Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

Ans.

Given $f(x) = \frac{|x-1|}{(x-1)}$

Obviously, $|x-1| = \begin{cases} (x-1) & \text{if } x-1>0 \text{ or } x>1 \\ -(x-1) & \text{if } x-1<0 \text{ or } x<1 \end{cases}$

Now, (*i*) $\forall\, x > 1$, $f(x) = \frac{(x-1)}{(x-1)} = 1$, (*ii*) $\forall\, x < 1$, $f(x) = \frac{-(x-1)}{(x-1)} = -1$,

i.e., $f(x) = -1, 1$

$\therefore$ Range of $f(x) = \{-1, 1\}$.

Q.2. If *f* is an invertible function, defined as $f(x) = \frac{3x-4}{5}$, write $f^{-1}(x)$.

Ans.

Since f^{-1} is inverse of f.

$\therefore \quad fof^{-1} = I \quad \Rightarrow \quad fof^{-1}(x) = I(x)$

$\Rightarrow \quad fof^{-1}(x) = x \quad \Rightarrow \quad f(f^{-1}(x)) = (x)$

$\Rightarrow \quad \frac{3(f^{-1}(x)) - 4}{5} = x \quad \Rightarrow \quad f^{-1}(x) = \frac{5x+4}{3}$

Q.3. If $f : R \rightarrow R$ is defined by $f(x) = 3x + 2$, define $f[f(x)]$.

Ans. $f(f(x)) = f(3x + 2) = 3(3x + 2) + 2$

$= 9x + 6 + 2 = 9x + 8$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. State the reason for following binary operation '*', defined on the set Z of integers, to be non-commutative $a * b = ab^3$. Also find 2 * 3.

Ans. Since $ab^3 \neq ba^3 \; \forall \; a, b \in Z$

$\Rightarrow \quad a * b \neq b * a$

Hence, '*' is not commutative.

Also, $2 * 3 = 2 \times 3^3 = 54$

Q.2. If $f : R \to R$ defined by $f(x) = \frac{2x-7}{4}$ is an invertible function then find f^{-1}.

Ans.

Let $f(x) = y \quad \Rightarrow \quad y = \frac{2x-7}{4}$

$\Rightarrow \quad 2x - 7 = 4y \quad \Rightarrow \quad 2x = 4y + 7 \quad \Rightarrow \quad x = \frac{4y+7}{2}$

Hence, $f^{-1}(x) = \frac{4x+7}{2}$

Q.3. Write the inverse relation corresponding to the relation R given by $R = \{(x, y): x \in N, x < 5, y = 3\}$. Also write the domain and range of inverse relation.

Ans.

Given, $R = \{(x, y) : x \in N, x < 5, y = 3\}$

$\Rightarrow \quad R = \{(1, 3), (2, 3), (3, 3), (4, 3)\}$

Hence, required inverse relation is

$R^{-1} = \{(3, 1), (3, 2), (3, 3), (3, 4)\}$

$\therefore \quad$ Domain of $R^{-1} = \{3\}$

And Range of $R^{-1} = \{1, 2, 3, 4\}$

Q.4. Let $A = \{1, 2, 3\}$. Write all one-one functions on A.

Ans. All one-one functions on A are as follows:

$f_1 = \{(1, 1), (2, 2), (3, 3)\}$; $\quad f_2 = \{(1, 1), (2, 3), (3, 2)\}$

$f_3 = \{(1, 2), (2, 1), (3, 3)\}$; $f_4 = \{(1, 3), (2, 2), (3, 1)\}$

$f_5 = \{(1, 3), (2, 1), (3, 2)\}$; $f_6 = \{(1, 2), (2, 3), (3, 1)\}$

Q.5. If $f : R \to R$ and $g : R \to R$ are given by

$f(x) = 3x + 1$ and $g(x) = x^2 + 2$

Find $fog(2)$.

Ans. $fog(x) = f(g(x)) = f(x^2 + 2) = 3(x^2 + 2) + 1 = 3x^2 + 6 + 1$

$\Rightarrow \quad fog(x) = 3x^2 + 7$

$\therefore \quad fog(2) = 3 \times 2^2 + 7 = 12 + 7 = 19$

Q.6. Let $A = \{1, 2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$. Let $f : A \to B$ and $g : B \to C$ be defined as $f(1) = 4$, $f(2) = 5$, $f(3) = 4$, $g(4) = 5$ and $g(5) = 6$. Find gof.

Ans. Obviously '*gof*' function is defined as

$gof : A \to C$ such that

$gof(1) = g(f(1)) = g(4) = 5$

$gof(2) = g(f(2)) = g(5) = 6$

$gof(3) = g(f(3)) = g(4) = 5$

Hence, $gof : A \to C$ is given by $gof = \{(1, 5), (2, 6), (3, 5)\}$

Q.7. Let * be the binary operation on the set {1, 2, 3, 4} defined by $a * b$ = HCF of a and b. Compute (2 * 3) * 4 and 2 * (3 * 4).

Ans. (2 * 3) * 4 = (HCF of 2 and 3) * 4 = (1 * 4) = 1

2 * (3 * 4) = 2 * (HCF of 3 and 4) = 2 * 1 = 1