towards the closed coil, the magnetic flux through the coil increases. This produces an induced emf which produces (or tend to produce if the coil is open) an induced current in the anti-clockwise sense. The anti-clockwise sense corresponds to the generation of North Pole which opposes the motion of the approaching N pole of the magnet. The face of the coil, facing the approaching magnet, then has the same polarity as that of the approaching pole of the magnet. The induced current, therefore, is seen to oppose the change of magnetic flux that produces it.

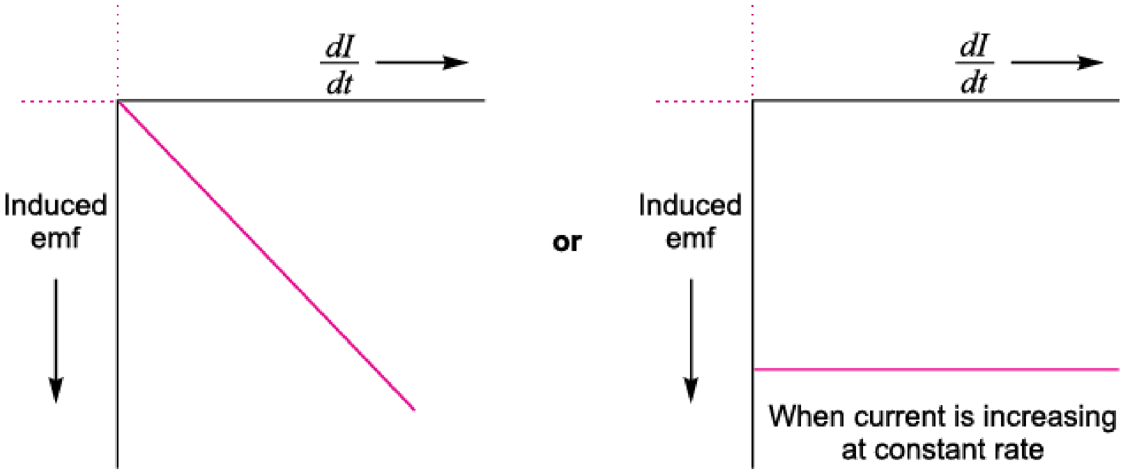
When a North Pole of a magnet is moved away from the coil, the current (I) flows in the clock-wise sense which corresponds to the generation of South Pole. The induced South Pole opposes the motion of the receding North Pole.



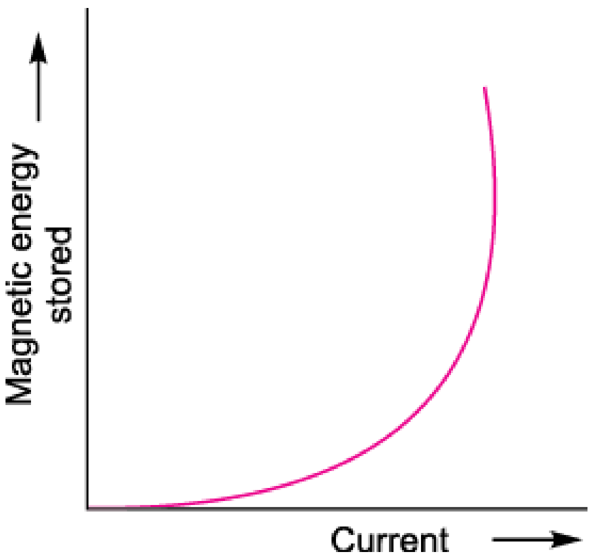
(ii) (i) Magnetic flux versus the current



(ii) Induced emf versus dI/dt



(iii) Magnetic energy stored



Q. 3. Derive an expression for (i) induced emf and (ii) induced current when a conductor of length l is moved with a uniform velocity, normal to a uniform magnetic field B. Assume the resistance of conductor to be R.

Ans. Expression for Induced emf: We know that if a charge q moves with velocity $\vec{V}$ in a magnetic field of strength $\vec{B}$, making an angle θ then magnetic Lorentz force

F = q vB sin θ

If $\vec{v}$ and $\vec{B}$ mutually perpendicular, then $\theta = 90°$

$$F = qvB \sin 90° = qvB$$

The direction of this force is perpendicular to both $\underset{V}{\rightarrow}$ and $\underset{B}{\rightarrow}$ and is given by Fleming's left hand rule.

Suppose a thin conducting rod PQ is placed on two parallel metallic rails CD and MN in a magnetic field of strength $\underset{B}{\rightarrow}$. The direction of magnetic field $\underset{B}{\rightarrow}$ is perpendicular to the plane of paper, downward. In fig $\underset{B}{\rightarrow}$ is represented by cross (×) marks. Suppose the rod is moving with velocity $\underset{V}{\rightarrow}$, perpendicular to its own length, towards the right. We know that metallic conductors contain free electrons, which can move within the metal. As charge on electron, q = – e therefore, each electron experiences a magnetic Lorentz force, F_m = evB, whose direction, according to Fleming's left hand rule, will be from P to Q Thus the electrons are displaced from end P towards end Q Consequently the end P of rod becomes positively charged and end Q negatively charged. Thus a potential difference is produced between the ends of the conductor. This is the **induced emf.**

Due to induced emf, an electric field is produced in the conducting rod. The strength of this electric field

$$E = \frac{V}{l} \qquad ...(i)$$

And its direction is from (+) to (–) charge, i.e., from P to Q.

The force on a free electron due to this electric field, $F_e = {}_eE$ ...(ii)

The direction of this force is from Q to P which is opposite to that of electric field. Thus the emf produced opposes the motion of electrons caused due to Lorentz force. This is in accordance with Lenz's law. As the number of electrons at end becomes more and more, the magnitude of electric force Fe goes on increasing, and a stage comes when electric force $\underset{Fe}{\rightarrow}$ and magnetic force $\underset{Fm}{\longrightarrow}$ become equal and opposite. In this situation the potential difference produced across the ends of rod becomes constant. In this condition

$F_e = F_m$

${}_eE$ = evB or E = B_v ...(iii)

∴ The potential difference produced,

V = El = B v l Volt

Also the induced current $I = \frac{V}{R} = \frac{Bvl}{R}$ ampere

Q. 4. Derive expression for self-inductance of a long air-cored solenoid of length l, cross-sectional area A and having number of turns N. [CBSE Delhi 2012, 2009]

Ans. Self-Inductance of a long air-cored solenoid:

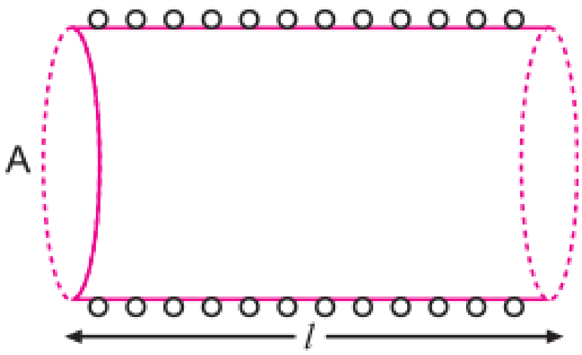
Consider a long air solenoid having ‘n’ number of turns per unit length. If current in solenoid is I, then magnetic field within the solenoid, $B = \mu_0 nI$ ...(i)

Where $\mu_0 = 4\pi \times 10^{-7}$ henry/metre is the permeability of free space.

If A is cross-sectional area of solenoid, then effective flux linked with solenoid of length l′ $\Phi = NBA$ where $N = nl$ is the number of turns in length ‘l’ of solenoid.

∴ $\Phi = (nl\, BA)$

Substituting the value of B from (i)



∴ $\Phi = (\mu_0 nI) A = \mu_0 n^2 AlI$ ...(ii)

∴ Self-inductance of air solenoid

$$L = \frac{\Phi}{I} = \mu_0 n^2 \text{ Al} \quad ...(iii)$$

If N is total number of turns in length l then

$$n = \frac{N}{l}$$

∴ Self-inductance $L = \mu_0 \left(\frac{N}{l}\right)^2 \text{ Al} \quad ..(iv)$

Remark: If solenoid contains a core of ferromagnetic substance of relative permeability μ_r then self inductance, $L = \frac{\mu_r \mu_0 N^2 A}{l}$.

Q. 5. Obtain the expression for the mutual inductance of two long co-axial solenoids S1 and S2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n2 be number of turns per unit length, when a current I is set up in the outer solenoid S_2.
[CBSE Delhi 2017]

OR

(a) Define mutual inductance and write its SI units.

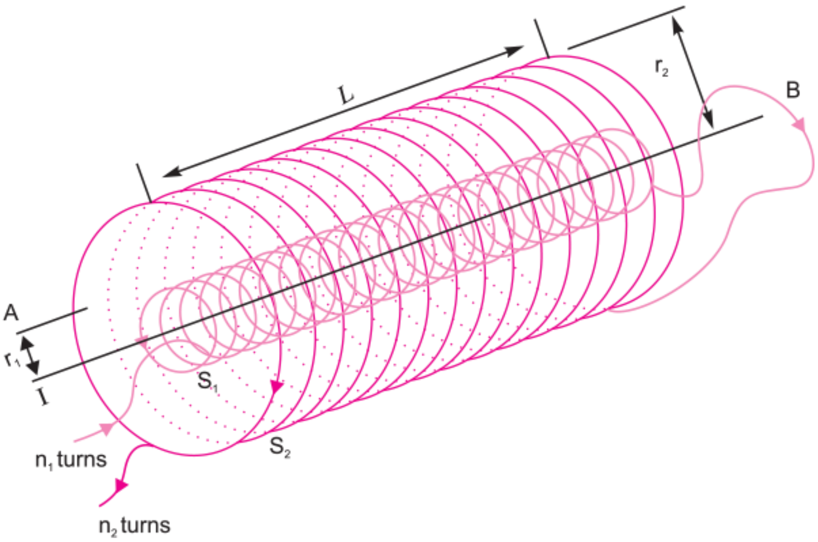
(b) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

(c) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced in the coil C_1 due to a change in the current through the coil C_2. [CBSE Delhi 2015]

Ans. (a) When current flowing in one of two nearby coils is changed, the magnetic flux linked with the other coil changes; due to which an emf is induced in it (other coil). This phenomenon of electromagnetic induction is called the mutual induction. The coil, in which current is changed is called the primary coil and the coil in which emf is induced is called the secondary coil.

The SI unit of mutual inductance is henry.

(b) Mutual inductance is numerically equal to the magnetic flux linked with one coil (secondary coil) when unit current flows through the other coil (primary coil).



Consider two long co-axial solenoids, each of length L. Let nl be the number of turns per unit length of the inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of the outer solenoid S_2 of radius r_2.

Imagine a time varying current I2 through S_2 which sets up a time varying magnetic flux φ1 through S_1.

∴ $\varphi 1 = M_{12}(I_2)$ ...(i)

Where, M_{12} = Coefficient of mutual inductance of solenoid S1 with respect to solenoid S_2

Magnetic field due to the current I_2 in S_2 is

∴ Magnetic flux through S_1 is

$\varphi_1 = B_2 A_1 N_1$

Where, $N_1 = n_1$ and L = length of the solenoid

$\varphi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 L)$ ...(ii)

$\varphi_1 = \mu_0 n_1 n_2 \pi r_1^2 \, \mathrm{LI}_2\,)$

From equations (i) and (ii), we get

$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 L$ ...(iii)

Let us consider the reverse case.

A time varying current I_1 through S_1 develops a flux φ_2 through S_2.

$\therefore \varphi_2 = M_{21}(I_1)$ ...(iv)

where, M_{12}= Coefficient of mutual inductance of solenoid S_2 with respect to solenoid S_1 Magnetic flux due to S_1 is confined solely in S_1 as the solenoids are assumed to be very long. There is no magnetic field outside S_1 due to current I in S_1.

The magnetic flux linked with S_2 is

$\therefore \quad \varphi_2 = B_1 A_1 N_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 L)$

$\varphi_2 = \mu_0 n_1 n_2 I_2 \pi r_1^2 \, \mathrm{LI}_1$ ...(v)

From equations (iv) and (v), we get

$M_{21} = \mu_0 n_1 n_2 \pi r_1^2$ ...(vi)

From equations (iii) and (vi), we get

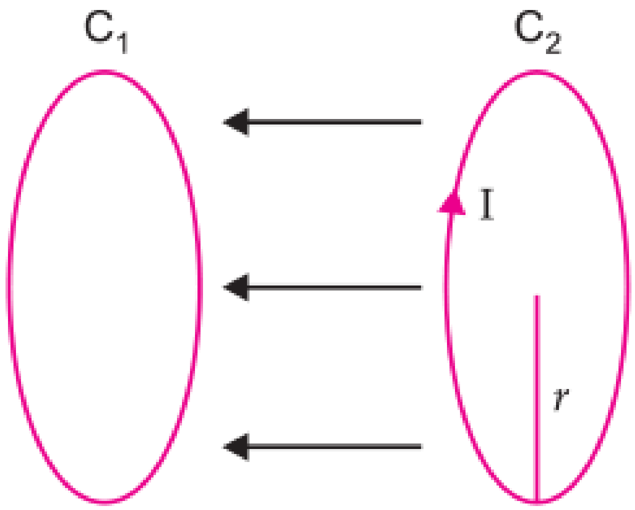
$M_{12} = M_{21} = M <= \mu_0 n_1 n_2 \pi r_1^2 L$

We can write the above equation as

$M = \mu_0 \left(\frac{N_1}{L}\right)\left(\frac{N_2}{L}\right)\pi r^2 \times L$

$M = \frac{\mu_0 N_1 N_2 \pi r^2}{L}$

When the current in coil C_2 changes, the flux linked with C_1 changes. This change in flux linked with C_1 induces emf in C_1.



A = area of coil

Flux linked with C_1 = flux of C_2

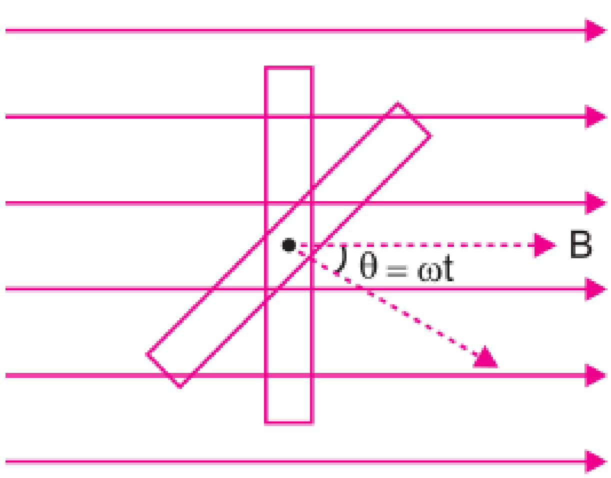
$$\varphi_{12} = B.A = \frac{\mu_0 I}{2r}.A$$

$$\text{emf in } C_1 = \frac{d\varphi_{12}}{dt} = \frac{d}{dt}\frac{\mu_0\,AI}{2r}$$

Q. 6. A coil of number of turns N, area A is rotated at a constant angular speed ω, in a uniform magnetic field B and connected to a resistor R. Deduce expression for

(i) Maximum emf induced in the coil.
(ii) Power dissipation in the coil.

Ans. (i) Suppose initially the plane of coil is perpendicular to the magnetic field B. When coil rotates with angular speed ω, then after time t, the angle between magnetic field $\vec{B}$ and normal to plane of coil is



θ = ωt

∴ At this instant magnetic flux linked with the coil $\varphi = BA \cos \omega t$

If coil constants, N-turns, then emf induced in the coil

$$\varepsilon = -N\frac{d\varphi}{dt} = -N\frac{d}{dt}(\mathrm{BA}\cos\omega t)$$

$$= +NBA\,\omega \sin \omega t \qquad \ldots(i)$$

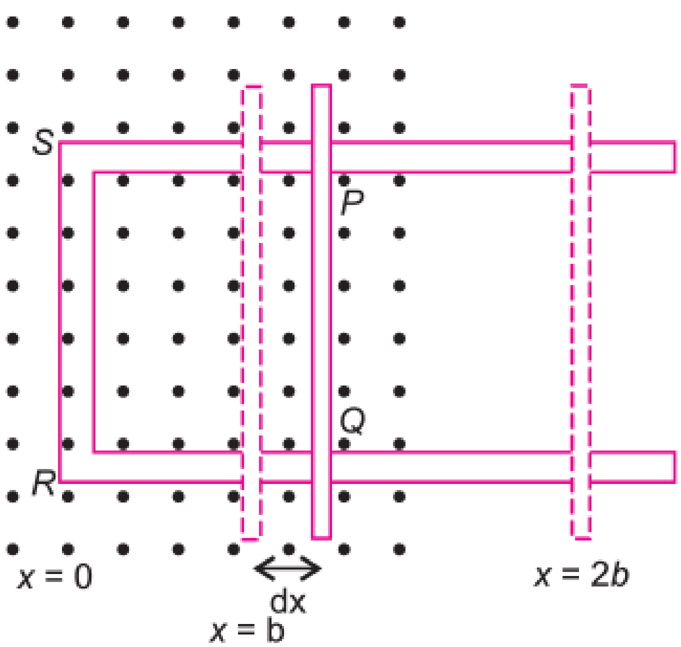
∴ For maximum value of emf ε,

$\sin \omega t = 1$

∴ Maximum emf induced, $\varepsilon_{max} = NBA\,\omega$

Q. 7. State Faraday's law of electromagnetic induction.

Figure shows a rectangular conductor PQRS in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from x = 0 to x = b and is zero for x > b. Assume that only the arm PQ possesses resistance r. When the arm PQ is pulled outward from x = 0 to x = 2b and is then moved backward to x =0 with constant speed v, obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance 0 ≤ x ≤ 2b
[CBSE (AI) 2010, (North) 2016]



Ans. Refer to Point 3 of Basic Concepts.

Let length of conductor PQ =1

As x = 0, magnetic flux φ = 1

When PQ moves a small distance from x to x + dx then magnetic flux linked= BdA=Bldx
The magnetic field is from x = 0 to x = b, to so final magnetic flux

= ∑Bldx = Bl ∑dx =Blb (increasing)

Mean magnetic flux from $x = 0$ to $x = b$ is $\frac{1}{2}$Blb

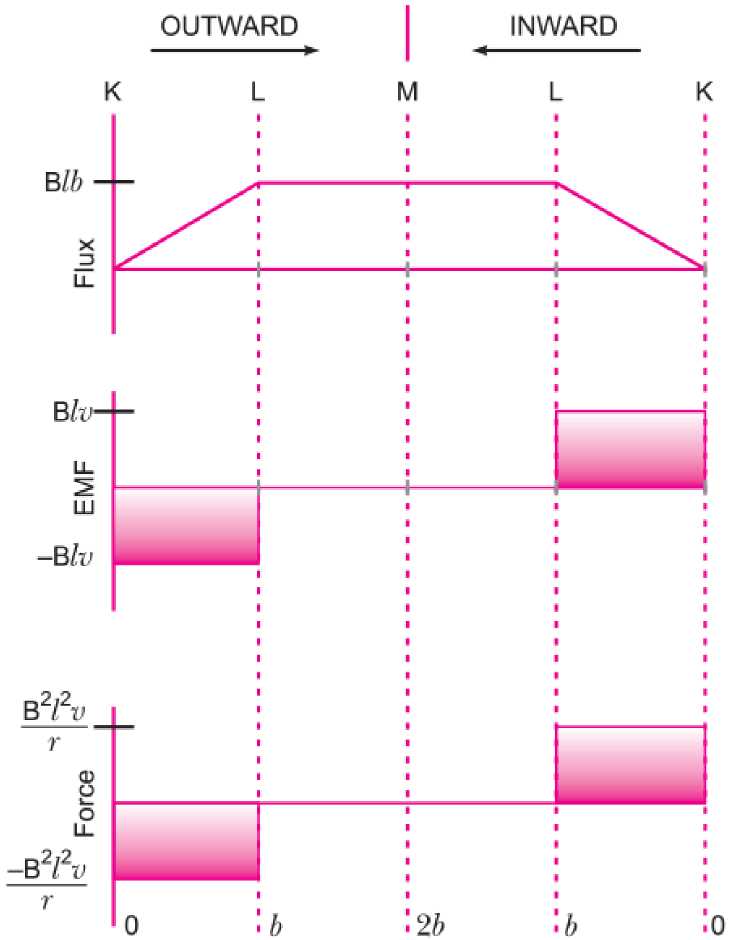
The magnetic flux from $x = b$ to $x = 2b$ is zero.

Induced emf, $\varepsilon = -\frac{d\varphi}{dt} = \frac{d}{dt}(\text{Bldx}) = -\text{Bl}\frac{dx}{dt} = -\text{Blv}$

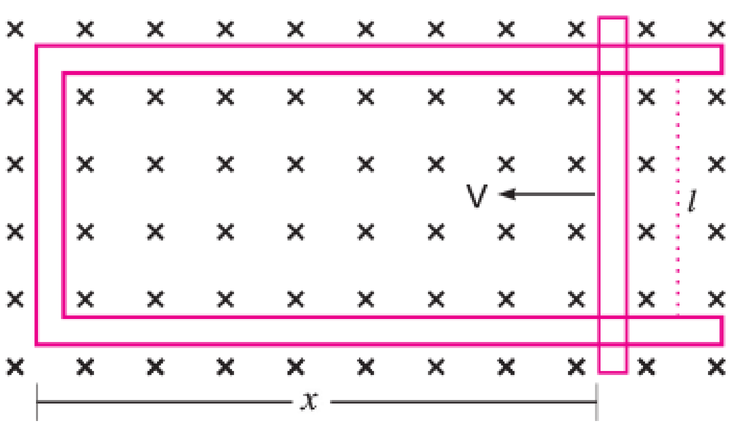
where $v = \frac{dx}{dt}$ velocity of arm PQ from $x = 0$ to $x = b$.

During return from $x = 2b$ to $x = b$ the induced emf is zero; but now area is decreasing so magnetic flux is decreasing, and induced emf will be in opposite direction.

$\varepsilon = Blv$



Q. 8. What are eddy currents? How are they produced? In what sense eddy currents are considered undesirable in a transformer? How can they be minimised? Give two applications of eddy currents. [CBSE (AI) 2011, (F) 2015]



Ans. Eddy currents: When a thick metallic piece is placed in a time varying magnetic field, the magnetic flux linked with the plate changes, the induced currents are set up in the conductor; these currents are called **eddy currents.** These currents are sometimes so strong, that the metallic plate becomes red hot.

Due to heavy eddy currents produced in the core of a transformer, large amount of energy is wasted in the form of undesirable heat.

Minimisation of Eddy Currents: Eddy currents may be minimised by using **laminated core** of soft iron. The resistance of the laminated core increases and the eddy currents are reduced and wastage of energy is also reduced.

Application of Eddy Currents:

(i) Induction Furnace: In induction furnance, the metal to be heated is placed in a rapidly varying magnetic field produced by high frequency alternating current. Strong eddy currents are set up in the metal produce so much heat that the metal melts. This process is used in extracting a metal from its ore. The arrangement of heating the metal by means of strong induced currents is called the induction furnace.

(ii) Induction Motor: The eddy currents may be used to rotate the rotor. Its principle is: When a metallic cylinder (or rotor) is placed in a rotating magnetic field, eddy currents are produced in it. According to Lenz's law, these currents tend to reduce to relative motion between the cylinder and the field. The cylinder, therefore, begins to rotate in the direction of the field. This is the principle of induction motion.