CBSE Test Paper 03

Chapter 2 Inverse Trigonometric Functions

1. If $\sin A + \cos A = 1$, then sin 2A is equal to **(1)**
   a. $\frac{1}{2}$
   b. 0
   c. 1
   d. 2
2. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$ holds true for all **(1)**
   a. $|x| \leqslant \frac{1}{2}$
   b. $|x| \leqslant 1$
   c. None of these
   d. $1 \geqslant x \geqslant 0$
3. $\tan(\sin^{-1}x)$ is equal to **(1)**
   a. None of these
   b. $\frac{|x|}{\sqrt{1-x^2}}$
   c. $\frac{x}{\sqrt{1-x^2}}$
   d. $\frac{-x}{\sqrt{1-x^2}}$
4. If and x + y + z = xyz, then a value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is **(1)**
   a. π
   b. $\frac{\pi}{2}$
   c. None of these
   d. $\frac{3\pi}{2}$
5. The value of $\cos^{-1}(-1) - \sin^{-1}(1)$ is **(1)**
   a. $\frac{3\pi}{2}$
   b. π

c. $-\frac{3\pi}{2}$
d. $\frac{\pi}{2}$

6. If 3tan^{-1}x + cot^{-1}x = π, then x equals ________.
7. If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then x is equal to ________.
8. The domain of the function defined by f(x) = sin$^{-1}\sqrt{x-1}$ is ________.
9. Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. **(1)**
10. Find the value of $\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$. **(1)**
11. Find the value of the following. $\cot\left[\frac{\pi}{2}-2\cot^{-1}(\sqrt{3})\right]$. **(1)**
12. Simplify $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$. **(2)**
13. If a>b>c>0 prove that
$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi$. **(2)**
14. Write the following function in the simplest form: $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}},\ x<\pi$. **(2)**
15. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}$. **(4)**
16. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$. **(4)**
17. Prove that $\sin^{-1}\left(\frac{63}{65}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$. **(4)**
18. Simplify $cos^{-1}\left(\frac{3}{5}cosx+\frac{4}{5}sinx\right)$. **(6)**

Solution

1. b. 0, **Explanation:** $(\sin A + \cos A)^2 = sin^2 A + cos^2 A + 2sinAcosA$
   1 = 1 + sin 2A
   So, sin 2A = 0
   Hence, A = 0
2. d. $1 \geqslant x \geqslant 0$, **Explanation:** let $\cos^{-1}(2x^2 - 1) \Rightarrow \cos^{-1}(2\cos^2\theta - 1)$
   $\Rightarrow \cos^{-1}(\cos 2\theta) = 2\theta = 2\cos^{-1}x$ if and only if
   $0 \leqslant 2\theta \leqslant \pi \Rightarrow 0 \leqslant \theta \leqslant \frac{x}{2}$
   $0 \leqslant \pi \leqslant 1$
   $\Rightarrow \cos 0 \geqslant \cos\theta \geqslant \cos\left(\frac{x}{2}\right)$ since cosine is decreasing function
   $\Rightarrow 1 \geqslant x \geqslant 0$
   This is true for all real values of x$\in$ [0,1]
3. c. $\frac{x}{\sqrt{1-x^2}}$, **Explanation:** Put $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$
   $\Rightarrow \sin\theta = \frac{x}{1} = \frac{Perp.}{Hyp.}$
   $\tan(\sin^{-1}x) = \tan\theta = \frac{Perp.}{Base.} = \frac{x}{\sqrt{1-x^2}}.$
4. a. π, **Explanation:** $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$
   $\Rightarrow \tan^{-1}\left[\frac{x+y}{1-xy}\right] + \tan^{-1}z$
   $\Rightarrow \tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\left(\frac{x+y}{1-xy}\right)z}\right]$
   $= \tan^{-1}\left[\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-xz-yz}{1-xy}}\right]$
   $= \tan^{-1}\left[\frac{xyz-xyz}{1-xy-xz-yz}\right] \quad x+y+z = xyz$
   $= \tan^{-1}(0) = \pi$
5. d. $\frac{\pi}{2}$, **Explanation:** Let $\cos^{-1}(-1) = A \Rightarrow \cos A = -1$
   $\Rightarrow \cos A = \cos\pi \therefore A = \pi$ and $\sin^{-1}(1) = B \Rightarrow \sin B = 1$
   $\Rightarrow \sin B = \sin\left(\frac{\pi}{2}\right)$

$\therefore B = \left(\frac{\pi}{2}\right)$

$\therefore \cos^{-1}(-1) - \sin^{-1}(1) = \pi - \frac{\pi}{2} \ = \frac{\pi}{2}$

6. 1
7. $\frac{2}{5}$
8. [1, 2]
9. Let $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \theta$

   $\cot\theta = \frac{-1}{\sqrt{3}}$

   We know that $\theta \in (0,\ \pi)$,

   $\cot\theta = \cot\left(\pi - \frac{\pi}{3}\right)$

   $\theta = \frac{2\pi}{3}$

   Therefore principle value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$
10. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$

    as $\frac{2\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

    $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right]$

    $= \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right] = \frac{\pi}{3}$
11. We have to find the value of $\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$

    Now, $\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$

    $= \tan\left(2\cot^{-1}\sqrt{3}\right)\ \left[\because \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta\right]$

    $= \tan\left(2 \times \frac{\pi}{6}\right)\ \left[\because \cot^{-1}\sqrt{3} = \cot^{-1}\left(\cot\frac{\pi}{6}\right) = \frac{\pi}{6}\right]$

    $= \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$
12. $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

    Dividing the N and D by b cosx

    $= \tan^{-1}\left(\frac{\frac{a\cos x}{b\cos x} - \frac{b\sin x}{b\cos x}}{\frac{b\cos x}{b\cos x} + \frac{a\sin x}{b\cos x}}\right)$

    $= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$

    $= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}(\tan x)$

    $= \tan^{-1}\left(\frac{a}{b}\right) - x$
13. $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$

$=\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\frac{(b-c)}{1+bc}+\pi+\tan^{-1}\left(\frac{c-a}{1+ca}\right)$

$= \tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c+\pi+(\tan^{-1}c-\tan^{-1}a)$

$=\pi$

$\left[\cot^{-1}c=\pi+\tan^{-1}\left(\frac{1}{x}\right) for\, x<0\right]$

14. $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$=\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$=\tan^{-1}\tan\frac{x}{2}=\frac{x}{2}$

15. $1\pm\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\pm 2\sin\frac{x}{2}\cos\frac{x}{2}$

$=\left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2$

LHS

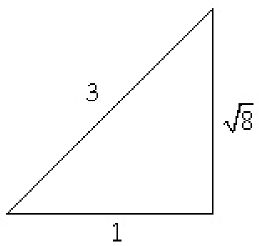
$= \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$

$= \cot^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$

$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

$= \cot^{-1}\left(\cot\frac{x}{2}\right)=\frac{x}{2}$

16. L.H.S.= $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right]$



$=\frac{9}{4}\cos^{-1}\frac{1}{3}\left[\because\sin^{-1}\frac{1}{3}+\cos^{-1}\frac{1}{3}=\frac{\pi}{2}\right]$

Let

$\cos^{-1}\frac{1}{3}=\theta$

$\implies \cos\theta=\frac{1}{3}$

$\implies \sin\theta=\frac{2\sqrt{2}}{3}$

$\implies \theta=\sin^{-1}\frac{2\sqrt{2}}{3}$

$\Rightarrow \cos^{-1}\frac{1}{3}=\sin^{-1}\frac{2\sqrt{2}}{3}$

$\therefore L.H.S = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} = R.H.S$

17. We have to prove that $\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$

Let us consider , $\sin^{-1}\frac{5}{13} = x$ and $\cos^{-1}(\frac{3}{5}) = y, \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $y \in [0, \pi]$ ....(1)

$\Rightarrow \quad \sin x = \frac{5}{13}$ and $\cos y = \frac{3}{5}$

$\Rightarrow \quad \cos x = \sqrt{1 - \sin^2 x}$ and $\sin y = \sqrt{1 - \cos^2 y}$

[taking positive sign as $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $y \in [0, \pi]$]

$\Rightarrow \quad \cos x = \sqrt{1 - \left(\frac{5}{13}\right)^2}$ and $\sin y = \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$\Rightarrow \cos x = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

and $\sin y = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Now, sin(x+ y) = sinx cosy + cosx siny

$\therefore \quad \sin(x+y) = \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$

$\Rightarrow \quad x + y = \sin^{-1}\left(\frac{63}{65}\right)$

$\therefore \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{63}{65}\right)$ [from Equation. (i)]

Hence proved.

18. $\frac{3}{5} = r\cos\theta, \ \frac{4}{5} = r\sin\theta$

Squaring and Adding both,

$r^2\cos^2\theta + r^2\sin^2\theta = \frac{9}{25} + \frac{16}{25}$

$r^2(1) = \frac{25}{25}$

r = 1

Now, $\frac{3}{5} = \cos\theta \ \frac{4}{5} = \sin\theta$

$\tan\theta = \frac{4}{3}$

$\cos^{-1}\left[\frac{3}{5}\cos x + \frac{4}{5}\sin x\right]$

$= \cos^{-1}[\cos\theta . \cos x + \sin\theta . \sin x]$

$= \cos^{-1}[\cos(x - \theta)]$

$= x - \theta$

$= x - \tan^{-1}\frac{4}{3}$