CHAPTER – 20

HEIGHT AND DISTANCE

Exercise - 20.1

1. An electric pole is 10 metres high. If its shadow is $10\sqrt{3}$ metres in length, find the elevation of the sum.

Solution:

Consider AB as the pole and OB as its shadow.

It is given that

AB = 10 m, OB = $10\sqrt{3}$ m and θ is the angle of elevation of the sum.

We know that

$\tan\theta = \frac{AB}{OB}$

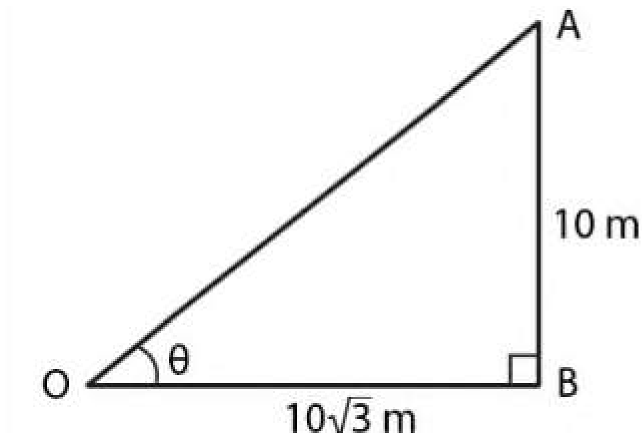
Substituting the values

$\tan\theta = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}}$

So we get

$\tan 30° = \frac{1}{\sqrt{3}}$

$\theta = 30°$

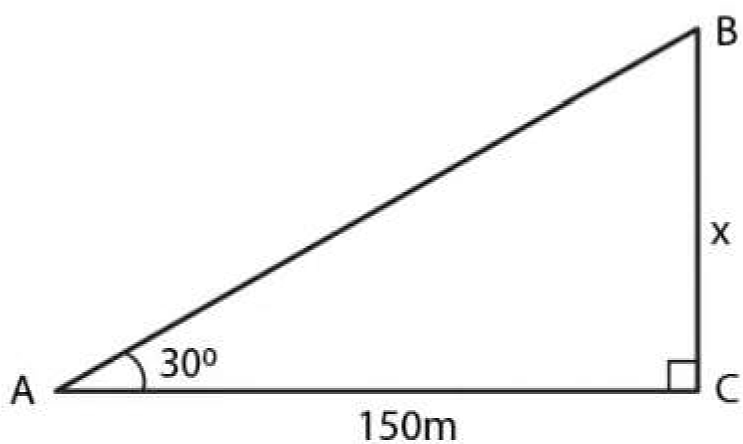


2. The angle of elevation of the top of a tower from a point on the ground and at a distance of 150 m from its foot is 30°. Find the height of the tower correct to one place of decimal.

Solution:

Consider BC as the tower and A as the point on the ground such that

$\angle A = 30°$ and AC 150 m



Take x m as the height of the tower

We know that

$\tan\theta = \frac{BC}{AC}$

Substituting the values

$\tan 30° = \frac{x}{150}$

By cross multiplication

$\frac{1}{\sqrt{3}} = \frac{x}{150}$

So we get

$x = \frac{150}{\sqrt{3}}$

Multiplying and dividing by $\sqrt{3}$

$x = \frac{(150\times\sqrt{3})}{(\sqrt{3}\times\sqrt{3})}$

By further calculation

$x = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$ m

Substituting the value of $\sqrt{3}$

$x = 50(1.732)$

$x = 86.600$ m

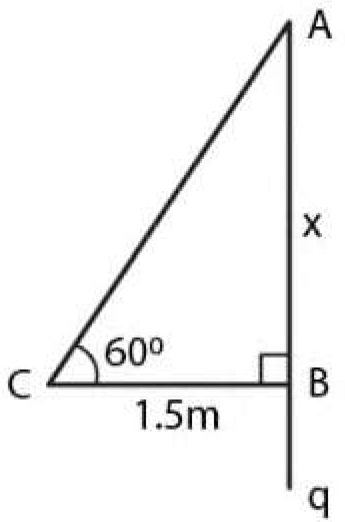
$x = 86.6$ m

3. A ladder is placed against a wall such that it reaches the top of the wall. The foot of the ladder is 1.5 metres away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.

Solution:

Consider AB as the wall and AC as the ladder whose foot C is 1.5 m awat from B

Take AB = x m and angle of inclination is 60°



We know that

$$\tan\theta = \frac{AB}{CB}$$

Substituting the values

$$\tan 60° = \frac{x}{1.5}$$

So we get

$$\sqrt{3} = \frac{x}{1.5}$$

By cross multiplication

$$x = \sqrt{3} \times 1.5 = 1.732 \times 1.5$$

$$x = 2.5980 = 2.6$$

Hence, the height of wall is 2.6 m.

4. What is the angle of elevation of the sun when the length of the shadow of a vertical pole is equal to its height?

Solution:

Consider AB as the pole and CB as its shadow

θ is the angle of elevation of the sun

Take AB = x m and BC = x m

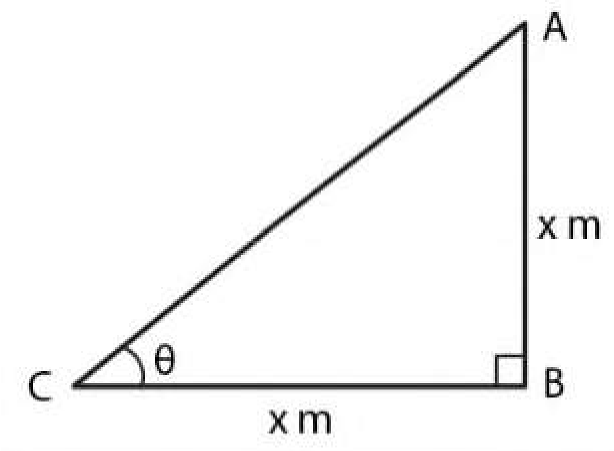
We know that

$\tan\theta = \frac{AB}{CB} = \frac{x}{x} = 1$

So we get

$\tan 45° = 1$

$\theta = 45°$



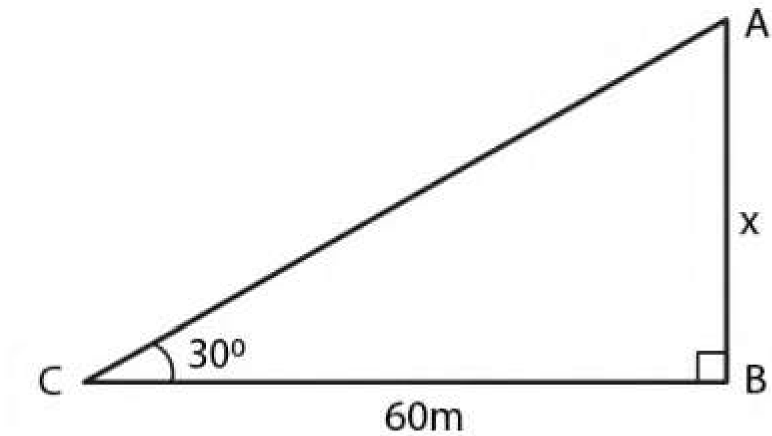
Hence, the angle of elevation of the sun 45°.

5. **A river is 60 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree on the other bank is 30°. Find the height of the tree.**

Solution:

Consider AB as the tree and BC as the width of the river

C is the point which is exactly opposite to B on the other bank and 30° is the angle of elevation



Take height of the tree AB = x m

Width of the river BC = 60 m

We know that

$\tan\theta = \frac{AB}{CB}$

Substituting the value

$\tan 30° = \frac{x}{60}$

So we get

$\frac{1}{\sqrt{3}} = \frac{x}{60}$

By cross multiplication

$x = \frac{60}{\sqrt{3}}$

Multiplying and dividing by $\sqrt{3}$

$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{60\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$

Substituting the value of $\sqrt{3}$

$x = 20(1.732)$

$x = 34.640$

$x = 34.64$ m

Hence, the height of the tree is 34.64 m

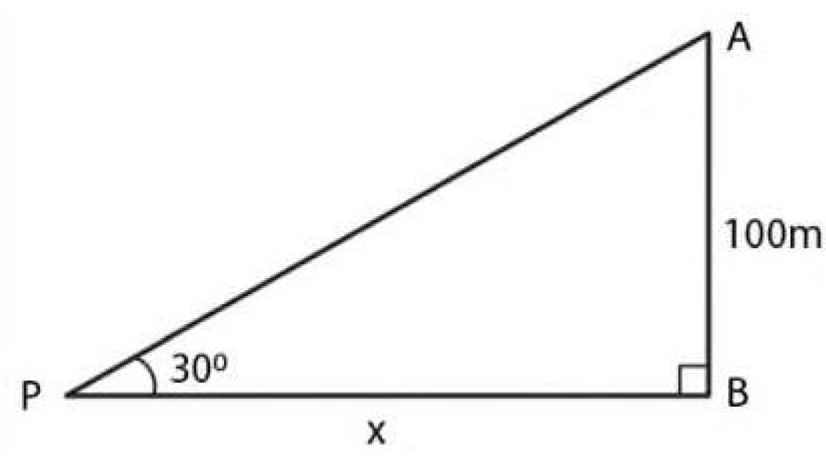
6. **From a point P on level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, How far is P from the foot of the tower?**

Solution:

Consider AB as the tower and P is at a distance of x m from B which is the foot of the tower.

Height of the tower = 100 m

Angle of elevation = 30°



We know that

$\tan\theta = \frac{AB}{CB}$

Substituting the value

$\tan 30° = \frac{100}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{100}{x}$

By cross multiplication

$x = 100\sqrt{3}$

$x = 100(1.732) = 173.2$ m

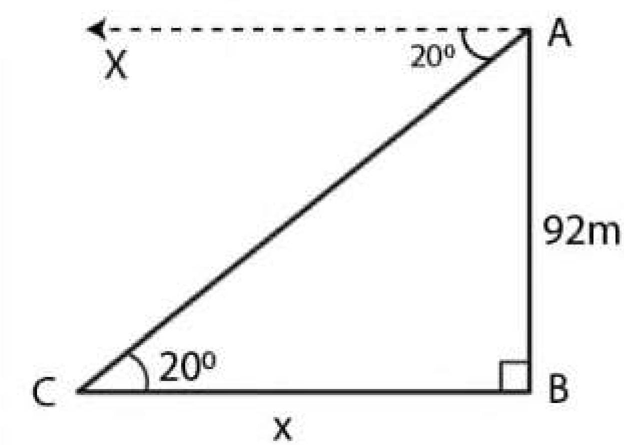
Hence, the distance of P from the foot of the tower is 173.2 m.

7. **From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate to the nearest metre, the distance of the buoy from the foot of the cliff.**

Solution:

Consider AB as the cliff whose height is 92 m

C is buoy making depression angle of 20°



We know that

$\angle ACB = 20°$

Take CB = x m

In a right angle triangle ABC

$\cot\theta = \frac{BC}{AB}$

Substituting the values

$\cot 20° = \frac{x}{92}$

By cross multiplication

$x = 92 \times \cot 20°$

So we get

$x = 92 \times 2.7475$

$x = 252.7700$ m

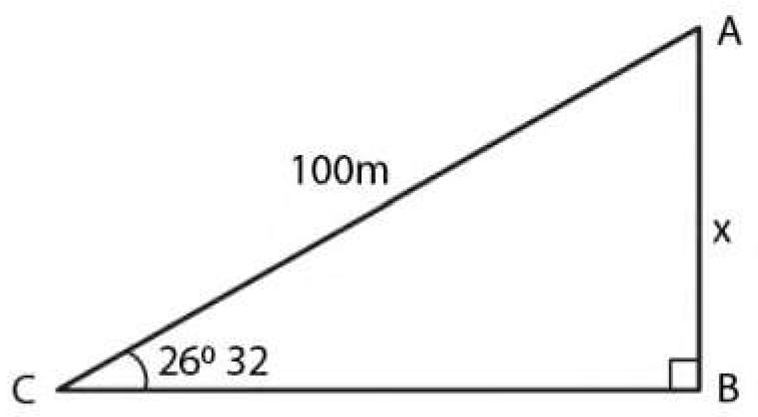
Hence, the distance of the buoy from the foot of the cliff is 252.77 m.

8. A boy is flying a kite with a string of length 100 m. If the string is tight and the angle of elevation of the kite is 26°32', find the height of the kite correct to one decimal place, (ignore the height of the boy).

Solution:

Consider AB as the height of the kite A and AC as the string

Angle of elevation of the kite = 26°32'



Take $AB = x$ m and AC = 100 m

We know that

$\sin\theta = \frac{AB}{AC}$

Substituting the values

$\sin 26°32' = \frac{x}{100}$

So we get

$0.4467 = \frac{x}{100}$

By further calculation

$x = 100 \times 0.4467$

$x = 44.67 = 44.7$ m

Hence, the height of the kite is 44.7 m.

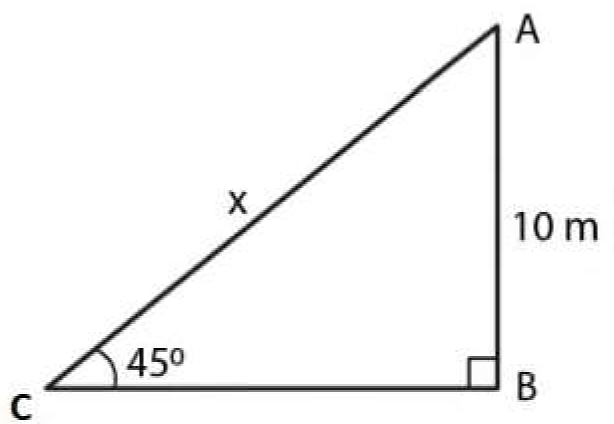
9. An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Solution:

Consider AB as the pole and AC as the wire which makes an angle of 45° with the ground.

Height of the pole AB = 10 m

Consider x m as the length of wire AC



We know that

$\sin\theta = \frac{AB}{AC}$

Substituting the values

$\sin 45° = \frac{10}{x}$

So we get

$\frac{1}{\sqrt{2}} = \frac{10}{x}$

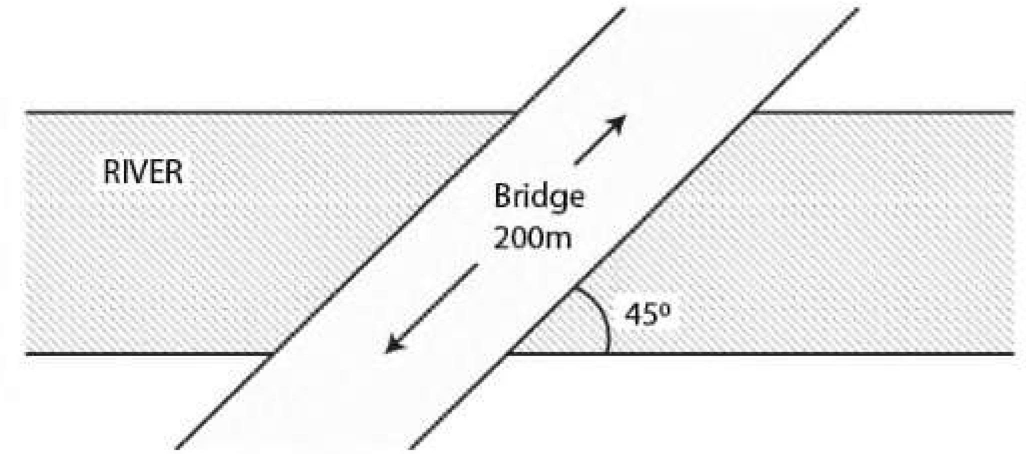
By cross multiplication

$x = \frac{10}{\sqrt{2}} = 10(1.414)$

$x = 14.14$

Hence, the length of the wire is 14.14 m.

10. A bridge across a river makes an angle of 45° with the river bank. If the length of the bridge across the river is 200 metres, what is the breadth of the river?



Solution:

Consider AB as the width of river = x m

Length of bridge AC = 200 m

Angle with the river bank = 45°

We know that

$\sin\theta = \frac{AB}{AC}$

Substituting the values

$\sin 45° = \frac{x}{200}$

So we get

$\frac{1}{\sqrt{2}} = \frac{x}{200}$

By cross multiplication

$x = \frac{200}{\sqrt{2}}$

Multiplying and dividing by $\sqrt{2}$

$x = \frac{200}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

By further calculation

$x = \frac{200(1.414)}{2}$

$x = 100(1.414)$

$x = 141.4$ m

Hence, the breadth of the river is 141.4 m.

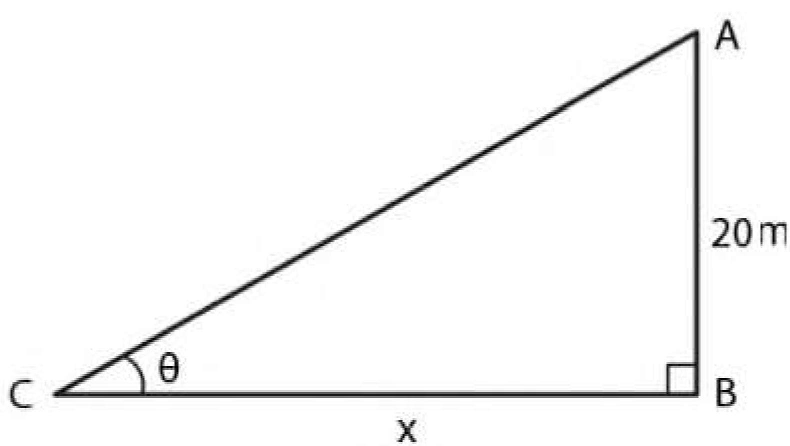
11. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower?

Solution:

Consider AB as the tower

Take a man C stands at a distance x m from the foot of the tower

$\cos \theta = 0.53$



We know that

Height of the tower AB = 20 m

$\cos\theta = 0.53$

So we get

$\theta = 58°$

Let us take

$\tan\theta = \frac{AB}{CB}$

Substituting the values

$\tan 58° = \frac{20}{x}$

So we get

$1.6003 = \frac{20}{x}$

By cross multiplication

$x = \frac{20}{1.6003}$

$x = 12.49 = 12.5$ m

Hence, the height of the tower is 12.5 m.

12. The upper part of a tree broken by wind falls to the ground without being detached. The top of the broken part touches the ground at an angle of 38°30' at a point 6 m from the foot of the tree. Calculate

(i) The height at which the tree is broken.

(ii) The original height of the tree correct to two decimal places.

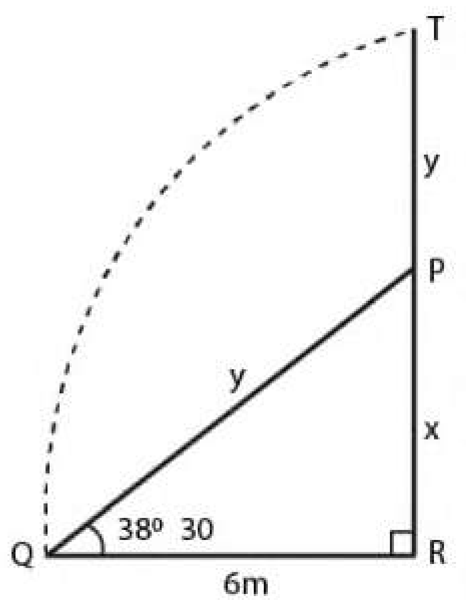
Solution:

Consider TR as the total height of the tree

TR as the broken part which touches the ground at a distance of 6 m from the foot of the tree which makes an angle of 38°30' with the ground

Take PR = x and TR = $x + y$

PQ = PT = y



In right triangle PQR

$\tan\theta = \frac{PR}{PQ}$

Substituting the values

$\tan 38°30' = \frac{x}{6}$

$\frac{x}{6} = 0.7954$

By cross multiplication

$x = 0.7954 \times 6 = 4.7724$

We know that

$\sin\theta = \frac{PR}{PQ}$

Substituting the values

$\sin 38°30' = \frac{x}{y}$

So we get

$0.6225 = \frac{4.7724}{y}$

$y = \frac{4.7724}{0.6225} = 7.6665$

Here

Height of the tree = $4.7724 + 7.6665 = 12.4389 = 12.44$ m

Height of the tree at which it is broken = 4.77 m

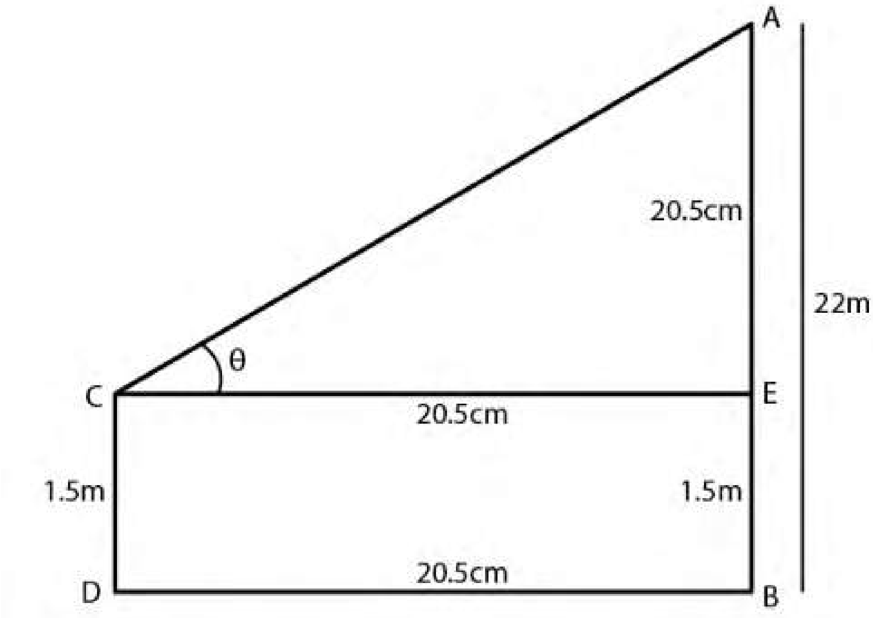
13. An observer 1.5 m tall is 20.5 metres away from a tower 22 metres high. Determine the angle of elevation of the top of the tower from the eye of the observer.

Solution:

In the figure,

AB is the tower and CD is ab observer

θ is the angle of observation



It is given that

AB = 22 m

CD = 1.5 m

Distance BD = 20.5 m

From the point C construct CE parallel to DB

AE = 22 – 15 = 20.5 m

CE = DB = 20.5 m

$\tan\theta = \frac{AE}{CE}$

Substituting the values

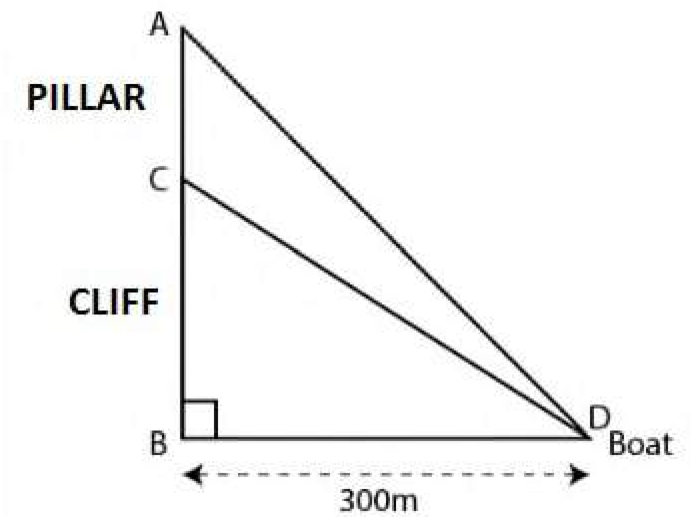
$\tan\theta = \frac{20.5}{20.5} = 1$

$\theta = 45°$

14.

(i) **In the adjoining figure, the angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre.**

(ii) **From a boat 300 metres away from a vertical cliff, the angles of elevation of the top and the foot of a vertical concrete pillar at the edge of the cliff are 55°40' and 54°20' respectively. Find the height of the pillar correct to the nearest metre.**



Solution:

Consider CB as the cliff and AC as the pillar

D as the boat which is 300 m away from the foot of the cliff BD = 300 m

Angle of elevation of the top and foot of the pillar are 55°40' and 54°20'

Take CB = x and AC = y

In a right triangle CBD

$\tan\theta = \frac{CB}{BD}$

Substituting the values

$\tan 54°20' = \frac{x}{300}$

So we get

$1.3933 = \frac{x}{300}$

By cross multiplication

$x = 300 \times 1.3933$

$x = 417.99$ m

In a right triangle ABD

$\tan\theta = \frac{AB}{BD}$

Substituting the values

$\frac{(x+y)}{300} = 1.4641$

By cross multiplication

$x + y = 1.4641 \times 300 = 439.23$

Substituting the value of x

$y = 439.23 - 417.99 = 21.24$ m $= 21$ m

Hence, the height of pillar is 21 m.

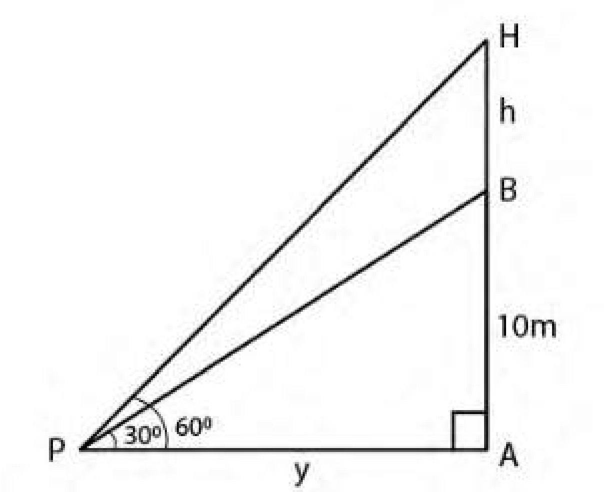
15. From a point P on the ground, the angle of elevation of the top of a 10 m tall building and a helicopter hovering over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Solution:

Consider AB as the building and H as the helicopter hovering over it

P is a point on the ground

Angle of elevation of the top of building and helicopter are 30° and 60°



We know that

Height of the building AB = 10 m

Take PA = x m and BH = h m

In right triangle ABP

$\tan\theta = \frac{P}{B}$

Substituting the values

$\tan 30° = \frac{AB}{PA} = \frac{10}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{10}{x}$

$x = 10\sqrt{3}$ m

In right triangle APH

$\tan 60° = \frac{AH}{PA}$

$\tan 60° = \frac{(10+h)}{x}$

So we get,

$\sqrt{3} = \frac{(10+h)}{10\sqrt{3}}$

By further calculation

$10\sqrt{3} \times \sqrt{3} = 10 + h$

$30 = 10 + h$

$h = 30 - 10 = 20$

Height of the helicopter from the ground = 10 + 20 = 30 m

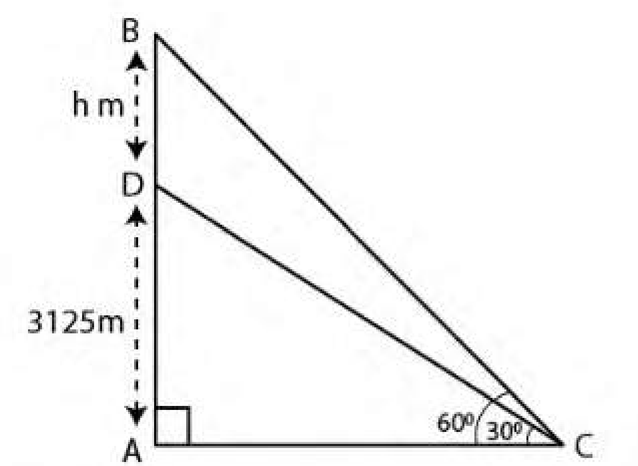
16. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at the instant.

Solution:

Consider the distance between two planes = h m

It is given that

AD = 3125 m, $\angle ACB = 60°$ and $\angle ACB = 30°$



In triangle ACD

$\tan 30° = \frac{AD}{AC}$

Substituting the values

$\frac{1}{\sqrt{3}} = \frac{3125}{AC}$

AC = $\frac{3125}{\sqrt{3}}$ … (1)

In triangle ABC

$\tan 60° = \frac{AB}{AC}$

Substituting the values

$\sqrt{3} = \frac{(AD+DB)}{AC}$

So we get

$\sqrt{3} = \frac{(3125+h)}{AC}$

$AC = \frac{(3125+h)}{\sqrt{3}}$ … (2)

Using both the equations

$\frac{(3125+h)}{\sqrt{3}} = 3125\sqrt{3}$

By further calculation

$h = \left(3125\sqrt{3} \times \sqrt{3}\right) - 3125$

$h = 3125 \times 3 - 3125$

$h = 9375 - 3125$

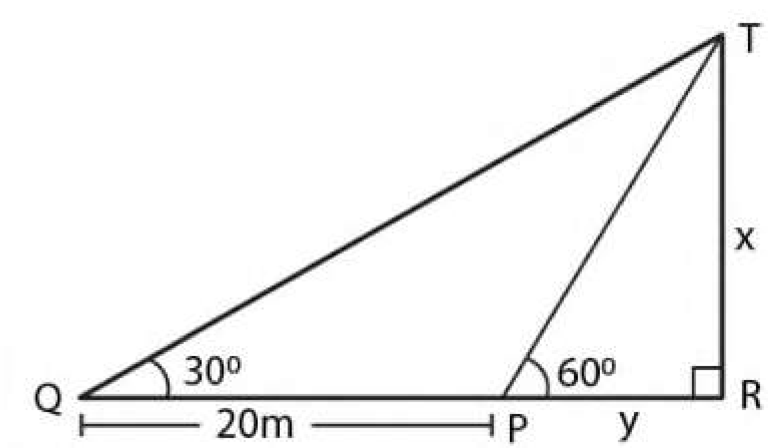
$h = 6250$ m

Therefore, the distance between two planes at the instant is 6250 m.

17. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retires 20 m from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river.

Solution:

Consider TR as the tree and PR as the width of the river.



Take TR = x and PR = y

In right triangle TPR

$\tan\theta = \frac{TR}{PR}$

Substituting the values

$\tan 60° = \frac{x}{y}$

So we get

$\sqrt{3} = \frac{x}{y}$

$x = y\sqrt{3}$ … (1)

In right triangle TQR

$\tan 30° = \frac{TR}{QR}$

$\tan 30° = \frac{x}{(y+20)}$

We get

$\frac{1}{\sqrt{3}} = \frac{x}{(y+20)}$

$x = \frac{(y+20)}{\sqrt{3}}$ … (2)

Using both the equations

$\frac{y}{\sqrt{3}} = \frac{(y+20)}{\sqrt{3}}$

So we get

$3y = y + 20$

$3y - y = 20$

$2y = 20$

$y = 10$

Now substituting the value of y in equation (1)

$x = 10 \times \sqrt{3} = 10(1.732) = 17.32$

Hence, the height of the tree is 17.32 m and the width of the river is 10 m.

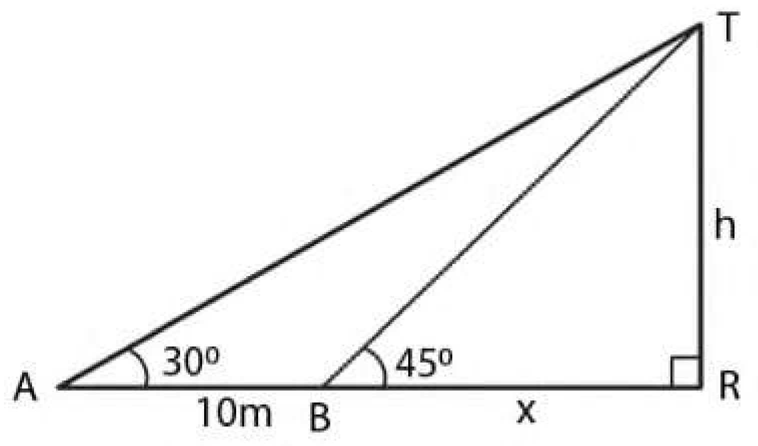
18. The shadow of a vertical tower on a level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower, correct to two decimal places.

Solution:

In the figure

AB is the tower BD and BC are the shadow of the tower in tow situations

Consider BD = x m and AB = h m



In triangle ABD

$\tan 45° = \frac{h}{x}$

So we get

$1 = \frac{h}{x}$

$h = x$ …. (1)

In triangle ABC

$\tan 30° = \frac{h}{(x+10)}$

So we get

$\frac{1}{\sqrt{3}} = \frac{h}{(x+10)}$

Using equation (1)

$h\sqrt{3} = h + 10$

$h\left(\sqrt{3} - 1\right) = 10$

We know that

$h = \frac{10}{(\sqrt{3}-1)}$

It can be written as

$h = \frac{[10(\sqrt{3}+1)]}{[(\sqrt{3}-1)(\sqrt{3}+1)]}$

By further calculation

$h = \frac{(10\sqrt{3}+1)}{2}$

So we get

$h = 5(1.73 + 1)$

$h = 5 \times 2.73$

$h = 13.65$ m

Therefore, the height of the tower is 13.65 m.

19. From the top of a hill, the angles of depression of two consecutive kilometer stones, due east are found to be 30° and 45° respectively. Find the distance of two stones from the foot of the hill.

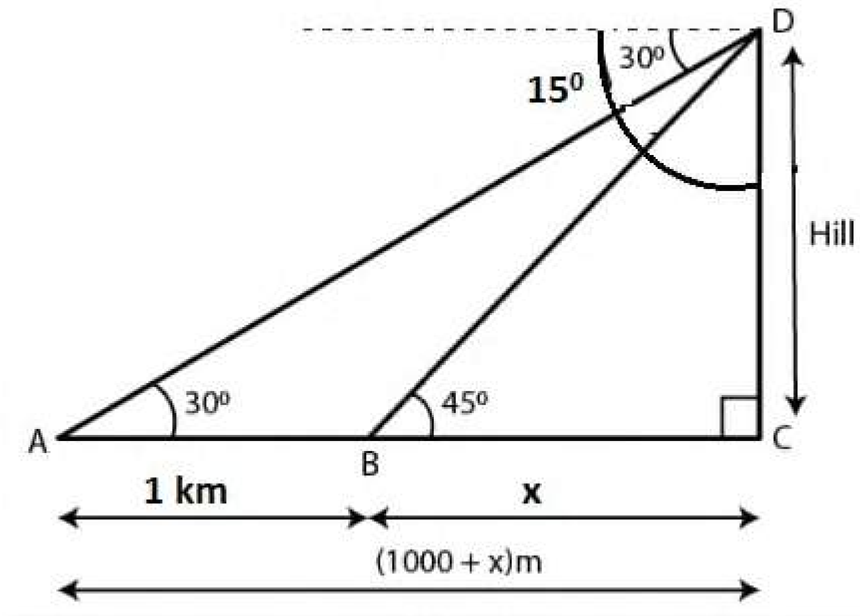
Solution:

Consider A and B as the position of two consecutive kilometer stones

Here AB = 1 km = 1000 m

Take distance BC = x m

Distance AC = $(1000 + x)$ m



In right angled triangle BCD

$\frac{CD}{BC} = \tan 45°$

So we get

$\frac{CD}{BC} = 1$

$CD = BC = x$

In right angled triangle ACD

$\frac{DC}{AC} = \tan 30°$

$\frac{x}{(x+1000)} = \frac{1}{\sqrt{3}}$

By cross multiplication

$\sqrt{3}x = x + 1000$

$(\sqrt{3} - 1)x = 1000$

$x = \frac{1000}{(\sqrt{3}-1)}$

We can write it as

$x = \left[\frac{1000}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}\right]$

$x = \left[\frac{1000(\sqrt{3}-1)}{(3-1)}\right]$

$x = 500(1.73 + 1)$

So we get

$x = 500 \times 2.73$

$x = 1365$ m

Here the distance of first stone from the foot of hill = 1365 m

Distance of the second stone from the foot of hill = 1000 + 1365 = 2365 m.

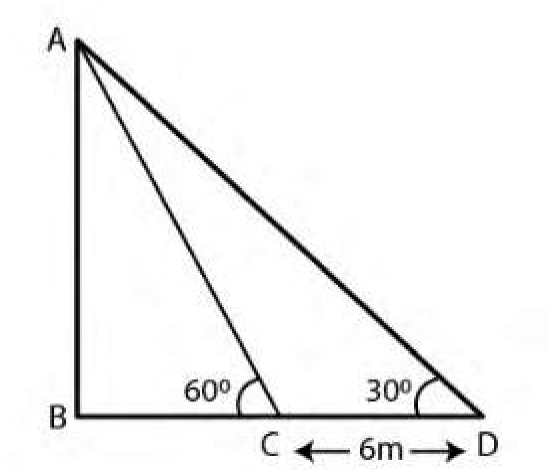
20. A man observes the angles of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest decimal place.

Solution:

It is given that

AB is a building

CD = 60 m



In triangle ABC

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC}$

So we get

$BC = \frac{AB}{\sqrt{3}}$

In triangle ABD

$\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{(BC+60)}$

By cross multiplication

$BC + 60 = \sqrt{3}\, AB$

$BC = \sqrt{3}\, AB - 60$

Using both the equations we get

$\frac{AB}{\sqrt{3}} = \sqrt{3}\, AB - 60$

By further calculation

$AB = 3AB - 60\sqrt{3}$

$3AB - AB = 60 \times 1.732$

So we get

$AB = \frac{(60 \times 1.732)}{2}$

$AB = 51.96$ m

21. At a point on level ground, the angle of elevation of a vertical lower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 m towards the tower, the tangent of the angle is found to be $\frac{3}{4}$. Find the height of the tower.

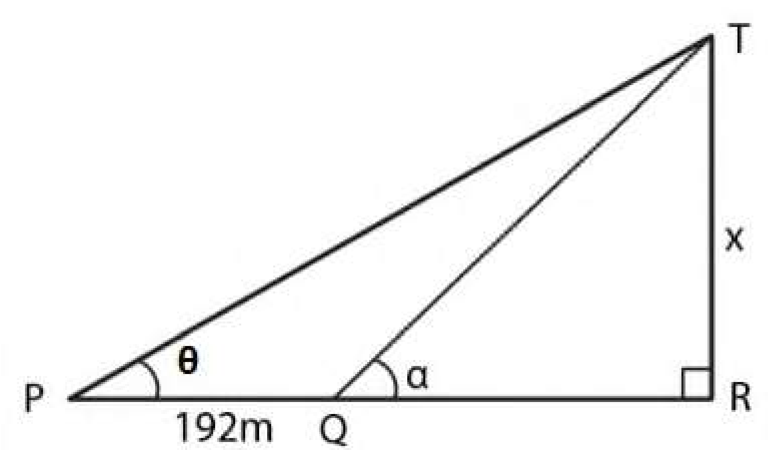
Solution:

Consider TR as the tower and P as the point on the ground such that $\tan\theta = \frac{5}{12}$

$\tan\alpha = \frac{3}{4}$

PQ = 192 m

Take TR = x and QR = y



In right triangle TQR

$$\tan\alpha = \frac{TR}{QR} = \frac{x}{y}$$

So we get

$$\frac{3}{4} = \frac{x}{y}$$

$$y = \frac{4}{3}x \qquad \text{..... (1)}$$

In right triangle TPR

$$\tan\theta = \frac{TR}{PR}$$

Substituting the values

$$\frac{5}{12} = \frac{x}{(y+192)}$$

$$x = \frac{(y+192)5}{12} \qquad \text{..... (2)}$$

Using both the equations

$$x = \frac{\left(\frac{4}{3}x+192\right)5}{12}$$

So we get

$$x = \frac{5}{9x+80}$$

$$x - \frac{5}{9}x = 80$$

$$\frac{4}{9}x = 80$$

By further calculation

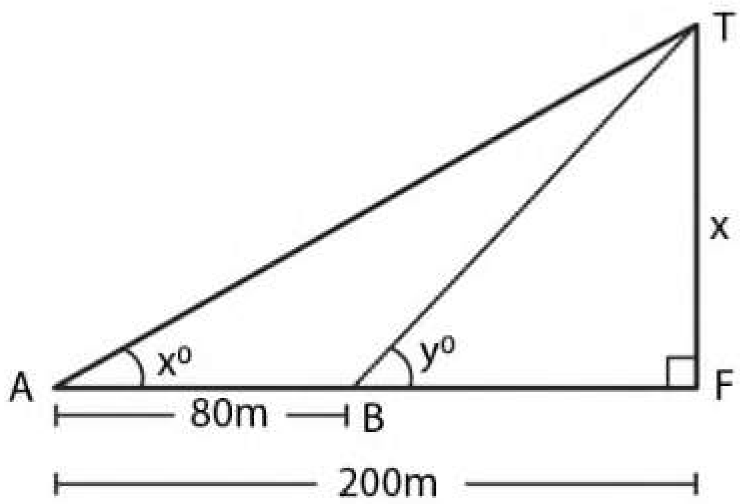
$$x = \frac{(80\times 9)}{4} = 180$$

Hence, the height of the tower is 180 m.

22. In the figure, not drawn to scale, TF is a tower. The elevation of T from A is x° where $\tan x = \frac{2}{5}$ and AF = 200 m. The elevation of T form B, where AB = 80 m, is y°. Calculate:

(i) The height of the tower TF.

(ii) The angle y, correct to the nearest degree.



Solution:

Consider the height of the tower TF = x

It is given that

$\tan x = \frac{2}{5}, AF = 200 \text{ m}. AB = 80 \text{ m}$

(i) In right triangle ATF

$\tan x° = \frac{TF}{AF}$

Substituting the values

$\frac{2}{5} = \frac{x}{200}$

So we get

$x = \frac{(2 \times 200)}{5}$

$x = \frac{400}{5}$

$x = 80$ m

Hence, the height of tower is 80 m.

(ii) In right triangle TBF.

$\tan y = \frac{TF}{BF}$

Substituting the values

$\tan y = \frac{80}{(200-80)}$

$\tan y = \frac{80}{120}$

$\tan y = \frac{2}{3} = 0.6667$

So we get

$y = 33°41' = 34°$

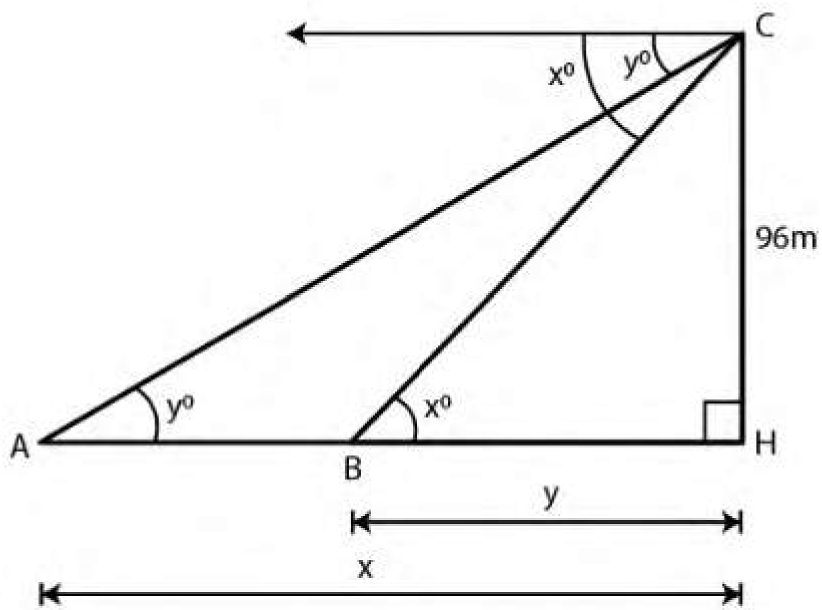
23. From the top of a church spire 96 m high, the angles of depression of two vehicles on a road, at the same level as the base of the spire and on the same side of it are x° and y°, where $\tan x° = \frac{1}{4}$ and $\tan y° = \frac{1}{7}$

Solution:

Consider CH as the height of the church

A and B are two vehicles which make an angles of depression x° and y° from C

Take AH = x and BH = y



In a right triangle CBH

$$\tan x° = \frac{CH}{AH} = \frac{96}{y}$$

Substituting the values

$$\frac{1}{4} = \frac{96}{y}$$

So we get

$$y = 96 \times 4 = 384 \text{ m}$$

In right triangle CAH

$$\tan y° = \frac{CH}{AH} = \frac{96}{x}$$

Substituting the values

$$\frac{1}{7} = \frac{96}{x}$$

So we get

$x = 96 \times 7 = 672$ m

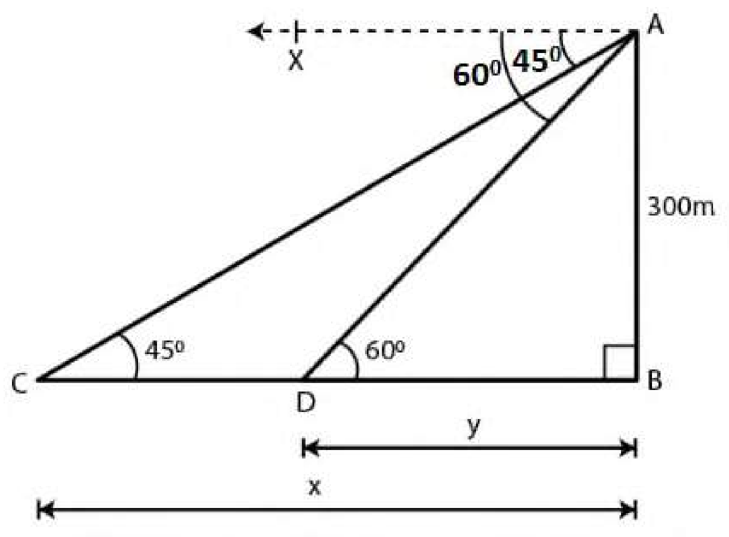
Here

AB = $x - y$

AB = 672 – 384

AB = 288 m

24. In the adjoining figure, not drawn to the scale, Ab is a tower and two object C and D are located on the ground, on the same side of AB. When observed from the top A of the tower, their angles of depression are 45° and 60°. Find the distance between the two objects. If the height of the tower is 300. Give your answer to the nearest metre.



Solution:

Consider CB = x and DB = y

AB = 300 m

In right triangle ACD

$\tan \theta = \frac{AB}{CB}$

Substituting the values

$\tan 45° = \frac{300}{x}$

$1 = \frac{300}{x}$

So we get

$x = 300$ m

In right triangle ADB

$\tan \theta = \frac{AB}{DB}$

Substituting the values

$\tan 60° = \frac{300}{y}$

$\sqrt{3} = \frac{300}{y}$

By further calculation

$y = \frac{300}{\sqrt{3}}$

Multiply and divide by $\sqrt{3}$

$y = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{300\sqrt{3}}{3}$

So we get

$y = 100 \times 1.732 = 173.2$ m

Here

CD = $x - y = 300 - 173.2 = 126.8 = 127$ m

Hence, the distance between two objects is 127 m.

25. The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, find the height of the first tower.

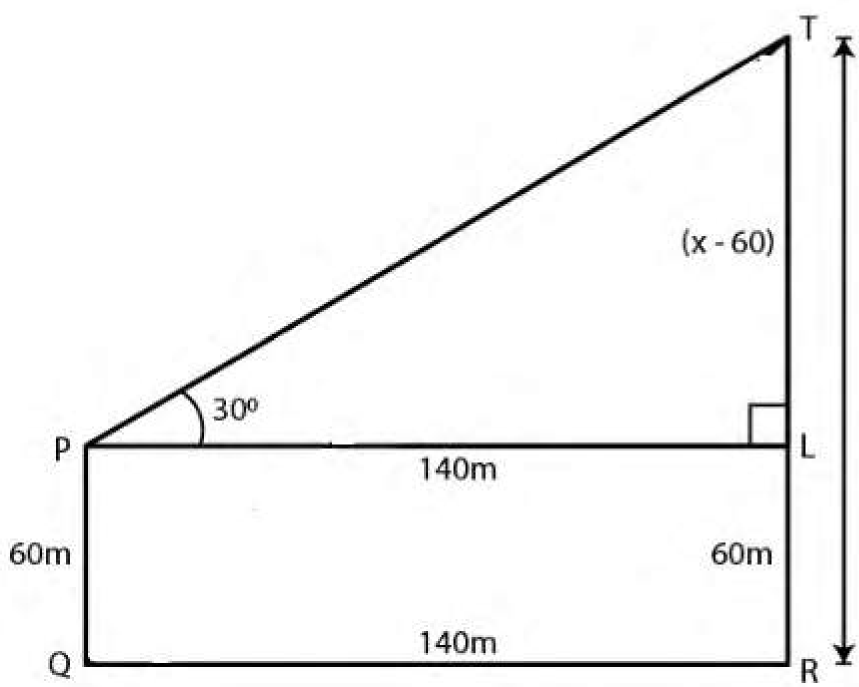
Solution:

Consider the height of the first tower TR = x

It is given that

Height of the second tower PQ = 60 m

Distance between the two towers QR = 140 m



Construct PL parallel to QR

LR = PQ = 60 m

PL = QR = 140 m

So we get

TL = $(x - 60)$ m

In right triangle TPL

$\tan\theta = \frac{TL}{PL}$

Substituting the values

$\tan 30° = \frac{(x-60)}{140}$

So we get

$\frac{1}{\sqrt{3}} = \frac{(x-60)}{140}$

By further calculation

$x - 60 = \frac{140}{\sqrt{3}}$

Multiply and divide by $\sqrt{3}$

$x - 60 = \frac{140}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{140\sqrt{3}}{3}$

We get

$x = \frac{140\sqrt{3}}{3} + 60$

$x = \frac{(140\times1.732)}{3} + 60$

$x = 80.83 + 60$

$x = 140.83$

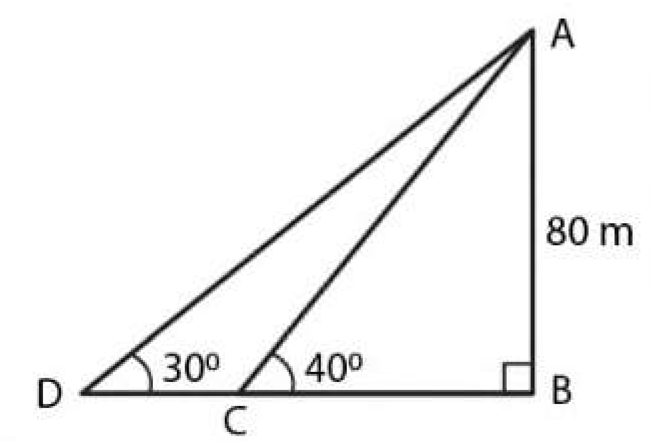
Hence, the height of first tower is 140.83 m.

26. As observed from the top of a 80 m tall light house, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the

distance between the two ships. Give your answer correct to the nearest metre.

Solution:

Consider AB as the light house and C and D as the two ships.



In triangle ADB

$\tan 30° = \frac{AB}{BD}$

Substituting the values

$\frac{1}{\sqrt{3}} = \frac{80}{BD}$

So we get

$BD = 80\sqrt{3}$ …. (1)

In triangle ACB

$\tan 40° = \frac{AB}{BC}$

Substituting the values

$0.84 = \frac{80}{BC}$

So we get

$BC = \frac{80}{0.04} = 95.25$

Here we get

DC = BD – BC

DC = 138.4 – 95.25 = 43.15

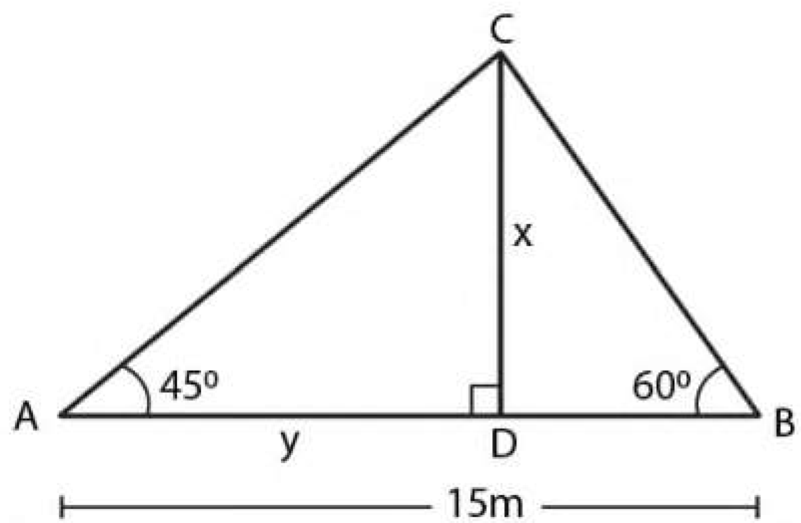
Therefore, the distance between the two ships is 43.15 m.

27. The angle of elevation of a pillar from a point A on the ground is 45° and from a point B diametrically opposite to A and on the other side of the pillar is 60°. Find the height of the pillar, given that the distance between A and B is 15 m.

Solution:

Consider CD as the pillar of x m

Angles of elevation of points A and B are 45° and 60°



It is given that

AB = 15 m

Take AD = y

DB = 15 – y

In right triangle CAD

$\tan\theta = \frac{CD}{AD}$

Substituting the values

$\tan 45° = \frac{x}{y}$

So we get

$1 = \frac{x}{y}$

$x = y$ …. (1)

In triangle CDB

$\tan 60° = \frac{x}{(15-y)}$

Substituting the values

$\sqrt{3} = \frac{x}{(15-y)}$

So we get

$x = \sqrt{3}(15 - y)$ …. (2)

Using both the equations

$x = \sqrt{3}(15 - y)$

$x = 15\sqrt{3} - \sqrt{3}x$

So we get

$x + \sqrt{3}x = 15\sqrt{3}$

$x\left(1 + \sqrt{3}\right) = 15\sqrt{3}$

$x = \frac{15\sqrt{3}}{(1+\sqrt{3})}$

We can write it as

$x = \frac{(15\times1.732)}{(1+1.732)}$

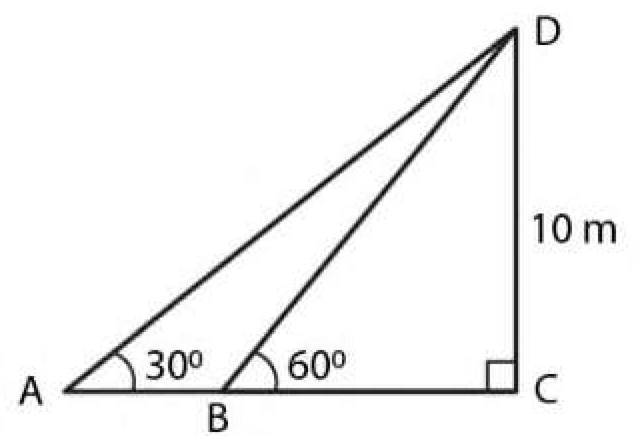
$x = \frac{25.98}{2.732}$

$x = 9.51$

Hence, the height of the pillar is 9.51 m.

28. From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10 m, find the distance between A and B correct to two decimal places.

Solution:



In triangle DBC

$\tan 60° = \frac{10}{BC}$

Substituting the values

$\sqrt{3} = \frac{10}{BC}$

$BC = \frac{10}{\sqrt{3}}$

In triangle DBC

$\tan 30° = \frac{10}{(BC+AB)}$

Substituting the values

$\frac{1}{\sqrt{3}} = 10\left[\frac{10}{\sqrt{3}} + AB\right]$

By further calculation

$\frac{1}{\sqrt{3}}\left[\frac{10}{\sqrt{3}} + AB\right] = 10$

So we get

$AB = 10\sqrt{3} - \frac{10}{\sqrt{3}}$

Talking LCM

$AB = \frac{(30-10)}{\sqrt{3}}$

$AB = \frac{20}{\sqrt{3}}$

$AB = \frac{20\sqrt{3}}{\sqrt{3}}$

So we get

$AB = \frac{(20\times1.732)}{3}$

$AB = 20 \times 0.577$

$AB = 11.540$ m

Hence, the distance between A and B is 11.54 m.

29.

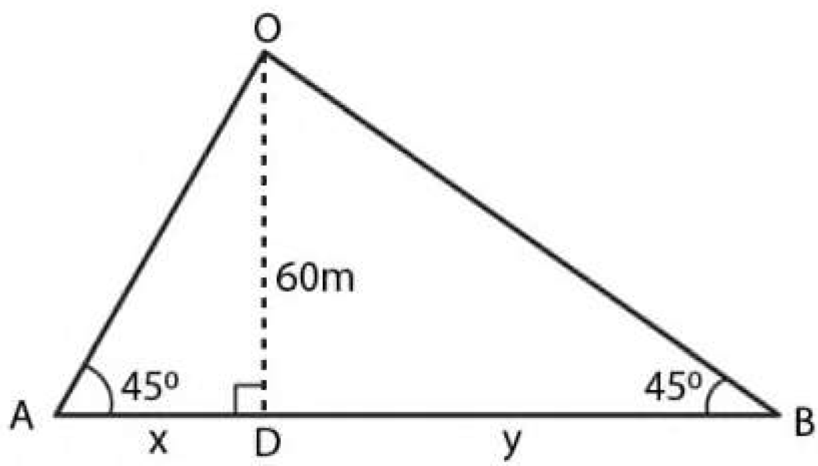
(i) **The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number.**

(ii) **An aeroplane at an altitude of 250 m observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number.**

Solution:

(i) Consider AD as the height of the light house CD = 60 m

Take AD = x m and BD = y m



In triangle ACD

$\tan 60° = \frac{CD}{AD}$

Substituting the values

$\sqrt{3} = \frac{60}{x}$

So we get

$x = \frac{60}{\sqrt{3}}$

Multiply and divide by $\sqrt{3}$

$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$

$x = 20 \times 1.732 = 34.64$ m

In triangle BCD

$\tan 45° = \frac{CD}{BD}$

Substituting the values

$1 = \frac{60}{y}$

$y = 60$ m

Here the distance between two ships = $x + y$

= $34.64 + 60$

= 94.64 m

= 95 m

(ii) In triangle OMA

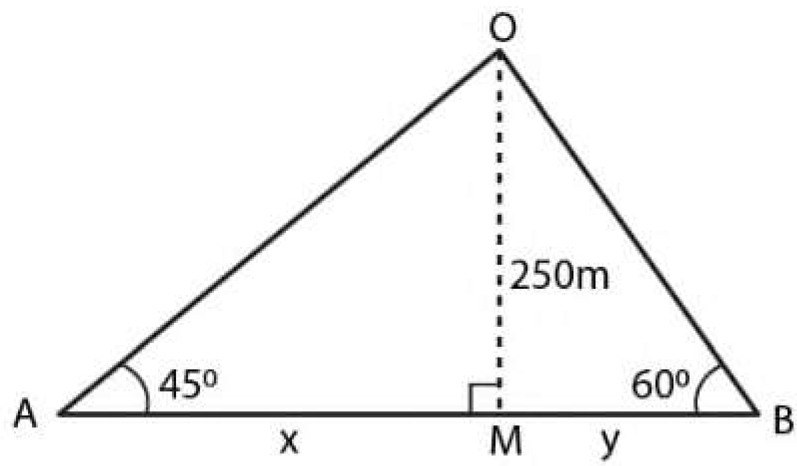
$\tan 45° = \frac{OM}{AM}$

Substituting the values

$1 = \frac{250}{x}$

So we get

$x = 250$ m



In triangle OMB

$\tan 60° = \frac{250}{y}$

Substituting the values

$\sqrt{3} = \frac{250}{y}$

So we get

$y = \frac{250}{\sqrt{3}} = \frac{250}{1.73}$

$y = 144.34$

Here

Width of the river = $x + y$

Substituting the values

= 250 + 144.34

= 394.34 m

30. From a tower 126 m high, the angles of depression of two rocks which are in a horizontal line through the base of the tower are 16° and 12°20'. Find the distance between the rocks if they are on

(i) The same side of the tower.

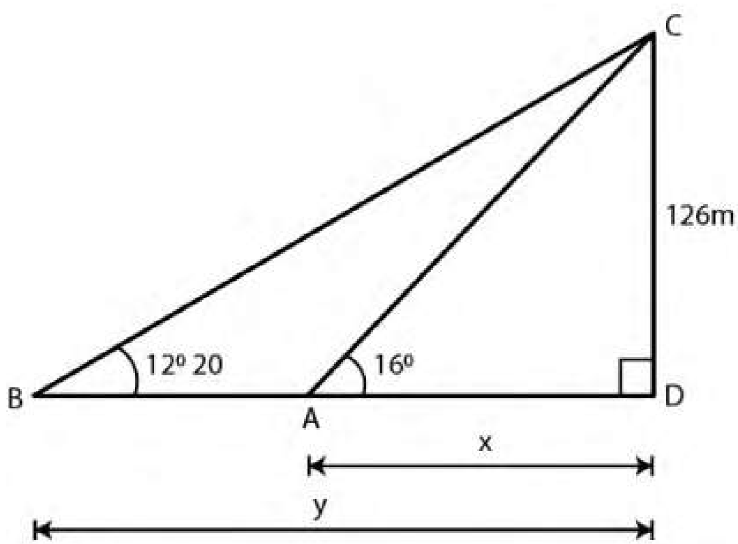
(ii) The opposite sides of the tower.

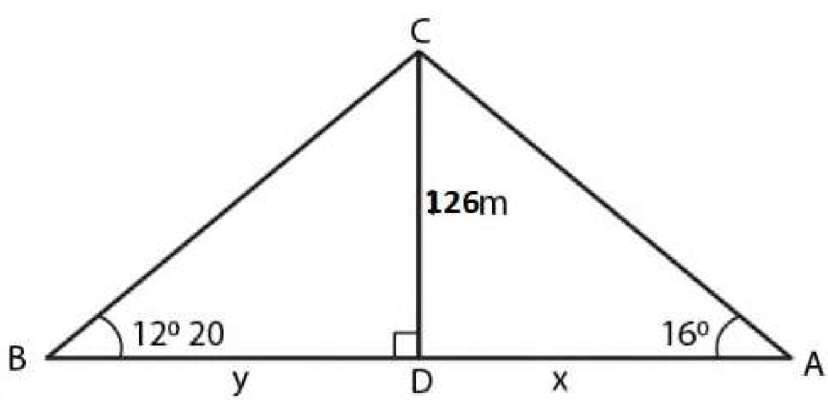
Solution:

Consider CD as the tower of height = 126 m

A and B are the two rocks on the same line

Angles of depression are 16° and 12°20'





In triangle CAD

$\tan\theta = \frac{CD}{AD}$

Substituting the values

$\tan 16° = \frac{126}{x}$

$0.2867 = \frac{126}{x}$

So we get

$x = \frac{126}{0.2867}$

$x = 439.48$

In right triangle CBD

$\tan 12°20' = \frac{126}{y}$

So we get

$0.2186 = \frac{126}{y}$

$y = \frac{126}{0.2186} = 576.40$

(i) In the first case

On the same side of the tower

AB = BD – AD

AB = $y - x$

Substituting the values

AB = BD + AD

AB = $y + x$

Substituting the values

AB = $576.40 + 439.48$

AB = 1015.88 m

31. A man 1.8 m high stands at a distance of 3.6 m from a lamp post and casts a shadow of 5.4 m on the ground. Find the height of the lamp post.

Solution:

Consider

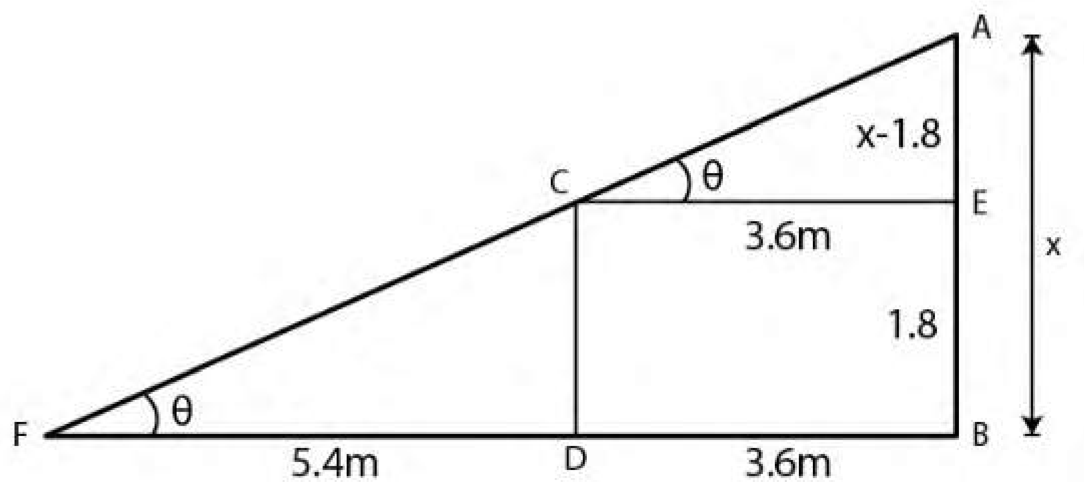
AB as the lamp post

CD is the height of man

BD is the distance of man from the foot of the lamp

FD is the shadow of man

Construct CE parallel to DB



Take AB = x and CD = 1.8 m

EB = CD = 1.8 m

AE = $x - 1.8$

Shadow FD = 5.4 m

In right triangle ACE

$\tan\theta = \frac{AE}{CE}$

Substituting the values

$\tan\theta = \frac{(x-1.8)}{3.6}$ ... (1)

In right triangle CFD

$\tan\theta = \frac{1.8}{5.4} = \frac{1}{3}$ ... (2)

Using both the equations

$\frac{(x-1.8)}{3.6} = \frac{1}{3}$

So we get

$3x - 5.4 = 3.6$

$3x = 3.6 + 5.4 = 9.0$

By division

$x = \frac{9}{3} = 3.0$

Hence, the height of lamp post is 3 m.

32. The angles fo depression of the top and the bottom of an 8 m tall building from the top of a multistoried building are 30° and 450◀ respectively. Find the height of tire multi-storied building and the distance between the two buildings, correct to two decimal places.

Solution:

Consider AB as the height and CD as the building

The angles of depression from A to C and D are 30° and 45°

$\angle ACE = 30°$ and $\angle ADB = 45°$

CD = 8 m

Take AB = h and BD = x

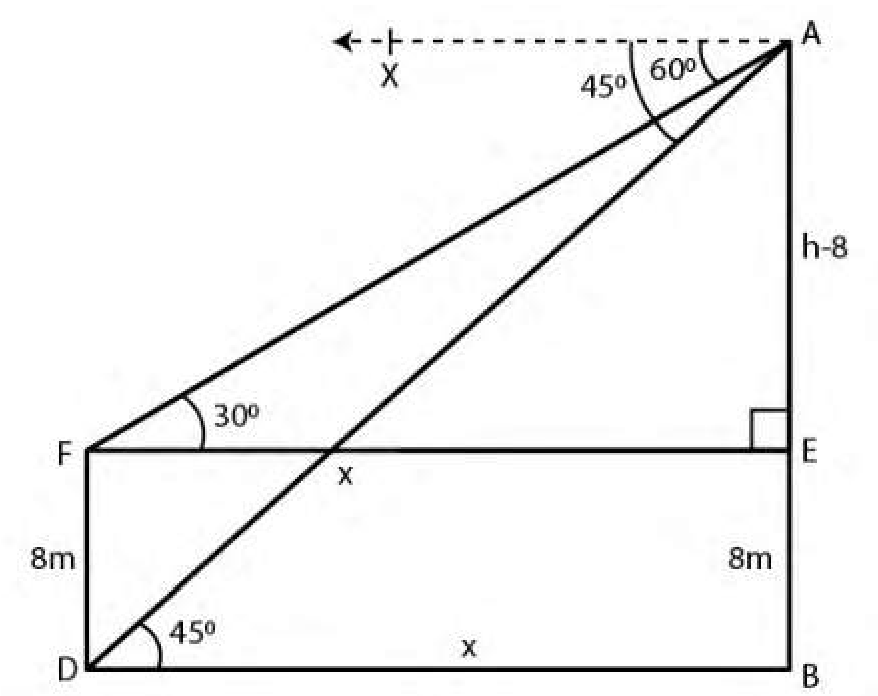
From the point C

Construct CE parallel to DB

CE = DB = x

EB = CD = 8 m

AR = AB – EB = $h-8$



In right triangle ADB

$\tan\theta = \frac{AB}{DB}$

Substituting the values

$\tan 45° = \frac{h}{x}$

So we get

$1 = \frac{h}{x}$

$x = h$

In right triangle ACE

$\tan 30° = \frac{AE}{CE}$

Substituting the values

$\frac{1}{\sqrt{3}} = \frac{(h-8)}{h}$

By further calculation

$h = \sqrt{3}h - 8\sqrt{3}$

So we get

$\sqrt{3}h - h = 8\sqrt{3}$

$h(\sqrt{3} - 1) = 8\sqrt{3}$

$h = \frac{8\sqrt{3}}{(\sqrt{3}-1)}$

Multiply and divide by $\sqrt{3} + 1$

$h = \frac{8\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}$

$h = \frac{8(3+\sqrt{3})}{(3-1)}$

Here

$h = \frac{8(3+1.732)}{2}$

$h = 4 \times 4.732$

$h = 18.928$

$h = 18.93$ m

Here

Height of multistoried building = 18.93 m

Distance between the two buildings = 18.93 m

33. A pole of height 5 m is fixed on the top of a tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle fo depression of the point A from the top of the tower is 45°. Find the height of the tower. (Take $\sqrt{3} = 1.732$)

Solution:

Consider QR as the tower

PQ as the pole on it

Angle of elevation from P to a point A is $\angle PAR = 60°$

Angle of depression from Q to A = 45°

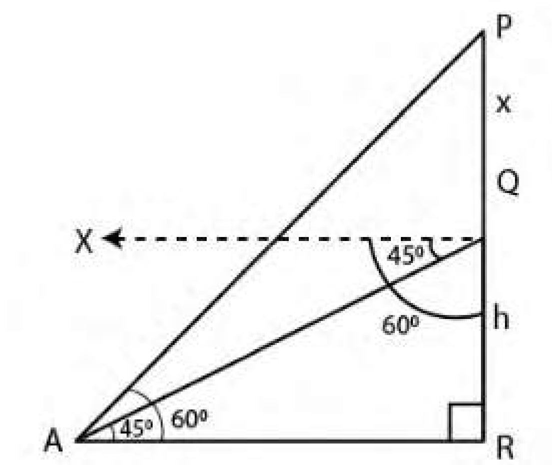
Here

$\angle QAR = 45°$ Which is the alternate angle

PQ = 5 m

Take QR = h m

PQ = 5 + h



In right triangle QAR

$\tan\theta = \frac{QR}{AR}$

Substituting the values

$\tan 45° = \frac{h}{AR}$

So we get

$1 = \frac{h}{AR}$

AR = h

In right triangle PAR

$\tan 60° = \frac{PR}{AR}$

Substituting the values

$\sqrt{3} = \frac{(5+h)}{h}$

So we get

$\sqrt{3}h = 5 + h$

$h(\sqrt{3} - 1) = 5$

$h(1.732 - 1) = 5$

By further calculation

$0.732\ h = 5$

$h = \frac{5}{0.732} = \frac{5000}{732}$

$h = 6.83$

Hence, the height of the tower is 6.83 m.

34. A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m.

Solution:

Consider TR as the tower

PL as the pole on the same level

Ground PL = 20 m

From the point P construct PQ parallel to LR

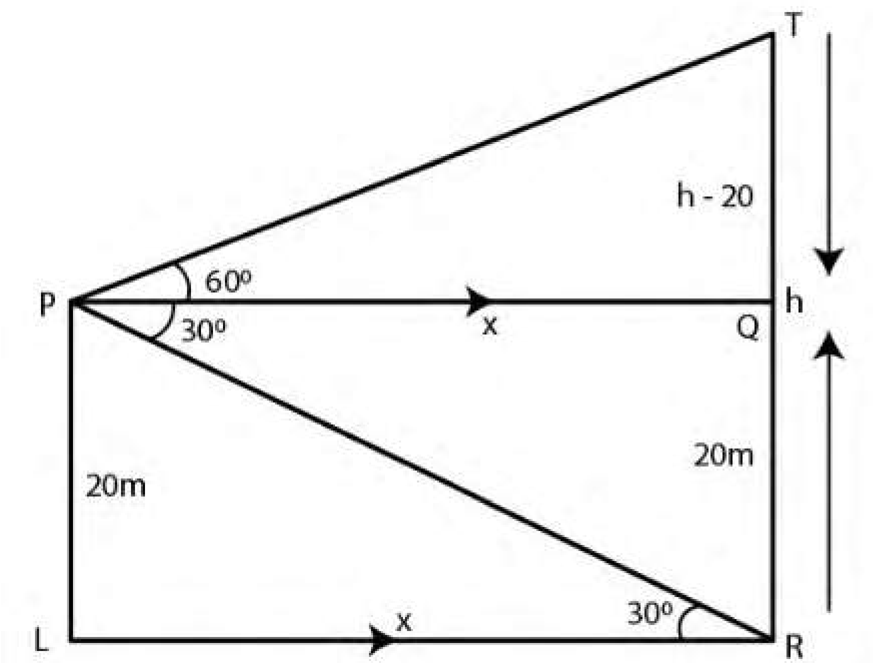
$\angle TPQ = 60°$ and $\angle QPR = 30°$

Here

$\angle PRL = \angle QPR = 30°$ Which are the alternate angles

Take LR = x and TR = h

TQ = TR – QR = (h – 20) m



In right triangle PRL

$\tan\theta = \frac{PL}{LR}$

Substituting the values

$\tan 30° = \frac{20}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{20}{x}$

$x = 20\sqrt{3}$ m

In right triangle PQT

$\tan 60° = \frac{TQ}{PQ}$

Substituting the values

$\sqrt{3} = \frac{(h-20)}{x}$

$\sqrt{3} = \frac{(h-20)}{20\sqrt{3}}$

By cross multiplication

$20\sqrt{3} \times \sqrt{3} = h - 20$

$20 \times 3 = h - 20$

$h = 60 + 20 = 80$ m

Hence, the height of the tower is 80 m.

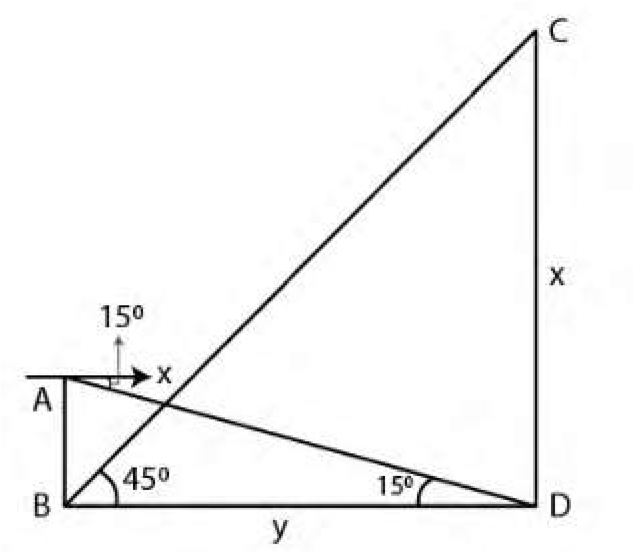
35. From the top of a building 20 m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°. Find the height of the monument.

Solution:

Consider AB as the building where AB = 20 m

CD as the monument where CD = x m

Take the distance between the building and the monument as y



In right triangle BCD

$\tan\theta = \frac{CD}{BD}$

Substituting the values

$\tan 45° = \frac{x}{y}$

$1 = \frac{x}{y}$

$x = y$ … (1)

In right triangle ABD

$\tan 15° = \frac{AB}{BD} = \frac{20}{x}$

Substituting the values

$0.2679 = \frac{20}{x}$

So we get

$x = \frac{20}{0.2679} = 74.65$ m

Hence, the height of the monument is 74.65 m.

36. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60°?

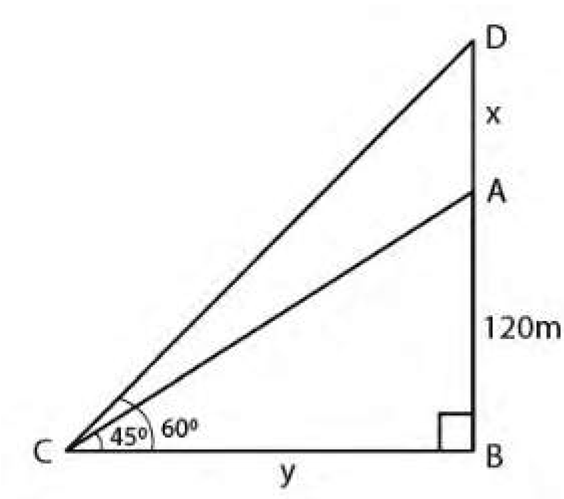
Solution:

Consider AB as the unfinished tower where AB = 120 m

Angle of elevation = 45°

Take x be higher raised so that the angle of elevation become 60°

BC = y



In right triangle ABC

$\tan\theta = \frac{AB}{CB}$

Substituting the values

$\tan 45° = \frac{AB}{CB} = \frac{120}{y}$

So we get

$1 = \frac{120}{y}$

$y = 120$ m

In right triangle DBC

$\tan 60° = \frac{DB}{CB}$

Substituting the values

$\sqrt{3} = \frac{(120+x)}{120}$

$120\sqrt{3} = 120 + x$

$x = 120\sqrt{3} - 120$

$x = 120\left(\sqrt{3} - 1\right)$

So we get

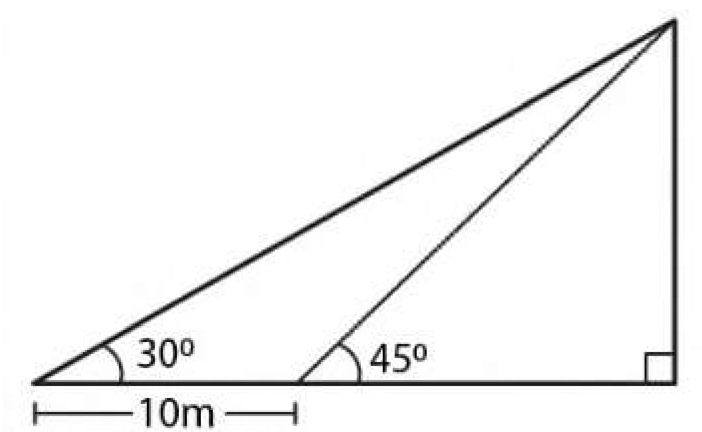
$x = 120(1.732 - 1)$

$x = 120(0.732)$

$x = 87.84$ m

Hence, the tower should be raised at 87.84 m.

37. In the adjoining figure, the shadow of a vertical tower on the level ground increases by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer, correct to $\frac{1}{10}$ of a metre.



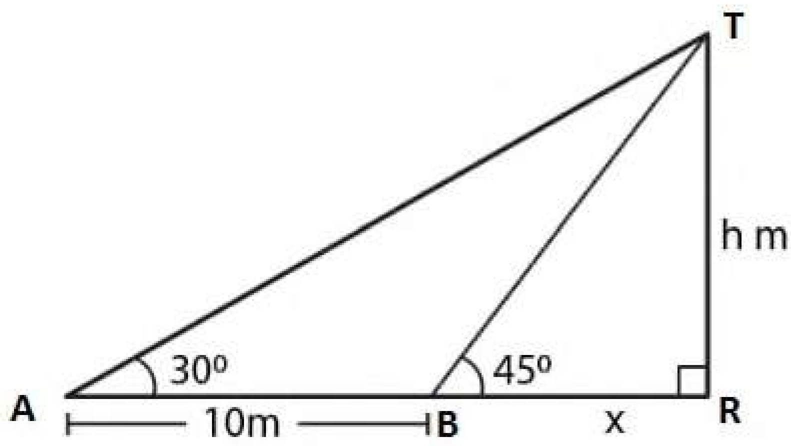
Solution:

Consider TR as the tower where TR = h

BR = x

AB = 10 m

Angles of elevation from the top of the tower at A and B are 30° and 45°



In right triangle TAR

$\tan\theta = \frac{TR}{AR}$

Substituting the values

$\tan 30° = \frac{h}{(10+x)}$

So we get

$\frac{1}{\sqrt{3}} = \frac{h}{(10+x)}$

$h = \frac{(10+x)}{\sqrt{3}}$ … (1)

In triangle TBR

$\tan 45° = \frac{TR}{BR} = \frac{h}{x}$

So we get

$1 = \frac{h}{x}$

$x = h$ … (2)

Using both the equation

$h = \frac{(10+x)}{\sqrt{3}}$

$\sqrt{3}h = 10 + h$

By further calculation

$\sqrt{3}h - h = 10$

$(1.732 - 1)h = 10$

$0.732 = 10$

$h = \frac{10}{0.732} = 13.66$

Hence, the height of the tower is 13.7 m.

38. An aircraft is flying at a constant height with a speed of 360 km/h. From a point on the ground, the angle of elevation of the aircraft at an instant was observed to be 45°. After 20 seconds, the angle of elevation was observed to be 30°. Determine the height at which the aircraft flying. (Use $\sqrt{3} = 1.732$)

Solution:

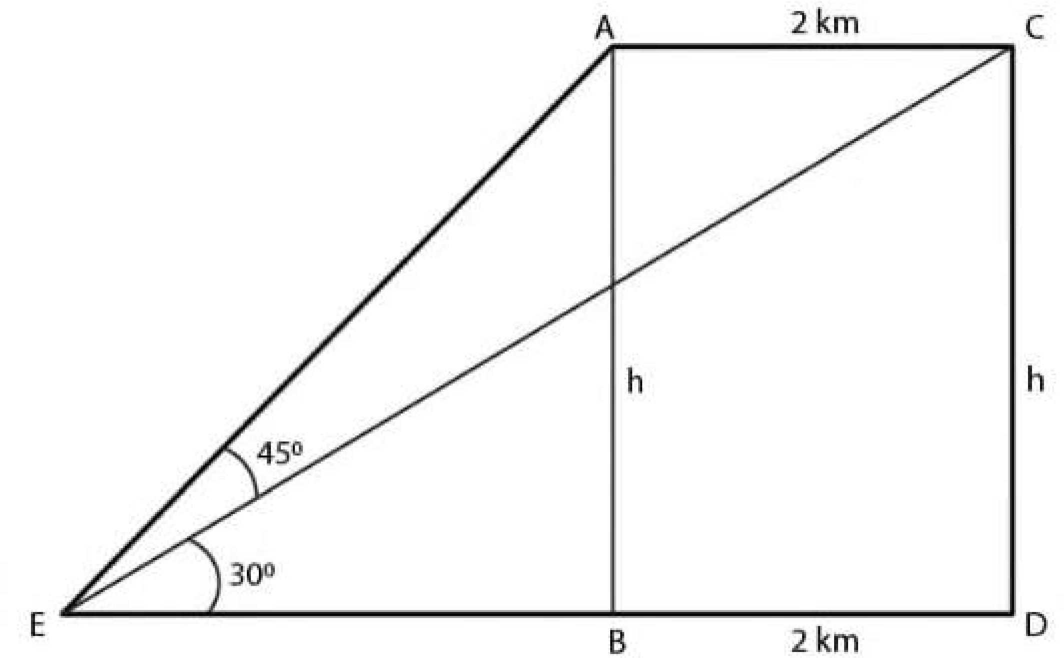
It is given that

Speed of aircraft = 360 km/h

Distance covered by the aircraft in 20 seconds = $\frac{(360 \times 20)}{(60 \times 60)} = 2$ km

Consider E as the fixed point on the ground

CD as the position of AB in height of aircraft



Take AB = CD = h km

In right triangle ARB

$\tan\theta = \frac{AB}{EB}$

Substituting the values

$\tan 45° = \frac{h}{EB}$

$EB = h$

Here

$ED = EB + BD = h + 2 \text{ km}$

In right triangle CED

$\tan 30° = \frac{CD}{ED}$

Substituting the values

$\frac{1}{\sqrt{3}} = \frac{h}{(h+2)}$

$\sqrt{3}h = h + 2$

$1.732h - h = 2$

$0.732h = 2$

We know that 2 km $= 2000$ m

$h = \frac{2000}{0.732}$

$h = \frac{(2000 \times 1000)}{732} = 2732$ m

CHAPTER TEST

1. **The angle of elevation of the top of a tower from a point A (on the ground) is 30°. On walking 50 m towards the tower, the angle of elevation is found to be 60°. Calculate**

(i) The height of the tower (correct to one decimal place)

(ii) The distance of the tower from A.

Solution:

Consider TR as the tower and A as the point on the ground

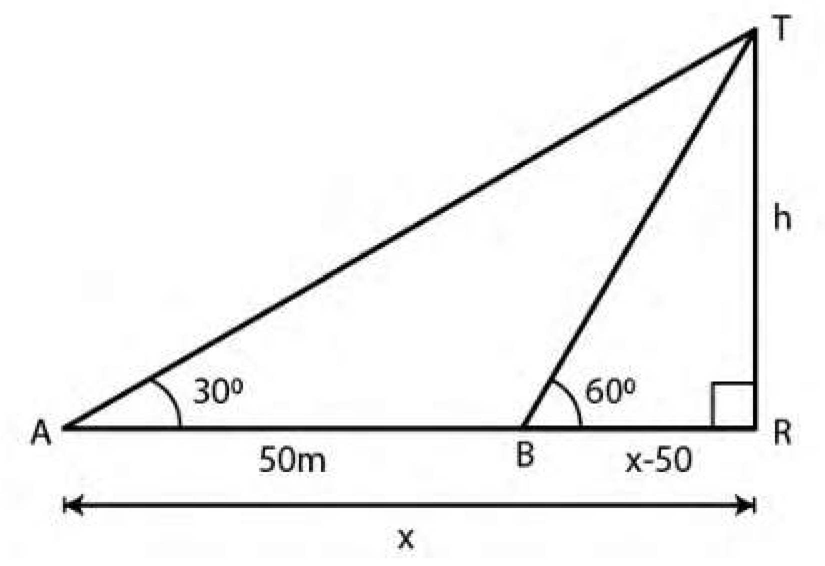
Angle of elevation of the top of tower = 30°

AB = 50 m

Angle of elevation form B = 60°

Take TR = h and AR = x

BR = $x - 50$



In right triangle ATR

$\tan\theta = \frac{TR}{AR}$

Substituting the values

$\tan 30° = \frac{h}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}h$ ... (1)

In triangle BTR

$\tan \theta = \frac{TR}{BR}$

Substituting the values

$\tan 60° = \frac{h}{(x-50)}$

So we get

$\sqrt{3} = \frac{h}{(x-50)}$

$h = \sqrt{3}(x - 50)$ ... (2)

Using both the equation

$h = \sqrt{3}(\sqrt{3}h - 50)$

By further calculation

$h = 3h - 50\sqrt{3}$

$2h = 50\sqrt{3}$

$h = 25\sqrt{3}$

So we get

$h = 25 \times 1.732 = 43.3$

Now substituting the values of h in equation (1)

$x = \sqrt{3} \times 25\sqrt{3}$

$x = 25 \times 3$

$x = 75$

Here

Height of the tower = 43.3 m

Distance of A from the foot of the tower = 75 m

2. An aeroplane 3000 m high passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplane from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

Solution:

Consider A and B as the two aeroplane

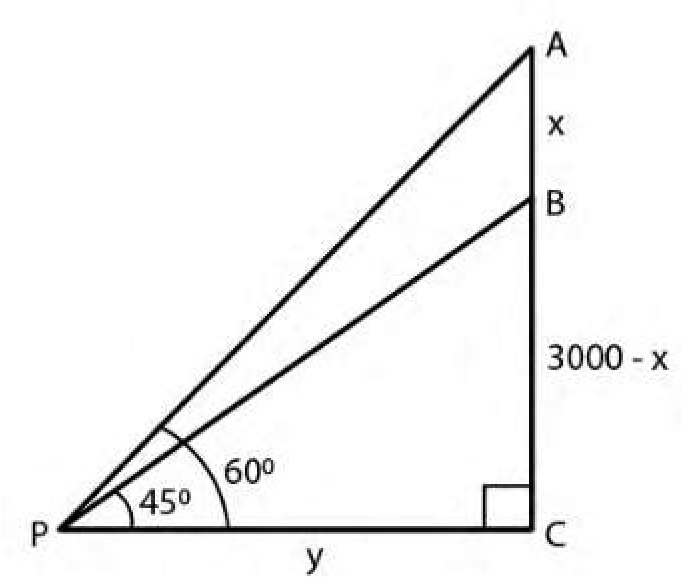
P is a point on the ground such that 60° and 45° are the angles of elevations from A and B

AC = 3000 m

Take AC = 3000 m

BC = 3000 – x

PC = y



In right triangle APC

$$\tan\theta = \frac{AC}{PC}$$

Substituting the values

$$\tan 60° = \frac{3000}{y}$$

So we get

$$\sqrt{3} = \frac{3000}{y}$$

$$y = \frac{3000}{\sqrt{3}} \qquad \dots (1)$$

In right triangle BPC

$$\tan\theta = \frac{BC}{PC}$$

Substituting the values

$$\tan 45° = \frac{(3000-x)}{y}$$

So we get

$1 = \frac{(3000-x)}{y}$

$y = 3000 - x$

Using equation (1)

$\frac{3000}{\sqrt{3}} = 3000 - x$

By further calculation

$x = 3000 - \frac{3000}{\sqrt{3}}$

Multiply and divide by $\sqrt{3}$

$x = 3000 - \frac{(3000\times\sqrt{3})}{(\sqrt{3}\times\sqrt{3})}$

So we get

$x = 3000 - 1000(1.732)$

$x = 3000 - 1732$

$x = 1268$

Hence, the distance between the two planes is 1268 m.

3. A 7 m long flagstaff is fixed on the top of a tower. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 45° and 36° respectively. Find the height of the tower correct to one place of decimal.

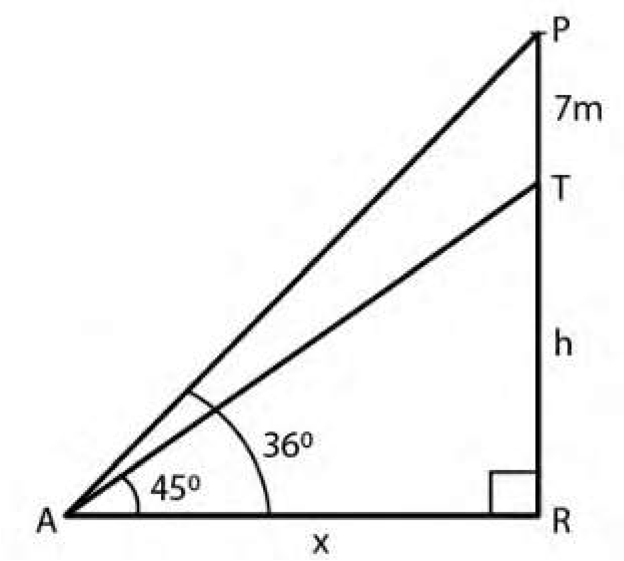
Solution:

Consider TR as the tower and PT as the flag on it

PT = 7 m

Take TR = h and AR = x

Angles of elevation from P and T are 45° and 36°



In right triangle PAR

$\tan\theta = \frac{PR}{AR}$

Substituting the values

$\tan 45° = \frac{(7+h)}{x}$

So we get

$1 = \frac{(7+h)}{x}$

$x = 7 + h$ … (1)

In right triangle TAR

$\tan\theta = \frac{TR}{AR}$

Substituting the values

$\tan 36° = \frac{h}{x}$

So we get

$0.7265 = \frac{h}{x}$

$h = x(0.7265)$ … (2)

Using both the equations

$h = (7 + h)(0.7265)$

By further calculation

$h = 7 \times 0.7265 + 0.7265h$

$h - 0.7265h = 7 \times 0.7265$

So we get

$0.2735h = 7 \times 0.7265$

By division

$h = \frac{(7 \times 0.7265)}{0.2735}$

We can write it as

$h = \frac{(7 \times 7265)}{2735}$

$h = 18.59 = 18.6$ m

Hence, the height of the tower is 18.6 m.

4. A boy 1.6 m tall is 20 m away from a tower and observes that the angles of elevation of the top of the tower is 60°. Find the height of the tower.

Solution:

Consider AB as the boy and TR as the tower

AB = 1.6 m

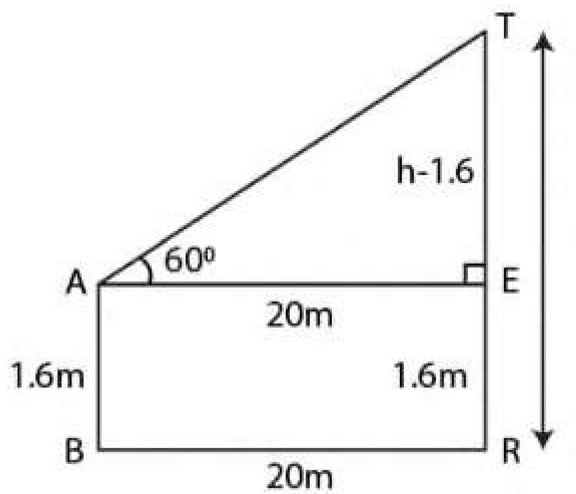
Take TR = h

From the point A construct AE parallel to BR

ER = AB = 1.6 m

TE = $h = 1.6$

AE = BR = 20 m



In right triangle TAE

$\tan\theta = \frac{TE}{AE}$

Substituting the values

$\tan 60° = \frac{(h-1.6)}{20}$

So we get

$\sqrt{3} = \frac{(h-1.6)}{20}$

$h - 1.6 = 20\sqrt{3}$

$h = 20\sqrt{3} + 1.6$

$h = 20(1.732) + 1.6$

By further calculation

$h = 34.640 + 1.6$

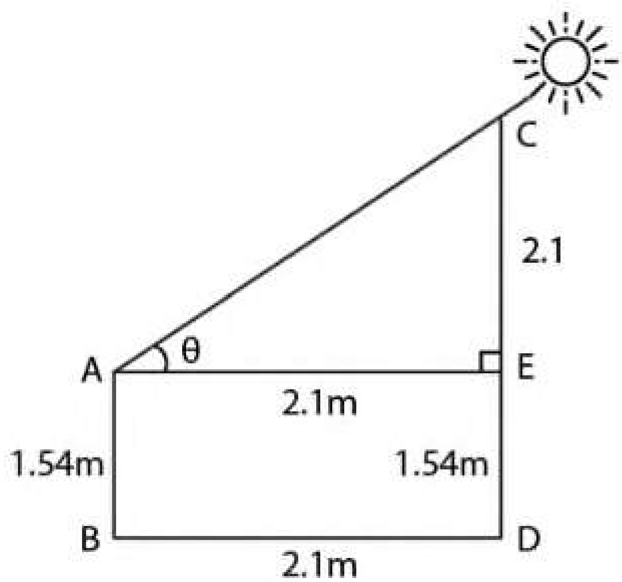
$h = 36.24$

Hence, the height of the tower is 36.24 m.

5. A boy 1.54 m tall can just see the sun over a wall 3.64 m high which is 2.1 m away from him. Find the angles of elevation of the sun.

Solution:

Consider AB as the boy and CD as the wall which is at a distance of 2.1 m.



AB = 1.54 m

CD = 3.64 m

BD = 2.1 m

Construct AE parallel to BD

ED = 1.54 m

CE = 3.64 – 1.54 = 2.1 m

AE = BD = 2.1 m

In right triangle CAE

$\tan\theta = \frac{CE}{AE}$

So we get

$\tan\theta = \frac{2.1}{2.1}$

$\tan\theta = \frac{21}{21}$

$\tan\theta = 1$

We know that

$\tan 45° = 1$

We get

$\theta = 45°$

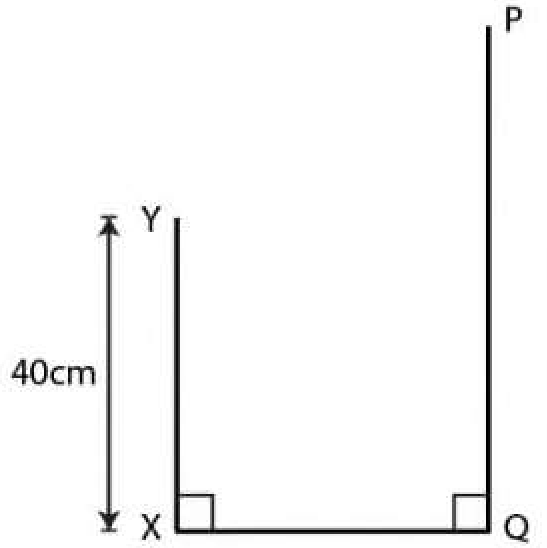
Hence, the angle of elevation of the sun is 45°.

6. In the adjoining figure, the angle of elevation of the top P of a vertical tower from a point X is 60°, at a point Y, 40 m vertically above X, the angle of elevation is 45°. Find

(i) The height of the tower PQ

(ii) The distance XQ

(Give your answer to the nearest metre)



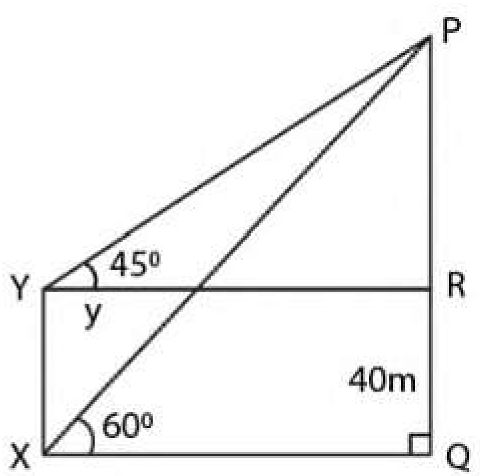
Solution:

Consider PQ as the tower = h

$XQ = YR = y$

$XY = 40$ m

$PR = h - 40$



In right triangle PXQ

$\tan\theta = \frac{PQ}{XQ}$

Substituting the values

$$\tan 60° = \frac{h}{y}$$

So we get

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}} \qquad \ldots (1)$$

In right triangle PYR

$$\tan\theta = \frac{PR}{YR}$$

Substituting the values

$$\tan 45° = \frac{(h-40)}{y}$$

So we get

$$1 = \frac{(h-40)}{y}$$

$$y = h - 40 \qquad \ldots (2)$$

Using both the equations

$$h - 40 = \frac{h}{\sqrt{3}}$$

By further calculation

$$\sqrt{3}h - 40\sqrt{3} = h$$

$$\sqrt{3}h - h = 40\sqrt{3}$$

So we get

$$(1.732 - 1)h = 40(1.732)$$

$$732h = 69.280$$

By division

$h = \frac{69.280}{0.732} = \frac{69280}{732} = 94.64$

Here

Height of the tower = 94.64 m = 95 m

Distance XQ = $h - y = 95 - 40 = 55$ m

7. An aeroplane is flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.

Solution:

Consider A and D as the two positions of the aeroplane

AB is the height and P is the point

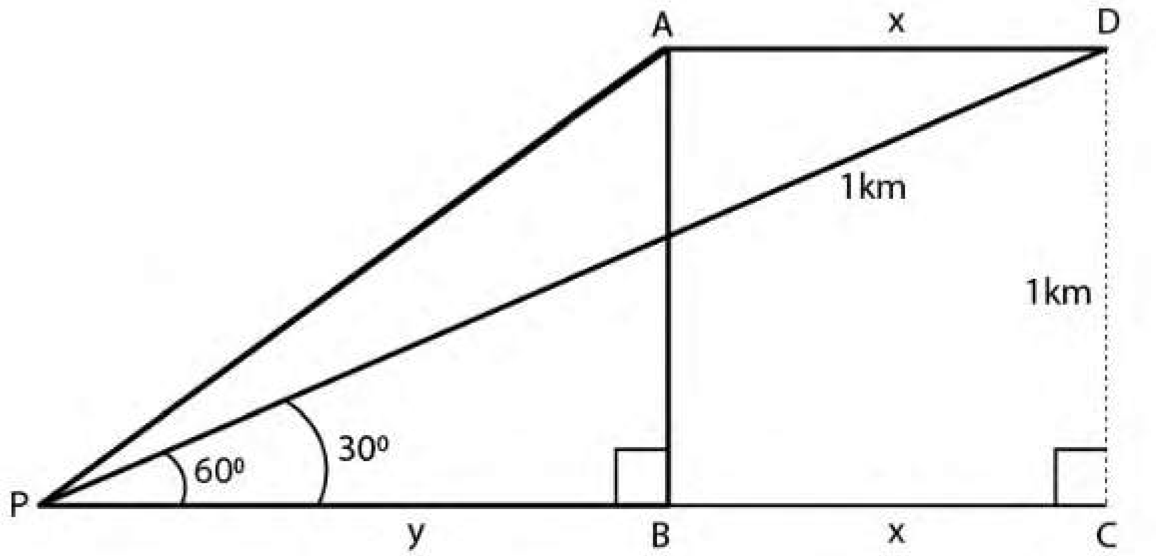
AB = 1 km

Take AD = x and PB = y

Angles of elevation from A and D at the point P are 60° and 30°

Construct DC perpendicular to PB

DC = AB = 1 km



In right triangle APB

$\tan\theta = \frac{AB}{PB}$

Substituting the values

$\tan 60° = \frac{1}{y}$

So we get

$\sqrt{3} = \frac{1}{y}$

$y = \frac{1}{\sqrt{3}} \qquad \ldots (1)$

In right triangle DPC

$\tan\theta = \frac{DC}{PC}$

Substituting the values

$\tan 30° = \frac{1}{(x+y)}$

So we get

$\frac{1}{\sqrt{3}} = \frac{1}{(x+y)}$

$x + y = \sqrt{3}$ … (2)

Using both the equations

$x + \frac{1}{\sqrt{3}} = \sqrt{3}$

By further calculation

$x = \sqrt{3} - \frac{1}{\sqrt{3}}$

$x = \frac{(3-1)}{\sqrt{3}}$

$x = \frac{2}{\sqrt{3}}$

Multiply and divide by $\sqrt{3}$

$x = \frac{(2\times\sqrt{3})}{(\sqrt{3}\times\sqrt{3})}$

So we get

$x = \frac{(2\times1.732)}{3}$

$x = \frac{3.464}{3}$ km

This distance is covered in 10 seconds

Speed of aeroplane (in km/hr) = $\frac{3.464}{3} \times \frac{(60\times60)}{10}$

By further calculation

$= \frac{3464}{(3\times1000)} \times \frac{3600}{10}$

So we get

$= \frac{(3646\times36)}{300}$

$= \frac{(3464 \times 12)}{100}$

$= \frac{41568}{100}$

= 415.68 km/hr

8. A man on the deck of a ship is 16 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angles of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Solution:

Consider A as the man on the deck of a ship B and CE is the cliff

AB = 16 m

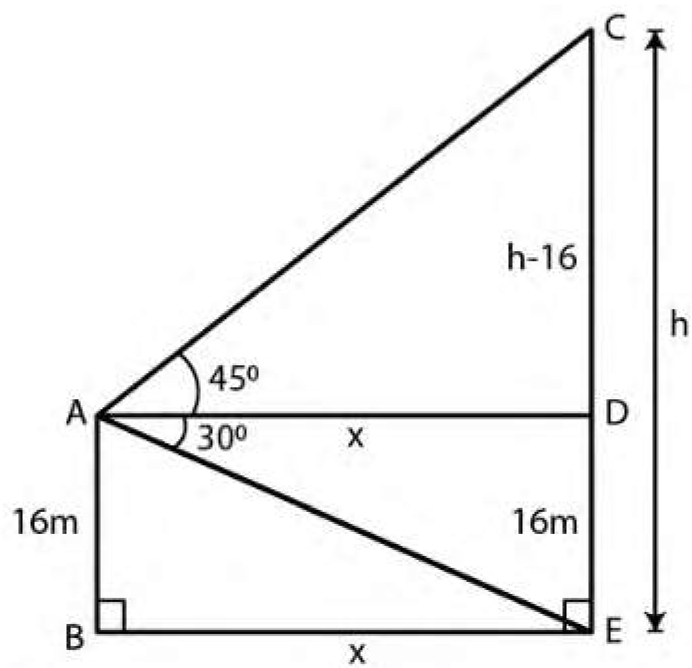
Angle of elevation from the top of the cliff = 45°

Angle of depression at the base of the cliff = 30°

Take CE = h, AD = x

CD = h – 16

AD = BE = x



In right triangle CAD

$\tan\theta = \frac{CD}{AD}$

Substituting the values

$\tan 45° = \frac{(h-16)}{x}$

So we get

$1 = \frac{(h-16)}{x}$

$x = h - 16$ … (1)

In right triangle ADE

$\tan\theta = \frac{DE}{AD}$

Substituting the values

$\tan 30° = \frac{16}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{16}{x}$

$x = 16\sqrt{3}$ … (2)

Using both the equations

$h - 16 = 16\sqrt{3}$

$h = 16\sqrt{3} + 16$

Taking out the common terms

$h = 16(1.732 + 1)$

$h = 16(2.732)$

$h = 43.712 = 43.71$ m

Substituting the value in equation (1)

$x = h - 16$

$x = 43.71 - 16$

$x = 27.71$

Here

Distance of cliff = 27.71 m

Height of cliff = 43.71 m

9. There is a small island in between a river 100 metres wide. A tall tree stands on the island. P and Q are points directly opposite to each other on the two banks and in the line with the tree. If the angles of elevation of the top of the tree from P and Q are 30° and 45° respectively. Find the height of the tree.

Solution:

Width of the river PQ = 100 m

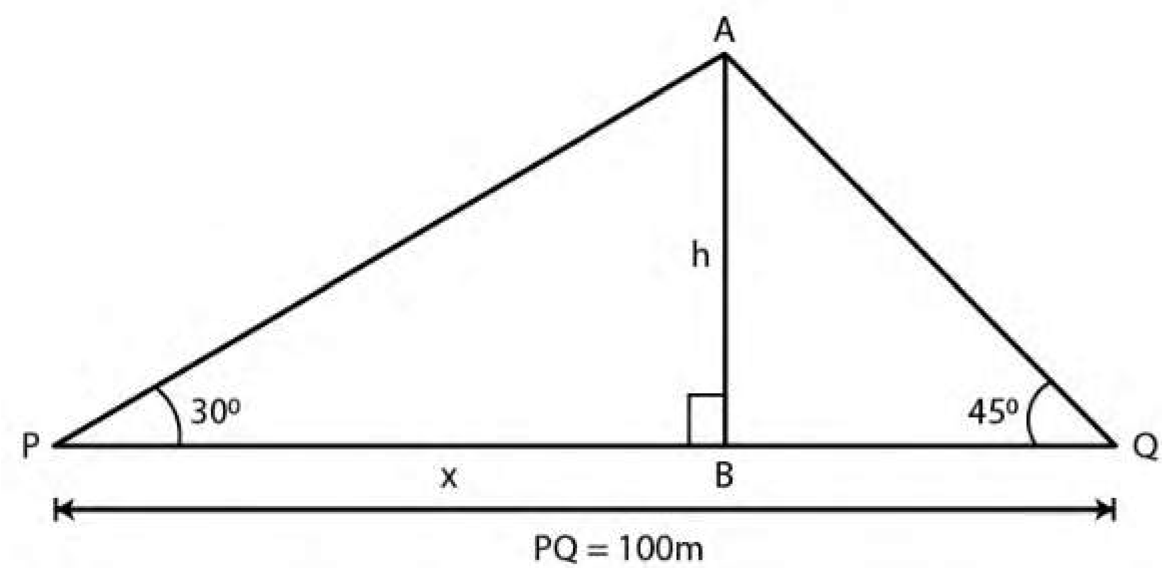
B is the island and AB is the tree on it

Angles of elevation from A to P and Q are 30° and 45°

Consider AB = h

PB = x

BQ = 100 – x



In right triangle APB

$\tan\theta = \frac{AB}{PB}$

Substituting the values

$\tan 30° = \frac{h}{x}$

So we get

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}h$ … (1)

In right triangle ABQ

$\tan\theta = \frac{AB}{BQ}$

Substituting the values

$\tan 45° = \frac{h}{(100-x)}$

So we get

$\frac{1}{\sqrt{3}} = \frac{h}{(100-x)}$

$x = 100 - x$ … (2)

Using both the equations

$h = 100 - \sqrt{3}h$

By further calculation

$h + \sqrt{3}h = 100$

So we get

$(1 + 1.732)h = 100$

$h = \frac{100}{2.732}$

Multiply and divide by 1000

$h = \frac{(100\times1000)}{2732}$

$h = \frac{100000}{2732}$

$h = 36.6$

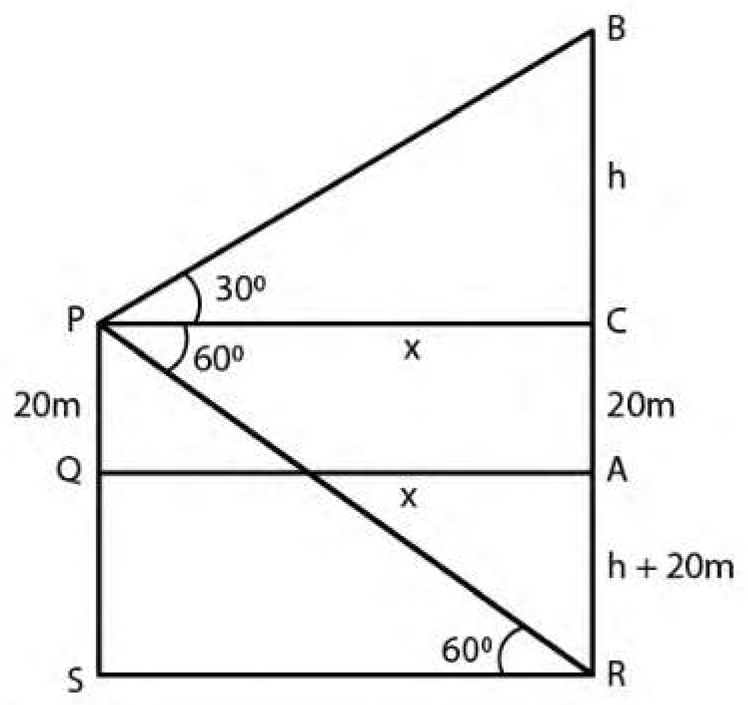
Hence, the height of the tree is 36.6 m.

10. A man standing on the deck of the ship which is 20 m above the sea-level, observes the angle of elevation of a bird as 30° and the angle fo depression of its reflection in the sea as 60°. Find the height of the bird.

Solution:

Consider P as the man standing on the deck of the ship which is 20 m above the sea level and B is the bird

Angle of elevation of the bird from P = 30° Angle of depression from P to shadow of the bird in the sea = 60°



Take BC = h

PQ = 20 m = CA

AR = $(h + 20)$ m

CE = $h + 20 + 20 = h + 40$ m

PC = CA = x

In right triangle PCB

$\tan 30° = \frac{BC}{PC}$

Substituting the values

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

So we get

$$x = \sqrt{3}h \text{ m} \qquad \dots (1)$$

In right triangle PCR

$$\tan 60° = \frac{CR}{PC}$$

Substituting the values

$$\sqrt{3} = \frac{(h+40)}{x}$$

Using equation (1)

$$\frac{(h+40)}{\sqrt{3}h} = \sqrt{3}$$

$$h + 40 = \sqrt{3} \times \sqrt{3}h = 3h$$

By further calculation

$$3h - h = 40$$

$$2h = 40$$

$$h = \frac{40}{2} = 20$$

From the sea level the height of the bird = $20 + h = 20 + 20 = 40$ m