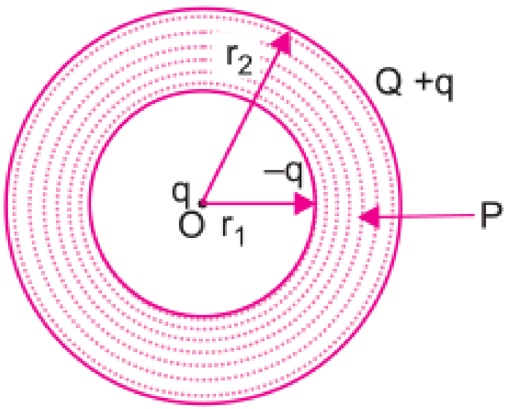
Short Answer Questions-II (PYQ)

Q. 1. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge ‘Q’. A charge ‘q’ is placed at the centre of the shell.

(a) What is the surface charge density on the (i) inner surface, (ii) outer surface of the shell?
(b) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell. [CBSE (AI) 2010]

Ans.



(a) Charge Q resides on outer surface of spherical conducting shell. Due to charge q placed at centre, charge induced on inner surface is –q and on outer surface it is +q. So, total charge on inner surface –q and on outer surface it is Q + q.

Surface charge density on inner surface = $-\frac{q}{4\pi r_1^2}$

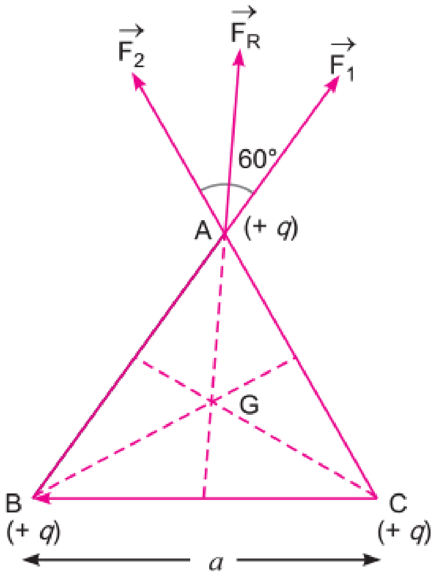
Surface charge density on outer surface = $\frac{Q+q}{4\pi r_2^2}$

(b) For external points, whole charge acts at centre, so electric field at distance x > r^2,

$$E(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{x^2}.$$

Q. 2. Three point electric charges +q each are kept at the vertices of an equilateral triangle of side a. Determine the magnitude and sign of the charge to be kept at the centroid of the triangle so that the charges at the vertices remain in equilibrium.
[CBSE (F) 2015] [HOTS]

Ans.



The charge at any vertex will remain in equilibrium if the net force experienced by this charge due to all other three charges is zero.

Let Q be the required charge to be kept at the centroid G.

Considering the charge at A,

Force $\vec{F}_1$ on charge at A due to charge at B

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{\mathrm{BA}}$$

Force $\vec{F}_2$ on charge at A due to charge at C

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{\mathrm{CA}}$$

Since angle between $\vec{F}_1$ and $\vec{F}_2$ is 60°.

$$\vec{F}_1 + \vec{F}_2 = \sqrt{3}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{\mathrm{GA}}$$

Also, the distance of centroid G from any vertex is $\frac{a}{\sqrt{3}}$

The nature of charge to be kept at G has to be opposite $(-\text{ ve})$ so that it exerts a force of attraction on charge $(+q)$ kept at A to balance the force $\vec{F}_1 + \vec{F}_2$

Force exerted by $(-\ Q)$ kept at G on charge $(+q)$ at $A = \frac{1}{4\pi\varepsilon_0}\ \frac{Qq}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{1}{4\pi\varepsilon_0}\ \frac{Q.3q}{a^2}$ along $\overrightarrow{AG}$

Equating the two forces, being equal and opposite

$$\sqrt{3}\frac{1}{4\pi\varepsilon_0}\ \frac{q^2}{a^2} = -\frac{1}{4\pi\varepsilon_0}\ \frac{3\,Qq}{a^2} \qquad \Rightarrow \qquad Q = -\frac{q}{\sqrt{3}}$$

Q. 3. (a) An infinitely long positively charged straight wire has a linear charge density λ Cm^{-1}. An electron is revolving around the wire as its centre with a constant velocity in a circular plane perpendicular to the wire. Deduce the expression for its kinetic energy.

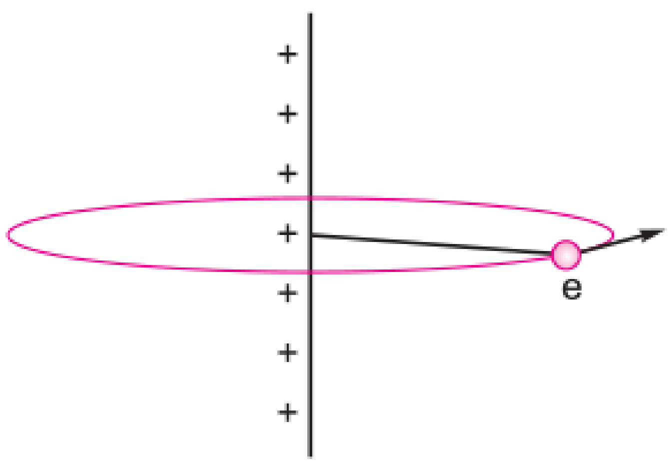
(b) Plot a graph of the kinetic energy as a function of charge density λ. [CBSE (F) 2013]

Ans. (a) infinitely long charged wire produces a radical electric field.

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \ldots (1)$$

The revolving electron experiences an electrostatic force and provides necessarily centripetal force.

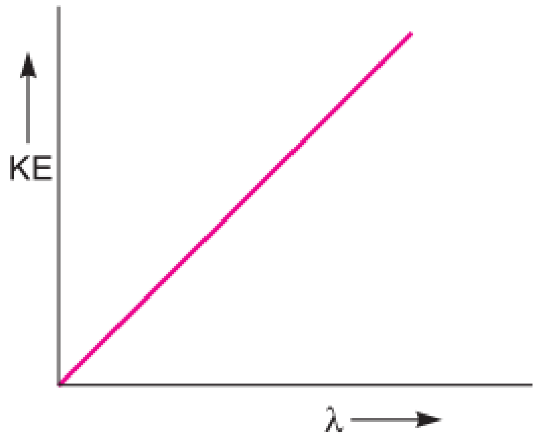
$$\mathrm{eE} = \frac{\mathrm{mv}^2}{r} \qquad \ldots (2)$$



$$\frac{e.\lambda}{2\pi\varepsilon_0 r} = \frac{\mathrm{mv}^2}{r} \qquad \Rightarrow \qquad \mathrm{mv}^2 = \frac{e\lambda}{2\pi\varepsilon_0}$$

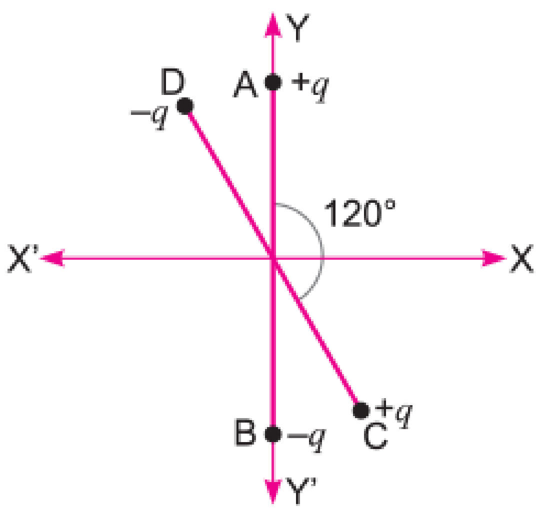
Kinetic energy of the electron, $K = \frac{1}{2}\ \mathrm{mv}^2 = \frac{e\lambda}{4\pi\varepsilon_0}$

(b)



Q. 4. Two small identical electrical dipoles AB and CD, each of dipole moment ‘p’ are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field $(\vec{E})$ directed along + X direction, what will be the magnitude and direction of the torque acting on this?
[CBSE Delhi 2011]

Ans.



Resultant dipole moment

$$\vec{p}_r = \sqrt{p^2 + p^2 + 2\,\mathrm{pp}\cos 120^\circ}$$

$$= \sqrt{2p^2 + 2p^2 \cos 120^\circ}$$

$$= \sqrt{2p^2 + (2p^2) \times \left(-\tfrac{1}{2}\right)} = \sqrt{2p^2 - p^2} = p,$$

Using law of addition of vectors, we can see that the resultant dipole makes an angle of 60° with the y axis or 30° with x - axis.

Torque, $\vec{\tau} = \vec{p} \times \vec{E}$ $(\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$

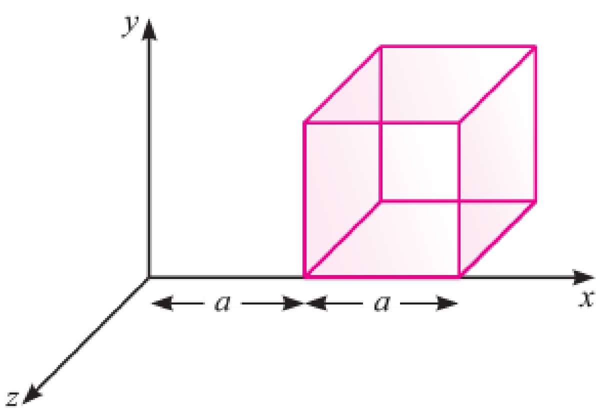
$= \mathrm{pE} \sin 30^{\circ} = \frac{1}{2} \mathrm{pE}$.

Direction of torque is along positive Z-direction.

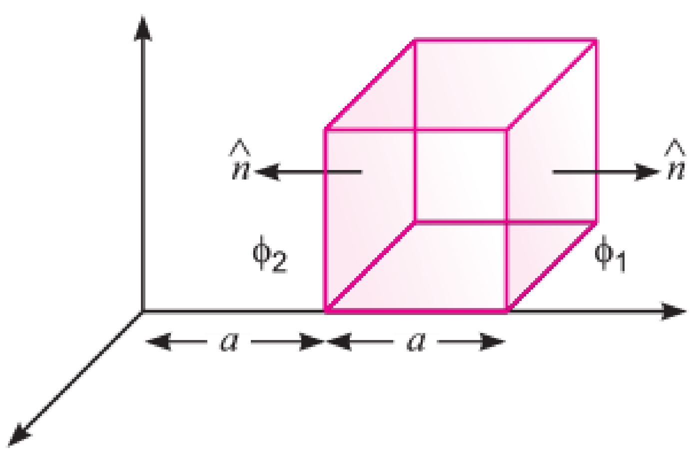
Q. 5. State Gauss's law in electrostatics. A cube with each side 'a' is kept in an electric field given by $\vec{E} = C \times \hat{r},$ **(as is shown in the figure) where C is a positive dimensional constant. Find out [CBSE (F) 2012]**

(i) The electric flux through the cube, and

(ii) The net charge inside the cube



Ans.



Gauss's Law in electrostatics states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of that charge.

$$\varphi = \oint_S \vec{E} . d\vec{S} = \frac{q}{\varepsilon_0}$$

(i) Net flux, $\varphi = \varphi_1 + \varphi_2$

where $\varphi_1 = \vec{E} . d\vec{S}$

$$= 2aC\, dS \cos 0^\circ$$

$$= 2aC \times a^2 = 2a^3\ C$$

$$\varphi_2 = aC \times a^2 \cos 180^\circ = -a^3 C$$

$$\varphi = 2a^3 C + (-a^3 C) = a^3 C\, \text{Nm}^2\ C^{-1}$$

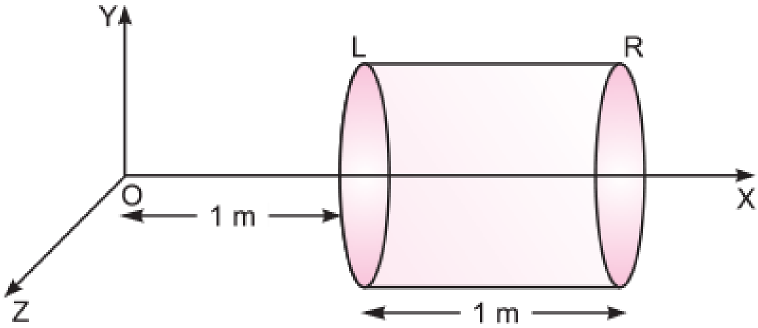
(ii) Net charge $(q) = \varepsilon_0 \times \varphi = a^3 \text{C}\, \varepsilon_0$ coulomb

$q = a^3 C\, \varepsilon_0$ coulomb.

Q. 6. A hollow cylindrical box of length 1 m and area of cross-section 25 cm2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50\, x\, \hat{i}$, where E is in NC^{-1} and x is in metres.

Find

(i) Net flux through the cylinder.
(ii) Charge enclosed by the cylinder. [CBSE Delhi 2013]



Ans.

(i) Electric flux through a surface, $\varphi = \vec{E} \cdot \vec{S}$

Flux through the left surface, $\varphi_L = -|E|\,|S| = -50x.\,|S|$

Since x = 1m, $\varphi_L = -50 \times 1 \times 25 \times 10^{-4}$

$= -1250 \times 10^{-4} = -0.125 \text{ N m}^2 \text{ C}^{-1}$

Flux through the right surface,

$\varphi_R = |E|\,|S|$

Since $x = 2$m, $\varphi_R = 50x\,|S|$

$= 50 \times 2 \times 25 \times 10^{-4} = 2500 \times 10^{-4} = 0.250 \text{ N m}^2 \text{ C}^{-1}$

Net flux through the cylinder, $\varphi_{net} = \varphi_R + \varphi_L$

$= 0.250 - 0.125 =$ $\mathbf{0.125 \text{ N m}^2 \text{ C}^{-1}}$

(ii) Charge inside the cylinder, by Gauss's Theorem

$\varphi_{net} = \frac{q}{\varepsilon_0} \Rightarrow q = \varepsilon_0\ \varphi_{Net}$

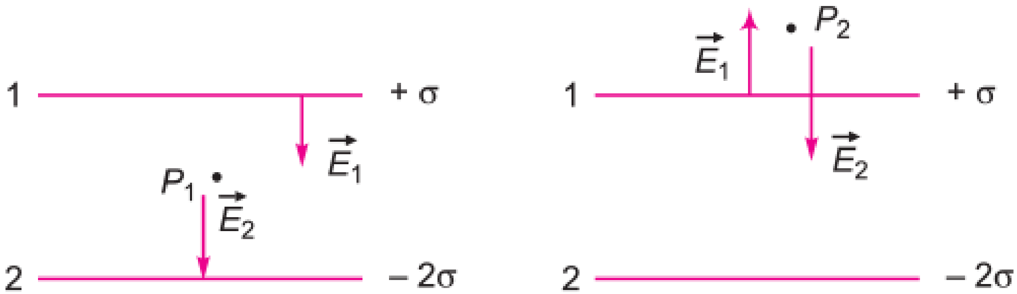
$= 8.854 \times 10^{-12} \times 0.125 = 8.854 \times 10^{-12} \times \frac{1}{8} = 1.107 \times 10^{-12} C$

Q. 7. Two parallel uniformly charged infinite plane sheets, '1' and '2', have charge densities + σ and –2 σ respectively. Give the magnitude and direction of the net electric field at a point.

(i) In between the two sheets and

(ii) Outside near the sheet '1'.   [CBSE Ajmer 2015]

Ans.



(i) Let $\overrightarrow{E_1}$ and $\overrightarrow{E}_2$ be the electric field intensity at the point P_1, between the plates. So,

$$|E_{P_1}| = |E_1| + |E_2|$$

$$= \frac{\sigma}{\varepsilon_0} + \frac{2\sigma}{\varepsilon_0}$$

$$= \frac{3\sigma}{\varepsilon_0} \qquad \text{(directed towards sheet 2)}$$

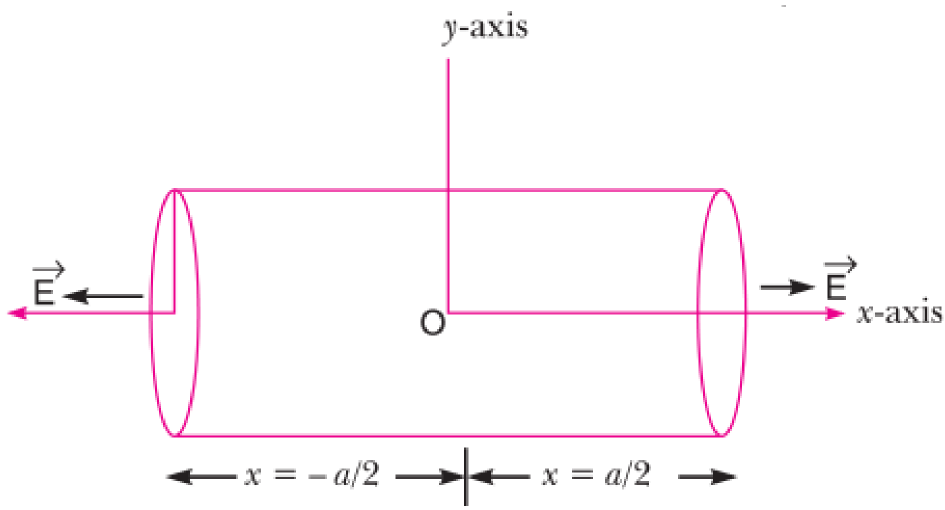
$$\overrightarrow{E}_{P_1} = \frac{3\sigma}{\varepsilon_0}(-\hat{j}) = -\frac{3\sigma}{\varepsilon_0}\hat{j}$$

(*ii*) Outside near the sheet '1', $|\overrightarrow{E}_{P_2}| = \overrightarrow{E}_2 - \overrightarrow{E}_1$

$$= \frac{2\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \qquad \text{(directed towards sheet 2)}$$

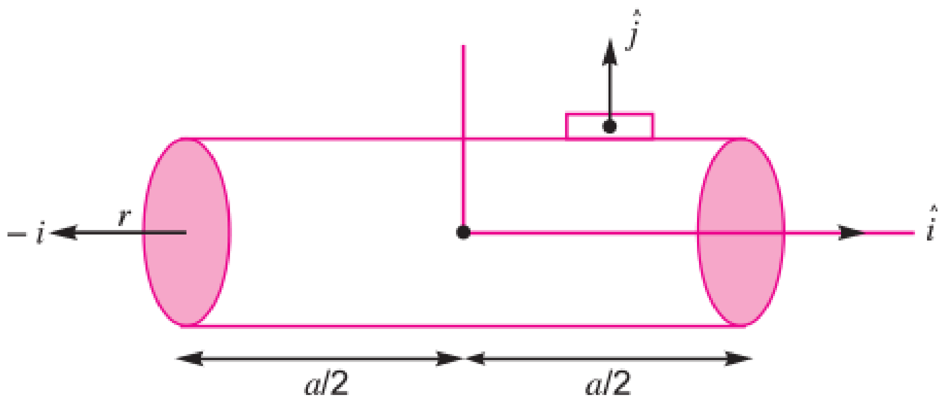
$$\overrightarrow{E}_{P_2} = \frac{\sigma}{2\varepsilon_0}(-\hat{j}) = -\frac{\sigma}{\varepsilon_0}\hat{j}$$

Q. 8. A right circular cylinder of length 'a' and radius 'r' has its centre at the origin and its axis along the x-axis so that one face is at x = + a/2 and the other at x = – a/2, as shown in the figure. A uniform electric field is acting parallel to the x-axis such that $\overrightarrow{E} = E_0\hat{i}$ for $x > 0$ and $\overrightarrow{E} = -E_0\hat{i}$ for $x > 0$.



Find out the flux (i) through the flat faces, (ii) through the curved surface of the cylinder. What is the net outward flux through the cylinder and the net charge inside the cylinder? [CBSE Chennai 2015]

Ans.



(i) Flux through the flat faces (both)

$\varphi_1 = E_0\hat{i}.\pi r^2\hat{i} = |E_0|\pi r^2$ $\quad [\because \hat{i}.\hat{i} = 1]$

(ii) Flux through the curved surface

$\varphi_2 = E_0\hat{i}.(2\pi\,\mathrm{ra}\,)\hat{j}$

$= 0 \quad [\because \hat{i}.\hat{j} = 0]$

(Field and area vector are perpendicular to each other)

Net outward flux through the cylinder,

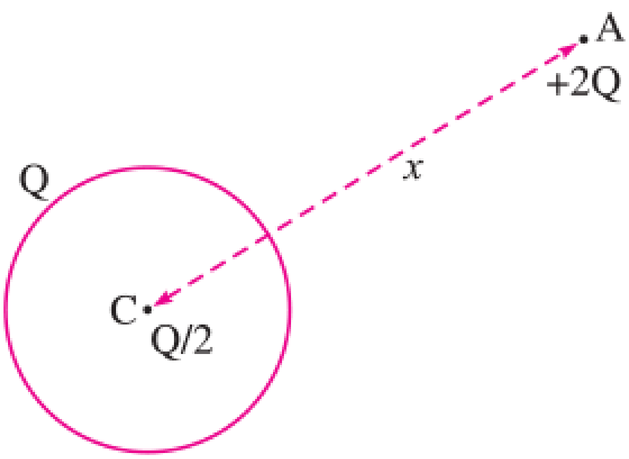
$\varphi_{\text{net}} = 2\varphi_1 + \varphi_2$

$= 2E_0\,\pi r^2$

According to Gauss's theorem, $\varphi_{\text{net}} = \frac{Q}{\varepsilon_0}$

$\therefore$ Charge inside the cylinder $Q = 2\pi\varepsilon_0 r^2\,E_0$

Q. 9. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge Q/2 is placed at the centre C and another charge +2Q is placed outside the shell at A at a distance x from the centre as shown in the figure.



(i) Find the electric flux through the shell.

State the law used.

(ii) Find the force on the charges at the centre C of the shell and at the point A. [CBSE East 2016]

Ans. (i)

(i) Electric flux through a Gaussian surface, $\varphi = \dfrac{\text{Total enclosed charge}}{\varepsilon_0}$

Net charge enclosed inside the shell, $q = 0$

$\therefore$ Electric flux through the shell $\dfrac{q}{\varepsilon_0} = 0$

(ii) **Gauss's Law:** Electric flux through a Gaussian surface is $\dfrac{1}{\varepsilon_0}$ times the net charge enclosed within it.

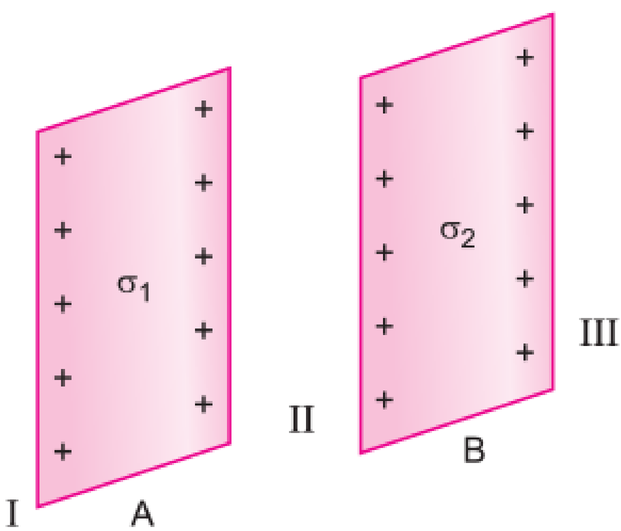
Mathematically, $\oint \vec{E} \cdot \vec{ds} = \dfrac{1}{\varepsilon_0} \times q$

(ii) We know that electric field or net charge inside the spherical conducting shell is zero. Hence, the force on charge Q/2 is zero.

Force on charge at A, $F_A = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2Q\left(Q + \frac{Q}{2}\right)}{x^2} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{3Q^2}{x^2}$

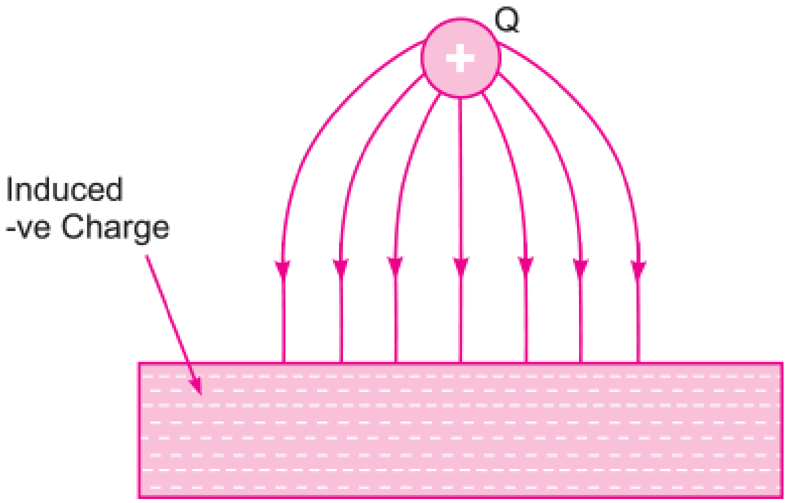
Q. 10. Answer the Following Questions.

(i) A point charge (+Q) is kept in the vicinity of uncharged conducting plate. Sketch electric field lines between the charge and the plate. [CBSE Bhubaneshwar 2015]



(ii) Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the net fields in the regions marked II and III. [CBSE (F) 2014]

Ans. (i) The lines of force start from + Q and terminate at metal place inducing negative charge on it. The lines of force will be perpendicular to the metal surface.



Ans.

$$(i)\ \text{Net electric field in region II} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$$

$$\text{Direction of electric field is from sheet } A \text{ to sheet } B.\ \text{III} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

(ii) Net electric field in region

Direction is away from the two sheets i.e., towards right side.

Q. 11. Answer the following Questions.

(i) ''The outward electric flux due to charge +Q is independent of the shape and size of the surface which encloses it.'' Give two reasons to justify this statement.

(ii) Two identical circular loops '1' and '2' of radius R each have linear charge densities –λ and +λ C/m respectively. The loops are placed coaxially with their centres R–√3 distance apart. Find the magnitude and direction of the net electric field at the centre of loop '1'. [CBSE Patna 2015]

Ans. Electric field at the centre O1 due to loop 1 is given by

$$\left|\vec{E}_1\right| = 0\ (\text{As } Z = 0)$$

Electric field at a point outside the loop 2 on the axis passing normally through O_2 of loop 2 is

$$\left|\vec{E}_2\right| = \frac{\lambda R}{2\varepsilon_0} \cdot \frac{Z}{(R^2+Z^2)^{3/2}}$$

Since $Z = R\sqrt{3}$

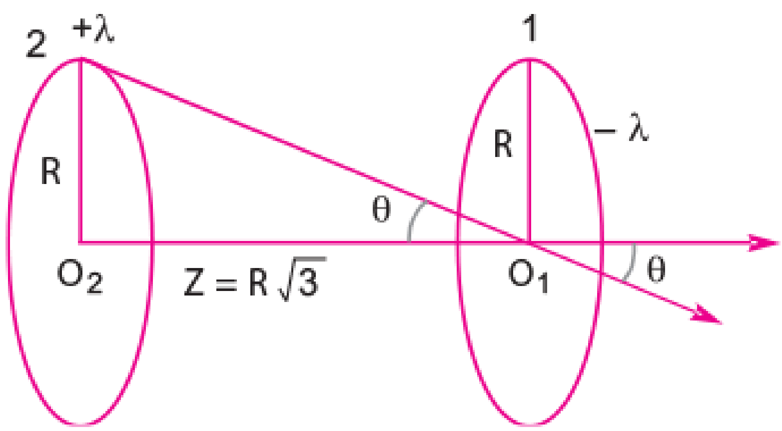
$$= \frac{\lambda R}{2\varepsilon_0} \cdot \frac{R\sqrt{3}}{(R^2+3R^2)^{3/2}}$$

$$= \frac{\lambda\sqrt{3}}{16\varepsilon_0 R} \text{ towards right (As } \lambda \text{ is positive)}$$

So, net electric field at the centre of loop 1

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= 0 + \frac{\lambda\sqrt{3}}{16\varepsilon_0 R} = \frac{\lambda\sqrt{3}}{16\varepsilon_0 R}$$



Short Answer Questions-II (OIQ)

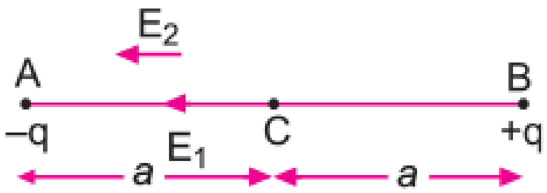
Q. 1. Define electric field intensity. Write its SI unit. Write the magnitude and direction of electric field intensity due to an electric dipole of length 2a at the midpoint of the line joining the two charges.

Ans. Electric Field Intensity : The electric field intensity at any point in an electric field is defined as the electric force per unit positive test charge placed at that point i.e.,

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

The test charge q_0 has to be vanishingly small so that it does not affect the electric field of the main charge.

The SI unit of electric field intensity is newton/coulomb.



Electric Field Strength at mid-point of dipole: The electric field strength at mid-point C due to charge + q is – q along the same direction.

$$E = E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}$$

Its direction is from +q to – q.

Q. 2. The electric field E due to any point charge near it is defined as $E = \lim_{q \to 0} \frac{\mathbf{F}}{\mathbf{q}}$ where q is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression?

Draw the electric lines of point charge Q when

(i) Q > 0 and (ii) Q < 0.

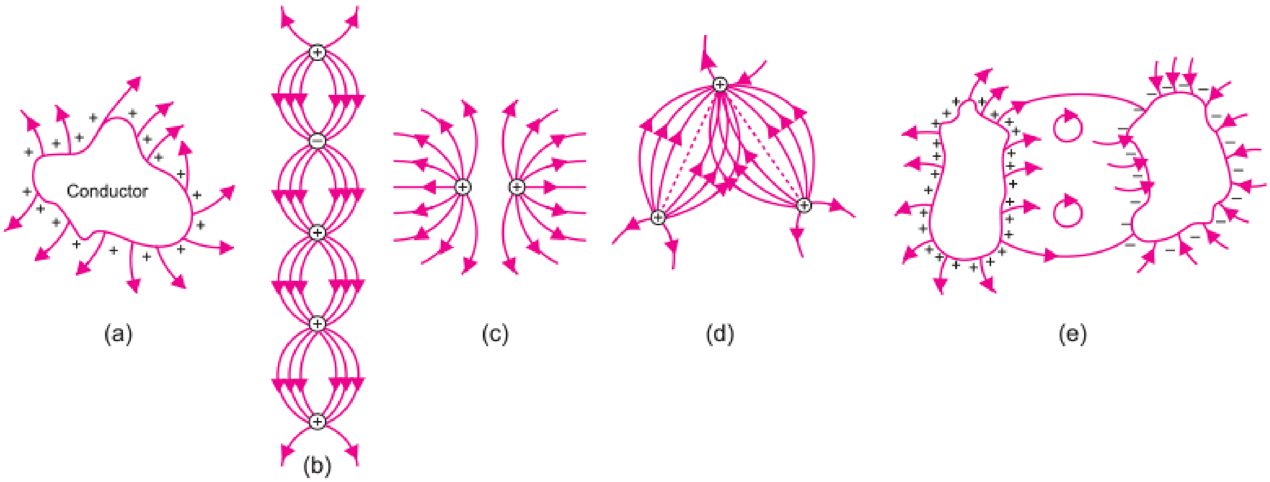
(i) Q > 0 (ii) Q < 0

Ans. The physical significance of $\lim_{q \to 0}$ in the definition of electric field $E = \lim_{q \to 0} \frac{F}{q}$

The point test charge q produces its own electric field, hence it will modify the electric field strength to be measured. Therefore, the test charge used to measure the electric field must be too small.

The electric lines of force are shown in figure above.

Q. 3. Which among the curves shown in figure cannot possibly represent electrostatic field lines?



Ans. (a) Field lines are wrongly drawn because electric field lines must be normal to the surface of the conductor at each point.

(b) Field lines are wrongly drawn because field lines cannot start from a negative charge.

(c) Field lines are correctly drawn, because they are originating from a positive charge.

(d) Field lines are wrongly drawn as the field lines cannot intersect.

(e) Field lines are wrongly drawn because they cannot form closed loops.

Q. 4. An electric dipole of dipole moment $\vec{P}$ is placed in a uniform electric field $\vec{E}$. Write the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. Show diagrammatically the orientation of the dipole in the field for which the torque is (i) Maximum (ii) Half the maximum value (iii) Zero.

Ans. Torque experienced by an electric dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Pairs of perpendicular vectors $(a)\ (\vec{\tau}, \vec{p})$ $(b)\ (\vec{\tau}, \vec{E})$

(i) Magnitude of torque $\tau = \mathbf{pE}\sin\theta$

For maximum torque $(\sin\theta)$ max $= 1 \Rightarrow \theta = 90^\circ$

Orientation is shown in figure (i)

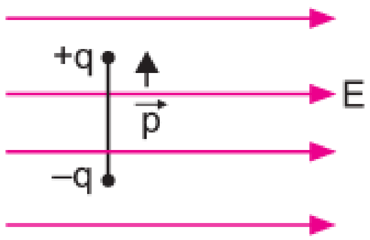
(ii) For $\tau = \frac{1}{2}\ \tau_{\max}$

$\mathbf{pE}\sin\theta = \frac{1}{2}\mathbf{pE} \quad \Rightarrow \quad \sin\theta = \frac{1}{2} \quad$ or $\quad \theta = 30^\circ$

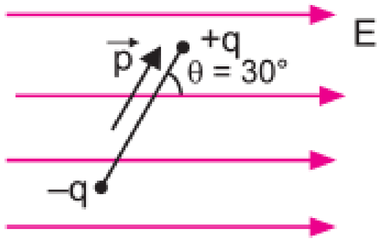
Orientation is shown in figure (ii)

(iii) For zero torque, $\sin\theta = 0 \Rightarrow \theta = 0$

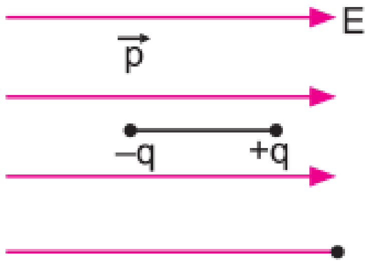
The orientations is shown in the figure (iii).



(i)

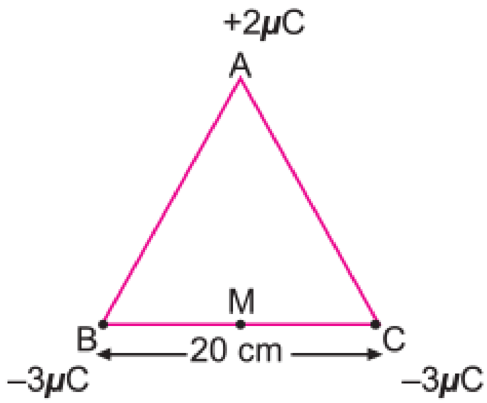


(ii)

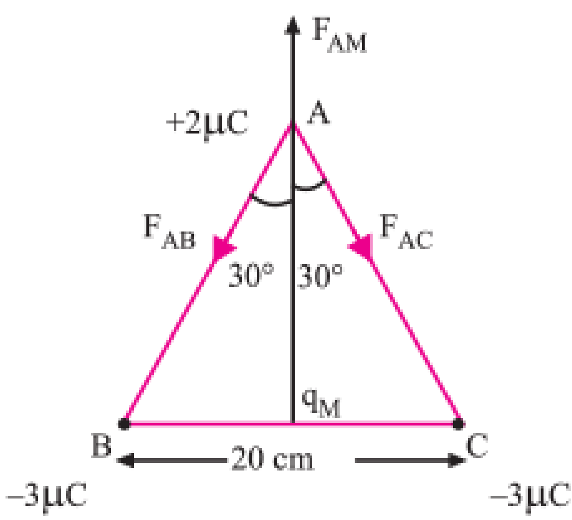


(iii)

Q. 5. Three point charges of + 2 μC, – 3 μC and – 3 μC are kept at the vertices A, B and C respectively of an equilateral triangle of side 20 cm as shown in figure. What should be the sign and magnitude of charge to be placed at the midpoint M of side BC so that charge at A remains in equilibrium?



Ans.



Let charge placed at M be q_M. The forces acting on charge (q_A = + 2μC) are F_{AB}, F_{AC} and F_{AM} as shown in figure.

$$\vec{F}_{AB} = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r^2_{AB}} \text{ along } \overrightarrow{AB}$$

$$= 9 \times 10^9 \times \frac{(2\times10^{-6})(3\times10^{-6})}{(0.20)^2} \text{ along } \overrightarrow{AB} = 1.35\ N \text{ along } \overrightarrow{AB}$$

$$\vec{F}_{AC} = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_C}{r^2_{AC}}$$

$$= 9 \times 10^9 \frac{(2\times10^{-6})(3\times10^{-6})}{(0.20)^2} = 1.35\ N \text{ along } \overrightarrow{AC}$$

$$\vec{F}_{AM} = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_M}{(r_{AM})^2} = 9 \times 10^9 \frac{(2\times10^{-6})(q_M)}{(\sqrt{3}\times10^{-1})^2}$$

$$= 6 \times 10^5 q_M\ N \text{ along } \overrightarrow{MA}$$

For equilibrium of charge q_A; the resultant of $\vec{F}_{AB}$ and $\vec{F}_{AC}$ must be equal and opposite to $\vec{F}_{AM}$.

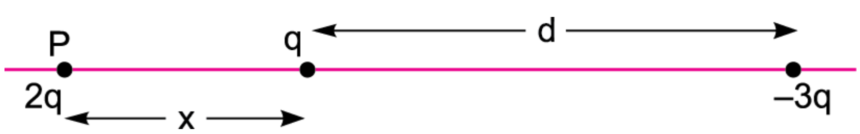
i.e., $F_{AB} \cos 30^\circ + F_{AC} \cos 30^\circ = F_{AM}$

$$\Rightarrow \quad 1.35 \times \frac{\sqrt{3}}{2} + 1.35 \frac{\sqrt{3}}{2} = 6 \times 10^5 q_M$$

$$\Rightarrow \quad q_M = \frac{1.35\sqrt{3}}{6\times10^5} = 0.225\sqrt{3} \times 10^{-5} C = 2.25\sqrt{3}\mu C$$

Q. 6. Two charges q and –3q are placed fixed on x-axis separated by distance ‘d’. Where should a third charge 2q be placed such that it will not experience any force?
[NCERT Exemplar]

Ans.



Let the charge 2q be placed at point P as shown. The force due to q is to the left and that due to –3q is to the right.

$$\therefore \quad \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0 (d+x)^2} \quad \Rightarrow \quad (d+x)^2 = 3x^2$$

$$\therefore \quad 2x^2 - 2\,dx - d^2 = 0 \quad \Rightarrow \quad x = \frac{d}{2} \pm \frac{\sqrt{3}\ d}{2}$$

($-$ve sign would be between q and $-3q$ and hence is unacceptable.)

$$\Rightarrow \quad x = \frac{d}{2} + \frac{\sqrt{3}\ d}{2} = \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q.$$

Q. 7. Two point charges of + 5 × 10^{-19} C and +20 × 10^{-19} C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.

Ans. Let charges q_1=+5×10^{-19} C and q_2=+20×10^{-19} C be placed at A and B respectively. Distance AB = 2 m.

As charges are similar, the electric field strength will be zero between the charges on the line joining them. Let P be the point (at a distance x from q_1) at which electric field intensity is zero. Then, AP = x metre, BP = (2 – x) metre. The electric field strength at P due to charge q_1 is

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{x^2} \text{ , along the direction } A \text{ to } P.$$

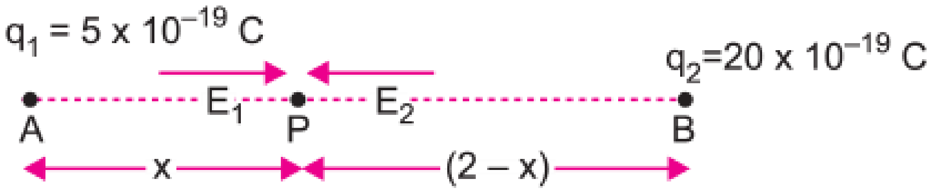
The electric field strength at P due to charge q_2 is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{(2-x)^2} \text{ , along the direction } B \text{ to } P.$$

Clearly, $\vec{E}_1$ and $\vec{E}_2$ and are opposite in direction and for net electric field at P to be zero, $\vec{E}_1$ and $\vec{E}_2$ and must be equal in magnitude.

So, $E_1 = E_2$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{(2-x)^2}$$



Given, $q_1 = 5\times10^{-19}$ C, $q_2 = 20\times10^{-19}$ C

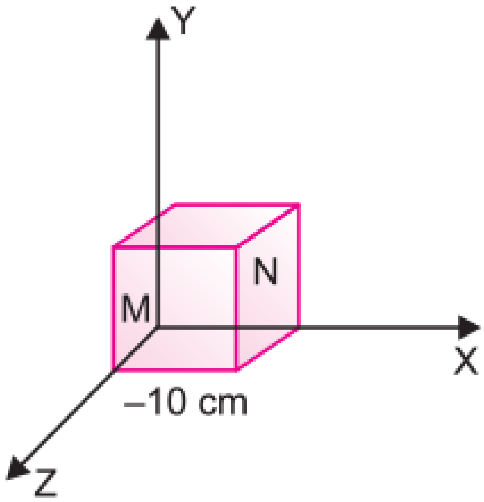
$$\text{Therefore,} \quad \frac{5\times10^{-19}}{x^2} = \frac{20\times10^{-19}}{(2-x)^2}$$

$$\text{or} \quad \frac{1}{2} = \frac{x}{2-x}$$

$$\text{or} \quad x = \frac{2}{3}m$$

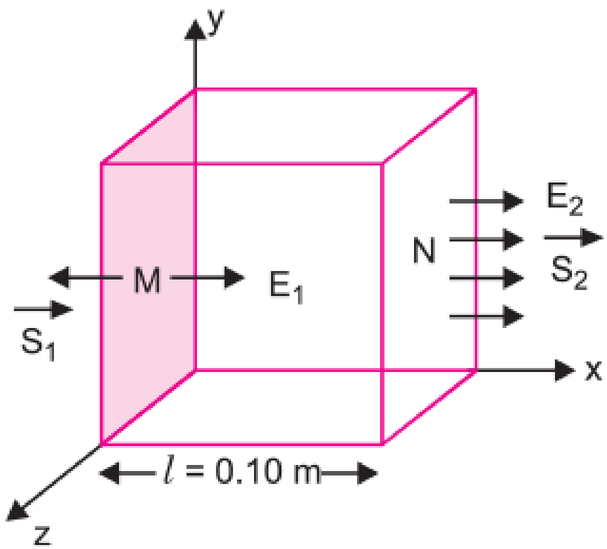
Q. 8. Electric field in the given figure is directed along + X direction and given by $E_x = 5A_x + 2B$, where E is in NC^{-1} and x is in metre, A and B are constants with dimensions. Taking A = 10 NC^{-1} m^{-1} and B = 5 NC^{-1}, calculate

(i) The electric flux through the cube.



(ii) Net charge enclosed within the cube.

Ans.



Given $E_x = 5A_x + 2B$

The electric field at face M where x = 0 is

E1 =2B

The electric field at face N where x = 10 cm = 0.10 m is

$E_2 = 5A \times 0.10 + 2B = 0.5A + 2B$

The electric flux through face M is

$$\varphi_1 = \vec{E}_1 . \vec{S}_1 = E_1 S_1 \cos \pi = -E_1 S_1$$
$$= -2B \times l^2 \text{ where } l = 10\text{cm} = 0.01m$$

The electric flux through face N

$$\varphi_2 = \vec{E}_2 . \vec{S}_2 = E_2 S_2 \cos 0 = (0.5A + 2B)l^2$$

Net electric flux, $\varphi = \varphi_1 + \varphi_2$

$$= -2\,\mathrm{Bl}^2 + (0.5A + 2B)l^2 = 0.5\ \ \mathrm{Al}^2$$

$$= 0.5 \times 10 \times (0.10)^2 = 5 \times 10^{-2} V\ m$$

Q. 9. A charge Q located at a point $\vec{r}$ is in equilibrium under the combined electric field of three charges q_1, q_2, q_3. If the charges q_1, q_2 are located at points

$\vec{r}_1$ **and** $\vec{r}_2$ **respectively, find the direction of the force on Q, due to q**$_3$ **in terms of q**$_1$**, q**$_2$**,** $\vec{r}_1$ $\vec{r}_2$ **and** $\vec{r}$ **[HOTS]**

Ans.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r} - \vec{r}_1) + \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2) + \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{|\vec{r} - \vec{r}_3|^3}(\vec{r} - \vec{r}_3) = 0$$

$$\Rightarrow \quad \frac{q_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r} - \vec{r}_1) + \frac{q_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2) = -\frac{q_3}{|\vec{r} - \vec{r}_3|^3}(\vec{r} - \vec{r}_3)$$

$$\Rightarrow \quad \vec{r} - \vec{r}_3 = -\frac{|\vec{r} - \vec{r}_3|^3}{q_3}\left[\frac{q_1(\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + \frac{q_2(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3}\right]$$

Direction of force on Q due to q_3 along $(\vec{r} - \vec{r}_3)$ is given by

$$\frac{q_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r}_1 - \vec{r}) + \frac{q_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r}_2 - \vec{r})$$