Chapter 1. Compound Interest

Formulae

1. Let Principal = P, Rate = R% per annum, Time = n years.

2. **When interest is compound Annually:**

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^{n}$$

3. **When interest is compounded Half-yearly:**

$$\text{Amount} = P\left[1 + \frac{(R/2)}{100}\right]^{2n}$$

4. **When interest is compounded Quarterly:**

$$\text{Amount} = P\left[1 + \frac{(R/4)}{100}\right]^{4n}$$

5. **When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.**

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^{3} \times \left(1 + \frac{\frac{2}{5}R}{100}\right)$$

6. **When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1^{st}, 2^{nd} and 3^{rd} year respectively.**

$$\text{Then, Amount} = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right).$$

7. **Present worth of Rs. x due n years hence is given by:**

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100}\right)^{n}}.$$

Future Value Formula (compound interest)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:
A = resulting amount (future value)
P = amount of principal (present value)
r = annual interest rate
n = number of compounding periods per year
t = time (in years)

Let Principal = P, Rate = R% per annum,
Time = n years.

1. When interest is compounded annually :

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

2. When interest is compounded half-yearly :

$$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

3. When interest is compounded quarterly :

$$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

4. When interest is compounded annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

5. When rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively. Then,

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

6. Growth : If the rate of growth is constant, then

$$V = V_0\left(1+\frac{r}{100}\right)^n$$

where $r\%$ is the rate of growth per year, n is the number of years, V_0 is the present measure of the quantity and V is the measure of the quantity after n years.

Similarly, if V_0 is the measure of the quantity n years ago and V is the present measure of the quantity, then

$$V = V_0\left(1+\frac{r}{100}\right)^n$$

7. Depreciation : If the rate of depreciation is constant, then

$$V = V_0\left(1-\frac{r}{100}\right)^n$$

where $r\%$ is the rate of depreciation per year, n is the number of years, V_0 is the present value and V is the value after n years.

Similarly, if V_0 is the value n years ago and V is the present value, then

$$V = V_0\left(1-\frac{r}{100}\right)^n$$

8. Population :

(i) Population after n years = Present population $\left(1+\frac{r}{100}\right)^n$

(ii) Present population = Population n years ago $\left(1+\frac{r}{100}\right)^n$

Formulae Based Questions

Question 1. Find the Compound Interest on Rs. 2,000 for 3 years at 15% per annum Compounded annually.

Solution : Amount at the end of the third year

$$A = P\left(1+\frac{r}{100}\right)^n$$

Given $P = ₹\,2{,}000, r = 15\%, n = 3$ years

$$A = ₹\,2{,}000\left(1+\frac{15}{100}\right)^3$$

$$= ₹\,2{,}000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{23}{20}$$

$$= ₹\,3{,}041{\cdot}75$$

Compound Interest = (A – P)

= ₹ (3,041·75 – 2,000)

= ₹ 1,041·75. *Ans.*

Question 2. If the interest is compounded half yearly, calculate the amount when the Principal is Rs. 7,400, the rate of interest is 5% per annum and the duration is one year.

Solution : For half yearly payable

$$A = P\left(1+\frac{r}{2\times 100}\right)^{2\times n}$$

where P is principal r is rate p.a. and n is time in years.

$$A = 7400\left(1+\frac{5}{200}\right)^2$$

$$= 7400 \times \frac{205}{200} \times \frac{205}{200}$$

= ₹ 7774·63. Ans.

Question 3. In how many years will Rs. 15,625 amount to Rs. 17,576 at 4% p.a., compound interest?

Solution : $A = ₹\,17,576,\ P = ₹\,15,625,$

$r = 4\%,\ n = ?$

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$17,576 = 15,625\left(1+\frac{4}{100}\right)^n$$

$$\frac{17,576}{15,625} = \left(1+\frac{1}{25}\right)^n$$

$$\frac{17,576}{15,625} = \left(\frac{26}{25}\right)^n$$

$$\left(\frac{26}{25}\right)^3 = \left(\frac{26}{25}\right)^n$$

$$n = 3 \text{ years.}$$

Question 4. The population of a city is 1,25,000. If the annual birth rate and death rate are 5.5% and 3.5% respectively. Calculate the population of the city after 3 years.

Solution : Here net growth

$$= (5.5 - 3.5)\% = 2\%$$

$$P = ₹\,1,25,000$$

$$n = 3 \text{ years}$$

$\therefore$ Population of the city after 3 years

$$= 1,25,000\left(1+\frac{2}{100}\right)^3$$

$$= 1,25,000\left(\frac{102}{100}\right)^3$$

$$= 1,25,000 \times \frac{102\times102\times102}{100\times100\times100}$$

$$= 1,32,651.$$

Question 5. There is a continuous growth in population of a village at the rate of 5% per annum. If its present population is 9,261, what population was 3 years ago?

Solution : Let P be the population 3 years ago.

Then, present population $= P \times \left(1 + \frac{5}{100}\right)^3$

$\therefore \quad 9{,}261 = \left(P \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right)$

or $\quad P = \left(9{,}261 \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21}\right)$

$= 8{,}000$

Hence, 3 years ago the population of village was

$= 8{,}000.$ Ans.

Question 6. The population of a town 2 years ago was 62,500. Due to migration to cities, it decreases at the rate of 4% per annum. Find its present population.

Solution : Present population

$= 62{,}500 \times \left(1 - \frac{4}{100}\right)^2$

$= \left(62{,}500 \times \frac{24}{25} \times \frac{24}{25}\right)$

$= 57{,}600$

$\therefore$ Present population

$= 57{,}600.$

Question. 7. The total number of industries in a particular portion of the country is approximately 1,600. If the government has decided to increase the number of industries in the area by 20% every year; find the approximate number of industries after 2 years.

Solution : Number of industries after 2 years

$= \text{Original number of industries} \left(1 + \frac{r}{100}\right)^n$

$= 1{,}600 \left(1 + \frac{20}{100}\right)^n$

$= 2{,}304$ Ans.

Question 8. The cost of a machine depreciates by 10% every year. If its present worth is Rs.18,000; what will be its value after three years?

Solution : Applying the forumula, we get;

$$\text{Value after 3 years} = ₹\,18{,}000\left(1-\frac{10}{100}\right)^3$$

$$= ₹\,13{,}122 \qquad \text{Ans.}$$

Question 9. 6000 workers were employed to construct a river bridge in four years. At the end of first year, 20% workers were retrenched; At the end of second year 5% of the workers at that time were retrenched. However, to complete the project in time, the number of workers was increased by 15% at the end of third year. How many workers were working during the fourth year?

Solution : The number of workers who were working during the fourth year

$$= 6000\left(1-\frac{20}{100}\right)\left(1-\frac{5}{100}\right)\left(1+\frac{15}{100}\right)$$

$$\left| V = V_0\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\right.$$

$$= 6000 \times \frac{4}{5} \times \frac{19}{20} \times \frac{23}{20} = 5244. \qquad \text{Ans.}$$

Concept Based Questions

Question 1. The S.I. and C.I. on a sum of money for 2 years is Rs. 200 and 210 respectively. If the rate of interest is the same. Find the sum and rate.

Solution : Let the sum be P and rate of interest be r % then.

$$\text{S.I.} = \frac{P \times r \times n}{100}$$

$$200 = \frac{P \times r \times 2}{100}$$

$$P.r = 10000 \qquad \ldots(i)$$

$$\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$210 = \frac{10000}{r}\left[\left(1+\frac{r}{100}\right)^2 - 1\right]$$

$$210 = \frac{10000}{r}\,\frac{(100+r)^2 - 100^2}{100^2}$$

$$210 = \frac{r^2 + 200r}{r}$$

$$r^2 + 200r = 210r$$

$$r^2 = 10r$$

$$r = 10\%$$

Using this in equation (1), we get

$$P = \frac{10000}{10}$$

$$= ₹\,1000$$

$$\therefore \quad P = ₹\,1{,}000$$

$$r = 10\%. \qquad \text{Ans.}$$

Question 2. Find the difference between the simple interest and compound interest on 2,500 for 2 years at 4% p.a., compound interest being reckoned semi-annualy.

Solution : $P = ₹\,2{,}500$

$r = 4\%$ p.a. $= \frac{4}{2} = 2\%$ half yearly

$T = 2$ year $= 4$ half years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= ₹\,2{,}500\left(1+\frac{2}{100}\right)^4$$

$$= ₹\,2{,}500\left(1+\frac{1}{50}\right)^4$$

$$= ₹\,2{,}500\left(\frac{51}{50}\right)^4$$

$$= ₹\,\frac{2{,}500\times 51\times 51\times 51\times 51}{50\times 50\times 50\times 50}$$

$$= ₹\,\frac{51\times 51\times 51\times 51}{50\times 50}$$

$$= ₹\,\frac{2{,}601\times 2{,}601}{2{,}500}$$

$$= ₹\,\frac{67{,}65{,}201}{2{,}500}$$

$$= ₹\,2{,}706{\cdot}08$$

$$C.I. = A - P$$

$$= ₹\,(2{,}706{\cdot}08 - ₹\,2{,}500)$$

$$= ₹\,206{\cdot}08$$

and $$S.I. = \frac{PRT}{100} = ₹\,\frac{2{,}500\times 4\times 2}{100}$$

$$= ₹\,200$$

Difference between C.I. and S.I.

$$= C.I. - S.I.$$

$$= ₹\,206{\cdot}08 - ₹\,200$$

$$= ₹\,6{\cdot}08.$$

Question 3. A sum of money is lent out at compound interest for two years at 20% p.a., being reckoned yearly. If the same sum of the money was lent Gut at compound interest of the same rate of percent per annum C.I., being reckoned half yearly would have fetched Rs. 482 more by way of interest. Calculate the sum of money lent out.

Solution : Let the principal be ₹ 100

For first case r = 20% p.a.

n = 2 years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= ₹\,100\left(1+\frac{20}{100}\right)^2$$

$$= ₹\,100\left(1+\frac{1}{5}\right)^2$$

$$= ₹\,100\left(\frac{6}{5}\right)^2$$

$$= ₹\,\frac{100\times6\times6}{5\times5}$$

$$= ₹\,\frac{3,600}{25}$$

$$= ₹\,144$$

$\therefore$ $CI = A - P$

$= ₹\,144 - ₹\,100 = ₹\,44$

For second case

r = 20% p.a. $= \frac{20}{2}$% half yearly

= 10% semi annual

Time = 2 years = 4 half years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= ₹\,100\left(1+\frac{10}{100}\right)^4$$

$$= ₹\,100\left(1+\frac{1}{10}\right)^4$$

$$= ₹\,100\left(\frac{11}{10}\right)^4$$

$$= ₹\frac{100 \times 11 \times 11 \times 11 \times 11}{10 \times 10 \times 10 \times 10}$$

$$= ₹\frac{121 \times 121}{10 \times 10}$$

$$= ₹\frac{14,641}{100}$$

$$= ₹ 146{\cdot}41$$

$$\text{C.I.} = A - P$$

$$= 146{\cdot}41 - 100$$

$$= ₹ 46{\cdot}41$$

Difference between two interest

$$= ₹ 46{\cdot}41 - ₹ 44{\cdot}00 = ₹ 2{\cdot}41$$

If difference is ₹ 2·41 then principal be ₹ 100

If difference is ₹ 482 then principal will be

$$= ₹\frac{100}{2{\cdot}41} \times 482$$

$$= ₹\frac{100 \times 482}{241} \times 100$$

$$= ₹ 100 \times 100 \times 2$$

$$= ₹ 20,000$$

∴ Sum is ₹ 20,000. Ans.

Question 4. A sum of money amounts to ₹ 2,240 at 4% p.a., simple interest in 3 years. Find the interest on the same sum for 6 months at $3\frac{1}{2}\%$ p.a.

Solution : Let P = ₹ 100

$$\text{S.I.} = \frac{PRT}{100}$$

$$= ₹\frac{100 \times 4 \times 3}{100}$$

$$= ₹ 12$$

$$A = P + \text{S.I.}$$

$$= ₹ 100 + ₹ 12 = ₹ 112$$

If A is ₹ 112 then P is ₹ 100

If A is ₹ 1 then P is $₹\frac{100}{112}$

If A is ₹ 2,240 then P be $\frac{100}{112} \times 2,240$

$$= ₹ 100 \times 20$$

$$= ₹ 2,000$$

So, principal is ₹ 2,000

Now, for second case

$$P = ₹ 2,000$$

$$\text{Time} = 6 \text{ months} = \frac{1}{2} \text{ year}$$

$$R = 3\frac{1}{2} = \frac{7}{2}\% \text{ p.a.}$$

$$\therefore \quad \text{S.I.} = ₹\frac{2{,}000 \times \frac{7}{2} \times \frac{1}{2}}{100}$$

$$= ₹\frac{2{,}000 \times 7 \times 1}{100 \times 2 \times 2}$$

$$= ₹\frac{20 \times 7}{4}$$

$$= ₹ 35.$$

Question 5. Mr. Kumar borrowed Rs. 15,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays Rs. 6,200 at the end of the first year, find the outstanding amount at the end of the second year.

Solution : $P = ₹ 15{,}000$

Interest for 1st year

$$= \frac{15{,}000 \times 8 \times 1}{100}$$

$$= ₹ 1{,}200$$

Amount after one year

$$= ₹ (15{,}000 + 1{,}200)$$

$$= ₹ 16{,}200$$

He repays ₹ 6,200 at the end of the 1st year

$\therefore$ Principal for 2nd year

$$= ₹ (16{,}200 - 6{,}200)$$

$$= ₹ 10{,}000$$

Now interest for the 2nd year

$$= \frac{10{,}000 \times 10 \times 1}{100}$$

$$= ₹ 1{,}000$$

$\therefore$ Amount outstanding at the end of 2nd year

$$= ₹ (10{,}000 + 1{,}000)$$

$$= ₹ 11{,}000.$$ Ans.

Question 6. The compound interest, calculated yearly, on a certain sum of money for the second year is Rs. 1320 and for the third year is Rs. 1452. Calculate the rate of interest and the original sum of money.

Solution : Compound Interest for second year

$= ₹\ 1320$

Compound Interest for third year

$= ₹\ 1452$

$r = ?, P = ?$

For third year : $P = ₹\ 1320,$

$I = 1452 - 1320 = ₹\ 132$

$T = 1$ year

$\therefore \quad I = \dfrac{P \times R \times T}{100}$

$132 = \dfrac{1320 \times R \times 1}{100}$

$R = \dfrac{132 \times 100}{1320}$

$R = 10\%$ p.a.

Now, let the principal for first year be x.

Then $I = \dfrac{P \times R \times T}{100} = \dfrac{10x}{100} = \dfrac{x}{10}$

Amount for first year $= x + \dfrac{x}{10} = \dfrac{11x}{10}$

For second year : $P = \dfrac{11x}{10},$

$R = 10\%$ p.a., $T = 1$ year

$I = \dfrac{\frac{11x}{10} \times 10 \times 1}{100} = \dfrac{11x}{100}$

From question,

$\dfrac{11x}{100} = 1320$

$\Rightarrow \quad x = \dfrac{1320 \times 100}{11} = 12000$

Thus the original sum of money is ₹ 12,000.