Chapter 3

Pair of Linear Equation in Two Variables

Exercise 3.1

Choose the correct answer from the given four options in the following questions:

Question 1.

Graphically, the pair of equations
$6x - 3y + 10 = 0$
$2x - y + 9 = 0$;
Represents two lines which are:

(a) intersecting at exactly one point
(b) intersecting at exactly two points
(c) coincident at exactly two points
(d) parallel

Solution:
(d) Parallel

$a_1 = 6$

$b_1 = -3$

$c_1 = 10$

$a_2 = 2$

$b_2 = -1$

$c_2 = 9$

On solving, we get,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{3}{1}$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

$$so, \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the linear equations has no solution and the two lines are parallel.

Question 2.

The pair of equationsx + 2y + 5 = 0 and –3x – 6y + 1 = 0 have

(a) a unique solution
(b) exactly two solution
(c) infinitely many solutions
(d) no solution

Solution:

On solving we get,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the pair of equations has no solution.

Question 3.

If a pair of linear equationsis consistent, then the lines will be,

(a) parallel
(b) always coincident
(c) intersecting or coincident
(d) always intersecting

Solution:

(c)Intersecting or coincident
For linear equations to be consistent, we have following conditions,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ for intersecting line (Unique solution)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, for coincident line

Question 4.

The pair of equations y = 0 and y = –7 has

(a) one solution
(b) two solutions
(c) infinitely many solutions
(d) no solution

Solution :
(d) no solution
The two equations above representsa pair of parallel lines. Therefore, there is nosolution.

Question 5.

The pair of equations x = a and y = b graphically represents lines which are
(a) parallel
(b) intersecting at (b, a)
(c)coincident
(d) intersecting at (a, b)

Solution:

(d) Intersecting at (a,b)

x = a is a straight line parallel to the y–axis.
Again,
y = b is a straight line parallel to the x–axis.

Therefore, the equations x = a and y = b graphically represents lines which intersects at (a, b).
Hence, (d) is the correct answer.

Question 6.

For what value of k do the equations 3x – y + 8 = 0 and 6x – ky = –16 represent coincident lines?

(a) $\frac{1}{2}$ **(b)** $\frac{-1}{2}$ **(c) 2(d) –2**

Solution:

(c) 2
3x – y = –18... (i)
6x – ky = –16 ... (ii)

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$$

On solving, we get,
k = 2.

Question 7.

If the line given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then the value of k is

(a) $\frac{-5}{4}$

(b) $\frac{2}{3}$

(c) $\frac{15}{4}$

(d) $\frac{3}{2}$

Solution:

(c) $\frac{15}{4}$

For parallel lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

On solving we get,

4k = 15

k = $\frac{15}{4}$

Question 8.

The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will haveinfinity many solutions is

(a) 3
(b) –3
(c) –12
(d) No value

Solution:

(d) no value

To have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

On solving, we get that there are no common values of c. Hence, there is no value of c for which the equations haveinfinitely many solutions.

Question 9.

One equation of a pair of dependent linear equations is –5x + 7y = 2. The secondequation can be:
(a) 10x + 14y + 4 = 0
(b) –10x – 14y + 4 = 0
(c)–10x + 14y + 4 = 0
(d) 10x – 14y = –4

Solution:

(d)
–5x + 7y – 2 = 0... (i)

Let the second equation be,
$a_2x + b_2y + c_2 = 0$ ...(ii)

∴For dependent system of linear equations,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
So,
$a_2 = –5k$,
$b_2 = 7k$,
$c_2 = –2k$

Putting value of k,
k = 0, and –1, it does not satisfy the required condition.

Let us take k = –2,

So,
$a_2 = +10$,
$b_2 = –14$
$c_2 = +4$
It satisfies the condition.

Question 10.
A pair of linear equations which has a unique solution x = 2, y = –3 is

(a) x + y = –1 and 2x – 3y = –5
(b) 2x + 5y = –11 and 4x + 10y = –22
(c) 2x – y = 1 and 3x + 2y = 0
(d) x – 4y –14 = 0 and 5x – y – 13 = 0

Solution:

b and d both.
As $x = 2$, $y = –3$ is unique solution of system of equations so these values of must satisfy both equations.

(a) $x + y = –1$ and $2x – 3y = –5$

Putting $x = 2$ and $y = 3$ in both the equations.

$LHS = x + y$
$2 – 3 = –1$ (RHS)
$LHS = 2x – 3y$
$= 2(2) –3(–3)$
$= 4 + 9$
$= 13$
$\neq$ RHS

(b) $2x + 5y = –11$ and $4x + 10y = –22$

Put $x = 2$ and $y = –3$ in both the equations.

$LHS = 2x + 5y$
$= 2 \times 2 + (–3)$
$= 4 – 15$
$= –11$
= RHS

$LHS = 4x + 10y$
$= 4(2) + 10(–3)$
$= 8 – 30$
$= –22$
= RHS

(c) $2x – y = 1$ and $3x + 2y = 0$

Put $x = 2$ and $y = –3$ in both the equations.

$LHS = 2x – y$
$2(2) + 3 = 7 \neq$ RHS
$LHS = 3x + 2y$
$= 3(2) + 1(–3)$
$= 6 – 6$
$= 0$
= RHS

(d) $x – 4y – 14 = 0$ and $5x – y – 13 = 0$
$x – 4y = 14$ and $5x – y = 13$

Put $x = 2$ and $y = –3$ in both the equations.

LHS = x – 4y
= 2 – 4(–3)
= 2 + 12
= 14 = RHS

LHS = 5x – y
= 5(2) – (–3)
= 10 + 3
= 13 = RHS

So, the pair of equations is (b) and (d).

Question 11.
If x = a, y = b, is the solution of the equations x – y = 2 and x + y = 4, then the values ofa and b are, respectively

(a) 3 and 5
(b) 5 and 3
(c) 3 and 1
(d) –1 and –3

Solution:
(c)
If (a, b) is the solution of the given equations, then it must satisfy the given equations so,
a – b = 2 ... (i)
a + b = 4 ... (ii)
2a = 6 [solving (i) and (ii)]
a = 3

Now,
3 + b = 4 [From (ii)]
b = 1
So,
(a, b) = (3, 1).

Question 12.
Aruna has only ₹1 and ₹2 coins with her. If the total number of coins that she has is50 and the amount of money with her is ₹75, then the number of ₹1 and ₹2 coins arerespectively:

(a) 35 and 15
(b) 35 and 20
(c) 15 and 35
(d) 25 and 25

Solution:
(d)
Let the number of ₹1 coins = x
and the number of ₹2 coins = y

So,
$x+y = 50$ ...... (i)
$1x+2y = 75$ .....(ii)

On subtracting equation (i)from (ii)
$y = 25$

Putting value of y in (i)
$x=25$
So,
$y = 25$ and $x = 25$.

Question 13.

The father's age is six times his son, age. Four years hence, the age of the father willbe four times his son's age. The present ages, (in years) of the son and the father are,respectively.

(a) 4 and 24
(b) 5 and 30
(c) 6 and 36
(d) 3 and 24

Solution:

(c)
Let,
Present age of father be x years
and
Present age of son be y years.

According to the question,
$x = 6y$ ... (i)

Age of the father after four years = $(x + 4)$ years
Similarly,
Age of son after four years = $(y + 4)$ years

Now,
$x + 4 = 4(y + 4)$ ... (ii)
$x + 4 = 4y + 16$

$6y - 4y = 16 - 4$ [as, $x = 6y$]

$2y = 12$

$y = 6$

So,

$x = 6 \times 6$

$x = 36$ years [From (i)]

Therefore, the present ages of the son and the father are 6 years and 36 years respectively.

Exercise No. 3.2

Question 1.

Do the following pair of linear equations have no solution? Justify your answer.

(i) 2*x* + 4*y* = 3

12*y* + 6*x* = 6

(ii) *x* = 2*y*

y* = 2*x

(iii) 3*x* + *y* – 3 = 0

2*x* + 2/3*y* = 2

Solution:

(i)

Yes

According to question,

$$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{3} = \frac{1}{3} \neq \frac{1}{2}$$

Hence the pair of linear equations has no solutions.

(ii)

No

According to question,

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-2}{-1} = \frac{2}{1}$$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{1}{2} \neq \frac{2}{1}$$

Hence the pair of linear equations have unique solutions.

(iii)

No

According to question,

$a_1 = 3$,

$b_1 = 1$,

$c_1 = -3$;

$a_2 = 2$,

$b_2 = 2/3$,

$c_2 = -2$;

$$\frac{a_1}{a_2} = \frac{3}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2/3} = \frac{3}{2}$$

$$\frac{c_1}{c_2} = \frac{3}{2}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the pair of linear equations are coincident.

Question 2.

Do the following equations represent a pair of coincident lines? Justify your answer.

(i) 3x + 1/7y = 3

7x + 3y = 7

(ii) –2x – 3y = 1

6y + 4x = – 2

(iii) x/2 + y + 2/5 = 0

4x + 8y + 5/16 = 0

Solution:

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(i) No.

Given pair of linear equations are:

3x + 1/7y = 3

7x + 3y = 7

We have

a_1 = 3,

b_1 = 1/7,

c_1 = – 3;

and,

a_2 = 7,

b_2 = 3,

c_2 = – 7;

$$\frac{a_1}{a_2} = \frac{3}{7}$$

$$\frac{b_1}{b_2} = \frac{1}{21}$$

$$\frac{c_1}{c_2} = \frac{3}{7}$$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The given pair of linear equations has unique solution.

(ii)

Yes,

The pair of linear equations are,

– 2x – 3y – 1 = 0 and 4x + 6y + 2 = 0;

We have

a_1 = – 2,

b_1 = – 3,

c_1 = – 1;

And

a_2 = 4,

b_2 = 6,

c_2 = 2;

$$\frac{a_1}{a_2} = \frac{-2}{4} = \frac{-1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{-1}{2}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The given pair of linear equations is coincident.

(iii)

No,

Pair of linear equations are:

x/2 + y + 2/5 = 0

4x + 8y + 5/16 = 0

We have,

a_1 = ½,

b_1 = 1,

c_1 = 2/5;

And

$a_2 = 4$,

$b_2 = 8$,

$c_2 = 5/16$;

$$\frac{a_1}{a_2} = \frac{1}{8}$$

$$\frac{b_1}{b_2} = \frac{1}{8}$$

$$\frac{c_1}{c_2} = \frac{32}{25}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given pair of linear equations has no solution.

Question 3.

Are the following pair of linear equations consistent? Justify your answer.

(i) –3x– 4y = 12

4y + 3x = 12

(ii) (3/5) x – y = ½

(1/5) x – 3y= 1/6

(iii) 2ax + by = a

ax + 2by – 2a = 0; a, b ≠ 0

(iv) x + 3y = 11

2 (2x + 6y) = 22

Solution:

For pair of linear equations to be consistent:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{(for unique solution)}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{(for coincident lines)}$$

(i)

No.

The given pair of linear equations

– 3x – 4y – 12 = 0

4y + 3x – 12 = 0

We get,

a_1 = – 3,

b_1 = – 4,

c_1 = – 12;

a_2 = 3,

b_2 = 4,

c_2 = – 12;

$$\frac{a_1}{a_2} = \frac{-3}{3} = -1$$

$$\frac{b_1}{b_2} = \frac{-4}{4} = -1$$

$$\frac{c_1}{c_2} = \frac{-12}{-12} = 1$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The pair of linear equations has no solution, i.e., inconsistent.

(ii)

Yes.

The given pair of linear equations

(3/5)x – y = ½

(1/5)x – 3y = 1/6

We have,

a_1 = 3/5,

b_1 = – 1,

c_1 = – ½;

$a_2 = 1/5$,

$b_2 = 3$,

$c_2 = -1/6$;

$$\frac{a_1}{a_2} = \frac{3}{1}$$

$$\frac{b_1}{b_2} = \frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{3}{1}$$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The given pair of linear equations has unique solution, i.e., consistent.

(iii)

Yes.

The given pair of linear equations –

$2ax + by - a = 0$

$4ax + 2by - 2a = 0$

We have,

$a_1 = 2a$,

$b_1 = b$,

$c_1 = -a$;

$a_2 = 4a$,

$b_2 = 2b$,

$c_2 = -2a$;

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{1}{2}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The given pair of linear equations has infinitely many solutions, i.e., consistent

(iv)

No.

The given pair of linear equations

x + 3y = 11

2x + 6y = 11

We have,

$a_1 = 1$,

$b_1 = 3$,

$c_1 = 11$

$a_2 = 2$,

$b_2 = 6$,

$c_2 = 11$

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{1}{1}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$$

The given pair of linear equations has no solution.

Question 4.

For the pair of equations λx + 3y = –7 , 2x + 6y = 14to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.

Solution:

λx + 3y + 7 = 0 .

2x + 6y – 14 = 0

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Now,

$$\frac{a_1}{a_2} = \frac{\lambda}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{7}{-14}$$

$$= \frac{1}{-2}$$

Here,

$\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (for any value of x)

So, the given statement is not true.

Question 5.

For all real values of c, the pair of equations x – 2y = 8 5x – 10y = c have a unique solution. Justify whether it is true or false.

Solution:

(Not true)

System of linear equations are

x – 2y = 8 ...(i)

5x – 10 = c ...(ii)

$$\frac{a_1}{a_2} = \frac{1}{5}$$

$$\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$$

$$\frac{c_1}{c_2} = \frac{-8}{-c}$$

$$= \frac{8}{c}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Hence, system of linear equations can never have unique solution.

So, the given statement is not true.

Question 6.

The line represented by x = 7 is parallel to the x–axis. Justify whether the statement is true or not.

Solution:

The line x = 7 is the form of x = a. The graph of the equation is a line parallel to the y–axis.

Hence, the given statement is false.

Exercise No. 3.3

1.

For which value(s) of λ, do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given equations are;

$\lambda x + y - \lambda^2 = 0$

$x + \lambda y - 1 = 0$

We have,

$a_1 = \lambda$,

$b_1 = 1$,

$c_1 = -\lambda^2$;

$a_2 = 1$,

$b_2 = \lambda$,

$c_2 = -1$;

$$\frac{a_1}{a_2} = \frac{\lambda}{1}$$

$$\frac{b_1}{b_2} = \frac{1}{\lambda}$$

$$\frac{c_1}{c_2} = \lambda^2$$

(i)

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\lambda = 1/\lambda \neq \lambda^2$

So, $\lambda^2 = 1$ and $\lambda^2 \neq \lambda$

We take only $\lambda = -1$,

Since the system of linear equations has infinitely many solutions at $\lambda = 1$,

(ii) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\lambda = 1/\lambda = \lambda^2$

So,

$\lambda = 1/\lambda$ gives $\lambda = +1$;

$\lambda = \lambda^2$ gives $\lambda = 1, 0$;

Satisfying both the equations,

$\lambda = 1$ is the answer.

(iii) For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So $\lambda \neq 1/\lambda$

Hence,

$\lambda^2 \neq 1$

$\lambda \neq +1$

So, all real values of λ except +1.

2.

For which value(s) of k will the pair of equations

$kx + 3y = k - 3$

$12x + ky = k$ have no solution?

Solution:

The given pair of linear equations is

$kx + 3y = k - 3$ …(i)

$12x + ky = k$ … (ii)

We have,

$a_1 = k$,

$b_1 = 3$,

$c_1 = -(k – 3)$

$a_2 = 12$,

$b_2 = k$,

$c_2 = – k$

Then,

$$\frac{a_1}{a_2} = \frac{k}{12}$$

$$\frac{b_1}{b_2} = \frac{3}{k}$$

$$\frac{c_1}{c_2} = \frac{k-3}{k}$$

For no solution of the pair of linear equations,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$k/12 = 3/k \neq (k-3)/k$

Taking first two parts, we get

$k/12 = 3/k$

$k^2 = 36$

$k = + 6$

Taking last two parts, we get

$$\frac{3}{k} = \frac{(k\text{-}3)}{k}$$

$$3k = k\ (k-3)$$

$$k^2 - 6k = 0$$

So, $k \neq 0,6$

Value of k for which the given pair of linear equations has no solution is $k = – 6$.

3.

For which values of *a* and *b,* will the following pair of linear equations have infinitely many solutions?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$

Solution:

The given pair of linear equations are:

$x + 2y = 1$ …(i)

$(a-b)x + (a + b)y = a + b - 2$ …(ii)

We have,

$a_1 = 1$,

$b_1 = 2$,

$c_1 = -1$

$a_2 = (a - b)$,

$b_2 = (a + b)$,

$c_2 = -(a + b - 2)$

$$\frac{a_1}{a_2} = \frac{1}{a-b}$$

$$\frac{b_1}{b_2} = \frac{2}{a+b}$$

$$\frac{c_1}{c_2} = \frac{1}{a+b-2}$$

For infinitely many solutions of the, pair of linear equations,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \text{ (Coincident lines)}$$

so,

$$\frac{1}{(a-b)} = \frac{2}{(a+b)} = \frac{1}{(a+b-2)}$$

Taking first two parts,

$$\frac{1}{(a-b)} = \frac{2}{(a+b)}$$

$$a + b = 2(a - b)$$

$$a = 3b \quad \ldots (iii)$$

Taking last two parts,

$$\frac{1}{(a+b-2)} = \frac{2}{(a+b)}$$

2(a + b – 2) = (a + b)

a + b = 4 … (iv)

Putting the value of a from Eq. (iii) in Eq. (iv), we get

3b + b = 4

4b = 4

b = 1

Put the value of b in Eq. (iii), we get

a = 3

So, the values (a,b) = (3,1) satisfies all the parts.

Required values of a and b are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

4.

Find the value(s) of *p* in (i) to (iv) and *p* and *q* in (v) for the following pair of equations:

(i) 3*x* – *y* – 5 = 0 and 6*x* – 2*y* – *p* = 0, if the lines represented by these equations are parallel.

Solution:

Given pair of linear equations is

3x – y – 5 = 0 … (i)

6x – 2y – p = 0 … (ii)

We have,

$a_1 = 3$,

$b_1 = -1$,

$c_1 = -5$;

$a_2 = 6$,

$b_2 = -2$,

$c_2 = -p$;

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{5}{p}$$

As the lines represented by these equations are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Taking last two parts, we get

$$\frac{1}{2} \neq \frac{5}{p}$$

So,

$p \neq 10$

Hence, the given pair of linear equations are parallel for all real values of p except 10.

(ii) – *x* + *py* = 1 and *px* – *y* = 1, if the pair of equations has no solution.

Solution:

Given pair of linear equations is

$-x + py = 1$ … (i)

$px - y - 1 = 0$ …(ii)

We have,

$a_1 = -1$,

$b_1 = p$,

$c_1 = -1$;

$a_2 = p$,

$b_2 = -1$,

$c_2 = -1$;

$$\frac{a_1}{a_2} = \frac{-1}{p}$$

$$\frac{b_1}{b_2} = \frac{-p}{1}$$

$$\frac{c_1}{c_2} = \frac{1}{1}$$

Since, the lines equations has no solution i.e., both lines are parallel to each other.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$-1/p = -p \neq 1$

Taking last two parts, we get

$p \neq -1$

Taking first two parts, we get

$p^2 = 1$

$p = +1$

So, the given pair of linear equations has no solution for p = 1.

(iii) – 3*x* + 5*y* = 7 and 2*px* – 3*y* = 1, if the lines represented by these equations are intersecting at a unique point.

Solution:

Given, pair of linear equations is

$-3x + 5y = 7$

$2px - 3y = 1$

We have,

$a_1 = -3$,

$b_1 = 5$,

$c_1 = -7$;

And

$a_2 = 2p$,

$b_2 = -3$,

$c_2 = -1$;

$\frac{a_1}{a_2} = \frac{-3}{2p}$

$\frac{b_1}{b_2} = \frac{-5}{3}$

$\frac{c_1}{c_2} = 7$

Since, the lines are intersecting at a unique point i.e., it has a unique solution

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

-3/2p ≠ -5/3

p ≠ 9/10

Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except 9/10

(iv) 2*x* + 3*y* – 5 = 0 and *px* – 6*y* – 8 = 0, if the pair of equations has a unique solution.

Solution:

Given, pair of linear equations is

2x + 3y – 5 = 0

px – 6y – 8 = 0

We have,

a_1 = 2,

b_1 = 3,

c_1 = – 5;

And

a_2 = p,

b_2 = – 6,

c_2 = – 8;

$$\frac{a_1}{a_2} = \frac{2}{p}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{5}{8}$$

Since, the pair of linear equations has a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So,

$$\frac{2}{p} \neq \frac{-1}{2}$$

$p \neq -4$

So, the pair of linear equations has a unique solution for all values of p except – 4.

(v) 2*x* + 3*y* = 7 and 2*px* + *p*y = 28 – *qy*, if the pair of equations have infinitely many solutions.

Solution:

Given pair of linear equations is

2x + 3y = 7

2px + py = 28 – qy

2px + (p + q)y – 28 = 0

We have,

$a_1 = 2$,

$b_1 = 3$,

$c_1 = -7$;

And

$a_2 = 2p$,

$b_2 = (p + q)$,

$c_2 = -28$;

$$\frac{a_1}{a_2} = \frac{2}{2p}$$

$$\frac{b_1}{b_2} = \frac{3}{p+q}$$

$$\frac{c_1}{c_2} = \frac{1}{4}$$

Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Taking first and third parts, we get

p = 4

Again, taking last two parts, we get

$$\frac{3}{p+q} = \frac{1}{4}$$

p + q = 12

Since, p = 4

So, q = 8

We get that the values of p = 4 and q = 8 satisfies all three parts.

So, the pair of equations has infinitely many solutions for all values of p = 4 and q = 8.

5.

Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

Solution:

Linear equations are

x – 3y – 2 = 0 … (i)

-2x + 6y – 5 = 0 … (ii)

We have,

a_1 =1,

b_1 =-3,

c_1 =- 2;

a_2 = -2,

$b_2 = 6$,

$c_2 = -5$;

$$\frac{a_1}{a_2} = \frac{-1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6}$$

$$= \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{2}{5}$$

i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [Parallel lines]

The, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

6.

Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

Solution:

Condition for the pair of system to have unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Let the equations be,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Since, x = – 1 and y = 3 is the unique solution of these two equations, then

It must satisfy the equations –

$a_1(-1) + b_1(3) + c_1 = 0$

$-a_1 + 3b_1 + c_1 = 0$ … (i)

And

$a_2(-1) + b_2(3) + c_2 = 0$

$-a_2 + 3b_2 + c_2 = 0$ … (ii)

As for the different values of a_1, b_1, c_1 and a_2, b_2, c_2 satisfy the Eq. (i) and (ii).

Therefore, infinitely many pairs of linear equations are possible.

7.

If 2*x* + *y* = 23 and 4*x* – *y* = 19, find the values of 5*y* – 2*x* and y/x – 2.

Solution:

Given equations are

2x + y = 23 … (i)

4x – y = 19 … (ii)

On adding both equations, we get

6x = 42

So, x = 7

Putting the value of x in Eq. (i), we get

2(7) + y = 23

y = 23 – 14

y = 9

And,

5y – 2x = 5(9) – 2(7)

= 45 – 14

= 31

y/x – 2 = 9/7 -2

= -5/7

So, the values of (5y – 2x) and y/x – 2 are 31 and -5/7 respectively.

8. Find the values of *x* and *y* in the following rectangle [see Fig. 3.2].

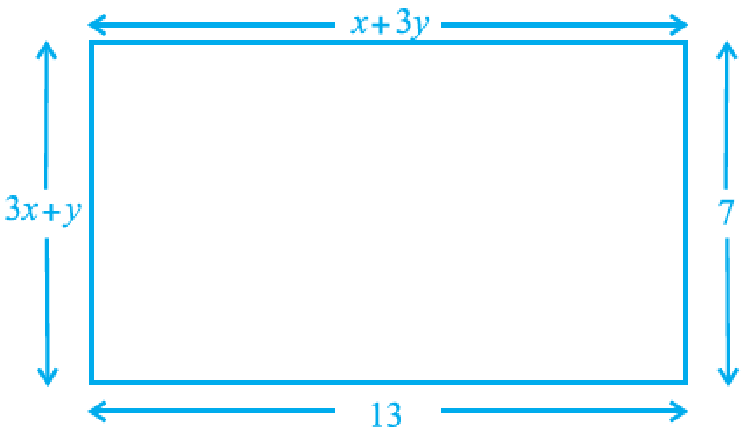


Fig. 3.2

Solution:

Using property of rectangle,

We know that, Lengths are equal,

CD = AB

Therefore,

$x + 3y = 13$ … (i)

Breadth are equal,

AD = BC

$3x + y = 7$ … (ii)

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),

We get,

$8x = 8$

$x = 1$

Putting $x = 1$ in Eq. (i),

$y = 4$

Therefore, the required values of x and y are 1 and 4, respectively.

9.

Solve the following pairs of equations:

i. $\mathbf{x + y = 3.3}$

$$\mathbf{\frac{0.6}{3x - 2y} = -1}$$

ii. $\mathbf{\frac{x}{3} + \frac{y}{4} = 4}$

$$\mathbf{\frac{5x}{6} - \frac{y}{8} = 4}$$

iii. $\mathbf{4x + \frac{6}{y} = 15}$

$$\mathbf{6x - \frac{8}{y} = 14}$$

iv. $\mathbf{\frac{1}{2x} - \frac{1}{y} = -1}$

$$\mathbf{\frac{1}{x} + \frac{1}{2y} = 8}$$

v. $\mathbf{43x + 67y = -24}$

$$\mathbf{67x + 43y = 24}$$

vi. $\mathbf{\frac{x}{a} + \frac{y}{b} = a + b}$

$$\mathbf{\frac{x}{a^2} + \frac{y}{b^2} = 2}$$

Solution:

(i)

x + y = 3.3

On multiplying eqn. by 20, we get

20x + 20y = 66 ... (i)

$$\frac{0.6}{3x - 2y} = -1$$

–3x + 2y = 0.6

On multiplying it by (–10), we get

30x – 20y = –6 .....(ii)

Now, adding (i) and (ii), we get

x = 1.2

Now,

10(1.2) + 10y = 33 [From (i)]

12 + 10y = 33

10y = 33 – 12

y = 2.1

So, the solution of the given system of equations is x = 1.2, and y = 2.1.

(ii)

$$\frac{x}{3}+\frac{y}{4}=4$$

$$\frac{5x}{6}-\frac{y}{8}=4$$

4x+3y = 48 .... (i)

20x-3y = 96 ... (ii)

24x=144 [adding both equations]

x = 6

Now, 4x + 3y = 48 [we have]

On putting the value of x = 6, we have

4(6) + 3y = 48

3y = 48 – 24

3y = 24

y = 8

So, the solution of the given equations is x = 6 and y = 8

(iii)

$$4x + \frac{6}{y} = 15$$

$$6x - \frac{8}{y} = 14$$

(As y is in denominators and symmetric so no need to remove denominator and 6 and 4 are

divisible by 2 so we can multiply (i), (ii) by 3 and 2 respectively) and subtracting both the equations,

We get, y = 2 andx = 3

So, the respective values are,

x = 3 and y = 2

(iv)

Given equations are;

$$\frac{1}{2x} - \frac{1}{y} = -1$$

$$\frac{1}{x} + \frac{1}{2y} = 8$$

[x, y both are in denominator and symmetric no need to convert into linear equation hence,

can be eliminated directly]

Multiplying eqn. (i) by the coefficient of 1/y in (ii) and vice versa, and adding both the equations,

We get, x = 1/6 and y= 1/4

(v)

Given pair of equations are

43x + 67y = –24 ... (i)

67x + 43y = 24 ... (ii)

110x + 110y = 0 [Adding (i) and (ii)]

x + y = 0 ....(iii)

Subtracting (ii) from (i), we have

–24x + 24y = –48

–x + y = –2 ...(iv)

x + y = 0 [From (iii)]

2y = –2 [Adding (iii) and (iv)]

y = –1

From (iii),

x + y = 0

x + (–1) = 0 [∵ y = –1]

x = 1and y = –1

Hence, the solution of the given equations is x = 1, y = –1.

(vi)

Pair of linear equations is,

$$\frac{x}{a}+\frac{y}{b}=a+b \quad ...(1)$$
$$\frac{x}{a^2}+\frac{y}{b^2}=2 \quad a,b\neq 0 \quad ...(2)$$

Multiplying (i) by 1/a and then subtracting from eq(ii), we get,

$$\frac{x}{a^2}+\frac{y}{b^2}=2$$
$$\frac{x}{a^2}+\frac{y}{ab}=1+\frac{b}{a}$$
$$- \quad - \quad -$$
$$y\left(\frac{1}{b^2}-\frac{1}{ab}\right)=2-1-\frac{b}{a}$$

On solving,

$$y\left(\frac{a-b}{ab^2}\right)=1-\frac{b}{a}=\left(\frac{a-b}{a}\right)$$

$$y=\frac{ab^2}{a}$$

$$y=b^2$$

Put the value of y in eq (2),

$$\frac{x}{a^2}+\frac{y}{b^2}=2$$

$$\frac{x}{a^2}=2-1$$

$$x=a^2$$

10.

Find the solution of the pair of equations x/10 + y/5 -1 = 0 and x/8 + y/6 =15. Hence, find λ, if y = λx + 5.

Solution:

Given equations are

x/10 + y/5 -1 = 0 (x 20)

2x + 4y = 20 ...(i)

x/8 + y/6 =15 (x 24)

3x + 4y = 360 ...(ii)

On solving, equation (i) and (ii),

x = +340

Now,

x + 2y = 10

340 + 2y = 10 [x = 340]

2y = 10 – 340

2y = –330

y = -165

Now,

y = λx + 5 [Given]

–165 = λ(340) + 5 [y = –165 and x = 340]

–λ(340) = 5 + 165

–λ(340) = 170

λ= -1/2

The solution of the given pair of equations is x = 340, y = –165 and λ= -1/2.

11.

By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(i) 3x + y + 4 = 0 and 6x – 2y + 4 = 0

(ii) x – 2y = 6 and 3x – 6y = 0

(iii) x + y = 3 and 3x + 3y = 9

Solution:

(i)
Given equations are
3x + y + 4 = 0 ... (i)
6x – 2y + 4 = 0 ... (ii)

The given pair of equations is consistent and has unique solution.

3x + y + 4 = 0 [From (i)]
Y= -3x-4

If
x = 0,
y = –3(0) – 4 = 0 – 4 = –4

x = 1,
y = –3(1) – 4 = –3 – 4 = –7

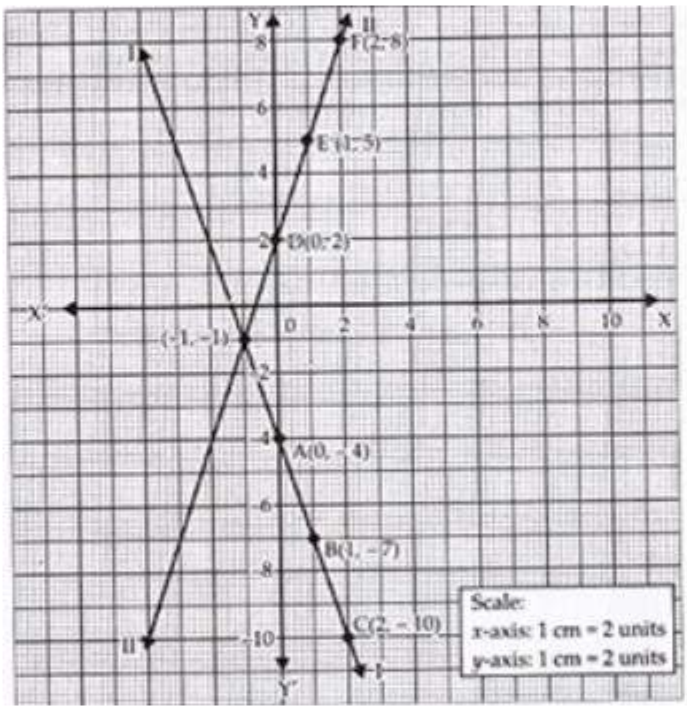
x = 2
y = –3(2) – 4 = –6 – 4 = –10

Now,
6x – 2y + 4 = 0 [From (ii)]
3x – y + 2 = 0
–y = –3x – 2

If x = 0,
y = 3(0) + 2 = 0 + 2 = 2

x = 1,
y = 3(1) + 2 = 3 + 2 = 5

x = 2,
y = 3(2) + 2 = 6 + 2 = 8



Intersecting point is (–1, –1) i.e., x = –1 and y = –1

(ii)
Given equations are,
x – 2y = 6 ...(i)
3x – 6y = 0 ...(ii)

System of equations is inconsistent. Hence, the lines represented by the given equations are parallel. So, the given equations have no solution.

(iii)

Pair of equations is

$x + y = 3$ .... (i)

$3x + 3y = 9$ ... (ii)

The system of given equations have infinitely many solutions. Graph will be overlapping so pair of equations is consistent.

As the lines are dependent so points on graph for both equations will be same. To draw, we can take any one equation.

$x + y = 3$

If $x = 0$,

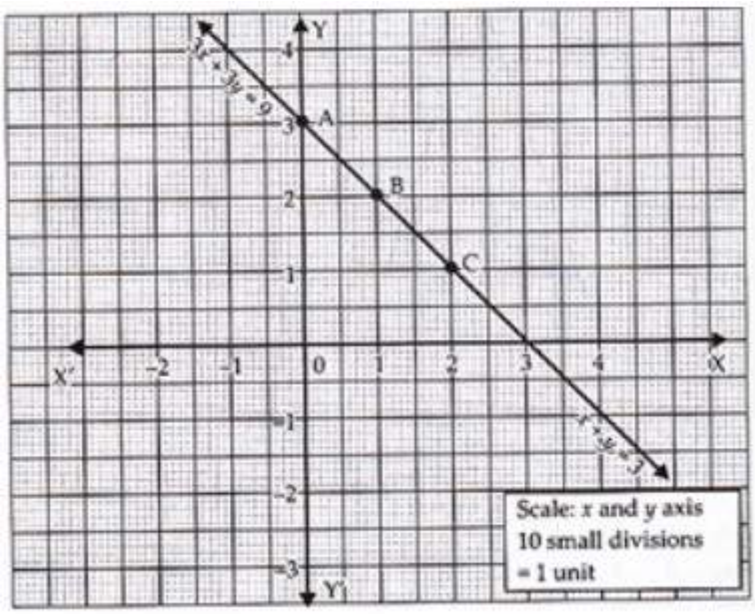
$y = 3 – 0 = 3$

$x = 1$,

$y = 3 – 1 = 2$

$x = 2$,

$y = 3 – 2 = 1$



So, the lines represented by the given equations are coinciding. Given equations are consistent.

Some solutions of system of equations are consistent.

Some solutions of system of equations are (0, 3), (1, 2), (2, 1), (3, 0), (4, –1) and (5, –2).

12.

Draw the graph of the pair of equations $2x + y = 4$ and $2x – y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

Solution:

$2x + y = 4$ ... (i)

$y = 4 - 2x$

If $x = 0$,

$y = 4 – 2(0)$

$= 4 – 0 = 4$

$x = 1$,

$y = 4 – 2(1)$

$= 4 – 2 = 2$

$x = 2$,

$y = 4 – 2(2)$

$= 4 – 4 = 0$

For,

$x = 0, 1,2, 3$

$y= 4, 2,0 ,–2$

Let the above four points be named as A, B, C, D

Now, $2x – y = 4$ ...(ii)

If, $x = 0$,

$y = 2(0) – 4$

$= 0 – 4 = –4$

$x = 1$,

$y = 2(1) – 4$

$= 2 - 4 = -2$

$x = 2$,

$y = 2(2) - 4$

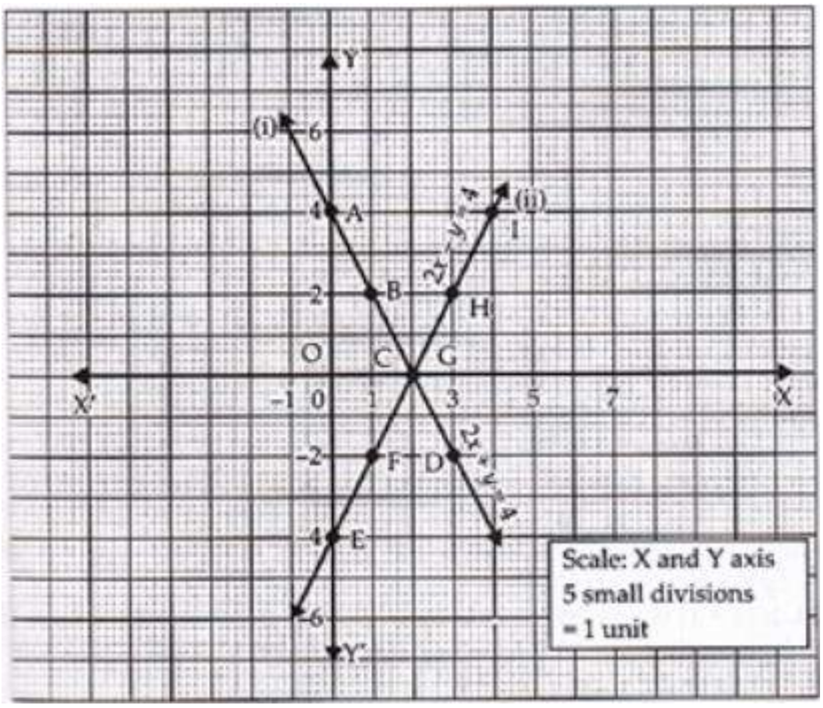
$= 4 - 4 = 0$

For,

$x = 0, 1, 2, 3, 4$

$y = -4, -2, 0, 2, 4$

Let the points be E, F, G, H, I



Triangle formed by the lines with y–axis is ΔAEC. Coordinates of vertices are A(0, 4), E(0, –4)and C(2, 0).

Area of ΔAEC = 8 square units

13.

Write an equation of a line passing through the point representing solution of the pair of linear equations $x+y = 2$ and $2x-y = 1$. How many such lines can we find?

Solution:

Given pair of linear equations is,

x + y = 2 ... (i)

2x – y = 1 ... (ii)

We have,

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{-1}$$

$$= -1$$

so,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the given pair of equations has unique solution.

For solution of equations, add (i) and (ii), we get

3x = 3

x = 1

Now, x + y = 2 [From (i)]

1 + y = 2

y = 1 [₹ x = 1]

So, the solution of the given equations is y = 1 and x = 1.

Now, we have to find a line passing through (1, 1). We can make infinite linear equations passing through (1, 1). Some of the linear equations are given below:

Step I:

Take any linear polynomial in x and y, let it 8x – 5y.

Step II:

Put x = 1 and y = 1 in the above polynomial, i.e.

8(1) – 5(1) = 8 – 5 = 3

Step III:

The required equations are 2x–3y=–1, 3x–2y=1 and 5x–2y=3 and x–y=0 etc.

14.

If x+1 is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that 2a–3b = 4.

Solution:

Let

$f(x) = 2x^3 + ax^2 + 2bx + 1$

If (x + 1) is a factor of f(x), then by factor theorem f(–1) = 0.

$f(-1) = 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$= -2 + a - 2b + 1 = 0$

So, a – 2b = 1 ... (i)

Also,

2a – 3b = 4 [Given]... (ii)

$$\begin{aligned} 2a - 4b &= 2 \quad [(i) \times 2] \\ 2a - 3b &= 4 \\ - \quad + \quad &\quad - \\ \hline -b &= -2 \end{aligned}$$

b = 2

Now,

a – 2b = 1 [From (i)]

a – 2(2) = 1 [₹ b = 2]

a = 1 + 4

a = 5,

b = 2

15.

The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.

Solution:

x, y and 40 are the measure of interior angles of a triangle.

x + y + 40° = 180°

x + y = 140° …..(i)

The difference between x and y is 30°,

$$\begin{aligned} x - y &= 30^o \\ x + y &= 140^o \\ \hline 2x &= 170^o \\ x &= \frac{170}{2} \\ x &= 85^0 \end{aligned}$$

Now,

x + y = 140° [From (i)]

85° + y = 140° [x = 85°]

y = 140° – 85°

y = 55°

and x = 85°

16.

Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?

Solution:

Let the present ago of Salim be x years.

Also, let the present age of his daughter be y years.

Age of Salim 2 years ago = (x – 2) years

Age of Salim's daughter 2 years ago = (y – 2) years

According to the question, we have

Age of Salim was = thrice × daughter [Given]

$x - 2 = 3 \times (y - 2)$

$x - 2 = 3y - 6$

$x - 3y = -4$

Age of Salim 6 years later = (x + 6) years

Age of Salim's daughter 6 years later = (y + 6) years

According to the question, we have

$x + 6 = 2(y + 6) + 4$

$x + 6 = 2y + 12 + 4$

$x - 2y = 16 - 6$

$x - 2y = 10$

so,

$$\begin{array}{rcl} x - 2y & = & 10 \\ x - 3y & = & -4 \\ - \quad + & & + \\ \hline y & = & 14 \end{array}$$

Now,

$x - 2y = 10$

$x - 2(14) = 10$ [₹ y = 14]

x = 10 + 28

x = 38

∴ Age of Salim at present = 38 years

and, age of Salim’s daughter at present = 14 years

17.

The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution:

Let the present age of father be x years.

Also, let the sum of present ages of two children by y years.

The age of father is (=) twice (× 2) the sum of the ages of two children

$x = 2 \times (y)$

$x = 2y$...(i)

$x – 2y = 0$

Age of father 20 years later = $(x + 20)$ years

Increase in age of first children in 20 years = 20 years

Increase in age of second children in 20 years = 20 years

∴ Increase in the age of both children in 20 years = 20 + 20 = 40 years

∴ Sum of ages of both children 20 years later = $(y + 40)$

Now, according to the question, we have

Father will be (=) sum of ages of two children [Given]

$x + 20 = y + 40$

$x – y = 20$ ...(ii)

$2y – y = 20$ [($x = 2y$) from (i)]

y = 20 years

Now,

x = 2y [From (i)]

$x = 2 \times 20$

x = 40

So, Age of father is 40 years.

18.

Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

Solution:

Let the numbers be 5x and 6x respectively. So, new numbers after subtracting 8 fromeach will be (5x – 8) and (6x – 8) respectively.

According to the question, ratio of new numbers is 4 : 5.

$$\frac{5x-8}{6x-5} = \frac{4}{5}$$

25x – 40 = 24x – 32

25x – 24x = 40 – 32

x = 8

Required numbers = 5x and 6x become 5×8, 6×8

Required numbers = 40 and 48

19.

There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Solution:

Let the number of students initially in hall A be x.

and the number of students initially in hall B be y.

Case I:

10 students of hall A shifted to B

Now, number of students in hall A = (x – 10)

Now, number of students in hall B = (y + 10)

According to the question, number of students in both halls are equal.

x – 10 = y + 10

x – y = 20 ...(i)

Case II:

20 students are shifted from hall B to A, then

Number of students in hall A becomes = x + 20

Number of students in hall B becomes = y – 20

According to the question, students in hall A becomes twice of students in hall B.

∴ x + 20 = 2(y – 20)

x + 20 = 2y – 40 .....(ii)

$$\begin{array}{rcl} x-2y & = & -60 \\ x-y & = & 20 \\ - \quad + & & - \\ \hline -y & = & -80 \end{array}$$

y = 80

Now,

x – y = 20 [From (i)]

$x - 80 = 20$

$x = 20 + 80$

$x = 100$

Number of students initially in hall A = 100

Number of students initially in hall B = 80

20.

A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs 22 for a book kept for six days, while Anand paid Rs 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Solution:

Let the fixed charges for first two days = ₹x

Let the additional charges per day after 2 days = ₹y

Latika paid ₹22 for six days. [Given]

2 days fixed charges + (6 – 2) days charges = 22

$x + 4y = 22$ ....(i)

Anand paid ₹16 for books kept for four days.

2 day's fixed charges + (4 – 2) day's additional charges = 16

$x + 2y = 16$ ...(ii)

$$\begin{array}{l} x + 2y \;\; = 16 \\ x + 4y \;\; = 22 \\ - \;\; - \;\;\;\; - \\ \hline -2y \;\;\; = -6 \end{array}$$

y = ₹ 3 per day

Now,

$x + 2y = 16$ [From (ii)]

$x + 2(3) = 16$

$x = 16 – 6$

$x = 10$

So, the fixed charges for first 2 days = ₹10

The additional charges per day after 2 days = ₹3 per day

21.

In a competitive examination, one mark is awarded for each correct answer while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Solution:

Taking the number of questions attempted correctly = x

Number of questions answered = 120

And, wrong answer attempted = (120 – x)

Marks awarded for right answer = 1 × x = x marks

Marks deducted for (120 – x) wrong answer = ½ (120 – x)

Therefore,

$$x - \frac{1}{2}(120 - x) = 90$$

$$x + \frac{x}{2} = 90 + 60$$

$$\frac{3x}{2} = 150$$

x = 100

Hence, Jayanti answered 100 questions correctly.

22.

The angles of a cyclic quadrilateral ABCD are ∠A = (6x + 10) °, ∠B = (5x)° ∠C = (x + y)°, ∠D = (3y – 10)°

Solution:

Sum of opposite angles of a cyclic quadrilateral ABCD is 180°,

∠A + ∠C = 180° [Opposite ∠s of cyclic quadrilateral)]

(6x + 10) + (x + y) = 180°

6x + 10 + x + y = 180°

7x + y = 170° ...(i)

Now,

∠B + ∠D = 180°

5x + (3y – 10) = 180°

5x + 3y = 180° + 10°

$$
\begin{aligned}
5x + 3y &= 190 \\
21x + 3y &= 510 \\
- \quad - \quad & \quad - \\
\hline
-16x &= -320 \\
x &= \frac{320}{16} \\
x &= 20
\end{aligned}
$$

x = 20

Now,

7x + y = 170 [From (i)]

7(20) + y = 170

$y = 170 – 140$

$y = 30$

and

$x = 20$

Therefore,

$\angle A = (6x + 10)°$

$= (6 \times 20 + 10)°$

$= (120 + 10)° = 130°$

$\angle B = (5x)°$

$= (5 \times 20)°$

$= 100°$

$\angle C = (x + y)°$

$= (20 + 30)°$

$= 50°$

$\angle D = (3y – 10)°$

$= (3 \times 30 – 10)°$

$= (90 – 10)°$

$= 80°$

So, the values of x and y are 20 and 30 respectively.

∠A, ∠B, ∠C, and ∠D are 130°, 100°,50°, 80° respectively.

Exercise No. 3.4

1. Graphically, solve the following pair of equations:

$2x + y = 6$

$2x - y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the *x*-axis and the lines with the *y*-axis.

Solution:

We have

$2x + y = 6$

$2x - y + 2 = 0$

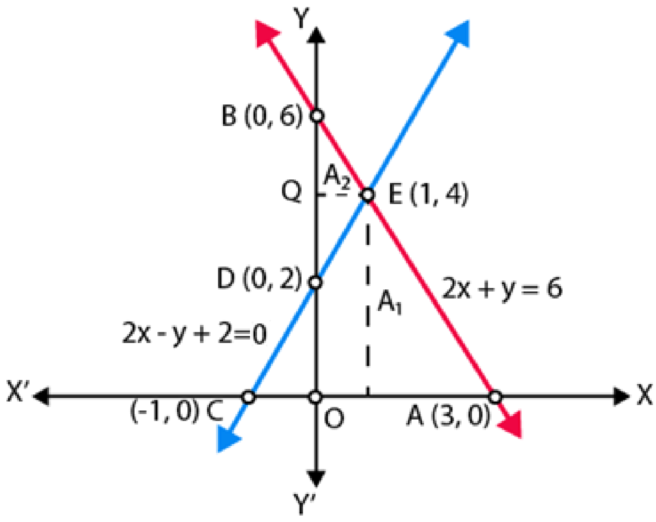
For $2x + y - 6 = 0$, values of x and y can be,

$x = 0, y = 6$ and $y = 0, x = 3$.

Also, for $2x - y + 2 = 0$, values of x and y can be,

$x = 0, y = 2$, and $y = 0, x = -1$

Taking A1 and A_2 as the areas of triangles ACE and BDE respectively.



If, Area of triangle formed with x -axis = T_1

Then,

T_1 = Area of $\triangle ACE$

$= ½ \times AC \times PE$

$T_1 = ½ \times 4 \times 4$

$= 8$

Also,

Area of triangle formed with y – axis = T_2

T_1 = Area of △BDE

$= ½ \times BD \times QE$

$T_1 = ½ \times 4 \times 1$

$= 2$

Therefore,

$T_1:T_2 = 8:2$

$= 4:1$

So, the pair of equations intersect graphically at point E(1,4)

x = 1 and y = 4.

2.

Determine, graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$, $x + y = 8$

Solution:

We have,

$y = x$ …(i)

$3y = x$ …(ii)

and $x + y = 8$ …(iii)

For line y = x,

x = 0, y = 0

x = 1, y = 1

x =2, y = 2

Also,

For line x = 3y,

x = 0, y = 0

x = 3, y = 1

x = 6, y = 2

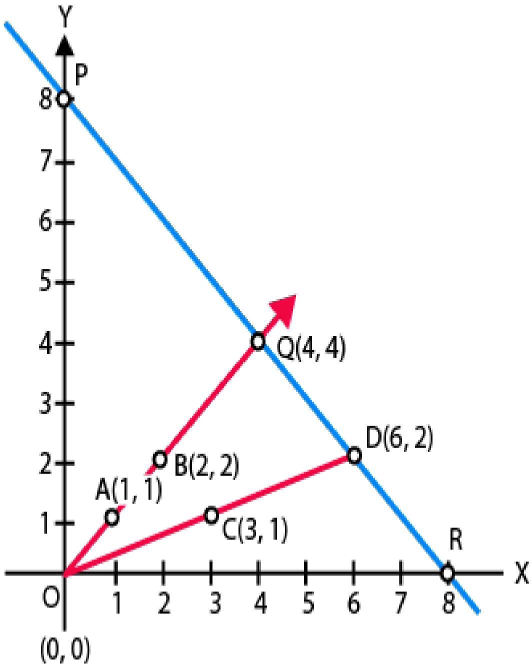
Now, for line x + y = 8

x = 0, y = 8

x = 4, y = 4

x = 8, y = 0

If we plot the graph of above three lines, we will get the following graph,



We can see that △OQD is formed by these lines.

Therefore, the vertices of the △OQD formed by the given lines are O(0, 0), Q(4, 4) and D(6,2).

3.

Draw the graphs of the equations *x* = 3, *x* = 5 and 2*x* – *y* – 4 = 0. Also find the area of the quadrilateral formed by the lines and the *x*–axis.

Solution:

We have,

x = 3,

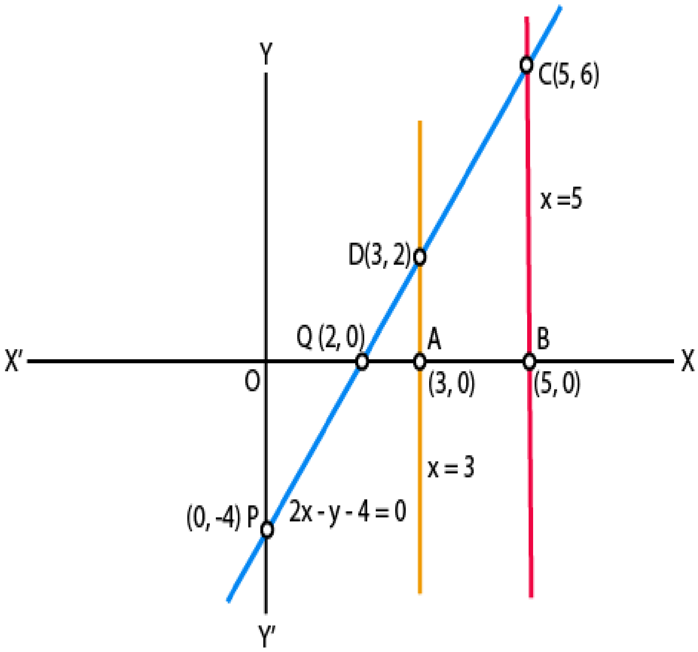
x = 5

2x-y-4 = 0.

For line 2x – y – 4 = 0, values of x and y can be,

x= 0, y = -4 and y = 0, x = 2

If we draw the graph, we get,



So,

AB = OB-OA

= 5-3

= 2

AD = 2

BC = 6

Therefore, quadrilateral ABCD is a trapezium.

Area of quadrilateral ABCD = half × (distance between parallel lines)

= half × (AB) × (AD + BC)

= 8 sq units

4.

The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Taking the cost of a pen and a pencil box be Rs x and Rs y respectively.

According to the question,

$4x + 4y = 100$

$x + y = 25$ ... (i)

and

$3x = y + 15$

$3x-y = 15$ ... (ii)

On adding Equation (i) and (ii), we get,

$4x = 40$

$x = 10$

Putting $x = 10$, in Eq. (i) we get

$y = 25-10$

$= 15$

So, the cost of a pen = Rs. 10,and,

The cost of a pencil box = Rs. 15

5. Determine, algebraically, the vertices of the triangle formed by the lines

$3x – y = 3$

$2x – 3y = 2$

$x + 2y = 8$

Solution:

$3x – y = 3$ …(i)

$2x -3y = 2$ …(ii)

$x + 2y = 8$ …(iii)

If the equations of the line (i), (ii) and (iii) represent the side of a ΔABC.

Then, on solving (i) and (ii),

[Multiplying Eq. (i) by 3 and then subtract]
$(9x-3y) - (2x-3y) = 9-2$

$7x = 7$

$x = 1$

Putting,

$x = 1$ in Eq. (i),

We get,

$3\times1-y = 3$

$y = 0$

Therefore, the coordinate of point B is (1, 0)

On solving lines (ii) and (iii),

[Multiplying Eq. (iii) by 2 and then subtract]
$(2x + 4y) - (2x-3y) = 16 - 2$

$7y = 14$

$y = 2$

Putting $y = 2$ in Eq. (iii),

$x + 2 \times 2 = 8$

$x + 4 = 8$

$x = 4$

Therefore, the coordinate of point C is (4, 2).

On solving lines (iii) and (i),

[Multiplying in Eq. (i) by 2 and then add]
$(6x-2y) + (x + 2y) = 6 + 8$

$7x = 14$

$x = 2$

Putting $x = 2$ in Eq. (i), we get

$3 \times 2 - y = 3$

$y = 3$

Hence, the coordinate of point A is (2, 3).

Therefore, the vertices of the ΔABC formed by the given lines are:

A (2, 3),

B (1, 0)

C (4, 2).

6.

Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus.

Solution:

Taking the speed of the rickshaw and the bus x and y km/h, respectively.

As, she has taken time to travel 2 km by rickshaw,

$t_1 = (2/x)$ hr

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Time taken to travel remaining distance. (14 – 2) = 12km is,

By bus

$t_2 = (12/y)$ hr

By first condition,

$$t_1 + t_2 = \frac{1}{2}$$

$$\frac{2}{x} + \frac{12}{y} = \frac{1}{2} \quad \ldots \text{(i)}$$

As, she has taken time to travel 4 km by rickshaw,

$t_3 = (4/x)$ hr

and

Time taken to travel remaining distance, (14 – 4) = 10km, by bus;

$t_4 = (10/y)$ hr

By second condition,

$t_3 + t_4 = ½ + 9/60$

$= ½ + 3/20$

Putting values of t_1 and t_2,

$\frac{4}{x}+\frac{10}{y}=\frac{13}{20}$ …(ii)

Let (1/x) = u and (1/y) = v

Then Equations. (i) and (ii) becomes

2u + 12v = $\frac{1}{2}$ …(iii)

4u + 10v = $\frac{13}{20}$ …(iv)

[Multiplying Eq. (iii) by 2 and then subtract]

(4u + 24v) – (4u + 10v) = $1-\frac{13}{20}$

v = $\frac{1}{40}$

Substituting the value of v in Eq. (iii),

2u + 12($\frac{1}{40}$) = $\frac{1}{2}$

u = 1/10

x = 1/u = 10km/hr

y = 1/v = 40km/hr

Therefore, the speed of rickshaw = 10 km/h

And the speed of bus = 40 km/h.

7.

A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

Solution:

Taking the speed of the stream be x km/hr.

Speed of the boat in still water = 5 km/hr

Speed of the boat upstream = (5 – x)km/hr

Speed of the boat downstream = (5 + x) km/hr

Time taken in rowing 40 km upstream = $\frac{40}{5-x}$

Time taken in rowing 40 km downstream = $\frac{40}{5+x}$

According to the question, we have

Time taken in 40 km upstream = 3 × Time taken in 40 km downstream

$\frac{40}{5-x}$ = 3 x ($\frac{40}{5+x}$)

–3x + 15 = x + 5

–3x – x = 5 – 15

–4x = –10

x = 2.5 km/hr

Hence, the speed of stream is 2.5 km/hr.

8.

A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Solution:

Taking speed of boat in still water = x km/hr

Speed of the stream = y km/hr

Speed of motor boat upstream = (x – y) km/hr

Speed of motor boat downstream = (x + y) km/hr

Case I: Time taken by motor boat in 30 km upstream = $\frac{30}{x-y}$

Time taken by motor boat in 28 km downstream = $\frac{28}{x+y}$

$$\frac{30}{x-y}+\frac{28}{x+y}=7$$

$$\frac{15}{x-y}+\frac{14}{x+y}=\frac{7}{2} \quad \text{.... (i)}$$

Case II: Time taken by motor boat in 21 km upstream = $\frac{21}{x-y}$

Time taken by motor boat to return 21 km downstream = $\frac{21}{x+y}$

$$\frac{21}{x-y}+\frac{21}{x+y}=5$$

$$\frac{1}{x-y}+\frac{1}{x+y}=\frac{5}{21} \quad \text{...(ii)}$$

As both the equations (i) and (ii) are symmetric to (x – y) and (x + y),

We can eliminate either (x–y) or (x+ y).

On solving equation (i) and (ii), we get,

(x – y) = 6 .....(iii)

Putting x – y = 6 in (ii),

x+y = 14 .....(iv)

Solving (iii) and (iv), we get,

x = 10 km/hr

Now,

x + y = 14 [From (iv)]

10 + y = 14

y = 4 km/hr

Therefore, the speed of motorboat and stream are 10 km/hr and 4 km/hr respectively.

9.

A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Solution:

Let the two digit number be 10x + y

Number = 8(x + y) – 5

10x + y = 8x + 8y – 5

10x – 8x + y – 8y = –5

2x – 7y = –5 ...(i)

Also,

Number = 16(x – y) + 3 = 10x + y

10x + y = 16x – 16y + 3

–6x + 17y = 3 ...(ii)

Multiplying (i) by 3, we get

6x – 21y = –15

Adding (iii) and (ii), we have

$$\begin{aligned} -6x + 17y &= 3 \\ 6x - 21y &= -15 \\ \hline -4y &= -12 \end{aligned}$$

So,y = 3

Now,

2x – 7y = –5 [From (i)]

2x – 7(3) = –5

2x = –5 + 21

2x = 16

x = 8

So, the number in xy = 83.

Therefore, the required number = 83

10.

A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs Rs 2530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs Rs 3810. Find the full first-class fare from station A to B, and also the reservation charges for a ticket.

Solution:

Taking the cost of full fare from station A to B = ₹ x

And,the reservation charges per ticket = ₹ y

Cost of one full ticket from A to B = ₹ 2530

So,

(1 fare + 1 reservation) charges = ₹ 2530

x + y = 2530

Cost of 1 full and one, half ticket from station A to B = ₹3810

(1 full ticket) + (1/2 ticket) charges = ₹3810

(x + y) + (1/2 fare + reservation) = 3810

$$(x+y)+\frac{1}{2}x+y=3810$$

$$\frac{3}{2}x+2y=3810$$

3x + 4y = 7620. ..(ii)

Multiplying (i), by 3, we get

3x + 3y = 7590 ... ...(iii)

Subtracting (iii) from (ii), we get

y = 30

Now, x + y = 2530 [From (i)]

x + 30 = 2530 (₹ y = 30)

x = 2530 – 30

x = ₹2500

So, full fare and reservation charges of a ticket station A to B are ₹2500 and ₹30 respectively.

11.

A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

Solution:

Taking the cost price of a saree = ₹x

and the list price of sweater = ₹y

Case I:

(S. P. of saree at 8% profit) + (S.P. of a sweater at 10% discount) = ₹1008

108x + 90y = 100800

6x + 5y = 5600 ...(i)

Case II:

(S.P. of saree at 10% profit) + (S.P. of a sweater at 8% discount) = ₹1028

$110x + 92y = 102800$ ...(ii)

On dividing (ii) by 2,

$55x + 46y = 51400$ ...(iii)

Multiplying (iii) by 5, we get

$275x + 230y = 257000$ ...(iv)

Multiplying (i) by 46, we get

$276x + 230y = 257600$ ...(v)

Subtracting (v) from (iv), we get

$-x = -600$

x = ₹ 600

Now,

$6x + 5y = 5600$ [From (i)]

$6 \times 600 + 5y = 5600$ [₹ x = 600]

$5y = 5600 - 3600$

$y = 400$

Therefore, the C.P. of a saree and L.P. of sweater are ₹ 600, ₹ 400 respectively.

12.

Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme?

Solution:

Taking, the money invested in scheme A = ₹x

The money invested in scheme B = ₹y

Case I: Susan invested ₹ x at 8% p.a. + Susan invested ₹y at 9% p.a. = 1860

$8x + 9y = 186000$ ...(i)

Case II: Interchanging the amount in schemes A and B, we have

$9x + 8y = 188000$ ...(ii)

Adding (i) and (ii), we get

$x + y = 22000$ ...(iii)

Subtracting (i) and (ii),

$x – y = 2000$ ...(iv)

$x = 12000$

Now,

$x + y = 22000$ [From (iii)]

$y = 22000 – 12000$

y = ₹10,000

Hence, the amount invested in schemes A and B are ₹12000 and ₹ 10,000 respectively.

13.

Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.

Solution:

Taking the number of bananas in lot A = x

Also, the number of bananas in lot B = y

Case I:

Taking S.P. of 3 bananas of lot A = 2

S.P. of 1 bananas of lot A = 2/3

S.P. of x bananas of lot = 2/3 x

Now, S.P. of 1 banana of lot B = 1

S.P. of y bananas of lot B = y

So,

$$\frac{2x}{3} + y = 400$$

2x + 3y =1200 ...(i)

Case II:

$$x + \frac{4}{5}y = 460$$

5x + 4y = 2300 ...(ii)

Multiplying (i) by 4, we get

8x + 12y = 4800 ...(iii)

Also, multiplying (ii) by 3, we get

15x + 12y = 6900

$$\begin{array}{l} 15x + 12y = 6900 \\ 8x + 12y = 4800 \\ - \quad - \quad - \\ \hline 7x = 2100 \\ x = \frac{2100}{7} \end{array}$$

So, x = 300

Now,

2x + 3y = 1200 [From (i)]

$2(300) + 3y = 1200$

$3y = 1200 - 600$

$y = 200$

So, the total number of bananas = $(x + y)$

$= (300 + 200)$

$= 500.$