Unit 4

Continuity and Differentiability

Teaching Learning points

● **Continuity and discontinuity of function:** A function y = f(x) is said to be continuous in an interval If for every value of x in that interval y exist. If we plot the points, the graph is drawn without lifting the pencil.

● **Continuity and discontinuity of a function at a point:** A function f(x) is said to be continuous at a point ‘a’ of its domain If

f(x), f(x), f(a) exist

$\lim_{x \to a^-}$ $\lim_{x \to a^{+1}}$

and f(x) f(x) f(a)

$\lim_{x \to a^+}$ $\lim_{x \to a^-}$

A function f(x) said to be descontinuous at x = a If it is not continous at x = a

● Let f(x) and g(x) are two real Continous functions at point x = a, then

(i) f(x) + g(x) is also continuous at x = a

(ii) f(x) · g(x) is also continuous at x = a

(iii) $\frac{f(x)}{g(x)}$ is also continuous at x = a, provided g(a) ≠ 0

(iv) λ · f(x) is also continuous at x = a, where λ is any scalar

(v) |f(x)| is also continuous at x = a

(vi) fog or gof is also continuous at x = a

● Before doing Exercise, the students must know the following facts

(i) Absolute value function is continuous for all real values of ‘x’.

(ii) A Polynomial function f(x) is continuous A x ∈ R

(iii) Any constant function f(x) = c and identity function f(x) = x is continuous A x ∈ R

(iv) Every Rational function $\frac{f(x)}{g(x)}$ is continuous for all values of x in the Domain, where the points at which g(x) is zero are not included in the Domain.

(v) The Trigonometric functions sin x, cos x, are continuous function every where.

(vi) f(x) = [x] is not continuous at any integral value of ‘x’.

● The process of finding derivative of a function is called differentiation.

● Let f(x) be any real valued function defined on an open interval (a, b), then f(x) is said to be differentiable at x = ∈ e (a, b) if = $\lim_{h \to 0^-} \frac{f(c+h)-f(c)}{h} = \lim_{h \to 0^+} \frac{f(c+h)-f(c)}{h}$ a finite number.

● A function f(x) is said to be differentiable in an interval (a, b) if it is differentiable at every point of (a, b).

● **Chain Rule:** If y = f(u) and u = g(x), then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

This process also known as Derivative of function of a function.

● **Derivative of implicit function:** When the variable x and y are related in such a way that y cannot be expressed as a function of 'x' in a easier way, then the function is known as implicit function.

So the derivative from implicit function is obtained by differentiating directly w.r.t. the suitable variable.

● While differentiating inverse trigonometric functions, first express it in simplest form by using suitable substitution and then differentiating the simplest form.

● **Some important substitutions:**

	Expression	Substitutions
(i)	$\sqrt{a^2 - x^2}$	$x = a \sin\theta$ or $x = a \cos\theta$
(ii)	$\sqrt{a^2 + x^2}$	$x = a \tan\theta$ or $x = a \cot\theta$
(iii)	$\sqrt{a^2 - a^2}$	$x = a \sec\theta$ or $x = a\ \text{cosec}\ \theta$
(iv)	$\sqrt{a \pm x}$	$x = a \cos\theta$
(v)	$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	$x = a \cos^2\theta$

● Logarithmic differentiation:

(i) This process is used when function is given in a complicated form

(ii) When y = $[f(x)]^{g(x)}$, then

$$\frac{dy}{dx} = [f(x)]^{g(x)} \cdot \frac{d}{dx}[g(x) \cdot \log\{f(x)\}]$$

● **Derivative of function in Parametric forms:** If y = f(x) be a function in which x and y are the variables, when the variables x and y are the functions of third variable 't' i.e., x = u(t) and y = v(t) then this form is called parametric form and 't' is called the parameter.

In order to find $\frac{dy}{dx}$, we use the following:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

● **Roll's theorem:** If f(x) be a real valued function defined in [a, b] such that

(i) f(x) is continuous in [a, b]

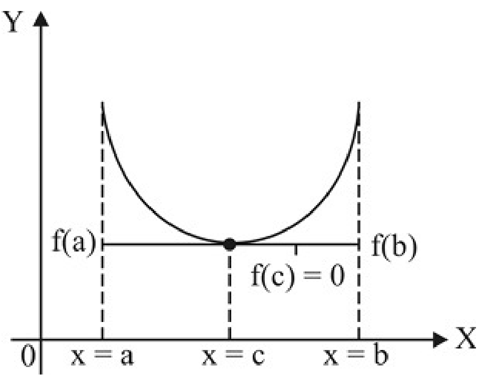
(ii) Derivable in (a, b)

(iii) f(a) = f(b)

Then, there exists at least one c∈ (a, b)

Such that $f'(c) = 0$

● **Geometrical interpretation:**



*in the interval (a, b) there must exist at least one point where the tangent is parallel to x-axis.

● **Lagrange's mean value theorem:** If 'f' be a function such that

(i) f(x) is defined in [a, b]

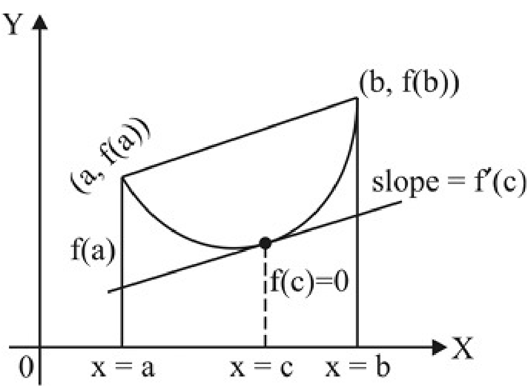
(ii) f(x) is continous in [a, b]

(iii) Derivable in (a, b)

Then there exists at least one $c \in (a, b)$

Such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

● **Geometrically:**



In the interval (a, b) there must exist at least one point 'c' where the tangent is parallel to the chord joining the end points.

Question for Practice

Very Short Answer Type Questions (1 Mark)

Q1. If $y = (1 + x^{1/3})(1 - x^{1/3})(1 + x^{2/3})$ find dy/dx

Q2. If $y = \tan^{-1}(\cot x)$ find dy/dx

Q3. $f(x) = 5^{3x}$ find $f'(2)$

Q4. $f(x) = \tan^{-1} x + \tan^{-1} 1/x$ find $f'(x)$ $x > 0$

Q5. f(x) = log10 x find f′(x)

Q6. If $y = \begin{vmatrix} x & & 1 \\ x & - & x \end{vmatrix}$ then find dy/dx

Q7. If y = e^x then what is the value of $\frac{1}{a}\left(2\frac{d^2y}{dx^2} - \frac{dy}{dx} - y\right)$

Q8. $x^{2/3} + y^{2/3} = 2$ find $\frac{dy}{dx}$ at (1, 1)

Q9. If f(x) = [x] write points where f(x) is not differentiable

Q10. For what value of λ the f(x) = sin (λx) is continous every where

Short Answer Type Questions (4 Marks)

Q1. f(x) = $\begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$

For what value of ‘a’ f(x) is continuous at x = 0

Q2. f(x) = $\begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}} & x > 0 \end{cases}$

Determine the value of a, b and c for which the function f(x) may be continuous at x = 0

Q3. Examine the following function for continuity at x = 0 where n ≥ 1

f(x) = $\begin{cases} x^n \sin\dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Q4. Determine the value of constants ‘a’ and ‘b’ such that the function defined as is continuous at x = 4 f(x) = $\begin{cases} \dfrac{x-4}{|x-4|}+a & \text{If } x<4 \\ a+b & x=4 \\ \dfrac{x-4}{|x-4|}+b & x>4 \end{cases}$

Q5. Show that the function f(x) defind by f(x) is continuous at x = 0

f(x) = $\begin{cases} \dfrac{\sin x}{x}+\cos x & x>0 \\ 2 & x=0 \\ \dfrac{4(1-\sqrt{1-x})}{x} & <0 \end{cases}$

Q6. If x = a(θ – sin θ) y = a(1 – cos θ)

Find $\dfrac{d^2y}{dx^2}$ at $\theta=\dfrac{\pi}{2}$

Q7. Diff. w.r.t. 'x'

$$\tan^{-1}\left[\frac{5x}{1-6x^2}\right]$$

Q8. If $x^m y^n = (x+y)^{m+n}$

Prove that $\dfrac{dy}{dx}=\dfrac{y}{x}$

Q9. $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, then show that

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

Q10. If $y=\log[x+\sqrt{1+x^2}]$ prove that

$$(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$$

Q11. y = $(x+\sqrt{x^2-1})^m$

Prove that $(x^2-1)y_2 + xy_1 = m^2y$

Q12. If $x = \tan\left(\frac{1}{a}\log y\right)$ show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

Q13. If x = log t and y = $\frac{1}{t}$ Prove that $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Q14. If y = $(\cos^{-1} x)^2$ Prove that $(1 - x^2)y_2 - xy_1 = 2$

Q15. If x = a $\sin^3\theta$ y = b $\cos^3\theta$ find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$

Q16. If y = $\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$ find dy/dx

Q17. Verify the applicablity of rolle's theorem for the following

1. f(x) = sin 2x [0, π/2]

2. f(x) = $x^{2/3}$ [–1, 1]

3. f(x) = $(x - 1)(x - 2)^2$ [1, 2]

Q18. Verity Lagrange's mean value theorem for the following functions in the given interval and find c of this theorem

f(n) = $x^3 - 5x^2 - 3x$ [1, 3]

f(n) = (x – 1) (x – 2) (x – 3) [0, 4]

f(n) = $x + \frac{1}{x}$ [1, 3]

Q19. Find $\frac{dy}{dx}$ If $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

Q20. If y = $\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

Prove that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

Q21. Differentiate $x^{x^2-3} + (x-3)^{x^2}$ w.r.t. 'x' for x > 3

Q22. Differentiate $x^{x\cos x} + (x\cos x)^x$ w.r.t. x

Q23. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ w.r.t. $\tan^{-1}\left[\frac{2x}{1-x^2}\right]$

Q24. Differentiate $\sin^2 x$ w.r.t. $e^{\sin x}$

Q25. f(n) = $\begin{bmatrix} x-3 & x<2 \\ 2x-5 & x \geq 2 \end{bmatrix}$ at x = 2

Show that f is not differentiable at x = 2

Q26. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Prove that $\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$

Q27. Differentiate the following w.r.t. 'x'

(i) $\sqrt{\frac{(x-3)(x^2+1)}{3x^2+4x+5}}$

Q28. If $(x-a)^2 + (x-b)^2 = c^2$ for some c > 0. Prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b

Q29. Find $\frac{dy}{dx}$ If $y^x + x^y + x^x = 10^{10}$

Q30. If $y = x^{y^x}$ show that $\frac{dy}{dx} = \frac{y \log y(x \log x \log y + 1)}{(1 - x \log y) x \log x}$

Q31. Discuss the differentiability and continuity of

f(x) = |x – 1| + |x – 2| at x = 1 and x = 2

Q32. f(x) = $\begin{cases} x-1 & x<2 \\ 2x-3 & x >, 2 \end{cases}$

Show that f(x) is continuous at x = 2 but not differentiable at x = 2

Q33. If the f(x) = $\begin{cases} 3ax+b & x>1 \\ 11 & \text{If } x=1 \\ 5ax-2b & x<1 \end{cases}$

is continuous at x = 1 find value of a and b

Q34. If sin y = x sin (a + y). Prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Q35. If y = sin log x

Prove that $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$

Q36. If y = $\tan^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ Prove that $\dfrac{dy}{dx}=\dfrac{-1}{2}$

Answers

1. $\dfrac{-4}{3}x^{1/3}$ 2. –1 3. 3log 5 × 5^6

4. 0 5. $\dfrac{1}{\log 10}\times\dfrac{1}{x}$ 6. –2x –1

7. 0 8. –1 9. for x ∈ I (I for integets)

10. λ ∈ R

Hints 2 Solutions of 4 Marks Questions

Answers

1. Now f (0) = a

LHL f(x) = $\dfrac{1-\cos 4x}{x^2}=\dfrac{2\sin^2 2x}{x^2}=8\left(\dfrac{\sin 2x}{2x}\right)^2$

$\underset{x\to 0^-}{\text{Lt}}$ $\underset{x\to 0^-}{\text{Lt}}$ $\underset{x\to 0^-}{\text{Lt}}$ $\underset{x\to 0^-}{\text{Lt}}$

$8(1)^2$ = 8 [x → 0^- ⇒ 2x → 0^-]

RHL f(x) = $\dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}=\dfrac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{(\sqrt{16+\sqrt{x}}-4)(\sqrt{16+\sqrt{x}}+4)}$

$\underset{x\to 0^+}{\text{Lt}}$ $\underset{x\to 0^+}{\text{Lt}}$ $\underset{x\to 0^+}{\text{Lt}}$

$\dfrac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{16+\sqrt{x}-16}$ = $\sqrt{16+\sqrt{x}}+4=8$

$\underset{x\to 0^+}{\text{Lt}}$ $\underset{x\to 0^+}{\text{Lt}}$

$\underset{x \to 0^+}{\text{Lt}}$ f(x) = $\underset{x \to 0^-}{\text{Lt}}$ f(x) = 8

∴ f(x) is continuous at x = 0 only. If a = 8

2. Here f(0) = c

LHL f(n) = $\underset{x \to 0^-}{\text{Lt}} \frac{\sin(a+1)x + \sin x}{x} = \underset{x \to 0^-}{\text{Lt}} \frac{\sin(a+1)x}{x} + \underset{x \to 0^-}{\text{Lt}} \frac{\sin x}{x}$

= $\underset{x \to 0^-}{\text{Lt}} \frac{\sin(a+1)x(a+1)}{x(a+1)} + \underset{x \to 0^-}{\text{Lt}} \frac{\sin x}{x}$

a + 1 + 1 = a + 2 [x → 0^- ⇒ (a + 1)x → 0^-]

RHL f(n) = $\underset{x \to 0^+}{\text{Lt}} \frac{\sqrt{x}[\sqrt{1+bx} - 1}{bx\sqrt{x}} = \underset{x \to 0^+}{\text{Lt}} \frac{\sqrt{x}(\sqrt{1+bx} - 1)}{bx\sqrt{x}} \times \underset{x \to 0^+}{\text{Lt}} \frac{\sqrt{1+bx}+1}{\sqrt{1+6x}+1} = \underset{x \to 0^+}{\text{Lt}} \frac{1+bx-1}{bx \times \sqrt{1+bx}+1)}$

= $\underset{x \to 0^+}{\text{Lt}} \frac{1}{\sqrt{1+bx}+1} = \frac{1}{2}$

Now f is continuous at x = 0. If $\underset{x \to 0^-}{\text{Lt}}$ f(x) = f(0) = f(x)

i.e., a + 2 = c = $\frac{1}{2}$ ⇒ a = $\frac{-3}{2}$ c = $\frac{1}{2}$ b ∈ R – {0}

(RHL is independent of b)

4. a = 1 b = –1

x = a(θ – sin θ) y = a(1 – cos θ)

Differentiating both w.r.t. 'θ' we get

$\frac{dy}{d\theta}$ = a(1 – cos θ) $\frac{dy}{d\theta}$ = a sin θ

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2} = \cot\theta/2$$

Differentiating w.r.t. 'x'

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\theta/2 \times \frac{1}{2}\frac{d\theta}{d\theta} = \frac{-1}{2}\operatorname{cosec}^2\theta/2\,\frac{1}{a(1-\cos\theta)}$$

$$= \frac{-1}{2a}\frac{\operatorname{cosec}^2\theta/2}{2\sin^2\theta/2} = \frac{-1}{4a}\operatorname{cosec}^4\theta/2$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\pi/2} = \frac{-1}{4a}\operatorname{cosec}^4\pi/4 = \frac{-1}{4a}\times 4 = \frac{-1}{a}$$

7. $\tan^{-1}\left[\frac{3x+2x}{1-3x\times 2x}\right]$

$\tan^{-1}3x + \tan^{-1}2x$

Differentiate w.r.t. (x)

$$\frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

8. $x^m y^n = (x + y)^{m+n}$

taking logarithm both side

m log x + n log y = (m + n) log (x + y)

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{x+y}\left[1+\frac{dy}{dx}\right]$$

$$\frac{dy}{dx}\left[\frac{n}{y} - \frac{m+n}{x+y}\right] = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\frac{dy}{dx}\left[\frac{nx+\cancel{ny}-my-\cancel{ny}}{y(x+y)}\right] = \frac{\cancel{mx}+nx-\cancel{mx}-my}{x(x+y)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

9. $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

x = cos A y = cos B

sin A + sin B = a(cos A – cos B)

$$\not{2}\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = -\not{2}a\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cot\left(\frac{A-B}{2}\right) = a$$

⇒ A – B = 2 $\cot^{-1}$a

$\cos^{-1}$x – $\cos^{-1}$y = 2 $\cot^{-1}$a

Differentiate w.r.t. (x)

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

12. Hint: x = $\tan\left(\frac{1}{a}\log y\right)$

a $\tan^{-1}$x = log y ⇒ $e^{a\tan^{-1}x}$

$$\frac{dy}{dx} = \frac{ae^{a\tan^{-1}x}}{1+x^2} = \frac{ay}{1+x^2}$$

$$(1+x^2)\frac{dy}{dx} = ay$$

Again Differentiate w.r.t. (x)

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\times 2x = \frac{ady}{dx}$$

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$

15. $\frac{4\sqrt{2}\,b}{3a^2}$.

16. y = $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

x = sin t t = $\sin^{-1}$ x

y = $\sin^{-1}$ $\left[\frac{5\sin t}{13}+\frac{12}{13}\cos t\right]$

5 = r cos d 12 = r sin d ⇒ r = 13

y = $\sin^{-1}\left[\frac{r\cos d\sin t + r\sin d\cos t}{13}\right]$

= $\sin^{-1}$ $\frac{r}{13}\sin(t+d) \Rightarrow t+d$

y = $\sin^{-1}x + \tan^{-1}\frac{12}{5}$

$\frac{dy}{dx}$ = $\frac{1}{\sqrt{1-x^2}}$

17. (ii) f(x) = $x^{2/3}$ [–1, 1]

clearly f(x) has definite and unique value for each x ∈ [–1, 1]

f(x) is continuous in [–1, 1]

f(x) = $\frac{2}{3}x^{-1/3}$ which does not exist for x = 0 (–1, 1)

Hence Roll's theorem is not applicable

18. (ii) f(x) = $x+\frac{1}{x}$ $[1,3]$

f(x) is defined in [1, 3]

f(x) is rational function such that denominator is not zero

for any value in [1, 3]

f(x) is continuous function in [1, 3]

f′(x) = $1-\frac{1}{x^2}$ which exist in (1, 3)

Hence all the condition of LMV are verified. Hence there exist atleast one value 'c' c ∈ (1, 3) such that f′(c) = $\dfrac{f(b)-f(a)}{b-a}$

$1-\dfrac{1}{c^2}$ = $\dfrac{\frac{10}{3}-2}{3-2}$ $\quad c=\pm\sqrt{3}\ \text{but} -\sqrt{3}\notin(1,3)$

only possible value c = $\sqrt{3}\in(1,3)$

19. y = $\sin^{-1}(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2})$

y = $\sin^{-1}(x\sqrt{1-(\sqrt{x})^2}-\sqrt{x}\sqrt{1-x^2})$ using $\sin^{-1}x - \sin^{-1}y$ = $\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$

y = $\sin^{-1}x-\sin^{-1}\sqrt{x}$

Differentiate w.r.t. (x)

$\dfrac{dy}{dx}$ = $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{\sqrt{1-x}}\times\dfrac{1}{2\sqrt{x}}$

20. **Hint:** Put x = cos θ

21. $x^{x^2-3}\left[\dfrac{x^2-3}{x}+2x\log x\right]+(x-3)^{x^2}\left[\dfrac{x^2}{x-3}+2x\log|x-3|\right]$

22. x ^[illegible]^ $[(1+\log x)\cos x - x\sin x\log x]+(x\cos x)^{\wedge}[\log(x\cos x)-x\tan x+1]$

24. **Hint :** y = $\sin^2 x$ z = $e^{\cos x}$

Diff w.r.t. ‘x’

$\dfrac{dy}{dx}$ = $2\sin\cos x$ $\quad\dfrac{dz}{dx}$ = $e^{\cos x}\times(\sin x)$

$\dfrac{dy}{dz}$ = $\dfrac{dy/dx}{dz/dx}$ = $\dfrac{-2\sin x\cos x}{\sin x\, e^{\cos x}}$ $\quad\dfrac{dz}{dx}\neq 0$

= $\dfrac{2\cos x}{e^{\cos x}}$

27. $\dfrac{1}{2}\sqrt{\dfrac{(x-3)(x^2+1)}{3x^2+4x+5}}\left[\dfrac{1}{x-3}+\dfrac{2x}{x^2+1}-\dfrac{6x+4}{3x^2+4x+5}\right]$

28. Given $(x-a)^2+(y-b)^2$ = c^2 ...(i)

Diff. twice w.r.t. (x)

$2(x-a)+2(y-b)y_1$ = 0 ⇒ $(x-a)+(y-b)y_1=0$ ...(ii)

$1+(y-b)y_2+y_1(y_1-0)$ = 0 ⇒ $(y-b)=-\frac{(1+y_1^2)}{y_2}$ ...(iii)

Put that value of (y – b) from (iii) to (ii)

x – a = $\frac{y_1(1+y_1^2)}{y_2}$ ...(iv)

Now put the value of (iii) and (iv) in (i) we get

$$\frac{y_1^2(1+y_1^2)^2}{y_2^2}+\frac{(1+y_1^2)^2}{y_2^2} = c^2$$

$$\frac{(1+y_1^2)^{3/2}}{y_2} = c$$

29. $\frac{-y^x\log y+yx^{y-1}+x^x(1+\log x)}{x\,y^{x-1}+x^y\log x}$

30. **Hint :** f(x) = $|x-1|+|x-2|$

$$f(x)=\begin{cases}-(x-1)-(x-2) & x<1\\ (x-1)-(x-2) & 1\le x<2\\ x-1+x-2 & x\ge 2\end{cases}$$

$$f(x)=\begin{cases}-2x+3 & x<1\\ 1 & 1\le x<2\\ 2x-3 & x\ge 2\end{cases}$$

Ans. f(x) is continuous at x = 1 & x = 2 lut not

Differentiable at x = 1 and x = 2

33. a = 3 b = 2