

## 40. Electromagnetic Waves

### Short Answer

#### 1. Question

In a microwave oven, the food is kept in a plastic container and the microwave is directed towards the food. The food is cooked without melting or igniting the plastic container. Explain.

#### Answer

The food which is being cooked contains water. The microwaves which are directed towards the food has a natural frequency that match with the frequency of water. The water molecules inside the food absorb energy from the microwaves and get heated up causing the food around it to heat up. Due to absorption of energy of the microwaves, the food gets cooked. Whereas the plastic does not have any water molecules which would absorb energy and the frequency also doesn't match with the natural frequency of the microwaves so the plastic container remains unaffected.

#### 2. Question

A metal rod is placed along the axis of a solenoid carrying a high-frequency alternating current. It is found that the rod gets heated. Explain why the rod gets heated?

#### Answer

The solenoid carrying a high frequency alternating current generates a continuously changing magnetic field across the rod placed along the axis. Due to this varying magnetic field across the rod, eddy currents are generated on the surface of the rod. The resistance of the rod and the eddy currents flowing on the rod are responsible for the generation of heat.

#### 3. Question

Can an electromagnetic wave be deflected by an electric field? By a magnetic field?

#### Answer

Electromagnetic waves are made up of electric and magnetic fields but not charges. When an electromagnetic wave passes from a region of electric or magnetic field, it will show no deflection as no charge is present which would experience a force due to these fields and be deflected.

#### 4. Question

A wire carries an alternating current  $i = i_0 \sin \omega t$ . Is there an

electric field in the vicinity of the wire?

### Answer

Given: Alternating current  $i = i_0 \sin(\omega t)$

The current is the rate of flow of charge per unit time. Mathematically  $i = \frac{dq}{dt}$

For a time,  $t$ , we can assume the charges in the wire are at rest,

then the charge will be given as

$$dq = i dt = i_0 \sin(\omega t) dt$$

Similarly, a no. of charges can be assumed to be at rest. These charges produce a time varying electric field.

### 5. Question

A capacitor is connected to an alternating-currents source. Is there a magnetic field between the plates?

### Answer

Yes, there is a magnetic field between the capacitor plates.

According to Ampere Maxwell's law, the magnetic field in a region is due to the current due to charge carriers and the changing electric field in a region. Mathematical form of the law is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{\text{enclosed}} + I_d)$$

where  $B$  is the magnetic field,  $dl$  is small element of length of an

Amperian loop,  $\mu_0$  is the magnetic permeability of free space and its value is  $4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $I_{\text{enclosed}}$  is the current due to charge carriers (conduction current) and  $I_d$  is the displacement current which

is related to changing electric field.

The displacement current is related to electric field as

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right|$$

where  $\phi_E$  is the time varying electric flux through the surface

and  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

As the capacitor is charging, the charges at different time create a time varying electric field in the space between the capacitor plates.

This time varying electric field further creates an alternating electric flux through a surface present in between the plates.

The time varying electric flux is

$$\phi_E = \oint E(t) dS \cos(\theta)$$

According to Gauss's law, the flux at time  $t$  is equal to  $1/\epsilon_0$  times the charge enclosed at that time

$$\phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

The charge enclosed at any time  $t$  will be

$Q = CV_0 e^{\omega t}$  where  $C$  is the capacitance of the capacitor and  $V_0$  is the amplitude of alternating voltage source and  $\omega$  is the angular frequency of the source.

The magnetic field in between the capacitor plates is given by

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I_{\text{enclosed}} + \epsilon_0 \frac{d}{dt} \left( \frac{CV_0 e^{\omega t}}{\epsilon_0} \right) \right)$$

## 6. Question

Can an electromagnetic wave be polarized?

### Answer

Electromagnetic waves consist of oscillating electric and magnetic field which are perpendicular to each other and the direction of propagation as well. Thus electromagnetic waves are transverse in nature and can be polarized. Polarization of electromagnetic waves will restrict the vibrations of electric and magnetic field vectors in one direction only.

## 7. Question

A plane electromagnetic wave is passing through a region. Consider the quantities

- (a) electric field,
- (b) magnetic field,
- (c) electrical energy in a small volume and
- (d) magnetic energy in a small volume.

Construct pairs of the quantities that oscillate with equal frequencies.

### Answer

Explanation: Let us consider an electromagnetic wave traveling in the z direction. The electric field E of the wave is assumed to be

$$E = E_0 \sin(kz - \omega t)$$

where  $E_0$  is the amplitude of the electric field, k is the wave number of the wave ( $k = \frac{2\pi}{\lambda}$ ),  $\lambda$  is the wavelength,  $\omega$  is the angular frequency of the wave ( $\omega = 2\pi\nu$ ) and t is the time. The frequency of the wave is

$$f = \frac{\omega}{2\pi}$$

Now the magnetic field can be considered as

$$B = B_0 \sin(kz - \omega t)$$

where  $B_0$  is the amplitude of the magnetic field, k is the wave number of the wave ( $k = \frac{2\pi}{\lambda}$ ),  $\lambda$  is the wavelength,  $\omega$  is the angular frequency of the wave ( $\omega = 2\pi\nu$ ) and t is the time. The frequency of the wave is

$$f = \frac{\omega}{2\pi}$$

The frequencies of electric and magnetic waves are same.

Hence (a) and (b) oscillate with the same frequency.

Now the electric field energy density of the electromagnetic wave is given by the relation

$$U_{de} = \frac{1}{2} \epsilon_0 E^2$$

where  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $U_{de}$  is the energy density of electric field. Now the energy of the wave over small volume V of the region is the product of energy density and the small volume V. The energy of the electric field is

$$U_E = U_{de} \times V = \frac{1}{2} \epsilon_0 E^2 \times V$$

$$\begin{aligned} U_E &= \frac{1}{2} \epsilon_0 (E_0 \sin(kz - \omega t))^2 \times V = \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) \times V \\ &= \frac{1}{2} \epsilon_0 E_0^2 \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) \times V = \frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V \end{aligned}$$

$$U_E = \frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

The energy of the electric field is  $\frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V$ .

The angular frequency of the energy of the wave is  $2\omega$ , the

Corresponding frequency will be

$$f' = \frac{2\omega}{2\pi}$$

Now the magnetic field energy density of the electromagnetic wave

is given by the relation

$$U_{db} = \frac{1}{2} \frac{B^2}{\mu_0}$$

where  $\mu_0$  is the magnetic permeability of free space and its value is

$4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $U_{db}$  is the magnetic field energy density. Now the energy of the wave over small volume  $V$  of the region is the product of energy density and the small volume  $V$ . The energy of the magnetic field is

$$U_B = U_{db} \times V = \frac{1}{2} \frac{B^2}{\mu_0} \times V$$

$$\begin{aligned} U_B &= \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2 \times V = \frac{1}{2\mu_0} B_0^2 \sin^2(kz - \omega t) \times V \\ &= \frac{1}{2\mu_0} B_0^2 \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) \times V = \frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V \end{aligned}$$

$$U_B = \frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

The energy of the magnetic field is  $\frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V$ .

The angular frequency of the energy of the wave is  $2\omega$ , the

Corresponding frequency will be

$$f' = \frac{2\omega}{2\pi}$$

The electric field energy and magnetic field energy have the same frequencies of oscillation.

So (c) and (d) form a pair.

## Objective I

### 1. Question

A magnetic field can be produced by

- A. a moving charge
- B. a changing electric field
- C. none of them
- D. both of them

### Answer

Explanation: A magnetic current can be produced by a moving charge. If the charge is moving uniformly, it will generate a constant current. If we assume a constant current is flowing through a conductor of length L, the magnetic field due to it at a distance R from it is given by Biot-Savart law. The field would be

$$\vec{B} = \frac{\mu_0}{4\pi} \times i \times \frac{(\vec{L} \times \vec{R})}{R^3}$$

where i is the constant current and  $\mu_0$  is the magnetic permeability of free space and its value is  $4\pi \times 10^{-7} \text{ T m A}^{-1}$ . So A. is correct.

A changing electric field can also generate a magnetic field around it. According to Ampere-Maxwell law, the total magnetic field around a region is due to flow of charge carriers (called as conduction current)

and the time varying electric flux. Mathematically

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{\text{enclosed}} + I_d) \dots (i)$$

where B is the magnetic field, dl is small element of length of an

Amperian loop,  $\mu_0$  is the magnetic permeability of free space and its value is  $4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $I_{\text{enclosed}}$  is the current due to charge carriers (conduction current)

and  $I_d$  is the displacement current which

is related to changing electric field.

The displacement current is related to electric field as

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right| \dots (ii)$$

where  $\phi_E$  is the time varying electric flux through the surface

and  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal

to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

The electric flux is the amount of electric field lines passing

through a surface normally. Mathematically

$$\phi_E = \oint E(t) dS \cos(\theta) \dots (iii)$$

where  $E(t)$  is the time varying electric field,  $dS$  are a small area

element on the surface and  $\theta$  is the angle between the electric field

vector and area vector (if the field lines are not falling normally).

From (ii) and (iii)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{\text{enclosed}} + \epsilon_0 \left| \frac{d(\oint E(t) dS \cos(\theta))}{dt} \right|)$$

If the conduction current is zero, so the magnetic field will entirely be

produced by the changing electric field and would be produced by it.

Hence B. is correct

Hence D. is correct

## 2. Question

A compass needle is placed in the gap of a parallel plate capacitor. The capacitor is connected to a battery through a resistance. The compass needle

A. does not deflect

B. deflects for a very short time and then comes back to the original position

C. deflects and remains deflected as long as the battery is connected.

D. deflects and gradually comes to the original position in a time which is large compared to the time constant.

**Answer**

deflects and gradually comes to the original position in a time, which is large compared to the time constant.

Explanation: The deflection in a compass needle are because of magnetic field around the region of the compass. The magnetic field arises due to charging of the capacitor which creates a varying electric field. This varying electric field produces a changing electric field through the capacitor plates. Due to this changing electric flux, a current called the displacement current arises in the capacitor. The current  $I_d$  is related to the electric flux as

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right| \dots (i)$$

where  $\phi_E$  is the time varying electric flux through the plane surface and  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

According to Ampere-Maxwell's law of electromagnetism, the total magnetic field is due to current due to charge carriers and the current due to time varying electric flux. Ampere-Maxwell law is the extension of Ampere Circuital Law. The mathematical relation is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{\text{enclosed}} + I_d) \dots (ii)$$

where B is the magnetic field, dl is small element of length of an

Amperian loop,  $\mu_0$  is the magnetic permeability of free space and its value is  $4\pi \times 10^{-7} \text{ T m A}^{-1}$ , and  $I_{\text{enclosed}}$  is the current due to charge carriers.

From (i) and (ii)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I_{\text{enclosed}} + \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right| \right) \dots (iii)$$

According to Gauss's Law of electrostatics, the electric flux through a surface is equal to  $1/\epsilon_0$  times the charge enclosed by it.

$$\phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



The charge enclosed by the capacitor plates is changing with time as the capacitor is charging. The charge is given as

$$Q = CV(1 - e^{\frac{-t}{RC}})$$

where

C=capacitance of the capacitor

V=potential of the battery

R=resistance of the resistor connected in series

Using the above two relations in (iii)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{enclosed} + \epsilon_0 \left| \frac{d(CV(1 - e^{\frac{-t}{RC}}))}{dt} \right|) = \mu_0 (I_{enclosed} + \left| \frac{d(CV(1 - e^{\frac{-t}{RC}}))}{dt} \right|)$$

So the magnetic field depends on the charge of the capacitor. The needle will show deflection till the capacitor gets charged to a specific value at a time  $\tau$  called the time constant.  $\tau = RC$  is the time constant of the capacitor. Charging after this time will be gradual so the needle will come to its original position.

### 3. Question

Dimensions of  $1/(\mu_0\epsilon_0)$  is

- A. L/T
- B. T/L
- C.  $L^2/T^2$
- D.  $T^2/L^2$ .

### Answer

Explanation: The electrical permittivity of free space and magnetic permeability of free space is related with the speed of light in vacuum as

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

squaring the above relation

$$C^2 = \frac{1}{\epsilon_0 \mu_0}$$

The dimension of  $\frac{1}{\epsilon_0 \mu_0}$  will be the dimension of  $C^2$ .

The dimension of C is  $[L T^{-1}]$ , so the dimension of  $C^2$  will be  $[L T^{-1}]^2$

$$[L T^{-1}]^2 = [L^2 T^{-2}]$$

Therefore C. is the only correct option.

#### 4. Question

Electromagnetic waves are produced by

- A. a static charge
- B. a moving charge
- C. an accelerating charge
- D. charge less particles

#### Answer

Explanation: An accelerating charge produces a changing electric field which in turn produces a magnetic field. These alternatively changing magnetic and electric fields give rise to electromagnetic waves. A static charge gives rise to only an electric field and a moving charge creates a magnetic field so A. and B. are incorrect.

#### 5. Question

An electromagnetic wave going through vacuum is described by

$$E = E_0 \sin(\kappa x - \omega t); B = B_0 \sin(\kappa x - \omega t)$$

Then

- A.  $E_0 \kappa = B_0 \omega$
- B.  $E_0 B_0 = \omega \kappa$
- C.  $E_0 \omega = B_0 \kappa$
- D. None of these

#### Answer

Explanation: The amplitudes of the electric and magnetic fields are related as

$$\frac{E_0}{B_0} = c \dots (i)$$

where  $c$  is the speed of wave and  $E_0$  and  $B_0$  are the amplitudes of the fields.

The speed of wave is related with its frequency and wavenumber as

$$\frac{\omega}{k} = c \dots (ii)$$

where  $\omega$  is the angular frequency of the wave ( $\omega = 2\pi\nu$ ) and  $k$  is the wavenumber ( $k = \frac{2\pi}{\lambda}$ ) and  $\nu$  is the frequency and  $\lambda$  is the wavelength of the wave.

From (i) and (ii)

$$c = \frac{\omega}{k} = \frac{E_0}{B_0}$$

$$\frac{\omega}{k} = \frac{E_0}{B_0} \rightarrow E_0 k = B_0 \omega \text{ so A. is correct.}$$

B. and C. are invalid relations. Only A. is the correct option.

## 6. Question

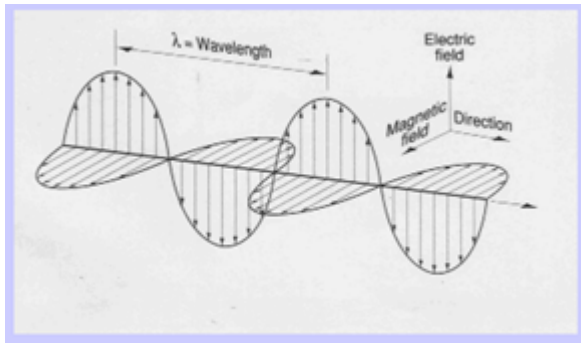
An electric field  $\vec{E}$  and a magnetic field  $\vec{B}$  exist in a region. The fields are not perpendicular to each other.

- A. This is not possible
- B. No electromagnetic wave is passing through the region.
- C. An electromagnetic wave may be passing through the region.
- D. An electromagnetic wave is certainly passing through the region.

## Answer

Explanation: There exists a mode of propagation of electromagnetic wave called the Transverse Electric and Magnetic (TEM) mode where both electric and magnetic field are moving transverse to the direction of propagation of wave. The fields are not in the direction of propagation.

It is possible that an electromagnetic wave may exist when electric and magnetic fields are not perpendicular to each other.



## 7. Question

Consider the following two statements regarding a linearly polarized, plane electromagnetic wave:

- (A) The electric field and the magnetic field have equal average values.  
 (B) The electric energy and the magnetic energy have equal average values.

- A. Both A and B are true  
 B. A is false but B is true  
 C. B is false but A is true  
 D. Both A and B are false

## Answer

Explanation: Let us assume the linearly polarized plane wave travels in z direction so the electric and magnetic fields of the wave are given by

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

Their average values over one cycle would be the average from time  $t=0$  to  $t=T$ .

Average of electric field

$$\begin{aligned} \langle E_x \rangle &= \frac{\int_0^T E_x dt}{\int_0^T dt} = \frac{\int_0^T E_0 \sin(kz - \omega t) dt}{\int_0^T dt} \\ &= \frac{1}{T} E_0 \int_0^T \sin(kz - \omega t) dt = \frac{E_0}{T} \left[ \frac{\cos(kz - \omega t)}{\omega} \right]_0^T \\ &= \frac{E_0}{T \times \frac{2\pi}{T}} \left[ \cos\left(kz - \frac{2\pi}{T} T\right) - \cos\left(kz - \frac{2\pi}{T} 0\right) \right] \end{aligned}$$

$$= \frac{E_0}{T \times \frac{2\pi}{T}} [\cos(kz - 2\pi) - \cos(kz)] = \frac{E_0}{T \times \frac{2\pi}{T}} [\cos(kz) - \cos(kz)] = 0$$

(because  $\omega = \frac{2\pi}{T}$  and  $\cos(\theta - 2\pi) = \cos(\theta)$  )

Therefore, the average of electric field is zero.

Similarly average of magnetic field is also zero as

$$\begin{aligned} \langle B_y \rangle &= \frac{\int_0^T B_y dt}{\int_0^T dt} = \frac{\int_0^T B_0 \sin(kz - \omega t) dt}{\int_0^T dt} \\ &= \frac{1}{T} B_0 \int_0^T \sin(kz - \omega t) dt = \frac{B_0}{T} \left[ \frac{\cos(kz - \omega t)}{\omega} \right]_0^T \\ &= \frac{B_0}{T \times \frac{2\pi}{T}} \left[ \cos\left(kz - \frac{2\pi}{T} T\right) - \cos\left(kz - \frac{2\pi}{T} 0\right) \right] \\ &= \frac{B_0}{T \times \frac{2\pi}{T}} [\cos(kz - 2\pi) - \cos(kz)] = \frac{B_0}{T \times \frac{2\pi}{T}} [\cos(kz) - \cos(kz)] = 0 \end{aligned}$$

So electric and magnetic field have their average value zero.

Therefore (A) is correct.

Now the electric energy associated with the electric field is given by

$$U_{de} = \frac{1}{2} \epsilon_0 E^2 \dots (i)$$

where  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is

equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $U_{de}$  is the energy density of electric field.

Similarly, the magnetic energy associated with the magnetic field is given by

$$U_{db} = \frac{1}{2} \frac{B^2}{\mu_0} \dots (ii)$$

where  $\mu_0$  is the magnetic permeability of free space and its value is

$4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $U_{db}$  is the magnetic field energy density.

Also the amplitudes of electric and magnetic fields are related as

$$\frac{E_0}{B_0} = C \dots \text{(iii)}$$

where C is the speed of light in vacuum.

$$U_{de} = \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) \text{ and } U_{db} = \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2$$

Using (iii) in (i)

$$U_{de} = \frac{\epsilon_0 B_0^2 C^2}{2} \sin^2(kz - \omega t) \dots \text{(iv)}$$

The electrical permittivity and magnetic permeability of free space related as

$$C = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \Rightarrow C^2 = \frac{1}{\epsilon_0 \mu_0} \dots \text{(v)}$$

Using (v) in (iv)

$$U_{de} = \frac{\epsilon_0 B_0^2 C^2}{2} \sin^2(kz - \omega t) = \frac{\epsilon_0 B_0^2}{2\epsilon_0 \mu_0} \sin^2(kz - \omega t) = \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2$$

$$U_{de} = U_{db}$$

Therefore, the electric and magnetic energy densities are same in magnitude and so their average values will be same.

Average of electric energy over one cycle will be

$$\begin{aligned} \langle U_{de} \rangle &= \frac{\int_0^T \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) dt}{\int_0^T dt} = \frac{1}{2T} \epsilon_0 E_0^2 \int_0^T \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) dt \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \int_0^T (1 - \cos(2kz - 2\omega t)) dt = \frac{1}{4T} \epsilon_0 E_0^2 \left[ t - \frac{\sin(2kz - 2\omega t)}{2\omega} \right]_0^T \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \left[ (T - 0) - \frac{1}{2\omega} \left[ \left( \sin(2kz - \frac{4\pi}{T} T) \right) - \left( \sin\left(2kz - \frac{4\pi}{T} 0\right) \right) \right] \right] \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \left[ T - \frac{1}{2\omega} [\sin(2kz - 4\pi) - \sin(2kz)] \right] \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \left[ T - \frac{1}{2\omega} [\sin(2kz) - \sin(2kz)] \right] = \frac{1}{4} \epsilon_0 E_0^2 \neq 0 \end{aligned}$$

So electric and magnetic energy and have non zero average values.

Therefore (B) is also correct.

## 8. Question

A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving

- A. along the electric field
- B. along the magnetic field
- C. along the direction of propagation of the wave
- D. in a plane containing the magnetic field and the direction of propagation.

**Answer**

Explanation: A particle placed in and electromagnetic wave moving with velocity  $v$  experiences a force called Lorentz's force due to the electric and magnetic fields. The force is given as

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

where  $q$  is the charge of the particle,  $E$  is the electric field,  $v$  is the velocity of the particle and  $B$  is the magnetic field.

An electron at rest is placed in the path of a plane electromagnetic wave. The Lorentz force acting on the electron will be

$$\vec{F} = q(\vec{E} + 0 \times \vec{B})$$

where  $e$  is the charge of the electron. As the electron is at rest so the velocity will be 0. The net Lorentz's force will be

$$\vec{F} = q\vec{E}$$

The electron will move in the direction of the field.

**9. Question**

A plane electromagnetic wave is incident on a material surface. The wave delivers momentum  $p$  and energy  $E$ .

- A.  $p = 0, E \neq 0$
- B.  $p \neq 0, E = 0$
- C.  $p \neq 0, E \neq 0$
- D.  $p = 0, E = 0$

**Answer**

Explanation: An electromagnetic wave striking a material surface delivers both energy and momentum to the surface.

If the frequency of the wave is  $\nu$  then the energy imparted will be

$$E = h\nu \dots (i)$$

where  $h$  is the Planck's constant. ( $h = 6.63 \times 10^{-34} \text{ J s}^{-1}$ )

The energy is also related to the speed of the wave as

$$E = pc \dots (ii)$$

where  $p$  is the momentum and  $c$  is the speed of the wave.

From (i) and (ii)

$$E = h\nu = pc$$

$$p = \frac{h\nu}{c} = \frac{E}{c}$$

The energy and momentum both are non zero.

We can also define a quantity known as *radiation pressure* which is the pressure the radiation exerts on the material surface it strikes.

The radiation pressure is given as

$$\text{Pressure}(P) = \frac{\text{force}(F)}{\text{area}(A)} \dots (iii)$$

Also the electromagnetic wave has energy so there is an intensity related to it given by

$$\text{Intensity}(I) = \frac{\text{Energy}(E)}{\text{area}(A) \times \text{time}(t)} \dots (iv)$$

From (iii) and (iv)

$$I = \frac{E}{\frac{F}{P} \times t} = \frac{\text{Power}}{\frac{F}{P}} = \frac{\text{velocity}(c)}{1/P}$$

(because energy dissipated per unit time is the power and power = force  $\times$  velocity)

So  $P = \frac{I}{c}$  is the radiation pressure.

## Objective II

### 1. Question

An electromagnetic wave going through vacuum is described by

$$E = E_0 \sin (\kappa x - \omega t).$$



Which of the following is/are independent of the wavelength?

A.  $\kappa$

B.  $\omega$

C.  $\kappa/\omega$

D.  $\kappa\omega$

**Answer**

Explanation:

Given: The equation of an electromagnetic wave is given as

$$E = E_0 \sin(\kappa x - \omega t)$$

where  $E_0$  is the amplitude of the electric field,  $k$  is the wave number of the wave ( $k = \frac{2\pi}{\lambda}$ ),  $\lambda$  is the wavelength,  $\omega$  is the angular frequency of the wave ( $\omega = 2\pi\nu$ ) and  $t$  is the time. The frequency of the wave is  $\nu = \frac{\omega}{2\pi}$ .

A. is  $k$  which is equal to  $\frac{2\pi}{\lambda}$  and is dependent on  $\lambda$  so A. is incorrect.

B is  $\omega$  which is  $2\pi\nu$ . If the speed of the wave is  $c$ , then

$c = \nu \times \lambda$ , so  $\omega$  becomes  $\frac{2\pi c}{\lambda}$ , hence B. is incorrect.

C. is  $\frac{\omega}{k}$ , substituting the values of  $\omega$  and  $k$  we get  $\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{\frac{2\pi c}{\lambda}} = \frac{1}{c}$  which is

independent of wavelength so C. is correct.

D. is  $k\omega$ , using the values of  $k$  and  $\omega$ ,  $k\omega = \frac{2\pi}{\lambda} \times \frac{2\pi c}{\lambda}$  which is

dependent on wavelength so D. is incorrect.

## 2. Question

Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor

A. increases

B. decreases

C. does not change

D. is zero

### Answer

Explanation: The displacement current depends on the changing electric flux across the capacitor plates. Due to the charging of the capacitor, the charge varies on the capacitor plates with time. Due to this, the electric flux also varies with time. The relation between the displacement current and the electric flux is given as

$$I_d = \epsilon_0 \frac{d(\phi_E)}{dt}$$

where  $\phi_E$  is the time varying electric flux through the plane surface,  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $I_d$  is the displacement current.

The electric flux is given by Gauss's law as

$$\phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

The displacement current then becomes

$$I_d = \epsilon_0 \times \frac{d\left(\frac{q_{\text{enclosed}}}{\epsilon_0}\right)}{dt} = \epsilon_0 \times \frac{1}{\epsilon_0} \times \frac{dq_{\text{enclosed}}}{dt} = \frac{d(q_{\text{enclosed}})}{dt}$$
$$I_d = \frac{d(q_{\text{enclosed}})}{dt}$$

which is dependent on charge. If the charge is zero, then the R.H.S

would be zero so D. is incorrect. If the charge doesn't change with

time so its derivative w.r.t time will be zero and so will be the

current, so C. is zero. For R.H.S to be non-zero, the charge must vary with time i.e. increase or decrease, hence A. and B. are correct.

### 3. Question

Speed of electromagnetic waves is the same

A. for all wavelengths

B. in all media

- C. for all intensities
- D. for all frequencies

### Answer

Explanation: The speed of electromagnetic waves is given by

$$c = v \times \lambda$$

where  $v$  is the frequency and  $\lambda$  is the wavelength. Thus the speed will be not same for all frequencies and wavelengths so A. and D. are incorrect. Also as wave travels from one medium to another, the speed changes so B. is incorrect. The relation between speed and intensity of a wave is given by

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

where  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ ,  $c$  is the speed of wave and  $E_0$  is the amplitude of electric field. The speed will be same for all intensities in a given medium will be same so C. is correct.

### 4. Question

Which of the following have zero average value in a plane electromagnetic wave?

- A. electric field
- B. magnetic field
- C. electric energy
- D. magnetic energy

### Answer

Explanation: Let us assume the wave travels in  $z$  direction so the electric and magnetic fields of the wave are given by

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

Their average values over one cycle would be the average from time  $t=0$  to  $t=T$ .

Average of electric field

$$\begin{aligned}
\langle E_x \rangle &= \frac{\int_0^T E_x dt}{\int_0^T dt} = \frac{\int_0^T E_0 \sin(kz - \omega t) dt}{\int_0^T dt} \\
&= \frac{1}{T} E_0 \int_0^T \sin(kz - \omega t) dt = \frac{E_0}{T} \left[ \frac{\cos(kz - \omega t)}{\omega} \right]_0^T \\
&= \frac{E_0}{T \times \frac{2\pi}{T}} \left[ \cos\left(kz - \frac{2\pi}{T} T\right) - \cos\left(kz - \frac{2\pi}{T} 0\right) \right] \\
&= \frac{E_0}{T \times \frac{2\pi}{T}} [\cos(kz - 2\pi) - \cos(kz)] = \frac{E_0}{T \times \frac{2\pi}{T}} [\cos(kz) - \cos(kz)] = 0
\end{aligned}$$

(because  $\omega = \frac{2\pi}{T}$  and  $\cos(\theta - 2\pi) = \cos(\theta)$  )

Therefore, the average of electric field is zero.

Similarly average of magnetic field is also zero as

$$\begin{aligned}
\langle B_y \rangle &= \frac{\int_0^T B_y dt}{\int_0^T dt} = \frac{\int_0^T B_0 \sin(kz - \omega t) dt}{\int_0^T dt} \\
&= \frac{1}{T} B_0 \int_0^T \sin(kz - \omega t) dt = \frac{B_0}{T} \left[ \frac{\cos(kz - \omega t)}{\omega} \right]_0^T \\
&= \frac{B_0}{T \times \frac{2\pi}{T}} \left[ \cos\left(kz - \frac{2\pi}{T} T\right) - \cos\left(kz - \frac{2\pi}{T} 0\right) \right] \\
&= \frac{B_0}{T \times \frac{2\pi}{T}} [\cos(kz - 2\pi) - \cos(kz)] = \frac{B_0}{T \times \frac{2\pi}{T}} [\cos(kz) - \cos(kz)] = 0
\end{aligned}$$

So electric and magnetic field have their average value zero.

Now the electric energy associated with the electric field is given by

$$U_{de} = \frac{1}{2} \epsilon_0 E^2 \dots (i)$$

where  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $U_{de}$  is the energy density of electric field.

Similarly, the magnetic energy associated with the magnetic field is given by

$$U_{db} = \frac{1}{2} \frac{B^2}{\mu_0} \dots (ii)$$

where  $\mu_0$  is the magnetic permeability of free space and its value is

$4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $U_{db}$  is the magnetic field energy density.

Also the amplitudes of electric and magnetic fields are related as

$$\frac{E_0}{B_0} = C \dots (iii)$$

where  $C$  is the speed of light in vacuum.

$$U_{de} = \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) \text{ and } U_{db} = \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2$$

Using (iii) in (i)

$$U_{de} = \frac{\epsilon_0 B_0^2 C^2}{2} \sin^2(kz - \omega t) \dots (iv)$$

The electrical permittivity and magnetic permeability of free space related as

$$C = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \Rightarrow C^2 = \frac{1}{\epsilon_0 \mu_0} \dots (v)$$

Using (v) in (iv)

$$U_{de} = \frac{\epsilon_0 B_0^2 C^2}{2} \sin^2(kz - \omega t) = \frac{\epsilon_0 B_0^2}{2\epsilon_0 \mu_0} \sin^2(kz - \omega t) = \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2$$

$$U_{de} = U_{db}$$

Therefore, the electric and magnetic energy densities are same in magnitude and so their average values will be same.

Average of electric energy over one cycle will be

$$\begin{aligned} \langle U_{de} \rangle &= \frac{\int_0^T \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) dt}{\int_0^T dt} = \frac{1}{2T} \epsilon_0 E_0^2 \int_0^T \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) dt \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \int_0^T (1 - \cos(2kz - 2\omega t)) dt = \frac{1}{4T} \epsilon_0 E_0^2 \left[ t - \frac{\sin(2kz - 2\omega t)}{2\omega} \right]_0^T \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \left[ (T - 0) - \frac{1}{2\omega} \left[ \left( \sin(2kz - \frac{4\pi}{T} T) - \sin\left(2kz - \frac{4\pi}{T} 0\right) \right) \right] \right] \\ &= \frac{1}{4T} \epsilon_0 E_0^2 \left[ T - \frac{1}{2\omega} [\sin(2kz - 4\pi) - \sin(2kz)] \right] \end{aligned}$$

$$= \frac{1}{4T} \epsilon_0 E_0^2 \left[ T - \frac{1}{2\omega} [\sin(2kz) - \sin(2kz)] \right] = \frac{1}{4} \epsilon_0 E_0^2 \neq 0$$

So electric and magnetic energy and have non zero average values.

## 5. Question

The energy contained in a small volume through which an electromagnetic wave is passing oscillates with

- A. zero frequency
- B. the frequency of the wave
- C. half the frequency of the wave
- D. double the frequency of the wave

## Answer

Explanation: Let us consider an electromagnetic wave traveling in the z direction. The electric field E of the wave is assumed to be

$$E = E_0 \sin(kz - \omega t)$$

where  $E_0$  is the amplitude of the electric field, k is the wave number of the wave ( $k = \frac{2\pi}{\lambda}$ ),  $\lambda$  is the wavelength,  $\omega$  is the angular frequency of the wave ( $\omega = 2\pi\nu$ ) and t is the time. The frequency of the wave is

$$f = \frac{\omega}{2\pi}$$

Now the electric field energy density of the electromagnetic wave is given by the relation

$$U_{de} = \frac{1}{2} \epsilon_0 E^2$$

where  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $U_{de}$  is the energy density of electric field. Now the energy of the wave over volume V of the region is the product of energy density and the volume V. The energy of the electric field is

$$U_E = U_{de} \times V = \frac{1}{2} \epsilon_0 E^2 \times V$$

$$U_E = \frac{1}{2} \epsilon_0 (E_0 \sin(kz - \omega t))^2 \times V = \frac{1}{2} \epsilon_0 E_0^2 \sin^2(kz - \omega t) \times V$$

$$= \frac{1}{2} \epsilon_0 E_0^2 \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) \times V = \frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

$$U_E = \frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

The energy of the electric field is  $\frac{1}{4} \epsilon_0 E_0^2 (1 - \cos 2(kz - \omega t)) \times V$ .

The angular frequency of the energy of the wave is  $2\omega$ , the

Corresponding frequency will be

$$f' = \frac{2\omega}{2\pi}$$

This frequency  $f'$  is twice of  $f$ ,  $f' = 2f$ .

Now the magnetic field can be considered as

$$B = B_0 \sin(kz - \omega t)$$

where  $B_0$  is the amplitude of the magnetic field,  $k$  is the wave

number of the wave ( $k = \frac{2\pi}{\lambda}$ ),  $\lambda$  is the wavelength,  $\omega$  is the angular

frequency of the wave ( $\omega = 2\pi\nu$ ) and  $t$  is the time. The frequency of the wave is

$$f = \frac{\omega}{2\pi}$$

Now the magnetic field energy density of the electromagnetic wave is given by the relation

$$U_{db} = \frac{1}{2} \frac{B^2}{\mu_0}$$

where  $\mu_0$  is the magnetic permeability of free space and its value is

$4\pi \times 10^{-7} \text{ T m A}^{-1}$ ,  $U_{db}$  is the magnetic field energy density. Now the energy of the wave over volume  $V$  of the region is the product of energy density and the volume  $V$ . The energy of the magnetic field is

$$U_B = U_{db} \times V = \frac{1}{2} \frac{B^2}{\mu_0} \times V$$

$$U_B = \frac{1}{2\mu_0} (B_0 \sin(kz - \omega t))^2 \times V = \frac{1}{2\mu_0} B_0^2 \sin^2(kz - \omega t) \times V$$

$$= \frac{1}{2\mu_0} B_0^2 \left( \frac{1 - \cos 2(kz - \omega t)}{2} \right) \times V = \frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

$$U_B = \frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V$$

The energy of the magnetic field is  $\frac{1}{4\mu_0} B_0^2 (1 - \cos 2(kz - \omega t)) \times V$ .

The angular frequency of the energy of the wave is  $2\omega$ , the

Corresponding frequency will be

$$f' = \frac{2\omega}{2\pi}$$

This frequency  $f'$  is twice of  $f$ ,  $f' = 2f$ .

As for both, electric and magnetic, the energy of wave is double the frequency of their respective waves, so the overall energy of the wave has twice the frequency of oscillation of the wave itself.

## Exercises

### 1. Question

Show that the dimensions of the displacement current  $\epsilon_0 \frac{d\phi_E}{dt}$  are that of an electric current.

### Answer

The displacement current  $I_d$  is produced by a varying electric field. It is given by the relation

$$I_d = \epsilon_0 \times \frac{d(\phi_E)}{dt}$$

where  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $\phi_E$  is the electric flux produced by the time varying electric field.

To find the dimension of  $I_d$  and check whether it is the same as that of electric current, we need to find the dimension of  $\epsilon_0 \frac{d(\phi_E)}{dt}$ .

We can simplify  $\frac{d(\phi_E)}{dt}$  using Gauss's Law. According to Gauss's law



the electric flux  $\phi_E$  through a surface is given as

$$\phi_E = \frac{q_{enclosed}}{\epsilon_0}$$

Using the above relation, the displacement current becomes

$$I_d = \epsilon_0 \times \frac{d\left(\frac{q_{enclosed}}{\epsilon_0}\right)}{dt} = \epsilon_0 \times \frac{1}{\epsilon_0} \times \frac{dq_{enclosed}}{dt} = \frac{d(q_{enclosed})}{dt}$$

$$I_d = \frac{d(q_{enclosed})}{dt}$$

The dimension of displacement current is the dimension of the quantity in the R.H.S of the above equation

$$\text{dimension of } I_d = \frac{\text{dimension of } q}{\text{dimension of time}} = \frac{[T A]}{[T]} = [A]$$

As [A] is the dimension of electric current so displacement current has same dimension as that of electric current.

## 2. Question

A point charge is moving along a straight line with a constant

velocity  $v$ . Consider a small area  $A$  perpendicular to the direction of motion of the charge figure. Calculate the displacement current through the area when its distance from the charge is  $x$ . The value of  $x$  is not large so that the electric field at any instant is essentially given by Coulomb's law.



### Answer

Given: Velocity of charge =  $v$

Area of patch =  $A$

Distance of charge from the patch =  $x$

We have to find the displacement current through the area when it is at a distance  $x$  from the charge.

The displacement current arises due to changing electric field which in turn produces a varying electric flux through an area. The displacement current depends on the electric flux as

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right|$$

where  $I_d$  is the displacement current,  $\phi_E$  is the varying electric flux through the area and  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

The electric field produced by the charge when it is at a distance  $x$  from the surface is given by Coulomb's law and is equal to

$$E = \frac{q}{4\pi\epsilon_0 x^2}$$

This electric field produces an electric flux through the area whose magnitude is given by Gauss's law

$$\phi_E = \oint \vec{E} \cdot \vec{dS}$$

$\phi_E = \oint \vec{E} \cdot \vec{dS} = \oint E dS \cos\theta = \oint E dS$ . This is because the electric field lines are directed along the normal to the area vector of the surface.

The angle  $\theta$  between them is  $0^\circ$  so  $\cos 0^\circ = 1$ .

$$\phi_E = \oint E dS = E \oint dS = EA = \frac{qA}{4\pi\epsilon_0 x^2}$$

The displacement current will be

$$\begin{aligned} I_d &= \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right| = \epsilon_0 \left| \frac{d\left(\frac{qA}{4\pi\epsilon_0 x^2}\right)}{dt} \right| = \epsilon_0 \times \frac{qA}{4\pi\epsilon_0} \times \left| \frac{d(x^{-2})}{dt} \right| \\ &= \frac{qA}{4\pi} \left| (-2) \times (x^{-3}) \times \frac{dx}{dt} \right| \\ &= \frac{qA}{2\pi x^3} \times \frac{dx}{dt} \end{aligned}$$

As  $x$  is the distance of the charge from the area at different intervals of time so the rate at which the particle is changing its position is its velocity  $v$

$$I_d = \frac{qAv}{2\pi x^3}$$

Thus the displacement current through the area is  $\frac{qAv}{2\pi x^3}$ .

### 3. Question

A parallel-plate capacitor having plate-area  $A$  and plate separation

d is joined to a battery of emf  $\epsilon$  and internal resistance R at  $t = 0$ .

Consider a plane surface of area  $A/2$ , parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time.

### Answer

Given: Area of capacitor plates = A

Separation between the plates = d

Emf of the battery =  $\epsilon$

Internal resistance of the battery = R

Area of plane surface =  $A/2$

Displacement current is the current which is generated by a time varying electric field, not by the flow of charge carriers.

This current is also responsible for the generation of a time varying magnetic field. The displacement current  $I_d$  is generated due to the fact that the charge on capacitor plates is changing with time.

The displacement current is given by

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right|$$

where  $\phi_E$  is the time varying electric flux through the plane surface and  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

The electric field in the space between the plates can be given by Gauss's Law. If the charge on the capacitor plate is Q and the area of the plate is A (given), then by Gauss's law,

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

where E is the electric field and  $\epsilon_0$  is the electric permittivity of free space and dS is a small area element on the plate.

Further  $\oint_{\vec{E}} \vec{E} \cdot d\vec{S} = \oint E dS \cos \theta = \oint E dS = EA$ . (because the area vector and electric field lines are both normal to the surface and in same direction i.e.  $\theta=0^\circ$  so  $\cos \theta=1$ )

So  $EA = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{A\epsilon_0}$ , the electric field between the plates is  $\frac{Q}{A\epsilon_0}$ .

This electric field produces an electric flux through the plane surface given by

$$\phi_E = \oint_{\vec{E}} \vec{E} \cdot d\vec{S} = \oint E dS \cos \theta = \oint E dS$$

(because the area vector and electric field lines are both normal to the surface and in same direction i.e.  $\theta=0^\circ$  so  $\cos \theta=1$ )

$$\oint E dS = E \oint dS = \frac{EA}{2} = \frac{Q}{A\epsilon_0} \times \frac{A}{2} = \frac{Q}{2\epsilon_0} = \phi_E$$

Now the charge on the capacitor is changing with time as it is charging. If the capacitance of the capacitor is C, then the charge Q at time t will be

$Q = \epsilon C (1 - e^{-\frac{t}{RC}})$  where  $\epsilon$  is the potential between plates which is equal to the emf of battery and R is the resistance attached in series.

The displacement current  $I_d$  is given as

$$\begin{aligned} I_d &= \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right| = \epsilon_0 \left| \frac{d\left(\frac{Q}{2\epsilon_0}\right)}{dt} \right| = \frac{1}{2} \left| \frac{d(\epsilon C (1 - e^{-\frac{t}{RC}}))}{dt} \right| \\ &= \frac{1}{2} \left| \frac{d(\epsilon C (1 - e^{-\frac{t}{RC}}))}{dt} \right| \\ &= \frac{1}{2} \left| \epsilon C \times \left(-\frac{1}{RC}\right) \times \left(-e^{-\frac{t}{RC}}\right) \right| \\ &= \frac{1}{2} \times \frac{\epsilon}{R} \times \left(e^{-\frac{t}{RC}}\right) = \frac{\epsilon}{2R} e^{-\frac{t}{RC}} \end{aligned}$$

Thus the displacement current as a function of time is  $\frac{\epsilon}{2R} e^{-\frac{t}{RC}}$ .

#### 4. Question

Consider the situation of the previous problem. Define displacement resistance  $R_d = V/i_d$  of the space between the plates where  $V$  is the potential difference between the plates and  $i_d$  is the displacement current. Show that  $R_d$  varies with time as

$$R_d = R (e^{t/\tau} - 1)$$

### Answer

Given: The displacement resistance  $R_d = \frac{V}{I_d}$

We will first calculate the displacement current.

The displacement current  $I_d$  is generated due to the fact that the charge on capacitor plates is changing with time.

The displacement current is given by

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right|$$

where  $\phi_E$  is the time varying electric flux through the plane surface and  $\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

We need to calculate the electric flux through the capacitor plate.

As the charge on the capacitor plate at time  $t$  can be taken as  $Q$  so by using Gauss's law, we will calculate the electric flux.

According to Gauss's law

$$\phi_E = \frac{\text{charge enclosed}}{\epsilon_0}$$

so the electric flux would be

$$\phi_E = \frac{Q}{\epsilon_0}.$$

As the capacitor is charging, the charge will be a function of time given as

$$Q = CV = CV_0 e^{\frac{-t}{RC}}$$

where  $C$  is the capacitance of the capacitor,  $V$  is the potential drop at time  $t$ ,  $R$  is the series resistance and  $V_0$  is the potential at time

$t=0$ . Now the flux is  $\frac{CV_0 e^{-\frac{t}{RC}}}{\epsilon_0}$ .

Now by our definition, the displacement current is given by

$$I_d = \epsilon_0 \left| \frac{d(\phi_E)}{dt} \right|$$

which is

$$I_d = \epsilon_0 \left| \frac{d \left( \frac{CV_0 e^{-\frac{t}{RC}}}{\epsilon_0} \right)}{dt} \right| = \epsilon_0 \times \frac{CV_0}{\epsilon_0} \left| \frac{d \left( e^{-\frac{t}{RC}} \right)}{dt} \right| = CV_0 \left| \left( \frac{-1}{RC} \right) \times \left( e^{-t/RC} \right) \right|$$

$$= \frac{V_0}{R} e^{-\frac{t}{RC}}$$

The displacement current is  $\frac{V_0}{R} e^{-\frac{t}{RC}}$ .

We know that  $R_d = \frac{V_0}{I_d} - R$

$$R_d = \frac{V_0}{\frac{V_0 e^{-\frac{t}{RC}}}{R}} - R = R e^{\frac{t}{RC}} - R = R \left( e^{\frac{t}{RC}} - 1 \right) = R \left( e^{\frac{t}{\tau}} - 1 \right)$$

therefore  $R_d = R \left( e^{\frac{t}{\tau}} - 1 \right)$ , where  $\tau$  is the time constant.

## 5. Question

Using  $B = \mu_0 H$  find the ratio  $E_0/H_0$  for a plane electromagnetic wave propagating through vacuum. Show that it has the dimensions of electric resistance. This ratio is a universal constant called the impedance of free space.

## Answer

Given: The magnetic field density and the magnetic field strength

as related by the relation

$$B = \mu_m H$$

where  $B$  is the magnetic field density,  $\mu_m$  is the relative magnetic

permeability of the material and depends on the nature of the

material or the medium and  $H$  is the magnetic field strength which

tells upto what extent a material can be magnetized.

For free space, the relation becomes

$$B_0 = \mu_0 H_0 \dots (i)$$

where  $B_0$  is the field intensity in free space(vacuum),  $H_0$  is the strength in free space and  $\mu_0$  is the magnetic permeability of free space.  $\mu_0$  is a universal constant and its value is  $4\pi \times 10^{-7} \text{ T m A}^{-1}$ .

We can define a relation between the electric field and magnetic field of an electromagnetic wave travelling in vacuum. It is

$$C = \frac{E_0}{B_0} \dots (ii)$$

where  $E_0$  is the electric field density and  $C$  is the speed of light in vacuum.

From (i) and (ii)

$$B_0 = \mu_0 H_0 = \frac{E_0}{C}$$

$$\mu_0 H_0 = \frac{E_0}{C}$$

$$\frac{E_0}{H_0} = \mu_0 C \dots (iii)$$

To find the dimension of  $\frac{E_0}{H_0}$ , we will find the dimension of the quantity in the R.H.S.

The dimension of  $C$  is  $[L T^{-1}]$  because it is the speed.

To find the dimension of  $\mu_0$ , we need to consider the Biot Savart's law which gives the magnetic field due to a current carrying conductor. According to Biot Savart law

$$\vec{B} = \frac{\mu_0}{4\pi} \times i \times \frac{(\vec{L} \times \vec{R})}{R^3} \dots (iv)$$

where  $i$  is the current in the conductor,  $L$  is the length of conductor and  $R$  is the distance between conductor and point where field is to be found.

The scalar form of the above equation will be simply

$$B = \frac{\mu_0}{4\pi} \times \frac{iL}{R^2} \dots (v)$$

From (v)

$$\mu_0 = \frac{4\pi R^2 B}{iL}$$

the dimensions of  $\mu_0$  will be the dimensions of R.H.S

$$\text{dimension of } B = [M T^{-2} A^{-1}]$$

$$\text{dimension of } R^2 = [L^2]$$

$$\text{dimension of } L = [L]$$

$$\text{dimension of } i = [A]$$

$$\text{dimension of R.H.S} = \frac{[M T^{-2} A^{-1}][L^2]}{[L][A]} = [M L^1 T^{-2} A^{-2}]$$

$$\text{Therefore dimension of } \mu_0 \text{ is } [M L T^{-2} A^{-2}].$$

Now the dimension of R.H.S of eq(iii) will be dimension of  $C \times$  dimension of  $\mu_0$

$$\text{dimension of } \frac{E_0}{H_0} = [M L T^{-2} A^{-2}][L T^{-1}] = [M L^2 T^{-3} A^{-2}].$$

As the dimensions of electric resistance given by Ohm's Law ( $V=iR$ ) is

$$\frac{\text{dimension of Potential difference}(V)}{\text{dimension of current}(i)} = \frac{[M L^2 T^{-3} A^{-1}]}{[A]} = [M L^2 T^{-3} A^{-2}]$$

Therefore  $\frac{E_0}{H_0}$  has the same dimensions as that of electrical resistance.

Also the value of  $\frac{E_0}{H_0}$  is a constant because the R.H.S of eq(iii) is the

product of 2 universal constants.

## 6. Question

The sunlight reaching the earth has maximum electric field of  $810 \text{ V m}^{-1}$ . What is the maximum magnetic field in this light?

### Answer

Given: Maximum electric field  $E_0 = 810 \text{ V m}^{-1}$ .

The sunlight travels through vacuum in outer space to reach the earth. The magnetic field and electric field are related to each other and their relation is

$$B_0 = \frac{E_0}{C}$$



where  $E_0$  is the amplitude of the electric field,  $C$  is the speed of light in vacuum and  $B_0$  is the amplitude of maximum magnetic field.

Thus the maximum magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{810 \text{ Vm}^{-1}}{3 \times 10^8 \text{ ms}^{-1}} = 2.7 \times 10^{-6} \text{ T} = 2.7 \mu\text{T}.$$

The maximum magnetic field is  $2.7 \mu\text{T}$ .

## 7. Question

The magnetic field in a plane electromagnetic wave is given by

$$B = (200 \mu\text{T}) \sin [(4.0 \times 10^{15} \text{ s}^{-1}) (t - x/c)].$$

Find the maximum electric field and the average energy density corresponding to the electric field.

## Answer

Given: The equation of magnetic field of a plane electromagnetic wave

$$B = (200 \mu\text{T}) \sin [(4.0 \times 10^{15} \text{ s}^{-1}) (t - x/c)]$$

where the amplitude of magnetic field is  $B_0 = 200 \mu\text{T}$ .

The amplitude of the electric field is related with amplitude of magnetic field as

$$E_0 = C \times B_0$$

where  $C$  is the speed of light in free space and  $E_0$  is the amplitude of electric field.

Thus the electric field intensity is given as

$$\begin{aligned} E_0 &= C \times B_0 \\ &= (3 \times 10^8 \text{ ms}^{-1}) \times (200 \mu\text{T}) \\ E_0 &= 6 \times 10^4 \text{ NC}^{-1}. \end{aligned}$$

The energy density associated with an electric field is given as

$$U_d = \frac{1}{2} \epsilon_0 E_0^2$$

where  $U_d$  is the energy density,  $\epsilon_0$  is the electric permittivity of free space (vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $E_0$  is the amplitude of

electric field.

Thus the electric field energy density is given by

$$\begin{aligned}U_d &= \frac{1}{2}\epsilon_0 E_0^2 \\&= \frac{1}{2} \times (8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}) \times (6 \times 10^4 \text{ NC}^{-1})^2 \\&= 159.3 \times 10^{-4} \text{ J m}^{-3} \\U_d &= 1.593 \times 10^{-2} \text{ J m}^{-3}\end{aligned}$$

The maximum electric field is  $6 \times 10^4 \text{ NC}^{-1}$  and the corresponding electric field energy density is  $1.593 \times 10^{-2} \text{ J m}^{-3}$ .

### 8. Question

A laser beam has intensity  $2.5 \times 10^{14} \text{ W m}^{-2}$ . Find the amplitudes of electric and magnetic fields in the beam.

### Answer

Given: The intensity of laser beam  $= 2.5 \times 10^{14} \text{ W m}^{-2}$ .

We have to find the amplitudes of electric and magnetic field. The amplitude of the electric field is related to the intensity of the wave by the relation

$$I = \frac{1}{2} \epsilon_0 E_0^2 C$$

where

$\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ ,  $C$  is the speed of light in vacuum and is equal to  $3 \times 10^8 \text{ m s}^{-1}$  and  $E_0$  is the amplitude of electric field.

$$I = \frac{1}{2} \epsilon_0 E_0^2 C$$

$$E_0^2 = \frac{2I}{C\epsilon_0}$$

$$E_0 = \sqrt{\frac{2I}{C\epsilon_0}} = \sqrt{\frac{2 \times 2.5 \times 10^{14}}{3 \times 10^8 \times 8.85 \times 10^{-12}}}$$

$$E_0 = 4.339 \times 10^8 \text{ N/C}.$$

The amplitudes of electric and magnetic fields are related as

$$B_0 = \frac{E_0}{C}$$

where  $B_0$  is the amplitude of magnetic field in free space.

Thus the value of magnetic field will be

$$B_0 = \frac{E_0}{C} = \frac{4.339 \times 10^8 \text{ N/C}}{3 \times 10^8 \frac{\text{m}}{\text{s}}} = 1.43 \text{ T}.$$

The electric field amplitude is  $4.339 \times 10^8 \text{ N/C}$  and the magnetic field amplitude is  $1.43 \text{ T}$ .

## 9. Question

The intensity of the sunlight reaching Earth is  $1380 \text{ W m}^{-2}$ . Assume this light to be a plane, monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.

## Answer

Given: The intensity of sunlight reaching the earth =  $1380 \text{ W m}^{-2}$ .

We have to find the amplitudes of electric and magnetic field if the light is a plane electromagnetic wave. The amplitude of the electric field is related to the intensity of the wave by the relation

$$I = \frac{1}{2} \epsilon_0 E_0^2 C$$

where

$\epsilon_0$  is the electric permittivity of free space(vacuum) and is equal to  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

$C$  is the speed of light in vacuum and is equal to  $3 \times 10^8 \text{ m s}^{-1}$

and  $E_0$  is the amplitude of electric field.

$$I = \frac{1}{2} \epsilon_0 E_0^2 C$$

$$E_0^2 = \frac{2 I}{C \epsilon_0}$$

$$E_0 = \sqrt{\frac{2I}{C \epsilon_0}} = \sqrt{\frac{2 \times 1380}{3 \times 10^8 \times 8.85 \times 10^{-12}}}$$

$$E_0 = 1.02 \times 10^3 \text{ N/C}$$

The amplitudes of electric and magnetic fields in free space are related as

$$B_0 = \frac{E_0}{c}$$

where  $B_0$  is the amplitude of magnetic field in free space.

Thus the value of magnetic field will be

$$B_0 = \frac{E_0}{c} = \frac{1.02 \times 10^3 \text{ N/C}}{3 \times 10^8 \frac{\text{m}}{\text{s}}} = 3.4 \times 10^{-6} \text{ T} = 3.4 \mu\text{T}.$$

The electric field amplitude is  $1.02 \times 10^3 \frac{\text{N}}{\text{C}}$  and the magnetic field amplitude is  $3.4 \mu\text{T}$ .