

Coordinate Geometry

- **Distance formula**

The distance between the points P (x_1, y_1) and Q (x_2, y_2) is given by
 $PQ = \sqrt{x_2 - x_1^2 + y_2 - y_1^2}$

Example 1:

Find the values of l , if the distance between the points $(-5, 3)$ and $(l, 6)$ is 5 units.

Solution:

The given points are A $(-5, 3)$ and B $(l, 6)$.

It is given that $AB = 5$ units

By distance formula we have

$$\sqrt{l - (-5)^2 + 6 - 3^2} = 5 \Rightarrow \sqrt{l + 25 + 9} = 5 \Rightarrow \sqrt{l + 34} = 5 \Rightarrow l + 34 = 25 \Rightarrow l = 25 - 34 \Rightarrow l = -9$$

Required values of l are -1 or -9 .

- The distance of a point (x, y) from the origin O $(0, 0)$ is given by $OP = \sqrt{x^2 + y^2}$.

- **Section formula:**



The co-ordinates of the point P (x, y) , which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m:n$, are given by:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Example: In what ratio does the point $(-4, 7)$ divide the line segment joining the points P $(-1, 1)$ and Q $(-6, 11)$.

Solution: Let the point $(-4, 7)$ divide the line segment joining the points P $(-1, 1)$ and Q $(-6, 11)$ in the ratio $\lambda : 1$.

Thus, by section formula, we have:

$$\frac{-6\lambda + (-1)(1)}{\lambda + 1} = -4, \frac{11\lambda + 1(1)}{\lambda + 1} = 7 \Rightarrow \frac{-6\lambda - 1}{\lambda + 1} = -4, \frac{11\lambda + 1}{\lambda + 1} = 7 \Rightarrow -6\lambda - 1 = -4\lambda - 4 \Rightarrow 2\lambda = 3 \Rightarrow \lambda = \frac{3}{2}$$

Therefore, the required ratio is $3:2$.

- The **mid-point** of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

[Note: Here, $m = n = 1$]

- If A (x_1, y_1) , B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC , then the coordinates of its **centroid** are given by the point $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$.

- **Area of a triangle**

The area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by the numerical value of the expression
 $12x_1y_2 - y_3 + x_2y_3 - y_1 + x_3y_1 - y_2$

Example 1:

Find the area of the triangle whose vertices are P $(-2, 2)$, Q $(2, 0)$ and R $(8, 5)$.

Solution:

We have P $(-2, 2)$, Q $(2, 0)$ and R $(8, 5)$ as the given points.

Let $(x_1, y_1) = (-2, 2)$; $(x_2, y_2) = (2, 0)$; $(x_3, y_3) = (8, 5)$

area of $\Delta PQR = 12x_1y_2 - y_3 + x_2y_3 - y_1 + x_3y_1 - y_2 \Rightarrow$ area of $\Delta PQR = 12 - 20 - 5 + 25 - 2 + 8 -$

$0 \Rightarrow$ area of $\Delta PQR = 12 - 10 + 6 + 16 \Rightarrow$ area of $\Delta PQR = 12 \times 32 \Rightarrow$ area of $\Delta PQR = 16$ square units

Example 2:

If the points $(-4, 1)$, $(2, 4)$ and $(p, 6)$ are collinear, then find the value of p .

Solution:

Since $(-4, 1)$, $(2, 4)$, $(p, 6)$ are collinear, the area of the triangle formed by these points is zero.

$\therefore 12 - 44 - 6 + 26 - 1 + p - 4 = 0 \Rightarrow 8 + 10 - 3p = 0 \Rightarrow 18 - 3p = 0 \Rightarrow 3p = 18 \Rightarrow p = 6$