Closed line integral of electric field - Maxwell's II Eqn :-

$$\oint E \cdot dl = 0$$

→ Potential difference exists b/w 2 different points but b/w the same point potential difference is zero

→ Potential is unique at a point at a time

→ Workdone in moving a charge in any closed path is always zero

→ In any closed path energy acquired = energy lost

i.e Energy acquired in field direction = energy lost opposite to field direction.

→ Electric field is a conservative field & never forms closed loops.

i.e. Irrotational vector ($\nabla \times E = 0$)

→ Workdone in moving a charge b/w two points is independent of path of consideration.

→ This is KVL in electric fields.

Potential, Vector Potential and Ampere's Law in H field :-

→ Potential is called as MMF or Magneto Motive force and is a scalar measure of magnetic field strength at any point in the field.

$$V_m = MMF = \int \vec{H} \cdot \vec{dl} = Amp$$

$$H = \nabla V_m$$

$$\boxed{\oint H \cdot dl = I}$$

This is Ampere's law in Integral form

$$\boxed{\nabla \times H = J}$$

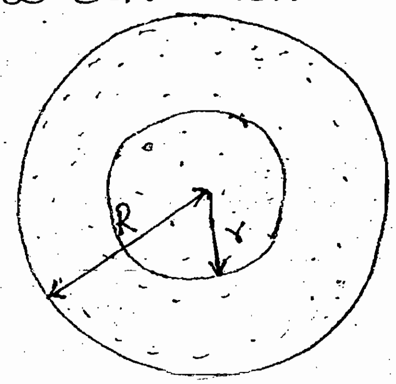
This is Ampere's law in point form

Statement :-

The net circulation ~~effects~~ of magnetic field in any closed line is always equal to the current crossing the surface enclosed.

Circulation = Strength × length
= current

consider a closed ~~ring~~ line symmetrical & cocentric with the I flow direction such that H = constant everywhere



$$\oint H \cdot dl = I$$

↓

constant

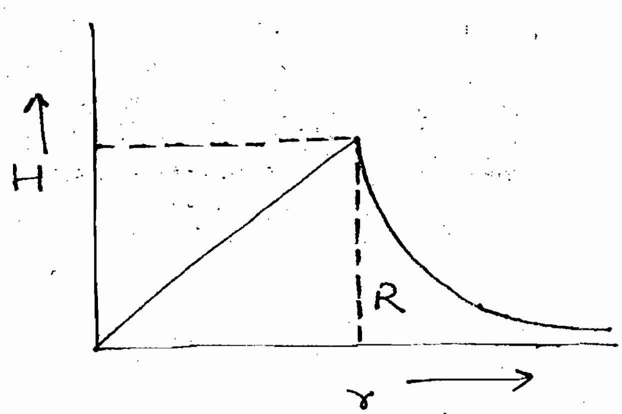
H · Length = current crossing

$r < R$

$$H = \frac{\frac{I}{\pi R^2} \cdot \pi r^2}{2\pi r} = \frac{Ir}{2\pi R^2}$$

$r > R$

$$H = \frac{I}{2\pi r}$$



Note :-

Point → $1/r^2$

Line → $1/\rho$

Sheet → Uniform

Solid → r

Vector Potential :-

Limitation of Scalar Potential (MMF) :-

→ The definition of potential in electric field is consistent with its nature i.e. irrotational nature

$$E = -\nabla V$$

$$\nabla \times E = 0$$

$$\nabla \times (-\nabla V) = 0$$

curl (Grad of scalar) = 0

→ A similar definition in magnetic field for MMF is not consistence with Ampere's law

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

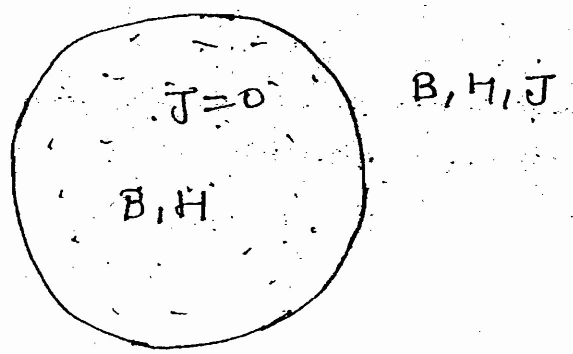
Hence MMF is defined and exists in only those regions where $J = 0$

i.e. outside current flowing regions

i.e. free space conditions and outside conductor

eg:- (i) MMF b/w windings of solenoids

(ii) MMF in air gaps in machines



Vector Potential A :-

→ The basic definition of vector potential is in accordance to nature of magnetic field i.e. solenoidal nature. Hence if curl of A = B then A is called as vector potential

$$B = \nabla \times \vec{A}$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

Div. (curl of vector) = 0

→ The term vector potential has a units weber/m which physically signifies the work for current element

$$\vec{A} = \text{Weber/m} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$= \frac{W}{I\,\vec{dl}}$$

and as $I\vec{dl}$ is a vector, $\vec{A}$ is a vector quantity.

Note :-

$$\frac{\text{Weber}}{\text{Second}} = \text{Volts}$$

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\frac{\text{Weber}}{\text{m}} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$V = \frac{W}{Q} = \frac{Q}{4\pi\epsilon r}$$

$$\boxed{\vec{A} = \frac{W}{I\vec{dl}} = \frac{\mu I \vec{dl}}{4\pi r}}$$

A's direction is along I flow direction & is a solenoidal vector.

Summary 1 :-

→ Q – coulomb – point charge – Basic cause (scalar) – E field

→ $I\vec{dl}$ – Amp-m – current element – H field (vector)

→ Q – $\rho_L\, dl$ – $\rho_s\, ds$ – $\rho_v\, dv$

→ $I\vec{dl}$ – $\vec{k}\, ds$ – $\vec{J}\, dv$

→ $\vec{E}$ – Intensity – Strength – Force – $\frac{\vec{F}}{Q}$ – $\frac{1}{\epsilon}$ dependent

→ $\vec{B}$ – Density – Strength – Force – $\frac{\vec{F}}{I\vec{dl}}$ – μ dependent

→ $\vec{D}$ – Density – Strength – $\frac{dQ}{ds} = c/m^2$

→ $\vec{H}$ – Intensity – Strength – $\frac{dI}{dl}$ = Amp/m

→ V – Potential – Strength – Work = W/Q → Scalar (Volts)

⇒ $\vec{A}$ – Potential – Strength – Work = $\frac{W}{Idl}$ → Vector. (Weber/m)

Summary :-

Points

Scalar function $\xrightarrow[\text{Gradient}]{\nabla}$ Vector f^n Intensity (per m)

Line

eg:- V vots $\longrightarrow$ ∇V $\longrightarrow$ E volts/m

Line

Vector function Intensity (per m) $\xrightarrow[\text{curl}]{\nabla \times}$ Vector f^n Density (per m^2)

Surface

eg:- Amp/m $\vec{H}$ $\longrightarrow$ $\nabla \times H$ $\longrightarrow$ J Amp/m^2

Web/m $\vec{A}$ $\longrightarrow$ $\nabla \times \vec{A}$ $\longrightarrow$ $\vec{B}$ Web/m^2

Surface

Vector function Density (per m^2) $\xrightarrow{\nabla}$ Scalar f^n Volume (per m^3)

Volume

eg:- $\vec{D}$ (C/m^2) $\longrightarrow$ $\nabla \cdot \vec{D}$ $\longrightarrow$ ρ_v C/m^3

33 $\oint A \cdot dl = \frac{\text{Weber}}{m} \times m = $ Webers

↓ flux

OR

$$\oint \vec{A} \cdot \vec{dl} = \int (\nabla \times A) \cdot ds = \int B \cdot ds = \psi_m$$

34.- Stoke's theorem

$\vec{V} = \nabla \times \vec{A}$

B

C $\rightarrow$ closed line

S_c $\rightarrow$ surface

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds = \int \vec{V} \cdot \vec{ds}$$

Laplace / Poisson's Equations :-

→ They are second order differential equations relating volume charge and potential developed inside / outside the charge

$$\nabla \cdot D = \rho_V$$

$$\nabla \cdot [\epsilon(-\nabla V)] = \rho_V$$

If ϵ = constant then

$$\boxed{\nabla^2 V = -\frac{\rho_V}{\epsilon}} \longrightarrow \text{Poisson's Equation}$$

For $\rho_V = 0$, i.e. outside the charge, charge free region

$$\boxed{\nabla^2 V = 0} \longrightarrow \text{Laplace Equation.}$$

Note :-

Inspite of being II order differential equation they always have a unique solution i.e. voltage is a single valued function of space

This is called as ~~UNQ~~ Uniqueness theorem

Mathematically

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

$$\nabla \cdot (\nabla V) = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2}\cdot\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

Extension :—

$\vec{A}$ Vs $\vec{J}$ in Magnetic Fields

$$\nabla \times H = \vec{J}$$

$$\Rightarrow \nabla \times \left(\frac{B}{\mu}\right) = J$$

$$\Rightarrow \nabla \times (\nabla \times A) = \mu J$$

$$\Rightarrow \nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

$$\Rightarrow \boxed{\nabla^2 A = -\mu J}$$

For current free region

$$\boxed{\nabla^2 A = 0}$$

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$$V = -\frac{6r^5}{\epsilon_0}$$

1. $\nabla^2 V = -\rho_v/\epsilon$ 2. $Q = \int \rho_v \, dv$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(-\frac{6r^5}{\epsilon}\right) = -\frac{\rho_v}{\epsilon}\right.$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot 6 \cdot 5 \cdot r^4\right) = \rho_v$$

$$\frac{30}{r^2} 6 \cdot r^5 = \rho_v$$

$$\Rightarrow \rho_v = 180 r^3$$

$$Q = \int_{r=0}^{1}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 180 r^3 \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 180 \frac{r^6}{6}\Big|_0^1 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = 120\pi C, \text{ Ans.}$$

<u>36.</u> $\rho_v = -10^{-8}(1+10\rho)$

V on $\rho = 5cm$

Given $\rho = 2cm$ V & $E = 0$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial V}{\partial \rho}\right) = \frac{10^{-8}(1+10\rho)}{\frac{1}{36\pi \times 10^9}}$$

$$= 360\pi(1+10\rho)$$

$$\rho\frac{\partial V}{\partial \rho} = 360\pi \int (\rho + 10\rho^2)\, d\rho$$

$$\rho\frac{\partial V}{\partial \rho} = 360\pi\left(\frac{\rho^2}{2} + \frac{10\rho^3}{3}\right)$$

$$\Rightarrow V = 360\pi \int \left(\frac{\rho}{2} + \frac{10\rho^2}{3}\right) d\rho$$

$$= 360\pi\left(\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right)_{2\times 10^{-3}}^{5\times 10^{-2}}$$

$$\Rightarrow \boxed{V = 0.5V}$$

<u>37.</u> $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho_v}{\epsilon}$$

$$= 20 \cdot 3 \cdot 2 \cdot x + 10 \cdot 4 \cdot 3\, y^2 \Big|_{(2,0)} = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \boxed{\rho_v = -240\epsilon_0}$$

38. ϕ — 1 dimension function — satisfying - laplace

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} = 0$$

$$\frac{\partial\phi}{\partial x} = c_1 = \text{Rate of change is constant}$$

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\phi = c_1x + c_2 \longrightarrow \text{Linear equation}$$

Ans - B

39. $V = \sinh x \cdot \cos ky \, e^{Pz}$

$$\frac{e^x - e^{-x}}{2} = \sinh x$$

$$\nabla^2 V = 0$$

$$\nabla^2 V = 1V - k^2V + P^2V = 0$$

$$\Rightarrow \boxed{k = \sqrt{1+P^2}}$$

Boundary Conditions for Electric fields

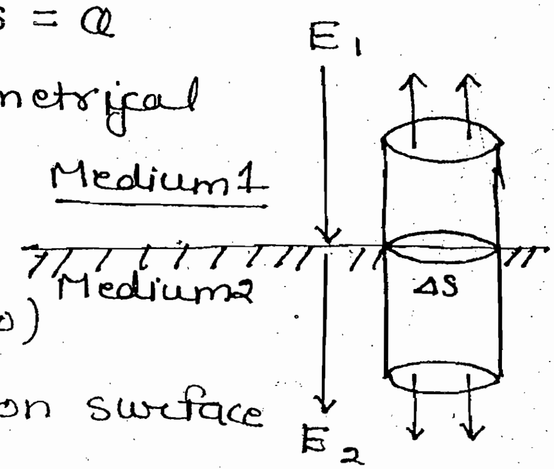
→ If a field is spread out into two different medium and known the field in one region. The field in the adjacent region can be calculated under two conditions

Case – (I) :–

Electric field is normal to the boundary.

Using Gauss law $\oint D \cdot ds = Q$

consider a surface symmetrical in both the medium



$$\boxed{D_2 \cdot \Delta S - D_1 \Delta S = 0}$$

$(\because Q = 0)$

If charge is present on surface then

$$\boxed{D_2 \Delta S - D_1 \Delta S = \rho_s \Delta S}$$

$D_{n_1} = D_{n_2}$

$D_{n_2} - D_{n_1} = \rho_s$

Statement :–

The normal components of electric flux density is same on either side but otherwise discontinous when a surface charge density exists on the boundary

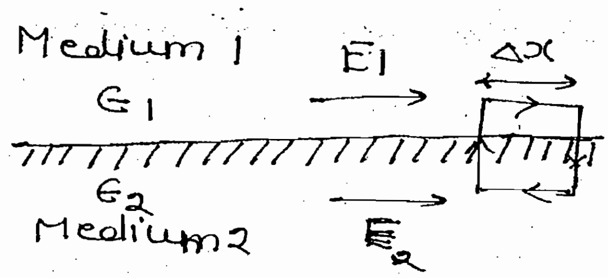
Case – (II) :–

Electric field is ~~parallel~~ tangential to boundary :–

Using $\oint E \cdot dl = 0$.

$E_1 \Delta x - E_2 \Delta x = 0$

$\Rightarrow \boxed{E_1 = E_2}$



$E_{t_1} = E_{t_2}$

The tangential components of electric field intensity is always continous

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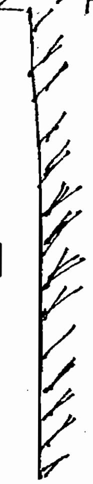
$x = 0$ Plane

$YZ \rightarrow$ Plane

$x < 0$ $\qquad$ $x > 0$

$\epsilon_{R_1} = 1.5$ $\qquad$ $\epsilon_{R_2} = 2.5$



$E_{n_1} = 2a_x$

$E_{t_1} = -3a_y + a_z$

$E_1 = 2a_x - 3a_y + a_z$

$E_{t_1} = E_{t_2} = -3a_y + a_z$

$D_{n_1} = 1.5 \times 2a_x \times \epsilon_0 \quad (D_{n_1} = \epsilon_0 \epsilon_R \times E_n)$

$\cancel{= 3a_x} \quad \cancel{= D_{n_2}}$

$= 3\epsilon_0 a_x \longrightarrow D_{n_2} = 3\epsilon_0 a_x$

$E_{n_2} = \dfrac{3\epsilon_0 a_x}{2.5\,\epsilon_0} = 1.2\,a_x$

$D_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z)$

$D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$

$\boxed{E_2 = 1.2a_x - 3a_y + a_z}$ Ans

Note :-

→ The value of vector dec. then its projection dec. i.e moves away from normal and moves towards the surface

→ The projection of normal component is dec. in E_2 when compare to E_1. Hence the field is shifting away from the normal. It can be verified with inc. tangential component when Comparing D_1 & D_2

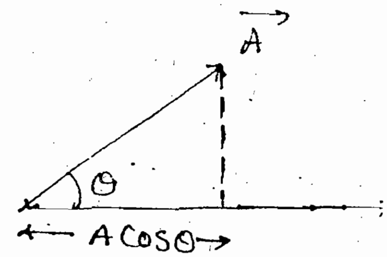
Sur

A's scalar projection on B

$$= A\cos\theta$$

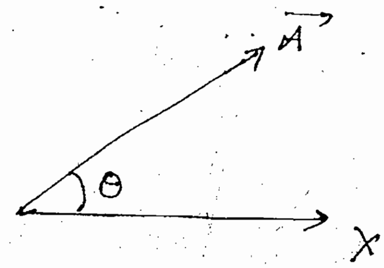
$$= \cancel{A}\,\frac{A\cdot B}{\cancel{A}\,B}$$

$$= \frac{A\cdot B}{|B|}$$



A's projection in x

$$= \frac{A\cdot a_x}{|a_x|}$$



A's vector projection on B

$$= (A\cos\theta)\cdot \hat{B}$$

$$= \frac{(A\cdot B)}{|B|}\,\hat{B}$$

$$= \frac{(A\cdot B)}{|B|^2}\,\vec{B}$$

of

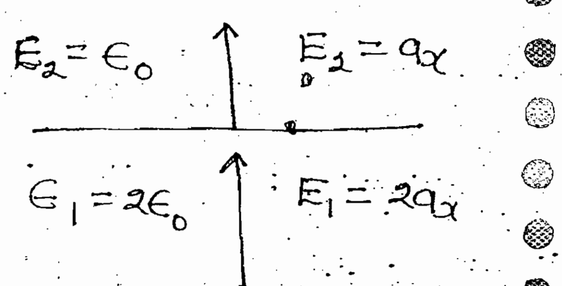
13.

41.

$$D_2 - D_1 = \rho_s$$

$$1\cdot \epsilon_0 - 2\cdot 2\epsilon_0 = \rho_s$$

$$\boxed{\rho_s = -3\epsilon_0}$$



42.

$$E_{t_1} = E_{t_2}$$

$$E_1 \cos 30 = E_{t_2}$$

$$\Rightarrow \quad E_{t_2} = 200\sqrt{3}$$

$$\epsilon_0 E_1 \sin 30^\circ = 2\epsilon_0 E_{n_2}$$

$$\Rightarrow \quad E_{n_2} = 10$$